

MODELS AND BETTER USE OF CONTROL THAT MAY BE AVAILABLE. IT IS RECOMMENDED THAT FURTHER STUDY BE GIVEN THE MATTER WITH THE VIEW OF DEFINITELY DETERMINING WHAT APPEARS TO BE MARKED ADVANTAGES.

THE INTERPRETATION OF LENS TESTS AND CAMERA CALIBRATIONS*

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I. INTRODUCTION

AIRPLANE MAPPING REQUIREMENTS ACCOUNT FOR MORE THAN HALF OF ALL THE TESTS MADE ON PHOTOGRAPHIC OBJECTIVES AT THE NATIONAL BUREAU OF STANDARDS. AS METHODS OF MAPPING BECOME MORE PRECISE AND, PARTICULARLY, AS THE STEREOSCOPIC METHOD OF PLOTTING CONTOURS BECOMES THE MORE USUAL METHOD OF TREATING A SERIES OF PHOTOGRAPHS, THE TESTING OF THE LENSES BECOMES MORE IMPORTANT AND GREATER PRECISION IS REQUIRED. THIS TENDENCY IS MADE VERY EVIDENT BY THE CHARACTER OF THE TESTS THAT ARE NOW BEING REQUESTED AND BY THE INCREASED INTEREST IN SPECIFICATIONS FOR LENSES AND CAMERAS. IN VIEW OF THIS IT HAS SEEMED ADVISABLE TO PRESENT A DETAILED DESCRIPTION OF THE LENS TESTS** REGULARLY CONDUCTED BY THE NATIONAL BUREAU OF STANDARDS AND TO DISCUSS, IN DETAIL, THE INTERPRETATION OF THE TESTS, THEIR SIGNIFICANCE, AND THEIR AVAILABILITY FOR DIFFERENT PURPOSES. A PRELIMINARY DISCUSSION OF CERTAIN ELEMENTARY OPTICAL PRINCIPLES IS INCLUDED BECAUSE THEIR USE IS NECESSARY IN AN ADEQUATE PRESENTATION.

II. METRICAL CHARACTERISTICS OF ABERRATION-FREE IMAGERY

1. OBJECT AT AN INFINITE DISTANCE

THE METRICAL CHARACTERISTICS OF AN IMAGE FORMED BY AN IDEAL LENS, ENTIRELY FREE FROM DISTORTION, ARE COMPLETELY DETERMINED BY THE EQUIVALENT FOCAL LENGTH AND THE LOCATION OF THE TWO NODAL POINTS. THESE METRICAL RELATIONS, FOR AN INFINITELY DISTANT OBJECT POINT, ARE ILLUSTRATED BY FIGURE 1, WHERE THE NODAL POINTS ARE INDICATED AT N AND N' AND THE FOCAL LENGTH HAS THE VALUE F. POINTS I, A, AND B ARE ASSUMED TO BE INFINITELY DISTANT. POINT I IS ON THE AXIS AND POINTS A AND B ARE LOCATED BY A AND B, THEIR ANGULAR DISTANCES FROM THE AXIS. THE IMAGES, FORMED IN THE FOCAL PLANE, ARE AT I', A', AND B' AND, AS INDICATED ON THE DIAGRAM, POINTS A' AND B' ARE DISTANT $F \tan A$ AND $F \tan B$ FROM THE AXIS.

THE NODAL POINTS OF A CAMERA LENS ARE THE POINTS OF UNIT ANGULAR MAGNIFICATION. A RAY, MAKING THE ANGLE A WITH THE AXIS AT N, THEREFORE, LEAVES THE FINAL SURFACE MAKING AN EQUAL ANGLE WITH THE AXIS AT N'. THESE RAYS, BEFORE AND AFTER PASSING THROUGH THE LENS, ARE INDICATED BY THE SOLID LINES IN THE DRAWING. SINCE, HOWEVER, THE OBJECT POINTS ARE INFINITELY DISTANT, THE DOTTED LINES DRAWN THROUGH N', PARALLEL TO THE FULL LINES, ALSO PASS THROUGH A AND B. HENCE, WHEN THE OBJECT IS INFINITELY DISTANT, WE HAVE A SPECIAL SIMPLIFIED TYPE OF IMAGERY IN WHICH THE POINT N NEED NOT ENTER THE CALCULATION AT ALL. WHEN A CAMERA IS USED IN AN AIRPLANE, THE SURFACE TO BE PHOTOGRAPHED IS AT A VERY LARGE BUT NEVERTHELESS FINITE DISTANCE FROM THE LENS. THE CONDITIONS OF FIGURE 1, HOWEVER, CAN BE UTILIZED. ASSUMING THAT THE OBJECT IS AT AN INFINITE DISTANCE IS EQUIVALENT TO ASSUMING THAT THE OBJECT IS SO FAR AWAY THAT A SHARP IMAGE IS OBTAINED IN THE FOCAL PLANE I' A' B'; AND THAT THE ANGLE BETWEEN LINES DRAWN FROM N AND N' TO A GIVEN OBJECT POINT IS NEGLIGIBLE. IF IT BE ASSUMED THAT THE PLANE OF THE FILM CAN FAIL TO COINCIDE WITH THE PLANE OF BEST FOCUS BY 0.05 MM WITHOUT HARMFUL RESULTS, IT CAN BE SHOWN THAT AN OBJECT DISTANT D FROM A LENS OF FOCAL LENGTH F WILL BE SATISFACTORILY IMAGED IN THE FOCAL PLANE, AND CAN THEREFORE BE ASSUMED TO BE INFINITELY DISTANT IF

$$D/F \approx F/0.05. \quad (1)$$

WHEN F EQUALS 150 MM (6 INCHES), D MUST EQUAL OR EXCEED 450 METERS (1500 FT.) TO

* PUBLICATION APPROVED BY THE DIRECTOR OF THE NATIONAL BUREAU OF STANDARDS OF THE U. S. DEPARTMENT OF COMMERCE.

** FOR A DESCRIPTION OF THE PRECISION LENS TESTING CAMERA, AN APPARATUS SPECIALLY DESIGNED FOR THE TESTING OF AIRPLANE CAMERA LENSES AND AIRPLANE CAMERAS, SEE RESEARCH PAPER RP984 IN THE APRIL, 1937 ISSUE OF THE NATIONAL BUREAU OF STANDARDS JOURNAL OF RESEARCH. SEPARATE REPRINTS WILL BE PURCHASABLE FROM THE SUPERINTENDENT OF DOCUMENTS, WASHINGTON, D. C. FOR FIVE CENTS (STAMPS NOT ACCEPTABLE).

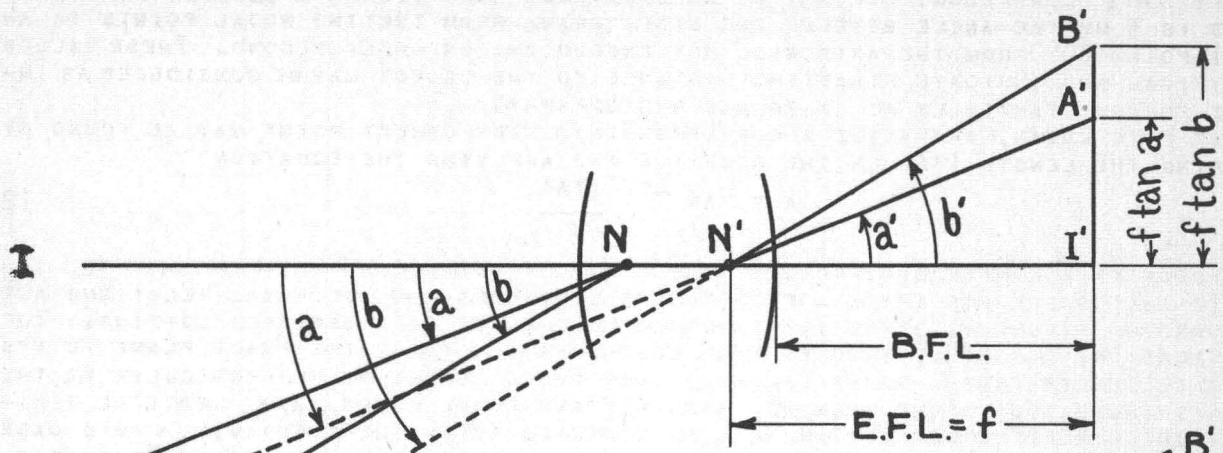


FIGURE 1
METRICAL RELATIONS WHEN THE OBJECT
IS AT AN INFINITE DISTANCE

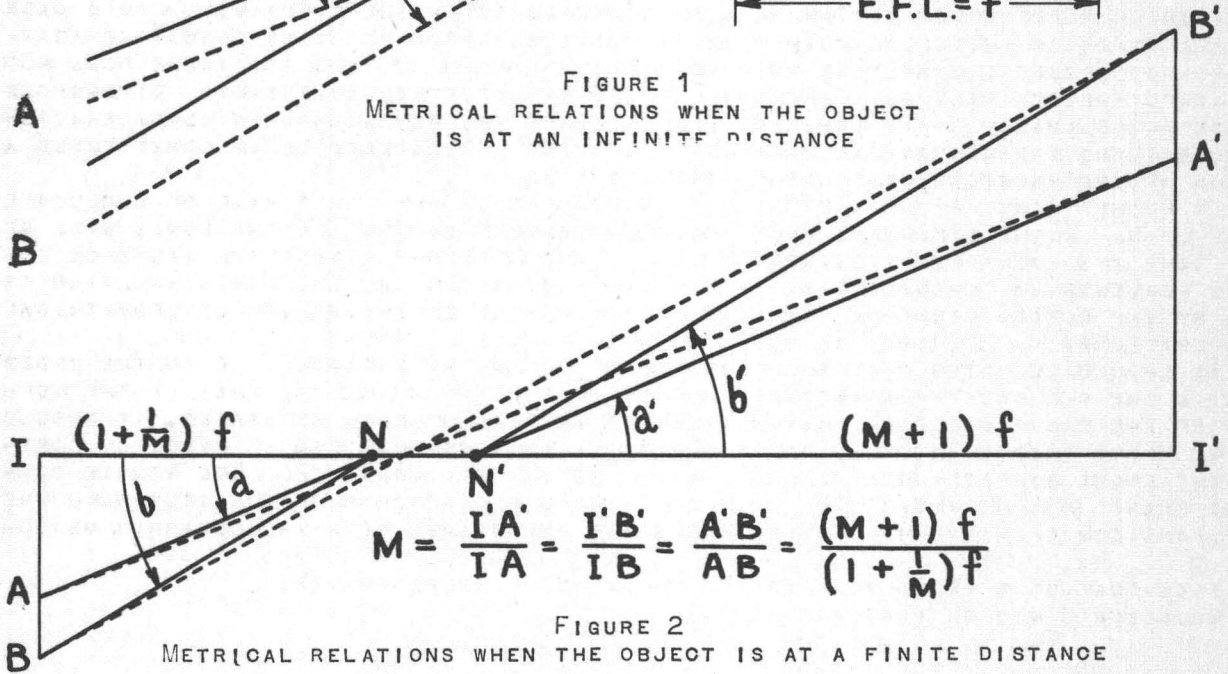


FIGURE 2
METRICAL RELATIONS WHEN THE OBJECT IS AT A FINITE DISTANCE

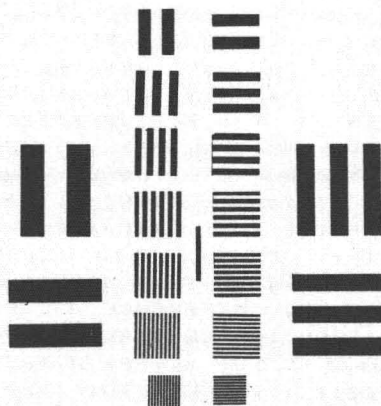


FIGURE 3

TEST CHART FOR DETERMINING THE RESOLVING POWER OF A LENS. THESE CHARTS ARE OF SUCH SIZE THAT THE NUMBER OF LINES TO THE MILLIMETER, AS RECORDED ON THE TEST PLATE, FORMS THE APPROXIMATELY GEOMETRIC SERIES 3.5, 5, 7, 10, 14, 20, 28, 40 AND 56. THE CENTRAL VERTICAL LINE IS THE FIDUCIAL MARK TO WHICH MEASUREMENTS ARE MADE FOR THE DETERMINATION OF FOCAL LENGTH AND DISTORTION.

SATISFY THIS CONDITION. IF IT BE ASSUMED THAT THE DISTANCE BETWEEN THE NODAL POINTS IS 3 MM THE ANGLE BETWEEN THE LINES DRAWN FROM THE TWO NODAL POINTS TO AN OBJECT POINT 30° FROM THE AXIS WILL NOT EXCEED ONE ONE-HALF SECOND. THESE VALUES ARE TYPICAL AND INDICATE THAT THE DISTANCE TO THE OBJECT MAY BE CONSIDERED AS INFINITE FOR SUBSTANTIALLY ALL AIRPLANE PHOTOGRAPHY.

IF F IS KNOWN, THE ANGLE A CORRESPONDING TO ANY OBJECT POINT MAY BE FOUND BY MEASURING THE LENGTH $l'A'$ ON THE NEGATIVE AND APPLYING THE EQUATION

$$A = \tan^{-1} \frac{l'A'}{F}. \quad (2)$$

2. OBJECT AT A FINITE DISTANCE.

IF THE OBJECT AND IMAGE ARE AT FINITE DISTANCES THE METRICAL RELATIONS ARE AS SHOWN IN FIGURE 2. THIS CORRESPONDS TO THE USE OF A LENS FOR COPYING. THE MAGNIFICATION, M , IS DEFINED AS THE RATIO OF A LENGTH IN THE IMAGE PLANE TO ITS CONJUGATE LENGTH IN THE OBJECT PLANE. WHEN THESE PLANES ARE PERPENDICULAR TO THE AXIS, AS INDICATED IN THE DRAWING, AND IF M AND F ARE GIVEN, THE METRICAL RELATIONS ARE COMPLETELY DEFINED AND ARE AS REPRESENTED IN THE DIAGRAM. IN THIS CASE THE SEPARATION OF THE NODAL POINTS MAY BE REGARDED AS A "NO MAN'S LAND" SEPARATING THE TWO PROJECTION SYSTEMS WITH VERTICES AT N AND N' AND ITS VALUE DOES NOT ENTER INTO THE STRICTLY OPTICAL FORMULAS GIVING THE RELATIONS BETWEEN DIMENSIONS OF OBJECT AND IMAGE. THIS SEPARATION MUST, HOWEVER, BE TAKEN INTO CONSIDERATION WHEN DESIGNING THE MECHANICAL PARTS OF A COPYING CAMERA BECAUSE IT CONSTITUTES A PORTION OF THE PHYSICAL DISTANCE SEPARATING l AND l' .

IF POINTS A AND A' , B AND B' , OR THE MEMBERS OF ANY OTHER PAIR OF CONJUGATE POINTS LYING IN THESE PLANES, ARE JOINED BY STRAIGHT LINES, ALL THE LINES WILL BE CONCURRENT AT A POINT LYING BETWEEN POINTS N AND N' AND DIVIDING THE DISTANCE INTO TWO SEGMENTS OF WHICH ONE IS M TIMES THE OTHER IN LENGTH. THIS RELATION IS SHOWN BY THE DOTTED LINES OF FIGURE 2 AND IT CAN BE DERIVED BY SIMPLE GEOMETRICAL CONSIDERATIONS.

IT SHOULD BE NOTED THAT POINTS N AND N' DO NOT NECESSARILY LIE IN THE ORDER SHOWN IN THE FIGURE. IT SOMETIMES HAPPENS THAT N' PRECEDES N , THAT IS THE VERTICES OF THE TWO PROJECTION SYSTEMS OVERLAP INSTEAD OF BEING SEPARATED. IT SHOULD ALSO BE NOTED THAT IF A LENS, NOT SYMMETRICAL IN CONSTRUCTION, IS USED FOR COPYING, THE FRONT SURFACE* OF THE LENS SHOULD BE TURNED TOWARD THE MORE REMOTE CONJUGATE PLANE BECAUSE THIS CONDITION OF USE CORRESPONDS MOST CLOSELY WITH THE CONDITIONS COMMONLY ASSUMED AS THE BASIS FOR THE DESIGN OF A PHOTOGRAPHIC OBJECTIVE.

3. DETERMINATION OF FOCAL LENGTH AND LOCATION OF NODAL POINTS.

EQUATION 2 MAY BE WRITTEN

$$F = \frac{l'A'}{\tan A},$$

AND THIS SUGGESTS A METHOD FOR THE DETERMINATION OF THE FOCAL LENGTH OF A LENS. BY OPTICAL METHODS TWO INFINITELY DISTANT TARGETS, ONE ON THE AXIS OF THE LENS AND THE OTHER AT A KNOWN ANGLE A FROM IT, ARE PROVIDED. THESE TARGETS ARE PHOTOGRAPHED BY THE LENS TO BE TESTED AND THE DISTANCE $l'A'$ MEASURED ON THE RESULTING NEGATIVE. BY SUBSTITUTION IN EQUATION 2 ONE OBTAINS THE VALUE OF F . REFERRING TO FIGURE 1, THE BACK FOCAL LENGTH (B.F.L.) IS MEASURED FROM THE REAR VERTEX OF THE LENS TO THE POSITION OCCUPIED BY THE PHOTOGRAPHIC PLATE. WHEN IT IS NECESSARY TO DISTINGUISH PRECISELY BETWEEN B.F.L. AND F , THE LATTER IS REFERRED TO AS THE EQUIVALENT FOCAL LENGTH, (E.F.L.). ALL LENSES HAVING THE SAME EQUIVALENT FOCAL LENGTH ARE EQUIVALENT IN THAT THEY WILL PRODUCE EQUAL IMAGES OF A GIVEN INFINITELY DISTANT OBJECT. LENSES OF THE SAME EQUIVALENT FOCAL LENGTH MAY HAVE BACK FOCAL LENGTHS THAT DIFFER GREATLY. A TELEPHOTO LENS, FOR EXAMPLE, MAY HAVE A BACK FOCAL LENGTH ONE HALF OR ONE QUARTER THAT OF A LENS OF THE MORE USUAL TYPE OF THE SAME EQUIVALENT FOCAL LENGTH. REFERENCE TO FIGURE 1 SHOWS THAT IF THE BACK FOCAL LENGTH IS SUBTRACTED FROM THE EQUIVALENT FOCAL LENGTH, ONE OBTAINS THE DISTANCE FROM THE SECOND NODAL POINT TO THE VERTEX OF THE LAST SURFACE OF THE LENS.

IN FIGURE 1, IT HAS BEEN TACITLY ASSUMED THAT THE LENS IS MOUNTED WITH THE FRONT SURFACE TURNED TOWARD THE INFINITELY DISTANT OBJECT AND THAT THE REAR

* THE FRONT SURFACE IS CONSIDERED TO BE THE ONE AT THE END OF THE BARREL THAT BEARS THE MANUFACTURER'S NAME, SERIAL NUMBER AND FOCAL LENGTH.

SURFACE OF THE LENS IS TURNED TOWARD THE PLANE I'B'. IF THE LENS IS REVERSED IN ORIENTATION, BRINGING THE FRONT SURFACE TOWARD THE PLANE I'B', A POSITION WILL BE FOUND FOR WHICH AN IMAGE IS FORMED IN THE PLANE I'B'. THIS IMAGE WILL BE THE SAME SIZE AS THAT FORMED WITH THE LENS IN ITS ORIGINAL POSITION. THEREFORE, IF EQUATION 2 IS APPLIED, I'A' AND THE ANGLE A WILL HAVE THE SAME VALUE AS BEFORE AND IT IS SEEN THAT THE EQUIVALENT FOCAL LENGTHS DETERMINED ON THE TWO SIDES OF THE LENS WILL BE EQUAL.* IN THIS SECOND POSITION OF THE LENS, IF IT IS OF UNSYMMETRICAL CONSTRUCTION, THE DISTANCE FROM THE FRONT SURFACE OF THE LENS TO THE PLANE I'B', TERMED THE FRONT FOCAL LENGTH (F.F.L.) MAY DIFFER GREATLY FROM THE BACK FOCAL LENGTH. THE DIFFERENCE OF EQUIVALENT AND FRONT FOCAL LENGTHS GIVES THE DISTANCE FROM THE FRONT SURFACE OF THE LENS TO THE FIRST NODAL POINT.

III. IMAGERY WITH ABERRATIONS PRESENT

IN THE FOREGOING PARAGRAPHS STATEMENTS HAVE, FROM TIME TO TIME, BEEN RESTRICTED IN THEIR APPLICATION BY THE PROVISIO "WHEN NO ABERRATIONS ARE PRESENT". THE EQUIVALENT FOCAL LENGTH AND THE LOCATION OF THE NODAL POINTS, WHEN DETERMINED, DEFINE AN IDEAL IMAGERY, FREE FROM DISTORTION AND OTHER ABERRATIONS, WHICH IS APPROACHED MORE OR LESS CLOSELY BY THE IMAGERY ACTUALLY PRODUCED BY THE LENS UNDER TEST. WHEN, HOWEVER, THE IMAGE PRODUCED IS CLOSELY EXAMINED, IT WILL IN GENERAL BE FOUND THAT THE RATIOS OF FIGURES 1 AND 2 ARE NOT EXACT BUT VARY SLIGHTLY AND IN DIFFERENT AMOUNTS FOR POINTS IN DIFFERENT PARTS OF THE FIELD. FURTHERMORE, THE IMAGE OF THE PLANE REPRESENTED BY IB DOES NOT COINCIDE WITH THE PLANE I'B' BUT MAY BE A CONVEX OR CONCAVE SURFACE TANGENT TO I'B' AT ITS AXIAL POINT. THE IMAGE WILL NOT BE PERFECTLY SHARP, WILL STAND ONLY A LIMITED AMOUNT OF ADDITIONAL MAGNIFICATION, AND THE METRICAL PROPERTIES OF THE IMAGE WILL BE DIFFERENT FOR OBJECTS DIFFERING IN COLOR. THESE DEPARTURES FROM THE SIMPLER IDEAL IMAGE ARISE FROM THE PRESENCE OF ABERRATIONS. IT IS IMPOSSIBLE TO ENTIRELY ELIMINATE THEM BUT IT IS THE FUNCTION OF THE LENS DESIGNER TO SO CHOOSE THE GLASSES TO BE USED AND TO SO ARRANGE THE SURFACES AS TO GIVE AN IMAGE SUBSTANTIALLY FREE FROM HARMFUL ABERRATIONS WHEN THE LENS IS PROPERLY USED. EACH LENS, THEREFORE, IS A COMPROMISE AND THE PREFERRED COMPROMISE IS DETERMINED BY THE PURPOSE OF THE LENS.

THIS NECESSITY FOR THE DIFFERENT FORMS OF COMPROMISE GIVES RISE TO THE DIFFERENT TYPES OF LENSES. TO ILLUSTRATE, FOR MILITARY PURPOSES, IT IS NECESSARY THAT THE LENS BE FAST WORKING IN ORDER THAT PICTURES MAY BE MADE UNDER ADVERSE CONDITIONS AND THE SPEED OF THE LENS MAY BE INCREASED TO SUCH AN EXTENT THAT IT IS NECESSARY TO LIMIT THE ANGLE OF VIEW OVER WHICH GOOD DEFINITION IS OBTAINED. FOR CIVILIAN PURPOSES, ON THE OTHER HAND, IT IS GOOD ECONOMY TO USE A LENS WITH A LARGE FIELD OF VIEW IN ORDER TO DIMINISH THE AMOUNT OF REQUIRED FLYING AND TO LESSEN THE LABOR OF INTERPRETING THE EXPOSURES EVEN THOUGH THIS NECESSITATES USING A SMALLER APERTURE OF THE LENS AND MAKES IT NECESSARY TO AWAIT RELATIVELY PERFECT DAYS FOR THE PHOTOGRAPHIC WORK.

FOR THE PHOTOGRAMMETRIST THE ABERRATIONS MAY BE CLASSIFIED UNDER TWO HEADS, (1) THOSE THAT AFFECT THE METRICAL CHARACTERISTICS OF THE IMAGE BY INTRODUCING SYSTEMATIC ERRORS OF DISTORTION THAT CAN BE COMPENSATED ONLY BY THE INTRODUCTION OF CORRECTION TERMS, AND (2) THOSE PRODUCING LACK OF DEFINITION. THIS SECOND CLASS OF ABERRATIONS DOES NOT INTRODUCE SYSTEMATIC ERRORS BUT CAUSES LACK OF PRECISION IN THE MEASUREMENTS AND MAY ALSO PREVENT THE RECOGNITION OF SMALL DETAIL ON THE GROUND.

1. DISTORTION

ABERRATION THAT PRODUCES A SYSTEMATIC DISPLACEMENT OF THE IMAGE POINTS IS TERMED DISTORTION. IF THE LENS IS ACCURATELY CONSTRUCTED SO THAT ALL POINTS ARE SYMMETRICAL ABOUT THE AXIS, THE DISTORTION ALSO, WILL NECESSARILY BE SYMMETRICAL ABOUT THE AXIS. IN SUCH A CASE, AS A RESULT OF DISTORTION, AN IMAGE POINT WILL BE DISPLACED FROM ITS DISTORTION-FREE POSITION ALONG A RADIUS, EITHER TOWARD OR FROM THE CENTER OF THE FIELD. THIS IS COMMONLY ASSUMED TO BE THE CASE AND THE DISTORTION, THEREFORE, IS USUALLY INVESTIGATED ALONG ONE OR PERHAPS TWO RADII OF THE FIELD.

IN THE PRESENCE OF DISTORTION, IF TWO DETERMINATIONS OF THE FOCAL LENGTH ARE MADE BY MEASURING A, B, I'A', AND I'B' OF FIGURE 1, AND BY SOLVING THE EQUATIONS

* FOR AN IDEAL LENS WITHOUT ABERRATIONS THIS CAN BE SHOWN TO BE STRICTLY TRUE. ACTUALLY, THE CHARACTER OF IMAGERY PRODUCED BY A LENS IN THE TWO POSITIONS, IN GENERAL, IS NOT THE SAME, AND SMALL DIFFERENCES BETWEEN THE TWO EQUIVALENT FOCAL LENGTHS WILL BE FOUND WHEN THEY ARE MEASURED ON AN OPTICAL BENCH.

$$F_1 = l'A'/\tan A \quad (3)$$

AND

$$F_2 = l'B'/\tan B, \quad (4)$$

IT WILL BE FOUND IN GENERAL THAT THE TWO VALUES OF F ARE DIFFERENT. IN SUCH A CASE, IT IS THE PRACTICE TO DEFINE THE EQUIVALENT FOCAL LENGTH F AS THE LIMITING VALUE OF F THAT IS OBTAINED AS THE ANGLE A APPROACHES ZERO AS A LIMIT.

WITH THIS DEFINITION OF F, DISTORTION BEING PRESENT, ONE MAY WRITE

$$l'A' = F \tan A + D_A \quad (5)$$

AND

$$l'B' = F \tan B + D_B, \quad (6)$$

WHERE D WILL HAVE DIFFERENT VALUES FOR POINTS AT DIFFERENT DISTANCES FROM THE AXIS. THE VALUE OF D IS THE DISTORTION AS GIVEN IN THE REPORTS OF THE NATIONAL BUREAU OF STANDARDS. WITH THIS DEFINITION POSITIVE DISTORTION CORRESPONDS TO A DISPLACEMENT OF THE IMAGE POINT OUTWARD FROM THE DISTORTION-FREE POSITION. WHEN ONLY ONE MEASUREMENT OF DISTORTION IS MADE, A POINT 30° FROM THE AXIS IS USUALLY SELECTED. BY MEANS OF THE BUREAU'S PRECISION LENS TESTING CAMERA IT IS A SIMPLE MATTER TO MEASURE THE DISTORTION AT FIVE DEGREE INTERVALS FROM 0 TO 30° OR EVEN TO 35° IF REQUIRED.

WITH THE FOCAL LENGTH AS DEFINED, THE DISTORTION WILL BE ZERO FOR POINTS NEAR THE CENTER OF THE FIELD AND WILL ORDINARILY INCREASE IN ABSOLUTE VALUE WITH INCREASE OF DISTANCE FROM THE CENTER OF THE FIELD. THE PHOTOGRAMMETRIST USES THE VALUE OF F, NOT AS AN OPTICAL CHARACTERISTIC OF THE LENS, BUT AS A SCALE FACTOR FOR INTERPRETING THE PICTURE. IN THE ABSENCE OF DISTORTION, THIS SCALE FACTOR IS CONSTANT OVER THE ENTIRE FIELD. IF POSITIVE DISTORTION IS PRESENT ALL CORRECTIONS WILL BE NEGATIVE, INCREASING FROM THE CENTER OUTWARD. HOWEVER, BY ARBITRARILY CHOOSING A SLIGHTLY LARGER VALUE OF F, THE CORRECTIONS MAY BE SO DISTRIBUTED THAT THOSE NEAR THE CENTER OF THE FIELD ARE NEGATIVE AND THOSE REMOTE FROM THE CENTER POSITIVE. BY THIS PROCEDURE IT MAY BE POSSIBLE TO MAKE THE CORRECTIONS SO SMALL THAT THEY CAN BE NEGLECTED. THIS IS A PERFECTLY LEGITIMATE AND PROPER PROCEDURE WHEN REQUIRED AND SUCH A CHANGE IN THE VALUE OF F MAY BE READILY DETERMINED AFTER THE DISTORTION AT FIVE DEGREE INTERVALS IS TABULATED. THE PROCESS, IN SOME RESPECTS, IS ANALOGOUS TO THAT USED BY THE SURVEYOR IN DISTRIBUTING ANGULAR ERRORS WHEN A POLYGON DOES NOT CLOSE. THIS PROCEDURE IS, HOWEVER, VERY MUCH LESS IMPORTANT THAT FORMERLY BECAUSE WITH MODERN LENSES THE TOTAL DISTORTION IS USUALLY LESS THAN 0.1 MM, EVEN NEAR THE EDGE OF THE PLATE. THE FOCAL LENGTH AS GIVEN IN THE BUREAU'S OFFICIAL REPORTS IS THE LIMITING VALUE OF F AT THE CENTER OF THE FIELD AND HAS NOT BEEN ALTERED TO DISTRIBUTE THE ERRORS ARISING FROM THE PRESENCE OF DISTORTION.

THE DISTORTION IS PARTICULARLY DEPENDENT ON THE MANNER IN WHICH A LENS IS USED. CONSEQUENTLY, WHEN A LENS IS TO BE TESTED FOR USE IN AN AIRPLANE CAMERA, THIS FACT SHOULD BE STATED AND THE DISTORTION WILL THEN BE MEASURED IN THE FOCAL PLANE FOR AN OBJECT AT AN INFINITE DISTANCE. WHEN A LENS DESTINED FOR USE IN A COPYING CAMERA OR TRANSFORMER IS TO BE TESTED, THE REQUEST SHOULD BE ACCOMPANIED BY A STATEMENT OF THE MAGNIFICATION AT WHICH IT IS TO BE USED. IF THE LENS IS TO BE USED IN A TRANSFORMER, THE MAGNIFICATION WILL VARY FOR DIFFERENT PARTS OF THE FIELD IN WHICH CASE THE MAGNIFICATION FOR THE CONJUGATE AXIAL POINTS IN OBJECT AND IMAGE PLANE SHOULD BE GIVEN. THE VALUES OF THE DISTORTION FOR AN INFINITELY DISTANT OBJECT AND FOR AN OBJECT AT A FINITE DISTANCE MAY DIFFER SO GREATLY THAT THE LENS IS SUITABLE FOR THE ONE PURPOSE AND QUITE UNSUITABLE FOR THE OTHER.

IN PHOTOGRAPHIC LITERATURE, THE DISTORTION IS SOMETIMES STATED AS A PERCENTAGE. REFERRING TO EQUATION 5, THE PERCENTAGE DISTORTION IS THE RATIO OF D_A TO $l'A'$, EXPRESSED AS A PERCENTAGE. THIS METHOD OF EXPRESSING DISTORTION IS PARTICULARLY CONVENIENT FOR THE LENS DESIGNER BECAUSE A SERIES OF LENSES OF DIFFERENT FOCAL LENGTHS MAY BE DESIGNED AND CONSTRUCTED OF THE SAME GLASS WITH THE DIFFERENT COMPONENTS GEOMETRICALLY SIMILAR. IN THIS CASE THE PERCENTAGE DISTORTION OF ALL THE LENSES, FOR A GIVEN DISTANCE FROM THE AXIS, WILL BE THE SAME. IN MOST CASES, HOWEVER, WHEN LENSES ARE SENT TO US FOR TEST, THE CLIENT IS INTERESTED IN COMPARING THE DISTORTION OF LENSES OF DIFFERENT MANUFACTURE HAVING APPROXIMATELY THE SAME FOCAL LENGTH. FURTHER, HE IS INTERESTED IN THE EXACT AMOUNT OF THE DISTORTION RATHER THAN IN A RATIO. FOR THIS REASON, THE NATIONAL BUREAU OF STANDARDS HAS DEEMED IT ADVISABLE TO GIVE THE DISTORTION DIRECTLY AND IN SUCH A MANNER THAT

IT CAN READILY BE CONVERTED TO THE PERCENTAGE SYSTEM IF DESIRED.

2. LACK OF DEFINITION ARISING FROM THE PRESENCE OF ABERRATIONS.

FOR CONVENIENCE THE IMPERFECTIONS OF IMAGERY THAT INTRODUCE LACK OF SHARPNESS ARE USUALLY REPRESENTED AS ARISING FROM THE COMBINED EFFECTS OF SEVERAL ABERRATIONS. A PARTIAL LIST OF THESE ABERRATIONS INCLUDES LONGITUDINAL CHROMATIC, LATERAL CHROMATIC, AND SPHERICAL ABERRATIONS; ALSO COMA, CURVATURE OF THE FIELD, AND ASTIGMATISM. FOR THE DESIGNER OR MANUFACTURER OF LENSES, THIS CLASSIFICATION OF THE ABERRATIONS IS ESPECIALLY USEFUL AND, WHEN CONSIDERING THE IMPROVEMENT OF A LENS SYSTEM, IT IS ADVANTAGEOUS TO DETERMINE THE VALUES OF THE DIFFERENT ABERRATIONS IN ORDER TO CHOOSE THE MODIFICATION OF DESIGN NECESSARY FOR THE IMPROVEMENT OF DEFINITION. THE NATIONAL BUREAU OF STANDARDS HAS MADE SUCH TESTS, BUT THEY ARE NECESSARILY EXPENSIVE AND ARE NOT OFTEN DESIRED BECAUSE LENS DESIGNERS AND MANUFACTURERS USUALLY PREFER TO MAKE THEIR OWN TESTS IN CONNECTION WITH THEIR DEVELOPMENT WORK.

THE USERS OF A LENS OR PARTIES CONTRACTING FOR PHOTOGRAMMETRIC WORK ARE NOT PRIMARILY CONCERNED WITH CAUSES OF IMPERFECT DEFINITION BUT ARE SOLELY INTERESTED IN KNOWING THE QUALITY OF DEFINITION IN ORDER THAT THEY MAY DECIDE WHETHER OR NOT THE LENS WILL BE SATISFACTORY FOR CERTAIN PURPOSES. FOR SUCH A REQUIREMENT, THE NATIONAL BUREAU OF STANDARDS MAKES A RESOLVING POWER TEST OF THE LENS AND, SINCE THE DEFINITION IS DIFFERENT IN DIFFERENT PARTS OF THE FIELD, THE RESOLUTION IS MEASURED AT 5 DEGREE INTERVALS FROM THE CENTER OF THE FIELD. FIGURE 3 SHOWS THE TARGET THAT IS USED FOR TESTING RESOLUTION. TARGETS OF DIFFERENT SIZES ARE USED FOR TESTING LENSES OF DIFFERENT FOCAL LENGTHS SO THAT THE RESULTING IMAGE ON THE TEST NEGATIVE WILL ALWAYS BE OF THE SAME SIZE. THE SPACINGS OF THE LINES ARE 3.5, 5, 7, 10, 14, 20, 28, 40 AND 56 TO THE MILLIMETER, THESE NUMBERS FORMING AN APPROXIMATELY GEOMETRICAL SERIES WITH THE RATIO OF ANY TWO SUCCESSIVE TERMS BEING THE SQUARE ROOT OF 2. ON THIS TARGET, WHICH IS ILLUMINATED BY TRANSMITTED LIGHT, THE LINES ARE OPAQUE, THE SPACES CLEAR, AND BOTH LINES AND SPACES ARE OF EQUAL WIDTH. FOR EACH SPACING TWO SETS OF PARALLEL LINES ARE PROVIDED, PERPENDICULAR TO EACH OTHER, AND THE TARGET IS PROJECTED OPTICALLY TO AN INFINITE DISTANCE. FOR MAKING THE TEST NEGATIVES, EASTMAN TYPE V-B SPECTROSCOPIC PLATES ARE USED. THESE PLATES ARE MUCH TOO SLOW FOR AIRPLANE MAPPING, BUT THEIR USE FOR TEST PURPOSES IS ADVANTAGEOUS BECAUSE THEY ARE VERY FINE GRAINED AND RECORD MUCH FINER DETAIL THAN THE FASTER EMULSIONS. CONSEQUENTLY, WITH THESE PLATES, WHEN A LENS SHOWS UNSATISFACTORY DEFINITION WE CAN BE CERTAIN THAT THE FAULT LIES IN THE LENS RATHER THAN IN OUR PHOTOGRAPHIC TECHNIQUE.

3. RESOLVING POWER AND DEFINITION.

IN SPECIFYING THE RESOLVING POWER, TWO VALUES ARE GIVEN, FOR EACH 5° POSITION, ONE APPLYING TO LINES PERPENDICULAR TO A RADIUS DRAWN FROM THE CENTER OF THE FIELD, THE OTHER TO LINES PARALLEL TO THE RADIUS. THESE VALUES ARE DESIGNATED RESPECTIVELY AS TANGENTIAL AND RADIAL RESOLVING POWERS. TO ILLUSTRATE, IF THE VALUES OF THE RESOLUTION FOR 20° ARE STATED TO BE TANGENTIAL 14, RADIAL 20, THIS SIGNIFIES THAT AT A POINT 20° FROM THE CENTER OF THE FIELD, A PATTERN OF PARALLEL LINES SPACED 14 TO THE MILLIMETER ON THE NEGATIVE AND LYING PERPENDICULAR TO THE RADIUS VECTOR FROM THE CENTER OF THE FIELD TO THE POINT UNDER CONSIDERATION IS RESOLVED INTO LINES THAT CAN BE SEPARATELY DISTINGUISHED ON THE NEGATIVE AND THE LINES OF THE NEXT GROUP, THAT IS 20 TO THE MILLIMETER, CANNOT BE CLEARLY RESOLVED. IF, AT THE SAME POINT IN THE FIELD, THE LINES ARE TURNED SO THAT THEY ARE PARALLEL TO THE RADIUS VECTOR THE GROUP OF LINES 20 TO THE MILLIMETER ARE THE FINEST THAT CAN BE RESOLVED. THE LOWER OF THIS PAIR OF VALUES IS THE MORE SIGNIFICANT BECAUSE THE LINES IN THE OBJECT PHOTOGRAPHED ARE IN RANDOM ORIENTATIONS AND THE LIMITING DETAIL CORRESPONDS TO THE SMALLER RESOLUTION. IN THE ABSENCE OF ASTIGMATISM, THE RESOLUTION IS INDEPENDENT OF THE ORIENTATION OF THE LINES.

ONE NATURALLY WISHES TO KNOW WHAT RESOLVING POWER CAN REASONABLY BE EXPECTED FROM AN AIRPLANE CAMERA LENS. THE BUREAU HAS MEASURED THE RESOLVING POWER OF A LARGE NUMBER OF OBJECTIVES, FOCAL LENGTH 150 MM, MAXIMUM APERTURE F/6.8, THAT ARE NOMINALLY IDENTICAL AND HAVE GIVEN SATISFACTORY PERFORMANCE IN CAMERAS USED IN THE FIELD. AN EXAMINATION OF THE PHOTOGRAPHIC RECORDS OF 50 LENSES, SELECTED AT RANDOM, SHOWS THAT THE RESOLVING POWER OF AT LEAST 50% OF THE LENSES WERE AS GOOD OR BETTER THAN THAT INDICATED IN THE FOLLOWING TABULATION.

TABLE 1. TYPICAL PERFORMANCE OF AN F/6.8 AIRPLANE CAMERA LENS.*

	0	5	10	15	20	25	30
TANGENTIAL	45	33	33	33	33	33	22
RADIAL	45	33	22	22	22	22	33

THE SECOND TABLE SHOWS THE PERFORMANCE OF A LENS SPECIALLY CONSTRUCTED TO COVER A LARGE FIELD WITH AN APERTURE RATIO OF F/11. IN THIS CASE, ALSO, THE FOCAL LENGTH WAS 150 MM.

TABLE 2. PERFORMANCE OF AN F/11 WIDE ANGLE LENS.

	0	5	10	15	20	25	30	35	40
TANGENTIAL	56	56	40	28	20	28	14	14	14
RADIAL	56	56	56	40	40	40	40	20	**

IV. GENERAL INFORMATION CONCERNING TESTS MADE BY THE NATIONAL BUREAU OF STANDARDS ON PHOTOGRAPHIC LENSES

EXTRACTS FROM THE TEST FEE SCHEDULE 442 LISTING THOSE TESTS REGULARLY MADE FOR PHOTOGRAMMETRIC PURPOSES ARE GIVEN BELOW:

442D - MEASUREMENT OF EQUIVALENT FOCAL LENGTH AND BACK FOCAL LENGTH..	\$5.00
442E - MEASUREMENT OF EQUIVALENT FOCAL LENGTH, BACK FOCAL LENGTH, SEPARATION OF NODAL POINTS, AND LENS THICKNESS.....	7.00
442F - DETERMINATION OF BACK FOCAL LENGTH, EQUIVALENT FOCAL LENGTH AND DISTORTION AT ONE POINT IN FIELD.....	20.00
442G - DETERMINATION OF BACK FOCAL LENGTH, EQUIVALENT FOCAL LENGTH, CURVATURE OF FIELD AND DISTORTION.....	40.00

TESTS 442D AND 442E ARE COMMONLY APPLIED TO LENSES FOR USE ON LESS PRECISE WORK FOR WHICH A DETERMINATION OF DISTORTION IS UNNECESSARY. IF THE LENS IS TO BE USED UNDER CONDITIONS FOR WHICH THE OBJECT MAY BE ASSUMED AT AN INFINITE DISTANCE, TEST 442D IS SUFFICIENT. IF THE OBJECT AND IMAGE ARE TO BE AT FINITE DISTANCES, TEST 442E WILL BE FOUND NECESSARY. TEST 442F IS THE TEST COMMONLY REQUIRED FOR AN AIRPLANE CAMERA LENS. USUALLY THE LENS CAN BE TESTED IN OUR PRECISION LENS TESTING CAMERA, AND IN THIS CASE, THE RESOLVING POWER FOR FIVE DEGREE INTERVALS FROM 0 TO 30° FROM THE AXIS WILL BE GIVEN WITHOUT EXTRA CHARGE. IF THE OBJECTIVE IS TO BE USED FOR COPYING OR IN A TRANSFORMER, 442F DOES NOT PROVIDE SUFFICIENT INFORMATION SINCE THE FRONT FOCAL LENGTH (OR LOCATION OF FIRST NODAL POINT) MUST BE KNOWN. THE WORK INVOLVED FOR THIS ADDITIONAL MEASUREMENT IS EQUIVALENT TO THAT OF TEST 442D AND, CONSEQUENTLY, FOR 442F WITH THIS ADDITIONAL DETERMINATION, THE TOTAL FEE IS \$25.00.

TEST 442G IS SOMETIMES REQUIRED BUT IT IS NOT RECOMMENDED IN CONNECTION WITH LENSES FOR MAPPING. TEST 442F, WHEN THE MEASUREMENT OF RESOLVING POWER IS INCLUDED, GIVES THE ESSENTIAL INFORMATION IN A FORM MORE READILY USED.

V. THE CALIBRATION OF A CAMERA

RECENTLY, AT THE REQUEST OF THE TENNESSEE VALLEY AUTHORITY, SPECIAL TESTS HAVE BEEN MADE ON A LIMITED NUMBER OF COMPLETED CAMERAS DESIGNED TO MAKE PHOTOGRAPHS FOR VERY PRECISE MAPPING. FOR THIS TEST, THE OBJECTIVE AND CAMERA CONE, ASSEMBLED AS A UNIT, MUST BE SUBMITTED. THIS UNIT MUST ALSO INCLUDE THE GUIDING SURFACE THAT DETERMINES THE PLANE OCCUPIED BY THE PHOTOGRAPHIC EMULSION WHEN AN EXPOSURE IS MADE AND THE COLLIMATION INDEX MARKS BY WHICH THE CENTRAL COLLIMATION POINT OF THE PHOTOGRAPH IS DETERMINED. IF THE CONE CARRYING THESE PARTS IS NOT REMOVABLE FROM THE CAMERA, THE TEST IN SOME CASES CAN BE MADE ON THE COMPLETE INSTRUMENT. IN GENERAL, THIS WILL BE POSSIBLE IF THE MAGAZINE CAN BE DETACHED WITHOUT DISTURBING THE GUIDES FOR THE FILM OR THE COLLIMATION INDEX MARKS.

THIS TEST, MADE WITHOUT REMOVING THE LENS FROM THE CAMERA, INCLUDES DETERMINATIONS OF DISTORTION, RESOLVING POWER, EQUIVALENT FOCAL LENGTH, AND CORRECT-

* THESE LENSES WERE TESTED WITH AN EARLIER FORM OF TEST CHART AND THE CHANGE OF SPACING, FROM PATTERN TO PATTERN, WAS NOT THE SAME AS IN THE TARGET OF FIGURE 3. THIS, HOWEVER, DOES NOT IMPAIR THE USEFULNESS OF THE RESULTS.

** THE RESOLUTION HERE IS LESS THAN THAT CORRESPONDING TO THE COARSEST PATTERN ON THE CHART, NAMELY, 3.5 LINES TO THE MILLIMETER.

NESS OF LOCATION OF THE COLLIMATION INDEX POINTS. THE EQUIVALENT FOCAL LENGTH, IN THIS CASE, IS ONE THAT CORRESPONDS TO THE BACK FOCAL LENGTH AS DETERMINED BY THE GUIDES THAT LOCATE THE FILM. THIS COMPLETE TEST IS A VERY USEFUL ONE AND IT APPEARS DESIRABLE THAT ALL CAMERAS MAKING PHOTOGRAPHS TO BE EVALUATED BY MULTIPLEX APPARATUS OR OTHER STEREOSCOPIC PLOTTING DEVICES SHOULD BE TESTED IN THIS MANNER. CAMERAS INTENDED FOR WORK OF THIS CHARACTER SHOULD BE SPECIALLY DESIGNED SO THAT THE CHARACTERISTICS MEASURED IN THIS TEST WILL REMAIN CONSTANT, NOT ONLY DURING USE OF THE CAMERA IN THE FIELD BUT ALSO DURING ANY PROCESS OF DISASSEMBLING AND REASSEMBLING TO WHICH IT IS LIKELY TO BE SUBJECTED. THIS TEST WILL BE DISCUSSED AT GREATER LENGTH AFTER IT BECOMES FURTHER DEVELOPED AND STANDARDIZED.

CAMERAS AND EQUIPMENT FOR USE IN PHOTOGRAMMETRY

BY
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BECAUSE OF THE GROWING USE OF AERIAL PHOTOGRAPHY AS A MEANS OF OBTAINING ACCURATE MAPPING RESULTS - RATHER THAN AS AN UNIMPORTANT ACCESSORY TO THE TIME-HONORED GROUND SURVEY METHODS - IT HAS BEEN NECESSARY FOR AMERICAN MANUFACTURERS TO IMPROVE EXISTING EQUIPMENT AND DEVELOP NEW PRODUCTS SO THAT THOSE ENGAGED IN AERIAL PHOTOGRAMMETRIC WORK CAN OBTAIN RESULTS OF THE REQUIRED ACCURACY AND UNIFORMITY.

AERIAL CAMERAS OF THE FAIRCHILD TYPE HAVE BEEN EMPLOYED FOR MANY YEARS WITHOUT RADICAL CHANGES IN DESIGN BY THE MILITARY AND NAVAL FORCES AND OTHER DEPARTMENTS OF THE UNITED STATES GOVERNMENT. THESE AGENCIES HAVE DONE MUCH TO ENCOURAGE PROGRESS IN THE PHOTOGRAMMETRIC APPLICATIONS OF AERIAL PHOTOGRAPHY, AND IT IS ONLY NATURAL THAT THE DEVELOPMENT OF IMPROVED APPARATUS HAS BEEN INFLUENCED GREATLY BY THESE INTERESTS. MUCH VALUABLE WORK HAS BEEN PERFORMED BY THE USE OF THESE ORIGINAL CAMERAS IN CONJUNCTION WITH STEREO PLOTTING MACHINES OF VARIOUS TYPES, ALTHOUGH THE CAMERAS THEMSELVES WERE DESIGNED PRIMARILY FOR MILITARY RECONNAISSANCE AND ROUGH MAPPING WORK. THE RESULTS THUS OBTAINED WERE VERY ENCOURAGING AND GRADUALLY THE REQUIREMENTS FOR SPECIAL CAMERA EQUIPMENT GAINED IN CLARITY AND MOMENTUM WITH THE RESULT THAT IT IS NOW POSSIBLE TO PRESENT A PHOTOGRAPHIC MAPPING INSTRUMENT WORTHY OF THE NAME AND YET DESIGNED TO FIT IN WITH PRESENT METHODS AND APPARATUS. BEFORE DESCRIBING THIS UNIT, IT MAY BE OF INTEREST TO MENTION BRIEFLY A FEW OTHER ITEMS WHICH HAVE BEEN PRODUCED FOR USE BY THE PHOTOGRAMMETRIST.

FOR GENERAL PURPOSES, IT IS NOW USUALLY RECOGNIZED THAT A SQUARE PHOTOGRAPH IS DESIRABLE AS SECURING OPTIMUM EFFICIENCY IN COVERAGE, SO 9 X 9 INCH ROLL FILM MAGAZINES ARE NOW AVAILABLE FOR USE ON STANDARD 7 X 9 INCH K-3B CAMERAS, THESE UNITS BEING FULLY INTERCHANGEABLE WITH THE ORIGINAL FILM UNITS. PARTICULARLY INTENDED FOR WIDE ANGLE WORK WITH 6 INCH TO $8\frac{1}{4}$ INCH LENSES, THE MAGAZINE HAS A CAPACITY OF OVER 250 EXPOSURES AT ONE LOADING AND USES FILM OF THE SAME WIDTH AS ITS PREDECESSOR. HOWEVER, ITS SIMILARITY TO THE ORIGINAL 7 X 9 MAGAZINE STOPS HERE. IN PLACE OF THE FOCAL PLANE GLASS, A VACUUM SUCTION PLATE IS UTILIZED TO SECURE ABSOLUTE FLATNESS OF THE FILM IN THE FOCAL PLANE. THE SUCTION PLATE IS AUTOMATICALLY RAISED BEFORE FRESH FILM IS BEING WOUND INTO PLACE, DESCENDING AGAIN, AFTER THIS MOTION CEASES, TO PRESS THE FILM ONCE MORE AGAINST A FIXED METAL FRAME SURROUNDING THE PICTURE AREA. THIS FRAME SERVES AS A SEAL TO PREVENT INGRESS OF UNWANTED AIR AS THE VACUUM LINE TO A SMALL VENTURI TUBE EXHAUSTS THE REMAINING SURPLUS BETWEEN FILM AND VACUUM PLATE. THE FRAME IS PROVIDED WITH COLLIMATING NOTCHES WHICH INDICATE THE PRINCIPAL POINT OF THE PHOTOGRAPH. A SIMPLIFIED AND ENTIRELY FOOLPROOF MECHANISM AUTOMATICALLY SYNCHRONIZES THE WINDING, METERING AND VALVE-OPERATING FUNCTIONS OF THE MAGAZINE WHICH CAN BE USED EITHER ON HAND OR ELECTRICALLY-OPERATED CAMERAS. IT IS POSSIBLE TO COLLIMATE THESE MAGAZINES WITHIN FAIRLY CLOSE LIMITS WITH REGARD TO THE LENS CONES ON WHICH THEY ARE EMPLOYED.

AMONG RECENT PRODUCTS, MENTION SHOULD BE MADE OF THE 9 LENS AERIAL CAMERA DEVELOPED IN COLLABORATION WITH THE U. S. COAST AND GEODETIC SURVEY, AND WITH WHICH THE MEMBERS OF THE AMERICAN SOCIETY OF PHOTOGRAMMETRY ARE MORE OR LESS FAMILIAR. THIS CAMERA PRODUCES ON A SINGLE FILM, 9 NEGATIVES WHICH ARE TRANSFORMED IN A SPECIAL PRINTER, TO PRODUCE A PRINT ABOUT THREE FEET SQUARE OF THE 130° AREA PHOTOGRAPHED. ALTHOUGH THIS PHOTOGRAPH IS MADE BY USING 9 SEPARATE