An Edge-Preserving Filter for Imagery Corrupted with Multiplicative Noise

H. C. North and Q. X. Wu

Abstract
In the segmentation of natural imagery, differentiation at feature boundaries is of crucial importance. The high-amplitude, multiplicative speckle noise present in synthetic aperture radar (SAR) data demands a high level of filtering, yet this noise must be removed without destroying the critical feature boundary information. We previously designed the minimum coefficient of variation (MCV) filter to meet the twin demands of noise removal and edge preservation in SAR imagery. MCV-filtered images exhibit clear feature boundaries, but the filter's strong edge-preserving nature also introduces step edge artifacts in a region of intensity gradient and texture. We present the modified MCV filter (MMCV) which is able to significantly reduce the occurrence of filtering artifacts, while retaining an edge-preserving character. The MMCV filter is compared to existing filters by operating them on SAR imagery and deriving edge maps from the filtered image. Though the MMCV-filtered image is not the most visually pleasing, the line work derived from it is the most useful in terms of clean, continuous feature boundaries.

Introduction
The random, signal-dependent speckle noise inherent in synthetic aperture radar (SAR) imagery renders any interpretation of the data a non-trivial problem. However, there are significant benefits in being able to fully utilize radar imagery. SAR image acquisition is weather- and light-independent. For a temperate, cloud-prone climate, such as that of New Zealand, this allows the acquisition of images on a regular, frequent basis for operational monitoring systems.

Briefly, SAR is an active imaging process where pulses of microwave radiation are transmitted, and the scattered return is received, by a sensor mounted on an aircraft or satellite platform. When coherent radiation is incident on a surface that is rough on the scale of a wavelength, it is scattered in a set direction by the various surface elements. Interference of this scattered radiation creates the phenomenon of speckle. A consequence of this noise generation process is that representative information about the surface cannot be obtained from a single measurement. Instead, one must characterize the signal by means of its expected value within a region of the image.

The statistics of coherent superposition are derived by a number of authors, notably, Goodman (1984) and Oliver (1991). They show that the speckle intensity field may be described by a negative exponential probability density function (PDF). In practice, SAR images are generally processed in a multiplicative form, where the noise process is described by a Rayleigh distribution. As with the negative exponential distribution, the Rayleigh distribution has a standard deviation that is linearly dependent on its mean, so the noise can be described as multiplicative. Furthermore, SAR data providers commonly reduce speckle noise by so-called multi-look processing. Usually this is done in the frequency domain by summing L sub-bands (assumed to be statistically independent) of the original data (Curlander and McDonough, 1991). Ulaby et al. (1982) consider the combined PDF of L Rayleigh-distributed variables and note that it quickly tends to a Gaussian distribution for larger L.

A multi-look amplitude image can be modeled as

\[ G(x) = S(x) \cdot F(x) \] 

for a two-dimensional image \( x = [x_1, x_2] \), where \( S(x) \) is the underlying radar backscatter amplitude, \( F(x) \) is a random process with unity mean and PDF derived from the combination of \( L \) independent Rayleigh distributions, and \( G(x) \) is the detected image.

A useful measure of multiplicative noise level in an image is the coefficient of variation (COV), \( \gamma_F \), which is defined as the ratio of standard deviation to mean of a random process. For the noise process alone we can write

\[ \gamma_F = \frac{\sigma_F}{\langle F \rangle} = \sigma_F \] 

because the noise process is modeled as having unity expected value (\( \langle F \rangle = 1 \)) and a standard deviation of \( \sigma_F \). In an area of constant signal,

\[ \langle G(x) \rangle = S(x) \] 
\[ \text{var}(G(x)) = \sigma^2 S^2(x) \]

We aim to segment a natural image into visually meaningful extended regions, which places stringent requirements on noise suppression and on edge preservation. Some form of smoothing is essential for this extremely noisy imagery, but the degree of smoothing that is required to suppress the noise to a satisfactory level may also blur feature boundaries to the point where they can no longer be easily differentiated. Filters that preserve edges are, therefore, particularly important for SAR imagery. The task is not straightforward; step edges of interest often have an amplitude which is comparable to the mean amplitude of speckle noise in which they are embedded. A second example of the non-ideal image characteristics that must be dealt with in a practice is that boundaries between features are often not sharp step edges, but take the form of brightness

Landcare Research, P.O. Box 69, Lincoln, Canterbury, New Zealand (NorthH@landcare.cri.nz).
ramps over a number of pixels. More gradual intensity gradients may be visually interpreted as a radiometrically changing region (perhaps due to topography), rather than as an edge.

Witken (1983) pointed out that a different representation of the image will be obtained, depending on the size of the smoothing operator applied to it. The smoothing operator width acts as a parameter of scale, creating an image representation that is band-limited in the spatial frequency domain. This concept of scale could be seen as a level of generalization. The inherent scale trade-off involved in image filtering is described by Canny's uncertainty principle (Canny, 1986). When an image is represented at a coarse scale (by processing it with a large operator), the noise is effectively reduced in the result but feature boundaries tend to be geometrically distorted. A smaller operator will provide a representation containing accurately located boundaries but also more noise. This is an inherent limitation of any filtering operation at a single scale. Canny proved that, for images corrupted with additive noise, smoothing with a Gaussian operator provided the optimum solution in this fundamental conflict between noise and localization. However, Gaussian smoothing is not appropriate for the removal of multiplicative noise, and filters have been designed specifically for this form of noise.

Use of an adaptive filter can allow a suitable matching of filter window characteristics to local image characteristics, such as scale, at each filtered point. For speckle suppression in SAR data, the adaptive filters of Lee (1986), Frost et al. (1982), and Kuan et al. (1985) are well known. The filtering window in these methods has a fixed size, but the pixel value is estimated using local statistics.

Another form of adaptation is utilized in the local region filter (Nagao and Matsuyama, 1979), where the position of the filter window is adapted to the local variance. The aim is to avoid smoothing over step edges by using low-variance window positions for pixel estimation. The filter window is divided into eight overlapping subwindows surrounding the central pixel. The mean of the lowest variance subwindow is the new estimate of the central pixel, because this window is least likely to cross a boundary. This filter is similar in concept to the minimum coefficient of variation filter discussed here, but is designed for additive noise, and the subwindows are defined differently.

Although a number of excellent schemes for SAR image smoothing (Kuan et al., 1985; Lee, 1986) and edge detection exist, these two processes are often considered in isolation. Few authors tie the two together to produce a final, practically usable image representation. We are attempting to produce boundary maps that represent the visually meaningful image regions and yet are sufficiently clean to use in a geographic information system (GIS). For this reason we emphasize a high level of speckle reduction, particularly in the vicinity of step edges. According to Canny's uncertainty principle, the price paid for clean, sharp edges is loss of spatial accuracy. This concern is addressed in other publications (North et al., 1998; Wu and North, 1999) where our filter is operated at a range of scales.

We begin by describing our minimum coefficient of variation (MCV) filter. This has been designed both to suppress multiplicative speckle noise and to preserve feature boundaries. However, its strong edge-preserving character can introduce filtering artifacts in areas of intensity gradient. To minimize this problem, we propose an enhancement to the MCV filter. The modified MCV filter (MMCV) is then described and its properties and performance discussed. Next we compare results from the MMCV filter with those of existing filters. Throughout the paper, the MMCV filter is operated on a number of subscenes from both the Radarsat and JERS-1 sensors to illustrate its utility for many types of imagery. Further examples are given, showing the filter's robustness to non-ideal image characteristics and its independence of image-specific a priori knowledge.

Minimum Coefficient of Variation Filter

The minimum coefficient of variation (MCV) filter was first described by Schulze and Wu (1995). It is adaptive to an estimate of the local CoV. In practice, the CoV is estimated from local assessments of standard deviation and mean, using Equation 3.

In areas where the signal is constant within the filtering window, we expect the local CoV estimate, $\gamma_{\text{loc}}$, to be equal to the theoretical $\gamma_F$ for the imagery being examined. However, if the filtering window subtends fluctuations in the underlying radar cross section, a further component of variance is introduced: i.e.,

$$\text{Constant } S(x) \Rightarrow \gamma_{\text{loc}} = 0$$

$$\gamma_{\text{loc}} > 0$$

where $\gamma_{\text{loc}}$ is the signal standard deviation within the filtering window about $x$. The expectations in Equation 4 provide us with a useful method of detecting non-homogeneity in a SAR image. The shape of the CoV surface, $\gamma_{\text{loc}}$, at a step edge is shown in one dimension in Figure 1. As stated in Equation 4, the CoV has its expected, constant value in areas of constant signal, and $\gamma_{\text{loc}} > \gamma_F$ across the step edge.

In order to avoid smoothing over boundaries, one approach is to adapt the filter to match the local statistics. The MCV filter is adaptive in the sense that it positions the filter window to operate in the more homogeneous areas rather than across boundaries (like the local region filter of Nagao and Matsuyama (1979)), using the local CoV to recognize such regions.

The MCV filter operates in two stages, using a circular window. To estimate the signal at any pixel $p_x$, we first construct a "first-stage" circular window, diameter $d$. The CoV is calculated in "second-stage" windows, also diameter $d$, which are constructed about each pixel in the first-stage window. Then $p_x$ is estimated by the mean of the second-stage window in which the CoV is minimum. This minimum position should correspond to a relatively uniform area on one side or the other of any boundary in the vicinity, rather than across it. A simple illustration is given in Figure 2, which shows the MCV filter operating on an image containing a pair of neighboring homogeneous regions. The first-stage window about point $p_x$ lies across the boundary, but the filtered value of $p_x$ is calculated as the mean of the second-stage window shown, which does not cross the boundary.

It is possible for multiple minimum CoV positions to exist in the first-stage window (there are three in the simple example of Figure 2). The position closest to the central pixel, $p_x$, is chosen, based on the preference for spatial localization expressed by Canny (1986).

Improving the MCV Filter

The MCV procedure yields a useful result, leading to very sharp step edges suitable for successful edge detection. But there is one drawback, which is illustrated in Figure 3b.

Any estimate of variance calculated within a finite window has a statistical distribution about the mean variance for the homogeneous region. The MCV filter, however, always selects the minimum value within its operating zone. This means that the lower tail of the CoV distribution attracts the filter window for all nearby pixels, so that a number of pixels adjacent to one another may take their estimates at this one locally minimum CoV position. The result is filtering artifacts in the smoothed image. These are not usually serious in a region of constant signal, but form distinct steps in regions that include an intensity gradient, such as that in Figure 3b.

The MCV filter was modified to provide a solution to the artifact problem. For each pixel in the original image, the first
Mean and standard deviation calculated at each point in 201-pixel windows. cov, defined in Equation 2, is used as a measure of variability in the presence of multiplicative noise.

Figure 2. MCV filtering at a step edge (two-dimensional illustration). First-stage window (diameter d) about p, crosses a boundary. A minimum cov-position of the second-stage window occurs about m, as shown. Two other minimum positions occur about the points marked with dots.

step is still to locate the minimum CoV position. The CoV at the minimum, CoVmin, is then compared with the other CoV values within the filter's first-stage window, CoVi, i = 1, ..., N, where N is the number of pixels in the filter window. The comparison is performed between each CoV and CoVmin using an F-test, because CoV is, in fact, a normalized variance (variance is proportional to the square of the mean (Equation 3)): i.e.,

\[
H_0 : \text{CoV}_i \leq \text{CoV}_{\text{min}} \\
H_1 : \text{CoV}_i > \text{CoV}_{\text{min}}
\]

Then, for a certain level of significance, \(\alpha\), we reject \(H_0\) if

\[
\frac{\text{CoV}_i^2}{\text{CoV}_{\text{min}}^2} \geq f_{\alpha/2, N-1, N-1}.
\]

The test statistic can only vary in the range

\[
1 < \frac{\text{CoV}_i}{\text{CoV}_{\text{min}}} < \infty,
\]

so only the right half of the F-distribution is used. Hence, the critical value is found at \(\alpha/2\) in the F-tables for a significance level of \(\alpha\). At \(\alpha\), we have a \((1 - \alpha) \times 100\%\) probability of accepting \(H_0\) if in fact \(\text{CoV}_i = \text{CoV}_{\text{min}}\).

From the set of positions whose CoVs are not significantly different from the minimum (\(H_0\) cannot be rejected), the position closest to the central pixel is chosen. Choice of the minimum closest to \(p_c\), is, as before, justified on the basis of spatial localization. The result obtained with the MMVC filter is shown in Figure 3c. Artifacts are no longer a major feature, and the intensity gradient is smooth rather than containing steps that give rise to edge segments.

This test is not statistically rigorous, but rather is used to provide a sensible and non-heuristic method of adjusting the decision threshold in response to filtering window size and local means. CoV is calculated from local estimates of variance and mean, and the distribution of the sample mean will cause the ratio of two squared CoVs to be not perfectly F-distributed. In addition, the F-test should be carried out using independent samples from normal populations. As previously mentioned, the noise in multi-look amplitude images tends quickly to Gaussian for large \((L)\) (Ulaby et al., 1982). The assumption of a normal population is reasonable for the three- and four-look imagery we commonly use, though will not be perfectly met for smaller \(L\).

The assumption of sample independence, however, is not complied with, because there is overlap of the second-stage filter window positions in which CoV is assessed. We disallow positions that are very close to the position of CoVmin (\(\pm 2\) pixels).
from this comparison procedure as they have a very high degree of overlap. This helps, but it does not altogether remove, the non-independence problem.

As \( \alpha \rightarrow 1 \), the probability of a false alarm increases. There is a higher probability that an upwards fluctuation in \( \text{COV} \) will be treated as significant and will, therefore, trigger adaptive action. In other words, as \( \alpha \rightarrow 1 \), the MMCV filter tends to the MCV filter, and more "edges" are preserved. Figure 4 shows a synthesized image, which has been filtered with the MMCV filter using a series of three \( \alpha \) values. The image contains a diffuse edge, which ramps up over a distance of ten pixels. The increasing significance attributed to a \( \text{COV} \) fluctuation, and thus to an edge or other signal structure, can be seen. The ramp is smoothed over for small \( \alpha \) but rendered as an edge for larger \( \alpha \). Note that this effect is scale-dependent. With a filtering window of comparable diameter to the ramp width (Figures 4a, 4b, and 4c), the filter window barely subtends the ramp, so perceives it as an intensity gradient. A larger filtering window (Figures 4d and 4e) is more able to perceive and locate such a diffuse edge.

This behavior allows us to choose \( \alpha \) according to our application. If there is a significant intensity gradient in the image, as in the example of Figure 3, we may choose a small \( \alpha \). However, if we have many low-amplitude, diffuse edges that we wish to preserve, then a large \( \alpha \) may be more suitable. We find a significance level of \( \alpha = 0.05 \) to be suitable for much of our work.

Properties of the MMCV Filter

The inherent trade-off between signal-to-noise ratio (SNR) and spatial localization, known as Canny's uncertainty principle (Canny, 1986), remains in force. In the effort to preserve edges and obtain a high level of noise suppression, the filter does create a greater level of geometric distortion of feature boundaries than do many other filters. A comparison between the MMCV filter and the Lee filter is presented in the next section, where this trade-off between noise and localization of feature boundaries is clearly seen. We address this issue in other aspects of our work, by operating the filter in multiscale mode (North et al., 1998; Wu and North, 1999). In this work, we initially obtain a reasonably clean, though geometrically distorted, edge map at a relatively coarse scale. We then correct the location of the edge elements by reference to finer-scale edge maps.

To understand the MMCV filter's response at different scales, Figure 5 shows a JERS-1 subscene filtered using four filt-
ter window diameters. The filter window diameters in Figure 5 are 23, 17, 11, and 5 pixels (listed from coarse to fine). The feature shape distortion at coarse scales is clear, as is the suppression of features that are small with respect to the filter window. Figure 5a displays the clean, continuous edges that can be obtained at coarse scale.

The issue of "visibility" of features that are small with respect to the filter window is discussed in North and Wu (1999). A large (compared to the feature) window will smooth over such features, leading to their suppression. However, a smaller window will perceive a relatively homogeneous region inside the feature, and so will preserve it. A further issue addressed in North and Wu (1999) was the asymmetry of the CoV surface about a step edge. This issue pertains to the computation of the CoV near a boundary. When the local standard deviation is normalized by the local mean, this operation is valid for the variance introduced by the speckle noise; however, it is not valid for the variance introduced by the step edge. The result is depressed CoV values on the brighter side of an edge (larger normalizing value), and elevated CoV values on the darker side, as seen in Figure 1. This leads to the MCV filter favoring filter positions on the brighter side of the edge, so that bright features are preserved in the filtered image down to a smaller size than is the case for dark features.

Modified MCV filtering is not tied to a comparison with the theoretical CoV for the imagery. Our experience is that the existence of texture (some distributed inhomogeneity in the signal whose period is smaller than the filter window) can increase the CoV in a region, but we do not want adaptive action to be triggered by this higher CoV. In our application, we are interested in the extent of the textured region, but not in the texture itself. We wish to smooth over it rather than allowing it to create filtering artefacts. The MMCV filter deals elegantly with this and will generally take a mean about the central pixel, because the CoV values in the first-stage window are all fairly similar to the minimum value, even though they may be higher than the theoretical CoV.

In our use of a higher-than-expected CoV to indicate areas where an edge may exist, we are making an important assumption: that normalized signal variance is approximately equal on either side of the step edge. When creating synthetic data for testing a filter, we tend to generate areas of constant signal ($\alpha = 0$) on either side of a step edge, but, unfortunately, this is often not the case in real data. If a region of high underlying signal variation exists on one side of an edge and not on the other, then the MMCV filter will tend to stay away from the high CoV values that the signal texture creates, and take a mean from the less variable side of the edge, leading to expansion of the less variable region in the filtered image.

Violation of the assumption of equal background signal homogeneity in non-boundary regions can result in geometric distortion of feature boundaries, as shown in Figure 6. In this JERS-1 subscene, the forest (bright) region is relatively homogeneous with a CoV near to the theoretical. But the pasture (dark) area around it is studded with shelter belts and isolated clumps of trees, setting up a region where the normalized variance is very high. Therefore, any filter window whose primary window includes a forest pixel will tend to find the minimum CoV position as far into the forest region as possible, even when $p_2$ is well outside the forest region.

The modifications to the MCV filter improve the situation for most of these issues, due, simply, to the preference for spatial localization it embodies. In most cases, the MMCV filter will provide feature boundaries having more accurate location than the MCV filter.

**Comparison with Existing Techniques**

Our comparison with existing methods is in the context of our application which is (1) imagery corrupted with multiplicative speckle noise; and (2) a requirement to derive an edge map that is usable in a GIS; that is, we wish to minimize inclusion of spurious edges and exclusion of feature boundaries.

Figure 7 is a direct comparison of a subscene filtered using (1) Lee's local statistics filter (Lee, 1986), (2) the local region filter (Nagao and Matsuyma, 1979), (3) the original MCV filter (Schulze and Wu, 1995), and (4) our modified MCV filter ($\alpha = 0.05$). All have been operated with a window diameter of 25 pixels. Edge maps derived from the filtered subsamples using a simple gradient operator are displayed in Figure 8. These have all been thresholded at a constant value so that a valid comparison can be made between the filters. Figure 9 contains edge maps extracted from small parts of the filtered area, which have been magnified so that boundary positions can be seen. The constant threshold used in Figure 8 is not in fact appropriate for an image corrupted with signal-dependent noise, so in Figure 9 we have used a statistically based thresholding.

Briefly, the thresholding is performed using a t-test on the difference between means on either side of the proposed edge. For the purposes of the test, the local variance on either side of the edge is estimated using the theoretical CoV ($\sigma_b$) and the local mean. The filtering window diameter is used to calculate the sample size.

The data (Figure 7e) are a subscene of Radarsat data (C-band, four-look) showing an area of pasture surrounded by forest. It would have to be said that the Lee filter produces a result that is more visually pleasing than the MCV or MMCV filters. More importantly, we observe that (1) Lee's filter retains small features but also more noise, and (2) edges in both the MCV- and MMCV-filtered images are sharper, cleaner, and more continuous but exhibit more distortion. Here we can clearly see the perennial trade-off between SNR and spatial localization. Lee's filter errs towards better spatial localization at the expense of edge clarity; the MMCV filter errs in the other direction in order to meet the needs of our application.

Comparing the MMCV filter (Figures 7d, 8d, and 9d) with the local region filter (Figures 7b, 8b, and 9b), it is clear that the

**Figure 6.** The effect of violating the assumption of equal normalized signal variance on both sides of a boundary. Boundary on the left of the bright forest area is misplaced due to high signal variance caused by small bright features (shelter belts) in left of image. Boundary on the right of the forest area is more accurately positioned because the CoV of the dark pasture area is similar to that of the forest. (a) Original JERS-1 data (L-band, three-look) overlaid with the edge map derived from (b). (b) MMCV ($\alpha = 0.05$) filtered using $d = 25$ pixels.
Figure 7. Comparison of four filters, all operated with \( d = 25 \) pixels. Data from Radarsat (C-band, four-look) of dark pasture area surrounded by forest. (a) Lee's local statistics filter. (b) Local region filter. (c) MCV filter. (d) MMCV filter \((a = 0.05)\). (e) Original data.

The bright linear features in the pasture area of Figure 7e are about 5 to 6 pixels wide. The 25-pixel filtering window cannot fall entirely within them, so pixel estimates in their vicinity are calculated in windows as far out from these bright features as possible. When \( p \) falls within these bright features, any second-stage window will still contain a few of these bright-feature pixels. Their effect can be seen in the filtered images.

**Further Examples**

We have applied the MMCV filter to two further examples, to show its utility for natural, non-ideal imagery. Figure 10 is a subscene from a JERS-1 (L-band, four-look) image showing an area of production conifer forestry surrounded by pasture. The preserved boundaries are a good representation of the interface between standing and felled timber.

The subscene in Figure 11 is also from the JERS-1 satellite. This area of coastline is part of the Marlborough Sounds, New Zealand. Here the hills create a strong radar return from the fore-slopes and a weak return from the back slopes. Again, intensity gradient is a strong feature of the image, and, yet, a rea-
Figure 9. Magnified edge maps from areas marked in Figure 8e as A1, A2, A3. Simple gradient operator was used, followed by statistical thresholding. (a) Original data. (b) Edge map after local region filtering. (c) Edge map after MCV filtering. (d) Edge map after MMCV filtering.

Figure 10. JERS-1 (L-band, three-look) data showing part of Santoft forest near Foxton, New Zealand. (a) Original data. (b) Filtered with MMCV ($\alpha = 0.05$) filter, $d = 19$ pixels. (c) Resulting edge map.

Figure 11. JERS-1 (L-band, three-look) data showing Tory Channel, Marlborough Sounds, New Zealand. (a) Original data. (b) Filtered with MMCV ($\alpha = 0.05$) filter, $d = 15$ pixels. (c) Resulting edge map.

Reasonable representation of coastlines and ridges can be obtained.

Conclusions
The minimum coefficient of variation (MCV) filter has been designed to effectively remove multiplicative speckle noise from SAR imagery, while retaining the critical boundaries between features. However, the strong edge-preserving nature of this filter can lead to step-edge artefacts in areas of intensity gradient. The modified MCV filter (MMCV) reduces the occurrence of such filtering artefacts without losing the edge-preserving characteristic. SAR subscenes have been smoothed with the MMCV filter and with several existing filters in order that the performance of this new filter may be assessed. Edge maps derived from these smoothed subscenes were also compared, because our aim is to produce clean, continuous linework suitable for input into a GIS. Results presented in this paper show that the MMCV filter is able to provide extremely good noise suppression and edge clarity, working from SAR imagery of natural targets. Further work is planned to minimize the filter's dependence on the assumption of equal normalized signal variance on either side of a boundary; the proposed approach is examination of Cov surface shape about step edges.

Acknowledgment
This work was supported by the New Zealand Foundation for Research, Science and Technology, contract number C09609.

References


(Received 29 June 1999; accepted 09 November 1999; revised 01 December 1999)