THIS WORK SO IT WAS NECESSARY TO PROCEED WITHOUT THESE TRANSFORMATIONS. WHILE NO CLAIMS ARE MADE FOR GREATER ACCURACY OF GRAPHICAL TRIANGULATION EXECUTED WITH THE AID OF THE PRIETO TILT FINDER, IT IS BELIEVED THAT THE REDUCTION IN TIME REQUIRED FOR MAKING THE LIST AND TILT ANALYSIS AS COMPARED WITH THE MCADAM METHOD IS OF NOTEWORTHY CONSIDERATION.

A DEVICE FOR GRAPHIC DETERMINATION OF TILT IN MULTI-LENS AERIAL PHOTOGRAPHS

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A TILT FINDING DEVICE IS USED TO DETERMINE THE LIST AND TIP COMPONENTS OF TILTED MULTIPLE-LENS PHOTOGRAPHS USED BY THE EL SALVADOR-GUATEMALA BOUNDARY COMMISSION. IT CONSISTS OF A POINTER ARM AND AN UPPER AND LOWER PLANE PIVOTED TOGETHER SO THAT THE POSITION OF ONE PART TO ANOTHER CAN BE ALTERED AS DESIRED AND THEN FIRMLY CLAMPED TOGETHER. PROVISION IS MADE FOR THE OPERATOR TO SET OFF ON EACH OF THESE PLANES MEASUREMENTS THAT HAVE BEEN TAKEN FROM TWO OVERLAPPING PHOTOGRAPHS AND THUS GRAPHICALLY REPRODUCE THE APPROXIMATE COMPONENTS OF THE TILT THAT EXISTED IN THE AERIAL CAMERA AT THE MOMENT OF EXPOSURE.

THE LINE OI ON THE POINTER ARM (FIG. 1) REPRESENTS ANY IMAGE RAY LINE, AND THIS LINE CAN BE SET ON EITHER OF THE TWO SCALES TO ACCORD WITH MEASUREMENTS THAT ARE MADE ON THE AERIAL PHOTOGRAPHS. A CLAMP IS PROVIDED FOR CLAMPING THE POINTER ARM TO THE UPPER PLANE AS SHOWN IN THE FIGURE. THE UPPER PLANE IS IN THE FORM OF A SEGMENT OF A CIRCLE AND CAN BE ROTATED AROUND THE PIVOT POINT WHICH IS FIXED TO THE LOWER PLANE. THE LINE OP' ON THE UPPER PLANE REPRESENTS THE OPTICAL AXIS OF THE CAMERA. THE SCALE A'B' IS DRAWN ON THIS SAME PLANE NORMAL TO THE LINE OP' AND AT A DISTANCE FROM THE PIVOT POINT EQUAL TO THE PRINCIPAL DISTANCE OF THE CAMERA. IT IS GRADUATED IN MILLIMETERS IN BOTH DIRECTIONS FROM THE POINT P' WHICH INDICATES THE PRINCIPAL POINT OF THE PHOTOGRAPH.

THE LOWER PLANE, TO WHICH THE PIVOT IS FIXED, REPRESENTS THE TILT-FREE POSITION OF THE CAMERA WHILE THE UPPER PLANE CORRESPONDS TO ITS ACTUAL TILTED POSITION AT THE TIME OF EXPOSURE. THE LINE OP ON THE LOWER PLANE CORRESPONDS TO THE VERTICAL FROM THE CAMERA TO THE GROUND SURFACE AND ALSO THE LENS AXIS IN A TILT-FREE POSITION. THE SLIDING, HORIZONTAL SCALE, MN, IS PLACED UNDER THE LOWER PLANE NORMAL TO THE OP LINE AT A DISTANCE FROM THE PIVOT EQUIVALENT TO THE PRINCIPAL DISTANCE OF THE CAMERA. THIS SCALE IS ALSO GRADUATED IN MILLIMETERS IN BOTH DIRECTIONS FROM ITS CENTRAL ZERO POINT AS INDICATED IN FIGURE 1 AND IS SO ARRANGED THAT IT CAN BE MOVED HORIZONTALLY AS NEEDED.

A NEW TILT FINDER MUST BE CONSTRUCTED FOR EACH AERIAL CAMERA AS THE OP AND OP' DISTANCES MUST ALWAYS EQUAL THE PRINCIPAL DISTANCE OF THE CAMERA. IT IS ESSENTIAL THAT CELLULOID, OR SOME OTHER TRANSPARENT MATERIAL, BE USED IN ITS CONSTRUCTION, TO PERMIT SETTING THE POINTER ARM ON BOTH SCALES AND READING THE DISTANCE ON THE A'B' SCALE BETWEEN THE OP AND OP' LINES.

MCADAM COMPARES MEASUREMENTS OF DISTANCES MADE ON SUCCESSIVE PHOTOGRAPHS FROM A COMMON IMAGE POINT TO THE OUTER FRAME OF THE A AND C WING PRINTS OF A FIVE-LENS COMPOSITE PHOTOGRAPH. HE GIVES A RELATIVELY SIMPLE FORMULA FOR CALCULATING THE SIDE SWING OF THE CAMERA BETWEEN SUCCESSIVE EXPOSURES AND THIS SWING IS THEN REDUCED BY A FACTOR TO DETERMINE THE RELATIVE LATERAL SHIFT IN POSITION OF THE PRINCIPAL POINTS AGAINST THE GROUND SURFACE. IN THE CASE OF THE NEW TILT FINDER THE IMAGE POINT COMMON TO TWO OVERLAPPING PHOTOGRAPHS

IS SELECTED AS IN THE McADAM METHOD AND PERPENDICULARS ARE DROPPED FROM ITS TWO POSITIONS TO THE CORRESPONDING COLLIMATING LINES PASSING THROUGH THE PRINCIPAL POINTS OF THE A, B, AND C PRINTS (FIG. 2). DISTANCES SCALED BETWEEN THESE PERPENDICULARS AND THE PRINCIPAL POINTS OF THE TWO COMPOSITE PHOTOGRAPHS ARE THE COMPLEMENTS OF THE MEASUREMENTS USED BY McADAM.

LET THE DISTANCE MEASURED ON PHOTOGRAPH NO. 1 BE INDICATED BY Y_1 AND ON PHOTOGRAPH NO. 2 BY Y_2 (FIG. 2). SWING THE POINTER ARM (FIG. 1) UNTIL THE OI LINE READS THE Y_1 DISTANCE ON THE A'B' SCALE AND CLAMP IT TO THE UPPER PLANE. HOLD THE ZERO OF THE MN SCALE ON THE OP LINE OF THE LOWER PLANE AND SWING THE UPPER PLANE UNTIL THE OI LINE OF THE POINTER ARM READS THE Y_2 DISTANCE ON THE MN SCALE. THE RELATIVE SIDE DISPLACEMENT OF THE PRINCIPAL POINTS OF THE TWO PHOTOGRAPHS WILL THEN BE REPRESENTED BY THE INTERVAL INTERCEPTED ON THE A'B' SCALE BY THE OP AND OP' LINES. CHECK MEASUREMENTS ARE MADE BY MEANS OF IMAGE POINTS SELECTED ON THE OUTER EDGE OF THE OPPOSITE WING PRINTS AND THE MEAN VALUE ACCEPTED AND RECORDED IN TABULAR FORM. THE PROCESS IS REPEATED FOR THE SECOND AND THIRD PHOTOGRAPHS, AND SIMILARLY FOR ALL PHOTOGRAPHS THROUGHOUT THE FLIGHT, THUS DETERMINING THE RELATIVE DISPLACEMENTS IN ONE DIRECTION OF ALL PRINCIPAL POINTS. THESE DISPLACEMENTS ARE NOT FROM THE HORIZONTAL PLANE, BUT ARE RELATIVE MEASUREMENTS BETWEEN SUCCESSIVE PHOTOGRAPHS. THE DISPLACEMENTS ARE PLOTTED ON A DIAGRAM, AS IN THE McADAM METHOD, AND THE LIST COMPONENT OF THE TILT TO BE APPLIED TO EACH PHOTOGRAPH IS DETERMINED BY MEANS OF A "MEAN BAND" AS USED IN THE McADAM METHOD. THESE COMPONENTS OF THE TILT ARE PLOTTED DIRECTLY ON THE Y AXES OF THE PHOTOGRAPHS, DUE REGARD BEING GIVEN TO THEIR SIGNS.

THE DETERMINATION OF THE TIP COMPONENT IS BASED ON MEASUREMENTS MADE BETWEEN THE PRINCIPAL POINTS OF SUCCESSIVE PHOTOGRAPHS. THIS IS ACCOMPLISHED BY IDENTIFYING THESE POINTS ON OVERLAPPING PHOTOGRAPHS AS INDICATED IN FIGURE 2. STARTING WITH THE FIRST PHOTOGRAPH IN THE STRIP, THE DISTANCE BETWEEN ITS PRINCIPAL POINT AND THE IMAGE OF THE PRINCIPAL POINT OF PHOTOGRAPH NO. 3 IS RECORDED AS X_1. A CORRESPONDING DISTANCE, X_2, IS THEN MEASURED ON PHOTOGRAPH NO. 2 AND THE PROCESS CONTINUED THROUGHOUT THE FLIGHT.

THE TILT AXIS OF PHOTOGRAPH NO. 2 MUST PASS REASONABLY CLOSE TO THE PRINCIPAL POINT OF THE PHOTOGRAPH FOR ALL TILTS ORDINARILY ENCOUNTERED. THE SCALE ON ONE SIDE OF THIS AXIS WILL BE ENLARGED, AND ON THE OTHER SIDE REDUCED, BY APPROXIMATELY EQUAL AMOUNTS, IF THE TILT DOES NOT GREATLY EXCEED FIVE DEGREES. FOR THIS REASON, THE DISTANCE X_2 CAN BE CONSIDERED AS DIVIDED APPROXIMATELY IN HALF BY THE TILT AXIS, FOR AN INCREASE IN LENGTH OF THIS LINE ON ONE SIDE OF THE TILT AXIS WILL BE OFFSET BY A CORRESPONDING DECREASE ON THE OTHER SIDE. WITH THIS ASSUMPTION THE LENGTH OF X_2 AS ACTUALLY MEASURED ON PHOTOGRAPH NO. 2, WHICH IS PRESUMABLY TILTED, WILL CLOSELY APPROXIMATE ITS TRUE AMOUNT HAD THE PHOTOGRAPH BEEN MADE WITHOUT TILT (SEE FIG. 3). THIS ASSUMPTION OF THE EQUALITY IN LENGTH OF X_2 VALUES AS MEASURED ON HORIZONTAL AND TILTED PLATES INTRODUCES ONLY SMALL ERRORS SO LONG AS THE TILT DOES NOT EXCEED FIVE DEGREES WHEN THE TILT AXIS IS NORMAL TO THE FLIGHT LINE. THE DIFFERENCES INCREASE RAPIDLY FOR GREATER TILTS BUT THIS SHOULD NOT HAVE AN IMPORTANT BEARING ON THE USE OF THIS METHOD BECAUSE SUCH FLYING SHOULD BE RARELY ENCOUNTERED. HOWEVER, RELIEF WILL AFFECT THE TIP COMPONENT MORE THAN IT AFFECTS THE LIST.

THE TILT-FINDER IS UTILIZED FOR TIP CORRECTIONS BY CLAMPING THE POINTER ARM SO THAT THE OI LINE READS THE X_1 DISTANCE ON THE A'B' SCALE AND THEN SWINGING THE UPPER PLANE AND SIMULTANEOUSLY MOVING THE MN SCALE UNTIL A POSITION IS FOUND WHERE THE OI LINE READS THE X_2 DISTANCE ON THE MN SCALE WHEN THE ZERO OF THIS SCALE IS ON THE OP' LINE. THE DISPLACEMENT OF THE PRINCIPAL POINT ALONG THE TIP COMPONENT WILL THEN BE INDICATED ON THE A'B' SCALE AS THE DISTANCE FROM P' TO THE FIXED VERTICAL LINE OP. THE TIP COMPONENT FOR PHOTOGRAPH NO. 1 IS FOUND BY COMPARISON WITH PHOTOGRAPH NO. 2, FOR PHOTOGRAPH NO. 2 BY COMPARISON WITH PHOTOGRAPH NO. 3 AND SO ON UNTIL THE END OF THE FLIGHT.

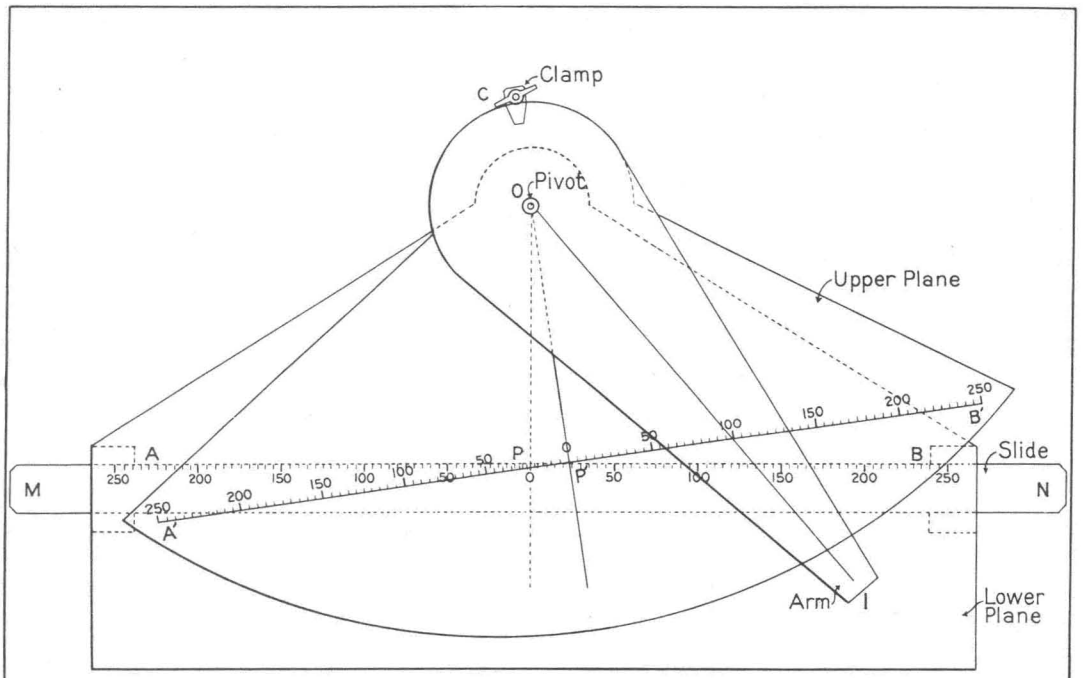


Fig. 1 - The tilt finder

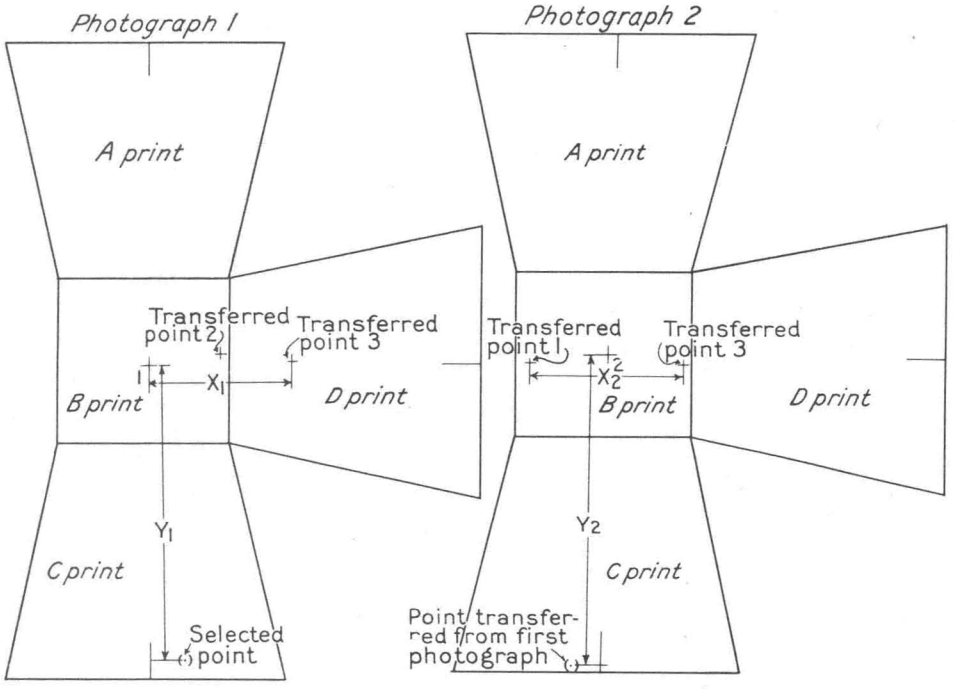


Fig. 2 - Distances between principal points and image points on overlapping four-lens photographs.

P_1, P_2, P_3 · principal points of photos 1, 2, 3 respectively.
V · Nadir (plumb) point of photo 2.
X_1 · Horizontal projection of distance between points P_1 and P_3
X_2 · Distance between points P_1 and P_3 as measured on photo 2

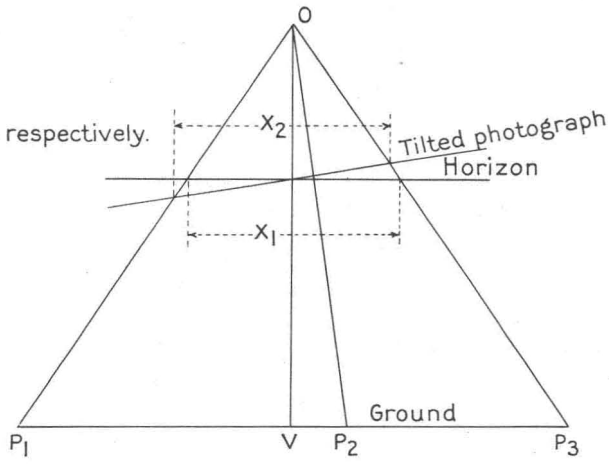
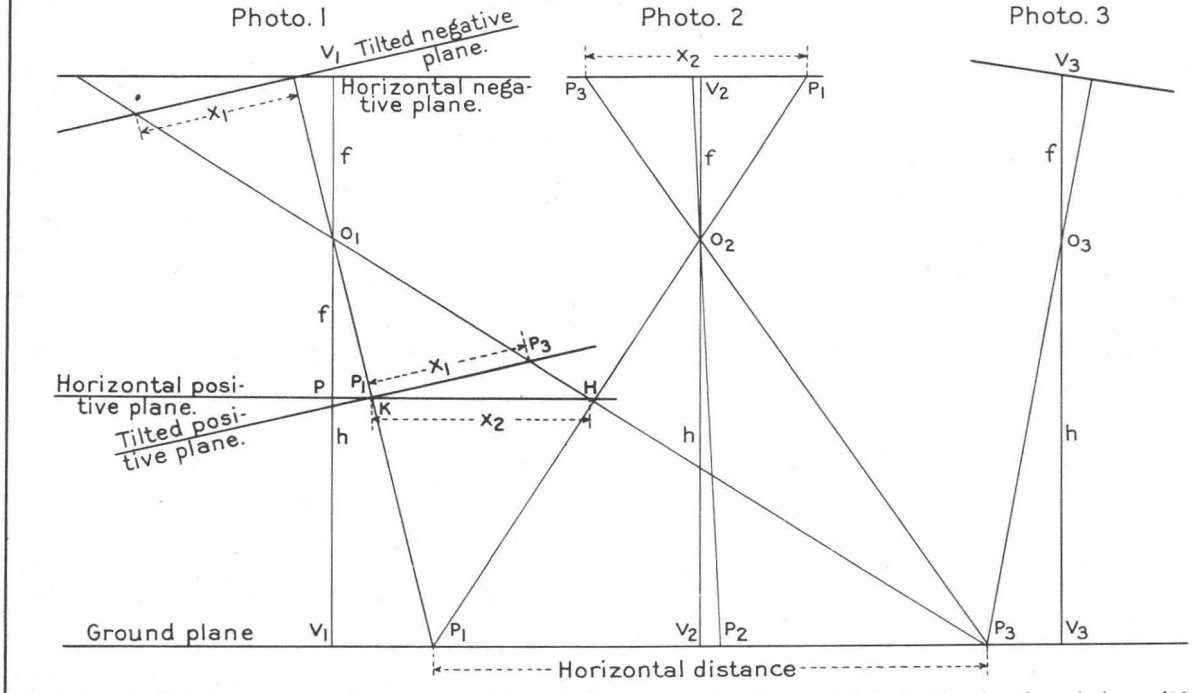


Fig. 3 - Tip component of tilt as measured on three successive photographs.



In triangle $P_1O_1P_3$-
$KH : P_1P_3 = O_1K : O_1P_1$
In triangle $P_1O_1V_1$-
$O_1K : O_1P_1 = f : h$
$\therefore KH : P_1P_3 = f : h$
In photo. 2-
$X_2 : P_1P_3 = f : h$
$\therefore KH = X_2$

By construction KH is the projection of P_1P_3 in the horizontal positive plane $\therefore X_2$ is the projection of X_1 in the horizontal positive plane. Therefore in placing the distance X_1 in the upper scale A'-B' of the Tilt Finder and then pivoting the upper plane so that the OP' line and the pointer arm line OI measure the distance X_2 on the sliding scale M-N, the original position of tilt in the vertical plane of the flight line at time of exposure has been recovered and the distance from P' to the vertical OP, as measured on the upper scale A'-B', is the measure of displacement of the principal point of Photo. 1 from the vertical.

Fig. 4 - Proof of tip component of tilt as measured on three successive photographs.

A BETTER UNDERSTANDING OF THE TIP RELATION CAN BE OBTAINED BY REFERENCE TO FIGURE 4 WHICH SHOWS PHOTOGRAPH NO. 1 SO TILTED THAT THE IMAGE RAY LINES TO THE TWO PRINCIPAL POINTS OF PHOTOGRAPHS NOS. 1 AND 3 INTERSECT THE CORRESPONDING OBJECT POINTS IN THE GROUND PLANE. THUS, THE ANGLE BETWEEN THE O1 LINE ON THE POINTER ARM AND THE OP' LINE (OPTICAL AXIS) INDICATED ON THE UPPER PLANE SUBTENDS ON THE HORIZONTAL SCALE MN THE X_2 DISTANCE, WHICH HAS BEEN ASSUMED EQUAL TO THE TRUE HORIZONTAL VALUE OF THE DISTANCE BETWEEN THE IMAGES CORRESPONDING TO THE TWO PRINCIPAL POINTS.

PHOTOGRAPH NO. 2 IS COMPARED WITH PHOTOGRAPH NO. 3 BY MEASUREMENTS MADE BETWEEN THE IMAGES OF THE PRINCIPAL POINTS OF PHOTOGRAPHS NOS. 2 AND 4. THE MEASUREMENT OF THE DISTANCE X_3 MADE ON PHOTOGRAPH NO. 3 IS AGAIN CONSIDERED AS THE TRUE HORIZONTAL VALUE FOR PHOTOGRAPH NO. 2.

ONE SHOULD ALWAYS PROCEED ALONG THE PHOTOGRAPHIC STRIP IN A DIRECTION WHICH WILL HAVE THE D PRINT OF A FOUR-LENS COMPOSITE PHOTOGRAPH IN ADVANCE. WHEN THE LAST PHOTOGRAPH IN THE STRIP IS REACHED IT WILL BE NECESSARY TO SELECT AN ARBITRARY POINT BEYOND THE LAST PRINCIPAL POINT AT A DISTANCE EQUAL TO THE INTERVAL BETWEEN THE TWO PRECEDING PRINCIPAL POINTS. THE MEASUREMENT MADE IN THE NEXT-TO-LAST PHOTOGRAPH (BETWEEN ITS PRINCIPAL POINT AND THE ARBITRARILY SELECTED POINT) IS THEN COMPARED TO THE DISTANCE MEASURED ON THE LAST PHOTOGRAPH. THE LAST PHOTOGRAPH IN A FLIGHT OF FOUR-LENS COMPOSITE PHOTOGRAPHS CANNOT BE CORRECTED, BUT IN BOTH FIVE-LENS OR NINE-LENS COMPOSITES, CALCULATIONS CAN BE MADE IN BOTH DIRECTIONS ALONG THE FLIGHT LINE AND A MEAN VALUE ADOPTED FOR THE TIP COMPONENT.

THE TIP AND LIST CORRECTIONS ARE TRANSFERRED TO EACH PHOTOGRAPH BY MEASUREMENTS ALONG THE **x** AND **y** AXES OF THE PHOTOGRAPHS, PROPER ATTENTION BEING GIVEN TO THE SIGNS OF THE MEASUREMENTS. THE INTERSECTION OF PERPENDICULARS FROM THESE CORRECTION POINTS WILL LOCATE THE APPROXIMATE POSITION OF THE NADIR (**v**) POINT ON THE PHOTOGRAPH AS IN THE MCADAM METHOD. THE POINT TO USE AS THE ORIGIN OF RADIAL LINES IS SELECTED ON THE LINE JOINING THE PRINCIPAL POINT AND THE NADIR AS DETERMINED BY THE TILT-FINDER. IN PHOTOGRAPHS OF FAIRLY LEVEL TERRAIN THE ORIGIN SHOULD BE PLACED NEAR THE ISOCENTER OR MID POINT OF THE LINE BUT FOR PHOTOGRAPHS OF TERRAIN WITH CONSIDERABLE RELIEF IT SHOULD BE PLACED CLOSE TO THE NADIR. IT IS RECOMMENDED THAT ALL PHOTOGRAPHS HAVING EXCESSIVE TILTS BE TRANSFORMED TO THE HORIZONTAL AS NEARLY AS POSSIBLE BEFORE USE IN THE PLOT.

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NOTE: IN ORDER TO COMPLETE A FULL SET OF ALL COPIES OF "NEWS NOTES" AND "PHOTOGRAMMETRIC-ENGINEERING" TO BE PLACED IN THE LIBRARY OF CONGRESS, THE SECRETARY MUST ACQUIRE A COPY OF "NEWS NOTES" FOR JANUARY-FEBRUARY-MARCH 1936 (VOL. II, NO. 1) AND APRIL-MAY-JUNE 1936 (VOL. II, NO. 2).

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