THE RADIAL-STEREOPLOTTER[1]

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Abstract

A machine for plotting from air photographs, by the radial line method, has been developed by the Associate Research Committee on Surveys. The purpose of the plotter is to increase the accuracy and speed of extending minor control through strips of photographs by this method which is so widely used in Canada.

A description of the machine is given, preceded by a brief explanation of the principles underlying its design.

The methods so far employed in using the plotter are explained together with an examination of the sources of error arising in these methods, some of which are of general application to radial line plotting. Owing to the short time that the radial-stereoplotter has been in operation, conclusive statements cannot be made as to the results it will give when handled by an experienced operator, but already there is evidence of a considerable saving of time and gain in accuracy as compared with hand graphical methods of extending radial line control.

Introduction

The Subcommittee on Mapping Methods of the Associate Research Committee on Surveys, in its efforts to develop those forms of air photogrammetry peculiarly applicable to Canadian conditions, *i.e.*, the need for small-scale maps produced as cheaply and efficiently as possible, has sponsored the construction of a machine for extending control by radial intersection through strips of air photographs. The principles of the plotter were developed by Colonel Burns, and the design and construction of the first model was undertaken by the staff of the National Research Council in co-operation with him.

Radial Intersection Control

The radial line method of extending control is now well established. It is discussed in References (1) and (2), and depends on the fact that for the case of vertical air photographs the distortion due to elevation differences and to small tilts may be regarded as radial from the principal point (2, p. 69). This permits nets of triangles to be extended through a strip of overlapping photographs, the corners of the triangles being formed by the ground plans (plotted to a uniform scale) of the principal points of consecutive photographs and of convenient details near the margins (minor control points). The points so plotted form a system of control for the transfer of detail from the photographs to the map.

Fundamental Considerations

Let Fig. 1 represent two consecutive vertical (untilted) photographs placed so that the principal point of the right-hand photograph, P_1, is placed over the image of the corresponding detail in the left-hand photograph, and one

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of the photographs is rotated about this point until the image of the detail corresponding to P, the principal point of the left-hand photograph, falls on the line PP_1; PP_1 is the projection of the air base, or line joining the two positions of the camera lens at the respective instants of exposure. Let A represent the image of a detail on the ground as registered on the left-hand photograph, and A_1 represent the image of the same detail on the right-hand photograph. If the photographs were untilted, undistorted, taken at the same altitude and the elevations of the three points P, P_1, and A were the same, A and A_1 would coincide. If this does not occur, and the tilt is small, the point a where AP and A_1P_1 intersect is the ground plan, to the scale set by the length of PP_1, of the detail represented by A with respect to PP_1. Perspective considerations show that A_1A is parallel to PP_1, for it is the trace on the photographs of a plane passing through the lens and parallel to PP_1. Relative tilt in the photographs will cause a departure from this parallelism.

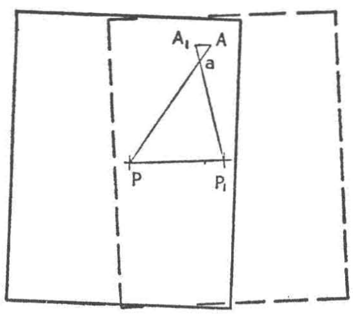


FIG. 1.

With the photographs placed as in Fig. 1, a linkage could be imagined consisting of two rods, pivoted respectively at P and P_1, which would serve to determine the true shape of triangles formed by the ground plans of the points P, P_1, and A. To make such a scheme feasible it would be necessary to separate the photographs so that both were visible at once, and conditions would become as indicated in Fig. 2. The right-hand photograph is moved

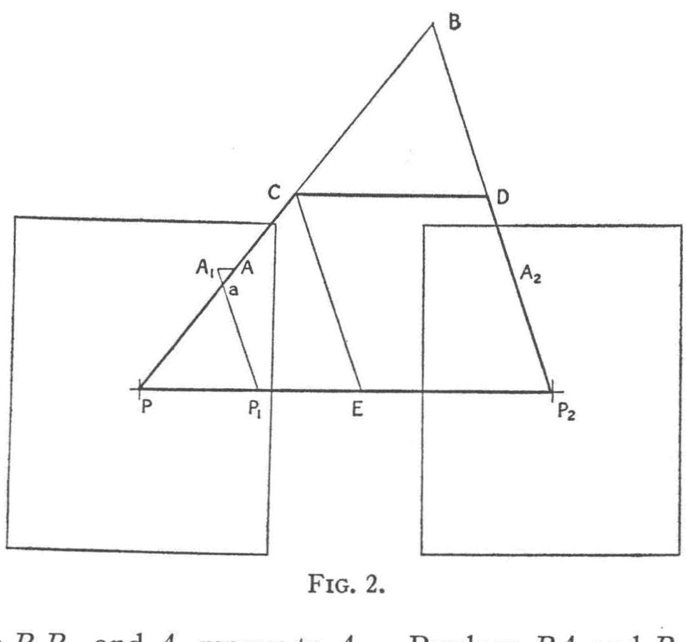


FIG. 2.

by an amount P_1P_2, and A_1 moves to A_2. Produce PA and P_2A_2 to intersect at B. The triangle BPP_2 is similar to the triangle aPP_1. This is quite general, and as the two radial arms PA and P_2A_2 were directed over the corresponding images of ground detail, the various positions of B would, if plotted, form a figure similar to that traced by the various positions of a, with the photographs superimposed as in Fig. 1. The scale of the plot of B's positions is obviously $\frac{PP_2}{PP_1}$ times the scale of the plot of a's positions. It follows, with the method visualized, that the scale can be adjusted by altering the length PP_2, but it is impracticable to reduce the scale to the extent that the photographs touch each other.

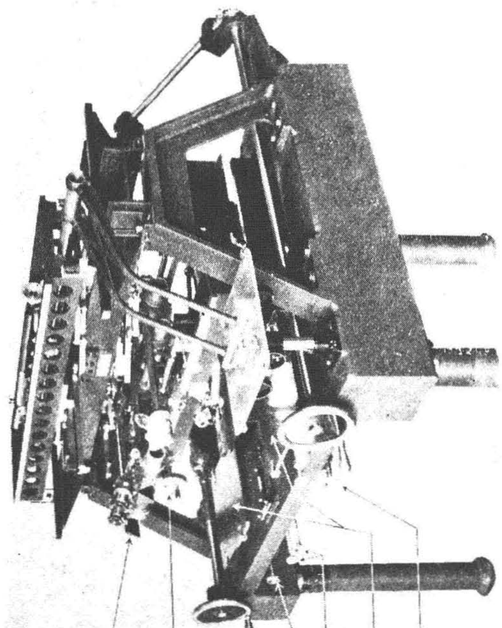
The scale may be reduced still further by the following modification. Draw CD parallel to PP_2, intersecting the radial lines in C and D. Draw CE parallel to P_2A_2, intersecting PP_2 in E. If CD remains parallel to PP_2 and of a constant length, as B moves from point to point, the triangle CPE will always be similar to BPP_2, and therefore C will trace a similar figure to B, but at a scale $\frac{PE}{PP_1}$ of the scale of the figure traced by a. In this way, by choosing the fixed length of CD we can arrange so that any convenient point in it (to which a pencil could be attached) would plot the plan positions of the detail registered on the overlapping photographs to a scale such that the air base was represented by the length $(PP_2 - CD)$.

In the radial-stereoplotter it is not practicable to use all parts of the figure CDP_2P in one plane, but the arms PA and P_2A_2, which determine the angles APP_1 and A_1P_1P, are in one plane, and in another, the arms CP and DP_2 move respectively parallel to them. The link or bridge CD is adjustable to determine the plotting scale, *i.e.*, by making the lines joining the plotted positions of selected points coincide with the lengths already plotted from the previous pair of photographs. The scale is chosen by setting the length of CD equal to a convenient amount when the first pair of a strip is plotted.

This device may be considered as an adaptation of the well known Zeiss parallelogram, so named about the time it was first used by Pulfrich and his assistants in the Zeiss Stereo-autograph for plotting terrestrial photographs. A further discussion of its properties will be found in References (2) and (5).

Principle of the Radial-stereoplotter

Imagine two consecutive photographs (a stereoscopic pair) oriented so that the air base lies along PP_1, Fig. 3, *i.e.*, set in the same relative orientation as at the respective instants of exposure, all tilts being ignored. In the plotter, the photographs are clipped on tables and have the principal points set over the centres of rotation, so that this adjustment is easily made (2, p. 34). A magnifying stereoscope D is provided with mirrors M and M_1, and by means of a right- and left-handed screw the distance MM_1 can be altered until it is possible to set the stereoscope so that the images, A and A_1, of a given detail are focused respectively on the reticules of the left- and right-hand telescopes of the stereoscope. The reticule marks (floating mark) of the stereoscope will then appear to lie on the ground at the point in question (2, p. 26) according to the well known principles of stereoscopy.



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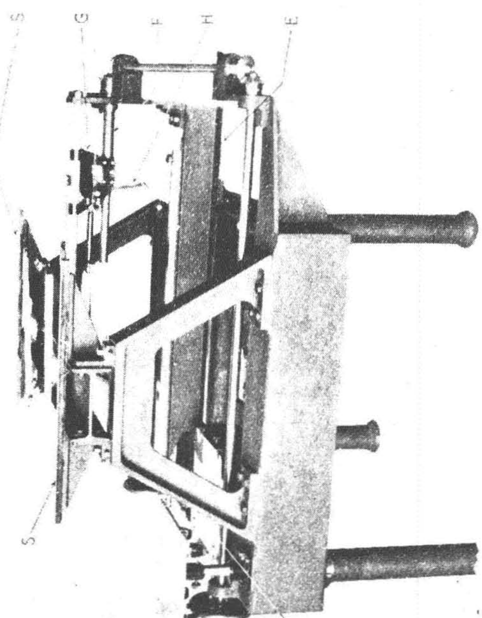

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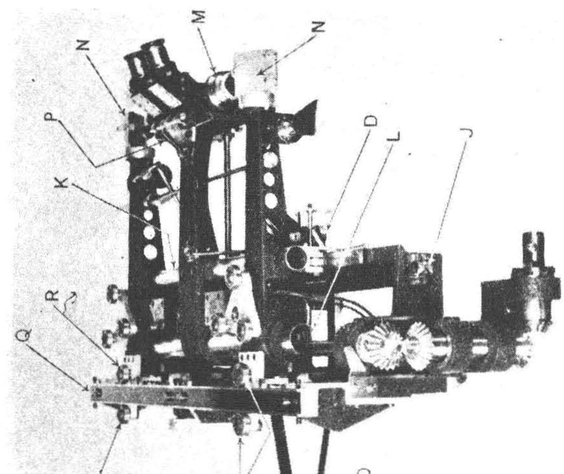

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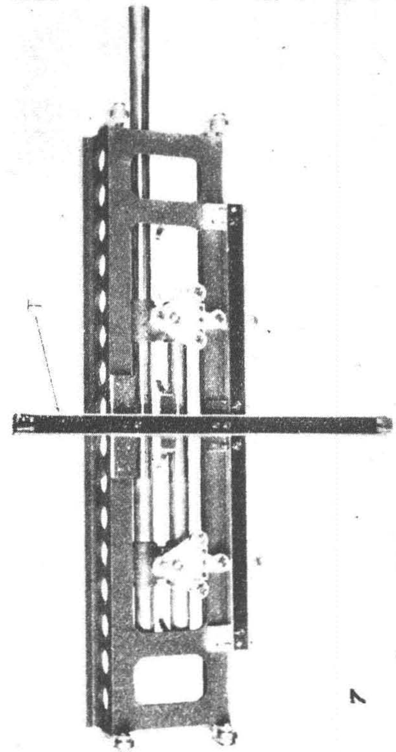

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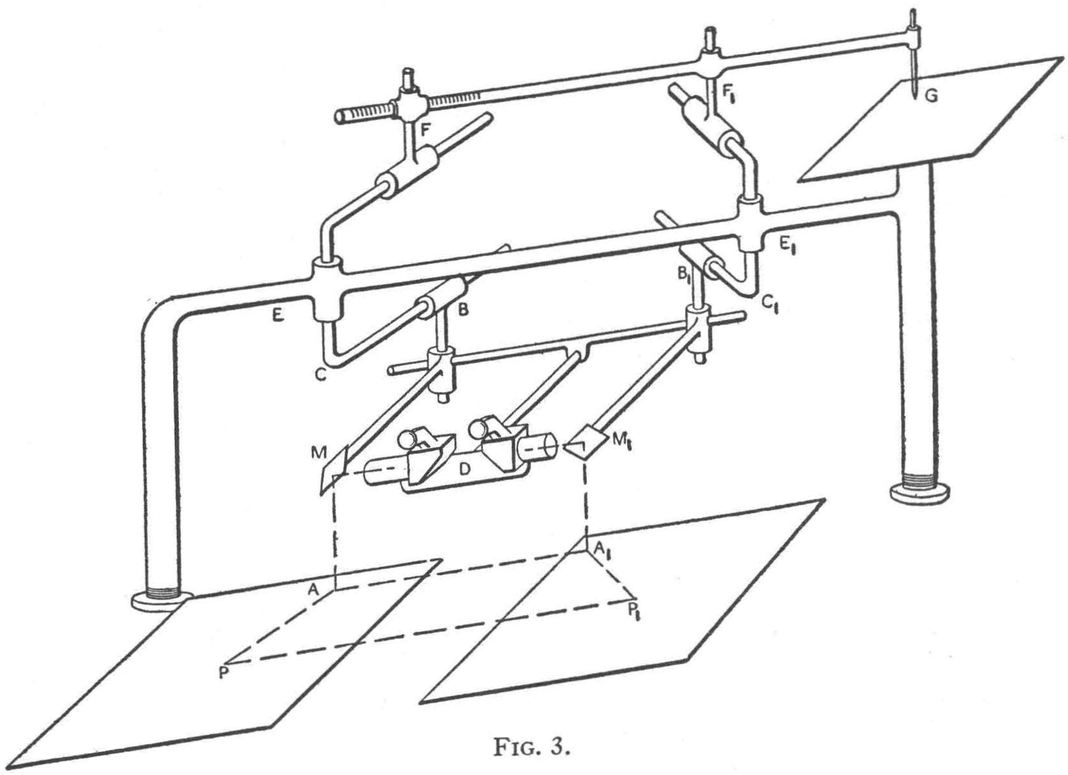

FIG. 3.

The brackets carrying the mirrors bear pivots at B and B_1, the upper parts of these pivots being fitted with slides on which engage arms CB and C_1B_1, which are free to swing about fixed pivots at E and E_1. The mirrors are adjusted until B is under E, when the left telescope is directed on P, and similarly for the right telescope. The horizontal projections of EE_1 and PP_1 then form a rectangle, and it is readily seen that the horizontal projection of EBB_1E_1 is similar and equal to that of PAA_1P_1, *i.e.*, we have the figure PAA_2P_2 in Fig. 2.

For plotting purposes two additional radial arms, above the pivots E and E_1, move parallel to the lower arms. A bridge FF_1G, constrained so as always to remain parallel to EE_1, is fitted with pivoted slides at F and F_1 in which the upper radial arms engage, thus forming a "Zeiss parallelogram". Any point on FF_1G, say G, will plot the relative plan positions of the points properly sighted on by the stereoscope to a scale at which the length, $(EE_1 - FF_1)$, represents the horizontal projected length of the air base PP_1. This scale may be quite different from that of the photographs.

PLATE I

FIG. 4. *The radial-stereoplotter.* FIG. 5. *The plotter without the upper and lower carriages. The upper radial arms, their fixed pivot bearings, and the plotting table are visible, as well as the handwheel and most of the mechanism for moving the lower carriage slowly in the Y direction.* FIG. 6. *The lower carriage, showing the mechanism for moving the stereoscope along the carriage in the X direction. The stereo telescopes and the mirror brackets, with the pivots which engage with the lower radial arms, are seen, as well as the two tubes along which the mirror brackets slide when parallax (Z direction) adjustments are made.* FIG. 7. *The upper carriage. The X and Y direction guide bars are clearly seen, as well as the triangular plates on the end of the pivots which engage with the upper radial arms. The spacing of these pivots determines the plotting scale. The plotting pencil is carried from the tube seen extending to the right.*

It should be observed that the radial intersection method of plotting fails when the two rays tend to approach the same direction, *i.e.*, along the air base. With the radial-stereoplotter it is possible to plot accurately the positions of the principal points themselves, but between them is a sort of lune within which points cannot be plotted with precision. This figure covers the area where the angle between the radial arms is 150° or greater.

Description of the Radial-stereoplotter

General

With the aid of the foregoing somewhat abbreviated explanation of the principles of the plotter, it will be possible to follow its construction. The main problem in design was to provide a number of sliding members without introducing backlash on the one hand, or, on the other, forces of sufficient magnitude to limit the free working of the machine or to strain any part to the extent that the accuracy would be impaired. This latter consideration, while to some extent self-evident, had been emphasized in some investigations on factors affecting the precision of first-order theodolites, carried out at the National Research Laboratories a short time ago (4).

The problem was met by constructing all moving parts as light as possible, and employing ball bearings to act as rollers at many of the sliding contacts. One also attempted to reduce the arms of couples to a minimum. When the machine had been constructed it was found that these efforts, coupled with the first-class workmanship contributed by the Instrument Shop, had been successful to the extent that operation is exceedingly free and smooth, with imperceptible backlash.

The machine is illustrated in Figs. 4, 5, 6 and 7. In Fig. 4 the complete plotter is shown, Fig. 5 is a view with the upper and lower carriages removed, Fig. 6 shows the lower carriage, bearing the stereoscope (seen from one end), and Fig. 7 is an underneath view of the upper, plotting carriage.

Two tables *A* (Figs. 4 and 5), mounted on pivots with centres 18.5 in. apart, hold the photographs. These tables can be clamped, released, and rotated freely by hand, or rotated slowly by screws *B*, acting on the clamping screws *C*, following somewhat the system used in transits. Fine needle holes indicate the centres of rotation of the tables, and spring clips serve to hold the photographs in place. This portion of the machine follows lines that have been successfully applied to simple stereoscopes designed for government survey offices at Ottawa.

The right-hand print table can be moved in a direction perpendicular to the base for the purpose of measuring want of correspondence in this direction. This is accomplished by mounting the pivot housing in slides, with gib and screws for taking up slackness. Movement of the table is controlled by a screw butting against the front of the housing and a spring and plunger at the rear. A scale and vernier measure displacements, and permit the table to be reset for plotting purposes.

An I beam, on its side, is used to support the main pivot bearing housings for the radial arms (E, Fig. 3). These bearings are also spaced at 18.5 in. centres and each consists of two torque-tube ball bearings, $2\frac{5}{16}$ in. bore. The radial arms are located by keys cut out of the solid ends of the pivots and are ground parallel. The centres of the pivots and of the tables are adjusted so that their horizontal projection forms a rectangle. This, as well as all the other adjustments necessary to secure accurate working, can be made directly by observations carried out with the aid of the stereoscope telescopes or other parts of the plotter. Adjusting screws are provided for bringing the different slides into parallelism and otherwise correcting the various geometrical relations of the elements of the machine to secure accurate performance.

Lower Carriage

The lower carriage, Fig. 6, is supported on three rollers, consisting of commercial ball bearings fitted on eccentric pins to permit adjustment. It is guided in the Y direction (*i.e.* perpendicular to the air base) by four similar rollers D, which engage with the sides of a fixed, rectangular section bar carried on a light I beam E (Fig. 5), and which also forms a rail for the central supporting roller of the carriage. The body of the carriage is an aluminium casting and is moved in the Y direction by means of a screw F. A sliding piece G, carried on two bars parallel to F, is fitted with a half nut which can be engaged with F or released by an arm H, linked to a lever J (Fig. 6) conveniently located at the end of the carriage. The coupling between G and the carriage is a strut fitted with spherical ends so that direct pull or push forces only can be transmitted.

The screw F is coupled to a handwheel, placed conveniently at the observer's right hand, by means of the shafts and bevel gears seen in Fig. 5, and which are mounted in ball bearings, with ball thrust for the screw, to permit smooth, easy operation of the carriage to its final setting during an observation.

The stereoscope may be considered as composed of three parts, mounted on two thin-walled steel tubes K and L (Fig. 6). The central portion, fixed to the middle of the tubes, bears the two telescopes. Optically, each telescope consists of an objective (25 mm. diam.), two prisms to erect the image and present it to the eye at a comfortable inclination (20° above horizontal), a glass diaphragm ruled with one-half the floating mark and an eyepiece giving a magnification of about 2.5×. The objective is mounted in a sleeve, focused by means of a nut, M, engaging with two portions of a screw fitted in slots on the telescope tube 180° apart. The eyepieces can also be focused and their spacing varied to suit individual observers. This latter adjustment is made by a right- and left-handed screw, and, when it is completed, clamping screws enable the telescopes to be clamped firmly on the slide carrying them.

Stainless steel mirrors, N, 50 by 50 mm., each spring-constrained against the points of three adjusting screws, are held on the two outer brackets of the stereoscope, which also bear the pivots corresponding to B and B_1 (Fig. 3). These pivots consist of spindles each carried in two self-aligning ball bearings, and fitted at its upper end with a triangular plate of duralumin on which are mounted three horizontal rollers (non self-aligning ball bearings). Eccentric pins are used to carry these rollers so that the axis of rotation of the pivot can be made to coincide with the axis of rotation of the lower radial arm with which the rollers engage. By adjustment the telescope diaphragms are brought to focus on points at the same spacing as the axes of the pivots, thus conforming to the relations illustrated in Fig. 3. Adjustment of this spacing (parallax change) is effected by means of a right- and left-handed screw, parallel to and placed between the tubes *K* and *L*. A quadruple-threaded wormwheel at the centre of this screw, mounted in the central bracket of the stereoscope assembly, engages with a worm on the same shaft as the handle *O* (Fig. 4). The right-left screw is cut with a vee thread, and the nuts on the outer brackets are split, so that these brackets, which are fitted with lapped bushings for the tubes *K* and *L*, slide along the tubes without backlash or the necessity of any clamping device, which, indeed, would introduce difficulty owing to the frequency with which changes in parallax settings have to be made.

Slow movement of the stereoscope as a whole in the *X* direction on the carriage is controlled by a screw with a disengaging half-nut on the central bracket, linked to the small lever *P* (Fig. 6). The gearing and quill seen at the end of the carriage in Fig. 6 permit the screw to be operated from a shaft with handle set at the observer's left hand. The quill transmits a torque only, so that non-alignment of the shaft does not introduce friction.

Motion of the stereoscope in the *X* direction is governed by a slide, *Q*, made for convenience in grinding, from two pieces, and carried on brackets mounted on the rear of the carriage. Extensions on the mirror brackets carry rollers *R* which engage with the two opposite ground faces of this slide, which is placed as closely as possible to the plane of the lower radial arms. The weight of the stereoscope is supported by two rollers beneath each of the mirror brackets, and it was found necessary to mount a small roller under each bracket, at the rear, to resist a lifting tendency when the machine was operated.

Graduated scales are provided for indicating the change in spacing between the mirror brackets (parallax)—which can be read to 0.02 mm. by means of a vernier—the displacement of each mirror bracket in the *X* direction, and the displacement of the carriage in the *Y* direction; these latter scales can be read to within 0.1 mm. Lenses mounted in front of the verniers permit the scales to be read comfortably from the observer's seat.

A constant illumination of the portion of the picture covered by the field of view of the stereoscope (65 mm. diameter) is obtained by mounting an automobile headlight bulb on each mirror bracket.

Upper Carriage

The upper, or plotting carriage, is simpler than the lower one. It is likewise a light aluminium casting and is supported on four rollers running on two rails, *S* (Fig. 5), carried on brackets bolted to the I beam flanges. Two additional brackets, at the centre of the beam, are mounted with horizontal rollers in which engages the parallel bar *T* (Fig. 7), rigidly attached to the bottom of the carriage, and serving to guide the motion in the *Y* direction. The upper radial arms engage with rollers carried on pivots similar to those used in the lower carriage, and housed in light castings fitted to two thin-walled steel tubes. The right casting is fixed to the tubes, while the left one can slide to alter the spacing, *i.e.*, set the plotting scale of the Zeiss parallelogram. This setting is made by a screw above the tubes, the right end of which fits in a journal bearing on the casting, while the left end threads in a split nut, which can be clamped when the setting is made. A scale and vernier indicate the inset, or base length, in millimetres (*i.e.*, the length *PE* in Fig. 2). Each casting is supported on two rollers, running on ground steel rails, as in the case of the lower carriage, and the *X* direction guides, seen in Fig. 7, are also similar.

One of the steel tubes extends through the end of the carriage, and to it is attached a light truss bearing the plotting point. The plotting pencil, at the lower end of this truss, was made to fit on a slide with a rack and pinion to adjust the pencil by aid of a scale and vernier. This arrangement was to permit plotting with the centre of either photograph table as origin, but in practice it has been found best always to use the right photograph.

A table, fitted in a pivoted support with a clamping screw, serves to hold the paper. This is used in the form of a roll, the unused portions being held in light brackets beneath the table. Spring clips hold the paper in position during plotting. The plotting table is placed conveniently to the observer's right, and is visible in Figs. 4 and 5.

Experience would indicate the desirability of fitting a sighting microscope, for use in place of the pencil, when adjusting the plot to the photographs.

Method of Plotting Control with the Radial-Stereoplotter

(a) Photographs are set by centering the floating marks in turn on the engraved crosses that mark the centres of rotation of the photo-carrying tables, and then placing the photograph principal points under the floating marks.

(b) The photographs are oriented along the line joining their principal points, which will hereafter be called the base line. This is done by observing points of detail near each principal point in turn, and rotating the other photograph until vertical parallax disappears. The instrument gives a sensitive determination of vertical parallax (or want of correspondence), owing to the form of its floating mark. The forms of the marks are as shown in Fig. 8, a. Observed stereoscopically, the vertical lines fuse, and, when there

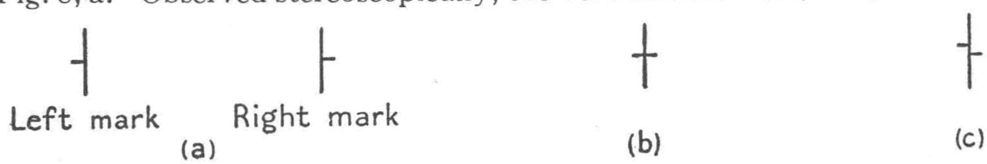

FIG. 8.

is no vertical parallax present in the plastic model (stereogram) against which the mark is viewed, the whole looks like a solid cross (Fig. 8, b). When there is any vertical parallax in the plastic model, the cross breaks up, and the horizontal lines appear separated in the *Y* direction (Fig. 8, c).

(c) The scale of the plot depends upon the ratio of the base-length (set on the upper carriage) to the natural distance between the objects on the ground on whose images the principal points of the photographs fall. If two points, whose distance apart is known, appear in the first overlap, the machine may be set to plot at any desired scale by first plotting the two points from the photograph with an arbitrary base-length setting. The arbitrary setting, B_1, is then corrected by multiplying it by the factor Dt/Dp, where Dt is the true distance at the desired scale, and Dp is the distance plotted at the first setting.*

(d) The machine works most conveniently when the plot proceeds from right to left of a series of photographs. When the scale has been set by adjusting the base-length as in *(c)*, two minor controls or scale control points are plotted. These should lie on a line at right angles to the base line, at a distance equal to the base above and below the left principal point. If there happens to be want of correspondence, owing to tilt, at these points, the arms of the cross constituting the floating mark will not appear collinear. When plotting the two scale control points, care must be taken to have the right arm of the cross in line with the right photograph image of the point of detail chosen, since we are performing a radial line intersection, and the angle between the line from the object to the principal point and the base line on the *right* photographs determines the *Y* co-ordinate of the plotted position.†

(e) The left principal point and the base line are plotted, and the base line on the plot is prolonged in both directions to the limit of the machine's movement.

(f) Points that are required as additional control for the subsequent tracing of detail are now plotted.

(g) When work with the first overlap is completed, the second photograph is transferred to the right photograph carrier, and the third photograph placed on the left one. The pair is then oriented as in *(b)* and the base lines ruled in.

(h) The plot is shifted so that the plotted principal point of the second photograph now lies on the centre of rotation of the plotting table, where the pencil rests when the floating mark is centered over the centre of rotation of the right photograph carrier. This is the origin for plotting in each overlap.

* *The base line is marked on each photograph by laying a straight-edge on stops which bring it exactly over the centres of rotation, and scoring lines with a needle point. This operation must be very carefully performed, since the azimuth of the strip depends on its accuracy, as will appear later. Accuracy may be easily checked by removing the straight-edge and moving the stereoscope along the base line, when the floating mark will show any deviation of the scored lines. The base lines are extended in both directions to the margins of the photographs.*

† *The point is identified by centering a piece of celluloid with a fine cross over it. The marker is fastened down by transparent adhesive tape. As the points are viewed stereoscopically in plotting successive overlaps, it is necessary only to identify them in this way. The actual plotting should be of the untouched point of photographic detail, as any attempt to mark it by pricking, pen, or pencil leads to loss of accuracy.*

(i) The plot must now be oriented. Regarding the successive base lines as the legs of a traverse, the angles between them will be shown on the photographs by the scored lines. We are now concerned with the angle between base 1–2 and base 2–3; this is shown on Photograph 2, and may be plotted. The plotting table is rotated through the appropriate angle, so that when the floating mark is placed on the prolongation of base line 1–2 on the plot the plotted base falls under the pencil point.

(j) It is now necessary to adjust the base-length so that the plot of the second overlap will be at the same scale as that of the first. This is done by setting the floating mark on one of the scale control points. If there is want of correspondence, the left arm of the floating mark must be in line with the left photograph image of the control point; for the same reason, *mutatis mutandis*, as given in *(d)* above. If the pencil now falls on the point as plotted from the previous overlap, the scale is correct. If there is a discrepancy, the base-length is altered until the pencil falls on the point.

(k) The scale is checked by plotting the other scale control point. If it plots in the same position as determined from the previous overlap, this proves the scale to be correct. Sometimes, however, there is a discrepancy, which may be due to some of the operations previously described not having been done with sufficient accuracy. These are checked, and if the discrepancy persists, it is due to the effect of tilt, combined with height differences, or distortion of the photograph materials. It may be observed that the instrument indicates clearly when relative tilt is present, by the want of correspondence (vertical parallax) shown by the floating mark; the magnitude and direction of tilt may also be deduced approximately.

Errors

The sources of error in plotting with the instrument, and by the radial line method generally will now be discussed, and methods will be suggested for eliminating them, or correcting the plot.

Sources of Error in Radial Line Plotting

In making a radial line plot of a series of aerial photographs, we are in effect making a graphical triangulation, using the angles between objects as shown on the photographs. The errors of the plot may be either in azimuth, or in scale, or in both. There may be an error in one azimuth angle, or a series of small errors having a cumulative bending effect on the chain of triangles; and there may be a change of scale in one overlap or a gradual change of scale, the cumulative effect of small inaccuracies. Sudden changes in azimuth or scale can be due only to mistakes, and can be detected by a simple check. The cumulative effects of small inaccuracies become apparent when the plotted strip is laid down in a compilation with the plots of adjoining photograph strips. If there are two ground control points in each end overlap of the strip, as is usually the case when control is by traverse, it is easy to determine whether a constant scale has been preserved throughout the length of the strip.

Azimuth Errors

Small errors in azimuth may be caused by:—

(a) Inaccurate centering of photographs on the carriers. The base line on the photographs will not be parallel to the base line as plotted.

(b) Inaccurate centering of principal point on plot on the centre of rotation (origin of plotting) of the table.

(c) Inaccurate orientation.

(d) Inaccurate ruling of principal point bases.

(e) Inaccurate setting off of change of azimuth on plot.

Errors due to perspective distortions in tilted photographs will be discussed later; their effects in flattish country, when tilts are less than 3°, are negligible.

The above sources of error can be largely eliminated by care on the part of the operator. The magnification of the photograph image, and the apparent magnitude of the floating mark, are such that it is possible to set the photograph in place to an accuracy of 0.03 mm.; base line orientation should be accurate to less than two minutes of arc. The thickness of the base lines scratched by needle point is about 0.10 mm., and any departure from straightness may be detected by moving the floating mark in the *X* direction over the *p.p.* base.

Errors *(b)* and *(e)* depend on the accuracy with which the pencil point can be placed over a previously drawn line. The thickness of the pencil lines is about 0.10 mm. With the aid of a magnifying glass, the pencil may be set to about this degree of accuracy. It is likely that the setting could be improved if a magnifying optical sight were available to replace the pencil for the adjustment, as in some continental plotting machines.

Experience shows that plotting by the machine holds azimuth accurately, as a rule, in flattish country. Long straight roads or railways, extending through several photographs, preserve their alignment. More difficulty has been found in keeping the scale constant throughout a strip; in many that were plotted in the early stages, variations of 0.025 between the beginning and end of a strip were found. Such errors are shown up by scaling on ground control at the ends of the strips, and by the fact that there are discrepancies between the positions of tie points plotted from adjoining strips. The same sort of errors have been met with in radial line plotting by ordinary graphical methods, and it was in the hope of eliminating these, and lessening the amount of adjustment necessary during compilation, that the radial-stereoplotter was devised.

Scale Errors

The scale is preserved, from overlap to overlap, by plotting the position of two lateral scale control points (minor control points) in one overlap, and then adjusting the base length of the succeeding overlap so that these points are plotted in identical positions. The accuracy with which this process can be carried out depends on the following factors:—

(a) Accurate identification, from overlap to overlap, of the points of detail serving as scale control points.

(b) Definition and magnitude of the points of detail.

(c) Accuracy of pointing from the principal point to control point: *i.e.*, accuracy of measurement of the angles used for intersection and resection.

(d) Accuracy of orientation, and fineness of the setting of the plotting pencil.

The effects of tilt on the preservation of scale will be discussed later.

An important advantage of the radial-stereoplotter is that the direction lines are fixed when the operator observes the images of the control points in both photographs stereoscopically. This eliminates the errors in identification, which occur when points in successive photographs are established by inspection. The next factor in securing accuracy of pointing is the definition of the point of detail. Artificial detail, such as the intersections of roads, corners of buildings, etc., is best; but much Canadian mapping is of areas where there is no artificial detail, and natural objects such as trees, rocks and so on, seldom show as sharply as one would wish. The operator must take care to choose an object on which he can point to within 0.10 mm. or less, if at all possible.

It is best not to actually mark the object by pricking or in any other manner, as this spoils the stereoscopic impression, as explained above. If the foregoing precautions are observed, it is possible to make pointings to an accuracy of 0.1 mm. or less.

If the floating mark is not brought to exactly the same parallax as the photograph image, the discrepancy in X is reproduced in the Y direction. It is, therefore, necessary to be careful in making coincidence; if want of correspondence is present, it should be eliminated by the differential Y motion of the right photograph carrier; after the floating mark is adjusted to the correct parallax, the difference in Y is restored, as pointings would otherwise be incorrect to any objects not on the ordinate through the right principal point.

With the usual conditions applying to Canadian air surveys—scale control points about 70 mm. from the principal point and a camera focal length of 210 mm.—an error of 0.1 mm. in plotting the scale control points will introduce a scale error of 1/700 or 0.0014. Unless the operator has some idiosyncrasy in pointing, the errors will be random, and tend to cancel out. A strip that showed a total scale change of more than 0.005 from one end to the other would, if reduced according to its over-all scale, put control points towards the centre out of position by about a millimetre—53 ft. at the usual scale of plotting, which is 1/15,840. While errors of this magnitude would not matter in mapping certain types of country, it is desirable to be able to eliminate them when greater accuracy is desired. In such a case, the strip would have to be plotted again; or a special adjustment made to correct for the effect of the scale variation.

The Effect of Tilt in Flat Country

If we take the case of three photographs of flat country, numbered 1, 2 and 3, of which 1 and 3 are untilted, and 2 is tilted to the right in the direction of the base line, the condition would be as shown in Fig. 9. V_2 is

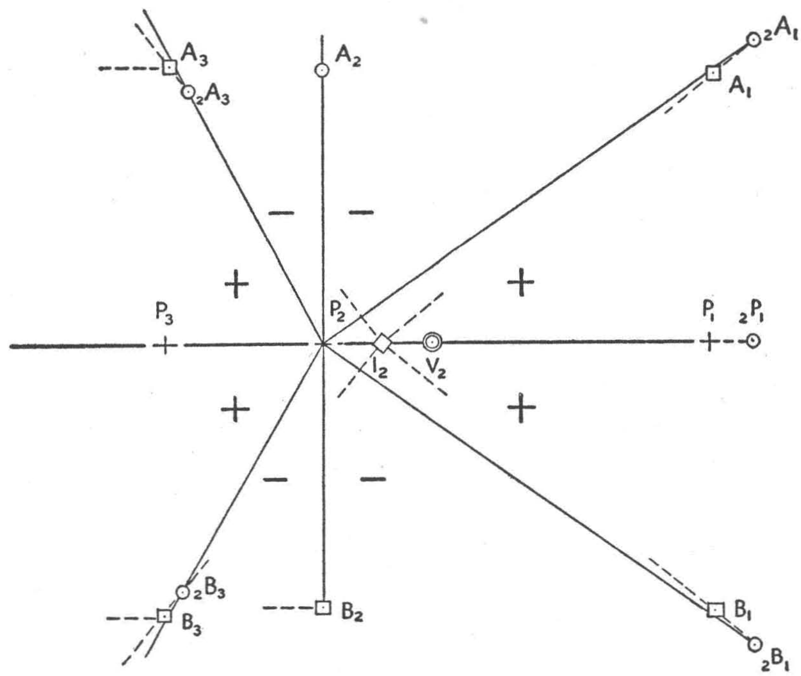

FIG. 9.

the plumb point of the second photograph, and I_2 its isocentre. Angles about its principal point will not be true. If the isocentre, I_2, could be found and used as the centre of plotting, angles would be true, but it is not practicable to do this.

It will be seen that the angles $_2A_1P_2P_1$ and $_2B_1P_2P_1$, and also $_2A_3P_2P_3$ and $_2B_3P_2P_3$ are larger than they should be. The ratio of the tangents of these angles to the tangents of true plan angles is sec θ, where θ is the tilt (3, Art. 17). The complementary angles will be too small, and in Fig. 9 a plus or minus sign has been placed to indicate the tilt effect on the angles in each of the eight sectors into which the photograph is divided by lines drawn to the surrounding control points.

It is clear that if the strip is oriented along the base lines there will be no error in strip azimuth; but a difference of scale will be introduced.

As the angles between A_1, and P_1 and B_1 are too large on photograph 2, when the position of P_2 is determined by resection it will be too close to P_1. The ratio of the plotted base line P_2P_1 to its true length will be cos θ. When A_2 and B_2 are intersected, the scale along A_2B_2 will be too small (in the same ratio) as it depends on the base P_2P_1; when P_3 is resected, the base P_3P_2 will be short, since it has been established from the line A_2B_2, which is short. But A_3 and B_3, which are intersected by the angles $A_3P_2P_3$ and $B_3P_2P_3$, which are too great, will be in their correct position and the scale will be re-established.

Tilts in gyropilot-controlled aircraft are seldom as great as 3°, and if the cosine of 3° is applied to 70 mm., the average base length is shortened by only 0.10 mm., a negligible amount, practically speaking.

If a line with an occasional larger tilt had to be plotted, the magnitude and direction of the tilt could be estimated (2, pp. 140–147), and after resection from the photograph, the base length could be increased by the amount required.

Inspection of Fig. 9 will show that if the direction of tilt were transverse to the line of flight, the angles with which resection and subsequent intersection are made would be too small, and the base lengths and scale generally in the two overlaps in which the tilted photograph figured would be too large, coming back to normal with the next untilted pair.

If the direction of tilt is at 45° to the base line, one of the resection angles will be too great, and the other will be too small. The position of P_2 will not be properly resected unless allowance is made for this, and the pointing to the photograph detail is slightly varied accordingly. There will be no error in azimuth introduced. This case of tilt is the most troublesome in practice.

Tilt and Elevation Differences

Fig. 10 is a diagram of the plan position of the principal points of three photographs, the centre one of which is tilted laterally. V_2 is its plumb point. If the point of detail upon which P_1 falls is higher than the plotting

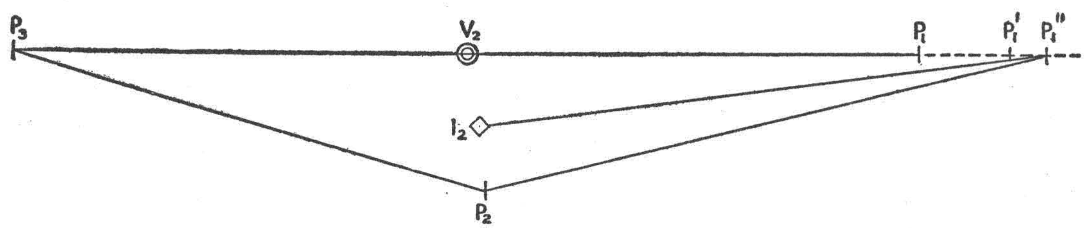

FIG. 10.

datum plane, its position will be distorted outwards to P_1'. Tilt will cause the principal point to move to P_2, and P_1' will move to P_1'' on a line radial from the isocentre. The line P_2P_1'' will therefore not give the true direction to the plan position of P_1 relative to other objects imaged in the photograph, and this discrepancy, the magnitude of which may be easily calculated, will seriously disorient the strip.

In such a case the trouble will soon show itself. The tilt will be indicated by want of correspondence; and its direction and magnitude, and hence the approximate plumb point, can be estimated by methods described by Major Hotine (2).

The overlaps where the difficulty occurs could then be plotted again, by use of the approximate plumb points so found, instead of the principal points.

Results Obtained with the Instrument

At the date of writing, the radial-stereoplotter has been in use for only about two months. Most photogrammetric machines can, in a special test, be made to give an output and accuracy beyond what would normally be obtained in routine mapping with them. On the other hand, when a new instrument is brought into use, operators gradually acquire skill with it and increase their output. It is, therefore, not possible to state finally to what degree the radial-stereoplotter will increase the accuracy and speed of radial line plotting. However, experience to date indicates that when plotting at 1/15,840 (4 in. to 1 mile) azimuth can be held correct in strips of photographs where the tilt is small so that the centre of a strip 750 mm. between ground control points will not be bent laterally by more than 1 mm. The scale can be held correct to about 1% from one end of the strip to the other.

In plotting a strip by machine, about 18 to 19 min. per photograph is required, including the plotting of roads and other subsidiary control for tracing. The experience in the Geographical Section, General Staff, has been that by the graphical method about 25 min. per photograph was required, time taken for plotting roads, etc., not included.

When strips are plotted to the accuracy mentioned above, an appreciable amount of time is saved in reduction, adjustment, and compilation, which, with the ordinary graphical method of plotting, takes about 16 min. per photograph.

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