BOOK REVIEWS

APPLIED PHOTOGRAMMETRY BY R. O. ANDERSON
EDWARDS BROTHERS, ANN ARBOR, MICHIGAN, 1937. 134 PAGES. ILLUSTRATED.
9" x 6". PEARL PRESSBOARD, $2.00.

(THE FOLLOWING REVIEW WAS FURNISHED BY THE AUTHOR.)

THIS PAPER WAS MOTIVATED BY THE URGENT NEED OF A DIRECT, TO THE POINT SYSTEM OF DETERMINING THE PROPERTIES OF THE AERIAL PHOTOGRAPH; I. E., SCALE, TILT, ELEVATION, DISTANCE, AREA, AND ORIENTATION, ALL OF WHICH ARE NECESSARY IN ORDER TO CONVERT THE PHOTOGRAPH INTO A MAP.

THE PRACTICAL THEORY OF PHOTOGRAMMETRY, AS PRESENTED IN THIS BOOK, TREATS PROGRESSIVELY: THE SINGLE POINT, THE LINE, THE ELEVATION AT WHICH THE PHOTOGRAPHIC SCALE PREVAILS, SLIDE RULE FOR DETERMINING THE ACTUAL FLYING HEIGHT, AT TIME OF EXPOSURE, AND CORRESPONDING UNIT SCALES AT ANY ELEVATION, UNIT SCALE AT ANY POINT ON A TILTED PHOTOGRAPH, LINE LENGTH AND AREA DETERMINATION, TILT DETERMINATION BY SCALE VARIATION, PHOTOGRAMMETRIC TRIANGULATION WITH ADJUSTMENTS AND COMPUTATIONS OF IMAGE COORDINATIONS, AND ELEVATION DETERMINATION OF IMAGES.

EVERY EFFORT WAS MADE TO COMPRESS THIS MATERIAL INTO PRACTICAL WORKING FORM, EXCLUDING GROUND SURVEY METHODS, THEORY OF PHOTOGRAPHY, APPLICATION TO SURVEYS OF SPECIFIC NATURE, ETC., WITH THE POINT IN MIND OF BRINGING TO THE FRONT THE PHOTOGRAMMETRIC SOLUTIONS. THE MATHEMATICS EMPLOYED SELDOM EXCEEDS SIMPLE PROPORTIONALITY.

THE MATERIAL IN THIS BOOK IS THE OUTGROWTH OF NEARLY FOUR YEARS OF CONTINUAL PHOTOGRAMMETRIC RESEARCH COVERING ABOUT 8,000 SQUARE MILES OF AERIAL SURVEYS. THE IMMENSE VOLUME OF WORK MADE NECESSARY MEANS AND METHODS OF THE GREATEST ECONOMY, AND SO, BY THAT TOKEN, A NEW SIMPLIFIED GEOMETRIC METHOD OF APPROACH WAS LAID DOWN AND REDUCED TO ITS LOWEST COMMON DENOMINATOR WITHOUT SACRIFICING A WELL DEFINED HIGH STANDARD OF ACCURACY.

ORIENTATION OF AERIAL PHOTOGRAPHS IN THE MULTIPLEX - PROJECTOR FROM COMPUTED DATA BY EARL CHURCH
PUBLISHED BY SYRACUSE UNIVERSITY, 1937. 27 PAGES.

THE SYRACUSE UNIVERSITY HAS JUST PUBLISHED THE EIGHTH OF A SERIES OF BULLETINS ON AERIAL PHOTOGRAPHY WRITTEN BY PROFESSOR EARL CHURCH. THIS LATEST PUBLICATION CONTAINS A FULL ANALYSIS OF THE METHODS FOR ORIENTING AERIAL PHOTOGRAPHS IN THE MULTIPLEX-PROJECTOR FROM COMPUTED DATA INCLUDING FORMULAE AND SAMPLE COMPUTATIONS. THE PAMPHLET HAS 27 PAGES. COPIES ARE BEING MAILED FROM SYRACUSE UNIVERSITY TO ALL MEMBERS OF THE SOCIETY.

RADIAL PLOTTING BY PRINCIPAL POINT RAYS

BY
WALTER SCHOLLA
SYRACUSE UNIVERSITY, 1937

THIS PAPER ON SOME THEORETICAL PHASES OF THE WIDELY USED METHOD OF RADIAL PLOTTING FOR CONSTRUCTING PLANIMETRIC MAPS FROM "VERTICAL" AERIAL PHOTOGRAPHS, HAS BEEN PREPARED AT THE SUGGESTION OF EARL CHURCH, ASSOCIATE PROFESSOR OF PHOTOGRAMMETRY AT SYRACUSE UNIVERSITY. THE DISCUSSION IS INTRODUCED BY THE STATEMENT OF THREE FUNDAMENTAL PRINCIPLES WITH WHICH MOST READERS ARE DOUBTLESS ALREADY FAMILIAR.

1. IF AN AERIAL PHOTOGRAPH IS EXPOSED WITH THE CAMERA AXIS TRULY VERTICAL, THEN IMAGE DISPLACEMENTS PRODUCED BY TOPOGRAPHIC RELIEF RADIATE FROM THE PRINCIPAL POINT, AND ANGLES BETWEEN PRINCIPAL POINT RAYS ARE EXACTLY EQUAL TO THE

CORRESPONDING GROUND ANGLES MEASURED AT THE GROUND PLUMB POINT. THIS SIMPLE PRINCIPLE IS SHOWN BY FIGURE 1, AND NEEDS NO PROOF.

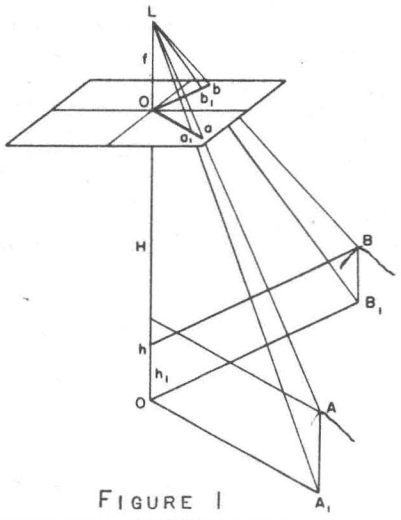

FIGURE 1
UNTILTED PHOTOGRAPH

IN THIS CASE THE IMAGE DISPLACEMENTS PRODUCED BY TOPOGRAPHIC RELIEF ARE ACTUALLY EQUAL TO

$$d \times h/H,$$ WHERE

h IS THE ALTITUDE OF THE POINT ON THE GROUND;

H IS THE ALTITUDE OF THE EXPOSURE STATION;

d IS THE RADIAL DISTANCE ON THE PHOTOGRAPH FROM THE PRINCIPAL POINT TO THE ACTUAL IMAGE OF THE POINT.

IF ONE IS THINKING OF A SINGLE VERTICAL PHOTOGRAPH AS A MAP TO THE SCALE **f/H**, WHEN **f** IS THE PRINCIPAL DISTANCE OF THE CAMERA LENS, THEN THE HORIZONTAL POSITION OF A POINT WHOSE ALTITUDE ON THE GROUND IS **h**, WILL BE IN ERROR BY THE AMOUNT:

$$d \times \frac{h}{H} \times \frac{H}{f} = h \times \frac{d}{f}.$$

THUS IN A 13 CM. X 18 CM. PHOTOGRAPH, TAKEN WITH A LENS WHOSE PRINCIPAL DISTANCE IS 18 CM., THE HORIZONTAL POSITION OF A POINT MIGHT BE IN ERROR BY AS MUCH AS $\frac{1}{2}$ **h**.

HOWEVER, THE METHOD OF RADIAL PLOTTING OR THE GRAPHICAL DETERMINATION OF PLANIMETRIC LOCATIONS BY THE USE OF PRINCIPAL POINT RAYS IN INTERSECTIONS AND RESECTIONS, ENTIRELY ELIMINATES THIS ERROR; FOR ONLY ANGLES AT THE PRINCIPAL POINTS ARE USED, AND THESE ANGLES ARE THE GROUND VALUES, INASMUCH AS IMAGE DISPLACEMENTS CAUSED BY TOPOGRAPHIC RELIEF RADIATE FROM THE PRINCIPAL POINT OF THE PHOTOGRAPH.

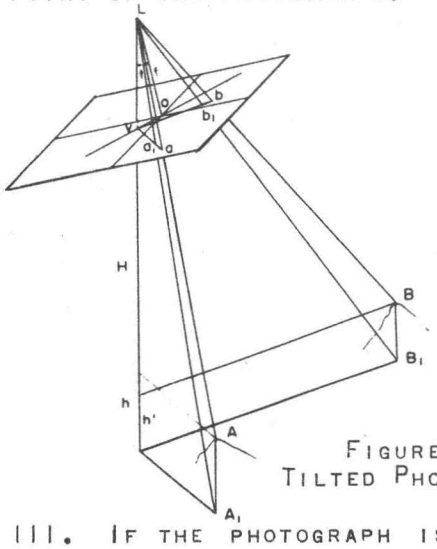

FIGURE 2
TILTED PHOTOGRAPH

II. IF A PHOTOGRAPH IS TAKEN WITH THE CAMERA AXIS INCLINED TO THE VERTICAL, AS IS ALWAYS THE CASE IN PRACTICE, DISPLACEMENTS OF IMAGES PRODUCED BY TOPOGRAPHIC RELIEF RADIATE FROM THE NADIR POINT OF THE PHOTOGRAPH. THIS IS SHOWN BY FIGURE 2. HOWEVER, BECAUSE OF TILT, ANGLES ON THE PHOTOGRAPH BETWEEN RAYS FROM THE NADIR POINT, ARE NOT TRUE MEASURES OF THE CORRESPONDING GROUND ANGLES; SO THAT EVEN IF THE TILT OF A PHOTOGRAPH WERE DETERMINED, IT WOULD NOT BE CORRECT, THEORETICALLY, TO DO RADIAL PLOTTING FOR PLANIMETRIC MAPS BY USING NADIR POINT RAYS FOR INTERSECTIONS AND RESECTIONS.

III. IF THE PHOTOGRAPH IS TILTED, BUT THE TERRAIN IS PERFECTLY FLAT, THEN ANOTHER GEOMETRIC CONDITION ARISES. FOR THIS CASE, ANGLES BETWEEN PRINCIPAL POINT RAYS ARE NO LONGER CORRECT MEASURES OF CORRESPONDING GROUND ANGLES, NOR IS THIS TRUE FOR ANGLES MEASURED BETWEEN NADIR POINT RAYS. HOWEVER, TRUE MEASURES OF GROUND ANGLES ARE FOUND FOR THIS CASE BETWEEN RAYS RADIATING FROM A POINT CALLED THE <u>ISOCENTER</u>. THIS POINT IS DEFINED AS THE INTERSECTION WITH THE PLANE OF THE PHOTOGRAPH, OF A LINE BISECTING THE ANGLE AT THE EMERGENT NODE OF THE CAMERA LENS BETWEEN THE CAMERA AXIS AND A TRULY VERTICAL LINE. WITH REFERENCE TO FIGURE 3, THE PROOF OF THIS GEOMETRIC PROPERTY FOLLOWS:

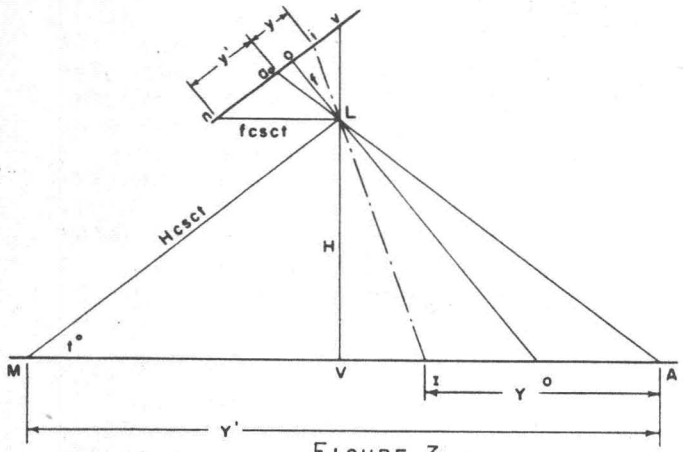

FIGURE 3
TILTED PHOTOGRAPH OF
FLAT TERRAIN

REFERRING TO FIGURE 3, THE PRINCIPAL PLANE IS SHOWN, **nv** BEING ITS INTERSECTION WITH THE PLANE OF THE PHOTOGRAPH, AND MA_o BEING ITS INTERSECTION WITH THE GROUND. **L** IS THE EXPOSURE STATION, **Lo** EQUALS **f**, AND **LV** EQUALS **H**. THE POINTS **o** AND **v** ARE THE PRINCIPAL AND NADIR POINTS, RESPECTIVELY, WITH **O** AND **V** THE CORRESPONDING GROUND POINTS.

ANGLE **oLv** IS THE TILT. THE LINE **iLi** BISECTS ANGLE **oLv**, AND HENCE **i** IS THE ISOCENTER OF THE PHOTOGRAPH BY DEFINITION. **Ln** IS HORIZONTAL, AND THEREFORE EQUAL TO **f csc t**. **LM** IS PARALLEL TO THE PLANE OF THE PHOTOGRAPH AND THEREFORE EQUAL TO **H csc t**.

FOR THE COORDINATE SYSTEMS, THE LINE MA_o IS USED AS THE **y**-AXIS ON THE GROUND, AND LINE **no** AS THE **y**-AXIS ON THE PHOTOGRAPH. A GROUND POINT **A** IS PROJECTED TO A_o ON THE GROUND **y**-AXIS, AND ITS IMAGE **a** IS PROJECTED TO a_o ON THE PHOTOGRAPHIC **y**-AXIS, THE PRINCIPAL LINE. THE PERPENDICULAR DISTANCES AA_o AND aa_o ARE **X** AND **x** ON THE GROUND AND ON THE PHOTOGRAPH, RESPECTIVELY. IA_o IS CALLED **Y** AND ia_o IS CALLED **y**.

NOW, IF $\frac{x}{y} = \frac{X}{Y}$, THEN THE GROUND DIRECTION OF THE LINE **IA** AND THE PHOTOGRAPHIC DIRECTION OF THE ISOCENTER RAY **ia** MEASURED FROM THE PRINCIPAL PLANE, ARE EXACTLY EQUAL. THEN ANGLES BETWEEN ISOCENTER RAYS ON THE PHOTOGRAPH WOULD BE EXACTLY EQUAL TO THE CORRESPONDING GROUND ANGLES. THIS IS WHAT WE WISH TO PROVE.

NOW, SINCE **iLI** BISECTS THE ANGLES **oLv** AND **OLV**, THE TRIANGLES **inL** AND **IML** ARE BOTH ISOSCELES.

$$\therefore\ y' + y = f \csc t \quad \text{or} \quad y' = f \csc t - y$$
$$\text{AND}\quad Y' - Y = H \csc t \quad \text{or} \quad Y' = H \csc t + Y$$

BY INSPECTION $\frac{X}{x} = \frac{A_oL}{a_oL}$

BY SIMILAR $\triangle$s $\frac{A_oL}{a_oL} = \frac{H \csc t}{y'}$ and $\frac{A_oL}{a_oL} = \frac{Y'}{f \csc t}$

$$\therefore\ \frac{X}{x} = \frac{Y'}{f \csc t} = \frac{H \csc t}{y'}$$

SUBSTITUTING THE ABOVE EXPRESSIONS FOR **Y'** AND **y'**:

$$\frac{X}{x} = \frac{H \csc t + Y}{f \csc t} = \frac{H \csc t}{f \csc t - y}$$

TAKING THE EQUALITY $\frac{H \csc t + Y}{f \csc t} = \frac{H \csc t}{f \csc t - y}$

THERE OBTAINS:

$$Hf \csc^2 t + Yf \csc t - Hy \csc t - Y y = Hf \csc^2 t$$
$$Y (f \csc t - y) = Hy \csc t$$
$$\frac{Y}{y} = \frac{H \csc t}{f \csc t - y}, \text{ WHICH EQUALS, SHOWN PREVIOUSLY,}$$

THE QUANTITY $\frac{X}{x}$

$$\therefore\ \frac{Y}{y} = \frac{X}{x} \text{ and } \frac{X}{Y} = \frac{x}{y} \quad \text{Q. E. D}$$

THIS PROVES THAT ANGLES BETWEEN ISOCENTER RAYS ON THE PHOTOGRAPH, ARE EQUAL TO THE CORRESPONDING ANGLES ON THE GROUND, REGARDLESS OF THE AMOUNT OF TILT IN THE PHOTOGRAPHS, PROVIDED THE GROUND HAS NO TOPOGRAPHIC RELIEF.

FROM THE ABOVE PRINCIPLES IT FOLLOWS THAT WITH TRULY VERTICAL PHOTOGRAPHS TAKEN OVER TERRAIN EITHER WITH OR WITHOUT TOPOGRAPHIC RELIEF, RADIAL PLOTTING OF PLANIMETRY, USING PRINCIPAL POINT RAYS IS THEORETICALLY CORRECT.

IT MAY ALSO BE OBSERVED, THAT FOR TILTED PHOTOGRAPHS TAKEN OVER TERRAIN

HAVING TOPOGRAPHIC RELIEF, PRINCIPAL POINT RAYS, NADIR POINT RAYS, AND ISOCENTER RAYS, ALL GIVE INCORRECT RESULTS THEORETICALLY IN RADIAL PLOTTING.

IN SOME PLANIMETRIC MAP CONSTRUCTION BY RADIAL PLOTTING, ATTEMPTS HAVE BEEN MADE TO DETERMINE THE TILTS OF THE PHOTOGRAPHS, IN ORDER TO UTILIZE ISOCENTER RAYS, WITH THE OBJECT OF ELIMINATING ERRORS FOUND IN RADIAL PLOTTING, WHICH HAVE BEEN ATTRIBUTED TO TILT. THE USE OF THESE ISOCENTER RAYS WOULD OF COURSE BE CORRECT IF THE COUNTRY WERE PERFECTLY FLAT. BUT WITH TOPOGRAPHIC RELIEF THE METHOD IS NOT CORRECT, FOR THE IMAGE DISPLACEMENTS DO NOT RADIATE FROM THE ISOCENTER. THE REASONING, HOWEVER, IS ERRONEOUSLY THAT "ISOCENTER RAYS ELIMINATE ERRORS DUE TO TILT." AND SINCE THE NADIR POINT FROM WHICH IMAGE DISPLACEMENTS CAUSED BY TOPOGRAPHIC RELIEF, ACTUALLY DO RADIATE ON A TILTED PHOTOGRAPH IS NEVER FAR FROM THE ISOCENTER, PERHAPS IN MOST CASES NO APPRECIABLE ERROR DUE TO DISPLACEMENTS OF IMAGES BY TOPOGRAPHIC RELIEF IS INTRODUCED.

THE ERRORS FOUND IN RADIAL PLOTTING BY PRINCIPAL POINT RAYS, WHICH HAVE BEEN ATTRIBUTED TO TILT AND WHICH HAVE PROMPTED THE ATTEMPT TO UTILIZE ISOCENTER RAYS, ARE NOT ACTUALLY CAUSED BY TILT. THEY ARISE FROM THE ASSUMPTION IN USING PRINCIPAL POINT RAYS, THAT ON A TILTED PHOTOGRAPH, TOPOGRAPHIC RELIEF DISPLACEMENTS RADIATE FROM THE PRINCIPAL POINT; AND THIS ASSUMPTION IS FALSE. ANY ERRORS FOUND IN GRAPHICAL WORK WILL THEREFORE NOT BE ELIMINATED BY THE SUBSTITUTION OF ISOCENTER RAYS. THE FOLLOWING DISCUSSION SHOULD VERIFY THESE STATEMENTS.

FIGURE 4 REPRESENTS A TILTED PHOTOGRAPH OVER TERRAIN HAVING NO TOPOGRAPHIC RELIEF. THE FOLLOWING RELATIONS REFER TO THIS FIGURE:

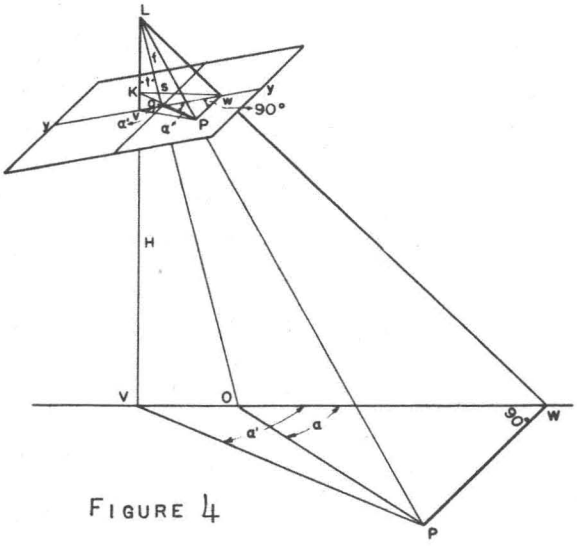

FIGURE 4

ANGLE **oLv** IS THE TILT, **t**, TAKEN FOR SIMPLICITY IN THE DIRECTION OF THE **y**-AXIS OF THE PHOTOGRAPH;
o IS THE PRINCIPAL POINT;
O IS THE CORRESPONDING GROUND POINT;
v IS THE NADIR POINT;
V IS THE CORRESPONDING GROUND POINT;
P IS ANY POINT ON THE GROUND;
p IS THE PHOTOGRAPHIC IMAGE OF **P**;
pw IS PERPENDICULAR TO THE **y**-AXIS, AND IS HORIZONTAL IN SPACE;
wk IS PERPENDICULAR TO **Lv**;
pwk IS A TRIANGLE IN A HORIZONTAL PLANE, AND IS THEREFORE SIMILAR TO TRIANGLE **VPW**;
H IS **LV**, ALTITUDE OF THE EXPOSURE STATION;
wp IS **x** AND **ow** IS **y**, THE PHOTOGRAPHIC COORDINATES OF **P**;
wop IS THE PHOTOGRAPHIC DIRECTION OF RAY **op** MEASURED FROM THE **y**-AXIS;
WOP IS α, THE CORRESPONDING GROUND ANGLE.

FROM THE FIGURE, IT MAY BE SEEN:

$$\frac{WP}{wp} = \frac{WO}{ws}$$

NOW, $ws = wo \sec t.$

$$\therefore \frac{WP}{WO} = \frac{wp}{wo \sec t.}$$

$$\tan \alpha = \frac{\tan \alpha}{\sec t} = \tan \alpha \cos t.$$

IF WE CONSIDER THAT $t = 3^\circ$, WHICH IS AS LARGE AS IT SHOULD EVER BECOME WITH GOOD FIELD WORK IN TAKING THE PHOTOGRAPHS, THEN

$$\tan \alpha = .99863 \tan \alpha$$

THIS SHOWS THAT α CAN NEVER DIFFER FROM α BY MORE THAN ABOUT 2 MINUTES FOR THE EXTREME CASES. EVEN FOR A TILT AS GREAT AS 5 OR 6 DEGREES, WHICH IN PRACTICE SHOULD NEVER BE ENCOUNTERED, THE DIFFERENCE BETWEEN α AND α WOULD STILL NOT BE DISCERNIBLE IN GRAPHICAL WORK. THEREFORE, IN RADIAL PLOTTING,

THERE IS NO ERROR CAUSED BY TILT ALONE IF PRINCIPAL POINT RAYS ARE USED; AND AS FAR AS TILT ONLY IS CONCERNED, NOTHING IS TO BE GAINED BY THE USE OF RAYS FROM THE ISOCENTER.

ALTHOUGH THIS DISCUSSION PROVES THAT ON A TILTED PHOTOGRAPH OF FLAT TERRAIN, THE ANGLES MEASURED ON THE PHOTOGRAPH AT THE PRINCIPAL POINT DIFFER FROM THE CORRESPONDING GROUND ANGLES BY ONLY THE FACTOR **cos t**, AND THAT THESE DIFFERENCES ARE INAPPRECIABLE FOR GRAPHICAL PLOTTING, NEVERTHELESS THE FACT REMAINS THAT WITH THE USE OF PRINCIPAL POINT RAYS FOR PLOTTING, ERRORS OF SERIOUS MAGNITUDE HAVE OCCURRED. THE EXPLANATION, AS MENTIONED, DOES NOT LIE IN THE CONSIDERATION OF THE TILT ALONE; BUT IT IS THE COMBINATION OF TILT AND TOPOGRAPHIC RELIEF WHICH PRODUCES THESE ERRORS.

FIGURES 5 AND 6 ARE PRESENTED TO ILLUSTRATE THE INVESTIGATION OF THIS PROBLEM. A SPECIAL CASE IS CONSIDERED, SPECIAL BECAUSE THE AXIS OF TILT IS ASSUMED PARALLEL TO THE **x - x** AXIS OF THE PHOTOGRAPH, AND BECAUSE **p** LIES ON A LINE PARALLEL TO THE **x**-AXIS, THROUGH **v**. A GENERAL CASE WOULD PERHAPS BE MORE DESIRABLE FROM A THEORETICAL POINT OF VIEW, BUT IT LEADS, IN THIS PROBLEM, TO INVOLVED AND CUMBERSOME MATHEMATICAL EXPRESSIONS. THE RESULTS FROM CONSIDERATION OF THE SPECIAL CASE FOLLOWING, SUFFICIENTLY ILLUSTRATE THE PRINCIPLES IN QUESTION.

THE FOLLOWING NOTATION REFERS TO BOTH FIGURES 5 AND 6:

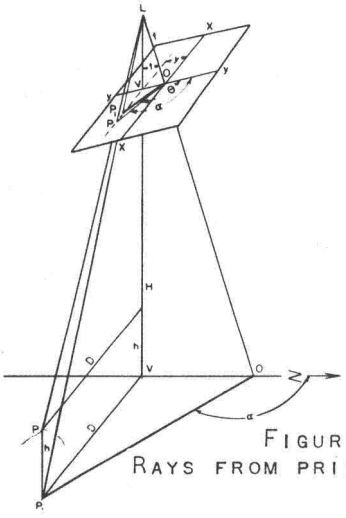

FIGURE 5
RAYS FROM PRINCIPAL POINT

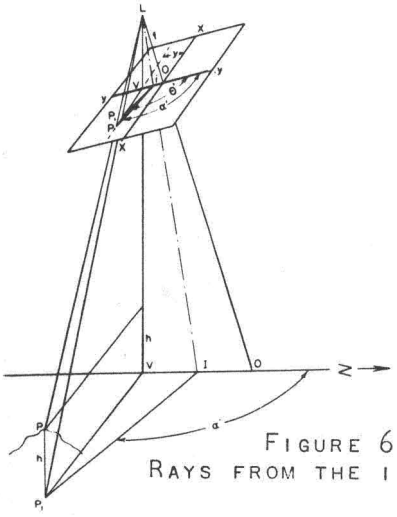

FIGURE 6
RAYS FROM THE ISOCENTER

L IS THE EMERGENT NODAL POINT OF THE LENS SYSTEM;
f IS THE PRINCIPAL DISTANCE;
o IS THE PRINCIPAL POINT, AND THE ORIGIN FOR THE PHOTOGRAPHIC AXES **x-x, y-y;**
O IS THE SEA LEVEL PIERCING POINT OF **Lo** PROJECTED;
v IS THE NADIR POINT;
V IS THE CORRESPONDING GROUND POINT;
p IS THE IMAGE OF **P**, WHICH LIES AT AN ELEVATION **h** ABOVE SEA LEVEL;
P_1 IS THE SEA LEVEL PROJECTION OF **P**;
P_1 IS THE FICTITIOUS IMAGE OF P_1;
H IS THE ALTITUDE ABOVE SEA LEVEL OF THE EXPOSURE STATION;
α IS THE AZIMUTH OF **OP**, (FIG. 5) AND α' OF **IP** (FIG. 6);
α IS THE CORRESPONDING PHOTOGRAPHIC ANGLE, (FIG. 5) AND α', CORRESPONDINGLY IN FIG. 6.
IS THE PHOTOGRAPHIC ANGLE CORRESPONDING TO α, AND θ' CORRESPONDS TO α', IF P_1 INSTEAD OF **P** WERE PHOTOGRAPHED;
t IS THE TILT.

NOW, LET $vp_1 = d_1$
AND $vp = d = x$

THEN $\frac{d_1}{D} = \frac{f \sec t}{H}$; and $\frac{d}{D} = \frac{f \sec t}{H - h}$.

$$d - d_1 = \frac{Df \sec t}{H - h} - \frac{Df \sec t}{H}.$$

$$= Df \sec t \left(\frac{H - (H - h)}{H(H - h)}\right).$$

$$d - d_1 = \frac{dh}{H} = \frac{xh}{H}$$

WHICH IS TRUE FOR BOTH FIGURE 5 AND FIGURE 6.

THE FOLLOWING DISCUSSION APPLIES TO FIGURE 5. IT MAY BE SEEN FROM INSPECTION THAT THE VALUE OF THE DIFFERENCES BETWEEN θ AND α WILL VARY FROM 0^o

FOR POINTS TAKEN ALONG vo PRODUCED, TO SOME MAXIMUM VALUE WHEN THE PERPENDICULAR BISECTOR OF pp_1 PASSES THROUGH o. THIS FACT SHOWS AT ONCE WHY ERRORS IN RADIAL PLOTTING WITH TILTED PHOTOGRAPHS CONTAINING TOPOGRAPHIC RELIEF HAVE BEEN FOUND TO BE LOCAL; FOR REGIONS OF ERROR WILL INCLUDE ONLY CERTAIN CRITICAL AREAS, AND IF TOPOGRAPHIC RELIEF SHOULD HAPPEN NOT TO BE PRESENT IN THESE AREAS, NO ERRORS IN PLOTTING WILL OCCUR.

REFERRING TO FIGURE 5:

$$\tan \alpha = x/y$$

$$\tan \theta = \frac{x - \frac{xH}{h}}{y} = \frac{xH - xh}{yH} = \frac{x}{y}\,\frac{(H - h)}{H}$$

SUPPOSE THAT IN PRACTICE, THE FOLLOWING VALUES SHOULD OCCUR:

H ---- 10,000 feet
h ---- 1,000 feet
t ---- 3^0
f ---- 18 cm. (about 7 inches)
x ---- 6 cm.

THEN $y = ov = f \tan 3^0$

$= 18 \times .05241 = .94338$ cm.

AND $\tan \alpha = \frac{6}{.94338} = 6.36011$; $\alpha = 98^0\ 56.1'$

$$\tan \theta = \frac{6}{.94338} \times \frac{10,000 - 1,000}{10,000} = 5.72410$$

$\theta = 99^0\ 54.6'$

IT WAS SHOWN THAT $\tan \alpha = \tan \theta \cos t$

$\tan \alpha = 5.7241 \cos 3^0$

$= 5.7241 \times .99863 = 5.7163$

$\alpha = 99^0\ 55.4'$, WHICH IS THE TRUE GROUND AZIMUTH OF P.

THE DIFFERENCE BETWEEN α AND α IS THE DIFFERENCE BETWEEN THE GROUND AZIMUTH OF P AND THE CORRESPONDING ANGLE MEASURED ON THE PHOTOGRAPH; THAT IS:

$99^0\ 55.4' - 98^0\ 56.1' = 0^0\ 59.3'$, which PRACTICALLY SPEAKING, IS 1 DEGREE, A DIFFERENCE IN ANGULAR VALUE FROM THE TRUE ANGLE, WHICH WILL CAUSE SERIOUS ERROR IN RADIAL PLOTTING.

IT IS THEREFORE APPARENT THAT UNDER CONDITIONS FREQUENTLY ENCOUNTERED IN PRACTICE, THE USE OF THE PRINCIPAL POINT FOR RADIAL PLOTTING WILL INTRODUCE, IN THE CRITICAL AREAS, INTOLERABLE ERRORS.

THE ERRORS MAY BE REDUCED IF, INSTEAD OF THE PRINCIPAL POINT, THE ISOCENTER BE USED AS THE ORIGIN FOR RAYS. LET US CONSIDER THE CASE WITH REFERENCE TO FIGURE 6:

$$oi = f \tan t/2$$

$$\tan \alpha' = \frac{x}{\frac{h}{y - f \tan t/2}}$$

$$\text{and } \tan \theta' = \frac{x - X_i H}{y - f \tan t/2} = \frac{x}{y - f \tan t/2} \cdot \frac{H - h}{h}$$

ASSUMING THE SAME PRACTICAL VALUES AS APPEAR ABOVE:

$$\tan \alpha' = \frac{6}{.(94338 - 18 \tan 1^0\ 30')} = 12.71294$$

and $\alpha' = 94^0\ 29.9'$

ALSO $\tan \theta' = \frac{6}{.94338 - 18 \tan 1^0\ 30'} \cdot \frac{10,000 - 1,000}{1,000} = 11.44165$

AND $\theta' = 94^0\ 59.7'$

SINCE θ IS, IN EFFECT, THE MEASURE OF AN ANGLE AT THE ISOCENTER, BETWEEN TWO POINTS AT EQUAL ELEVATION, THEN $\alpha = \theta'$ (PAGE 20) EQUALS $94^0\ 59.7'$

THE DIFFERENCE BETWEEN α AND α' IS THE DIFFERENCE BETWEEN THE GROUND AZIMUTH OF P AND THE CORRESPONDING ANGLE MEASURED ON THE PHOTOGRAPH. FOR THIS CASE THE DIFFERENCE IS:

$94^0\ 59.7' - 94^0\ 29.9' = 0^0\ 29.8'$

WHILE THIS IS ONLY HALF THE ERROR CAUSED BY USING PRINCIPAL POINT RAYS, UNDER THE SAME GIVEN PRACTICAL CONDITIONS, ANGULAR ERRORS OF THIS MAGNITUDE CAN HARDLY BE NEGLECTED.

THE ERRORS IN RADIAL PLOTTING MAY FURTHER BE REDUCED, INDEED BE MADE NEGLIGIBLE, IF THE NADIR POINT BE USED AS ORIGIN FOR THE RAYS. THIS MAY BE SHOWN BY THE FOLLOWING DISCUSSION:

SINCE TOPOGRAPHIC RELIEF DISPLACEMENTS ON A TILTED PHOTOGRAPH RADIATE FROM THE NADIR POINT (FIGURE 2) IT IS EVIDENT THAT THE PRESENCE OF TOPOGRAPHIC RELIEF WILL CAUSE NO ERRORS IN RADIAL PLOTTING IF RAYS ARE DRAWN FROM THE NADIR POINT. THE ONLY ERROR THAT REMAINS, IS THE ERROR INTRODUCED BY THE TILT OF THE PHOTOGRAPH.

REFERRING TO FIGURE 4, AND AS A COROLLARY TO THE PROOF THERE PRESENTED, IT CAN BE SHOWN THAT $\tan \alpha \cos t = \tan \alpha'$. THIS INDICATES THAT THE DIFFERENCE BETWEEN GROUND ANGLES AND CORRESPONDING PHOTOGRAPHIC ANGLES MEASURED AT THE PLUMB POINT AND THE NADIR POINT, RESPECTIVELY, WILL NOT BE MORE THAN ABOUT 2 MINUTES FOR A TILT OF 3 DEGREES. EVEN FOR A TILT AS GREAT AS 5 DEGREES, THE EXTREME CASE ENCOUNTERED IN PRACTICE, THE ERROR WILL STILL BE INAPPRECIABLE IN GRAPHICAL WORK.

WHILE FIGURE 4 AND THE ABOVE STATEMENTS REFER TO PHOTOGRAPHS OF FLAT TERRAIN, IT IS EVIDENT THAT THE SAME CONCLUSIONS ARE CORRECT FOR PHOTOGRAPHS OF COUNTRY CONTAINING TOPOGRAPHIC RELIEF, SINCE WE ARE CONSIDERING NADIR POINT RAYS, AND BECAUSE TOPOGRAPHIC RELIEF DISPLACEMENTS RADIATE FROM THE NADIR POINT. IT IS THUS APPARENT THAT THEORETICALLY, THE ONLY ERROR ENCOUNTERED IN RADIAL PLOTTING BY NADIR POINT RAYS IS THE ERROR OF TILT, WHICH IN GRAPHICAL PRACTICE, IS NOT DISCERNIBLE.

IT HAS BEEN SHOWN THAT RAYS FROM THE PRINCIPAL POINT WILL GIVE SATISFACTORY RESULTS FOR RADIAL PLOTTING WHEN THE TERRAIN APPEARING ON THE AERIAL PHOTOGRAPHS IS FLAT. BUT WHEN TOPOGRAPHIC RELIEF IS PRESENT, (THE CONDITION USUALLY ENCOUNTERED IN PRACTICE) OF THE THREE ORIGINS FOR RAYS FOR RADIAL PLOTTING THAT HAVE BEEN HERE CONSIDERED, THE NADIR POINT ALONE WILL GIVE RESULTS FREE FROM APPRECIABLE GRAPHICAL ERRORS.

IF IT IS DESIRED TO ELIMINATE COMPLETELY, THE ERRORS IN RADIAL PLOTTING, THERE IS A METHOD WHICH IS ABSOLUTELY CORRECT IN THEORY. LET US SUPPOSE THAT FIGURE 7 REPRESENTS A TILTED PHOTOGRAPH OF TERRAIN HAVING CONSIDERABLE TOPOGRAPHIC RELIEF.

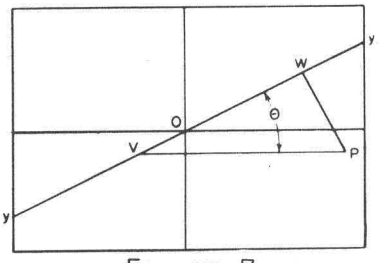

FIGURE 7
TILTED PHOTOGRAPH

THE NADIR POINT IS INDICATED BY **v**, AND THE PRINCIPAL POINT BY **o**, **vo** BEING THE PRINCIPAL LINE. THE POINT **p** IS AN IMAGE OF A GROUND POINT AT ANY ELEVATION. SUPPOSE **pw** IS DRAWN PERPENDICULAR TO THE PRINCIPAL LINE. LET US DESIGNATE BY **(x, y)** THE RECTANGULAR COORDINATES OF **p** BASED UPON A PAIR OF AXES FOR WHICH **v** IS THE ORIGIN AND **vo** THE **y**-AXIS; THAT IS, **wp** EQUALS **x**, AND **vw** EQUALS **y**. THE DIRECTION OF THE NADIR POINT RAY IS NOT AFFECTED BY THE ELEVATION OF THE POINT WHOSE IMAGE IS **p**, BECAUSE THE TOPOGRAPHIC RELIEF DISPLACEMENT OF THE IMAGE IS ALONG THE LINE **vp**. BUT θ IS NOT THE TRUE MEASURE OF THE GROUND ANGLE.

HOWEVER, LET US OONSIDER THE FOLLOWING:

$$\tan \theta = x/y$$

THIS WOULD BECOME THE TRUE MEASURE OF A GROUND ANGLE AT THE PLUMB (GROUND) POINT IF **y** WERE MULTIPLIED BY $\cos t$. THAT IS, IF THE ANGLE θ WERE CORRECTED TO ϕ, FOR WHICH $\tan \phi = x/y \cos t$, THEN ϕ IS A TRUE HORIZONTAL ANGLE ON THE GROUND.

FIGURE 8 SHOWS WHY $\phi = \tan^{-1} \frac{x}{y \cos t}$ IS ACTUALLY A TRUE HORIZONTAL ANGLE MEASURING EXACTLY THE CORRESPONDING GROUND ANGLE. THE PHOTOGRAPH IS SHOWN TILTED, THE PRINCIPAL POINT IS AT **o**, THE NADIR POINT IS AT **v**, AND THE ANGLE **oLv** IS EQUIVALENT TO THE TILT **t**. THE PRINCIPAL PLANE AT SEA LEVEL IS **VO**. A POINT **P** HAS ITS IMAGE AT **p**, AND IS PROJECTED VERTICALLY TO SEA LEVEL AT P_1. THE LINE **pw** IS DRAWN PERPENDICULAR TO THE PRINCIPAL LINE AND IS THEREFORE HORIZONTAL IN SPACE, **wk** IS PERPENDICULAR TO **Lv**, AND **k** IS JOINED TO **p** FORMING A HORIZONTAL TRIANGLE.

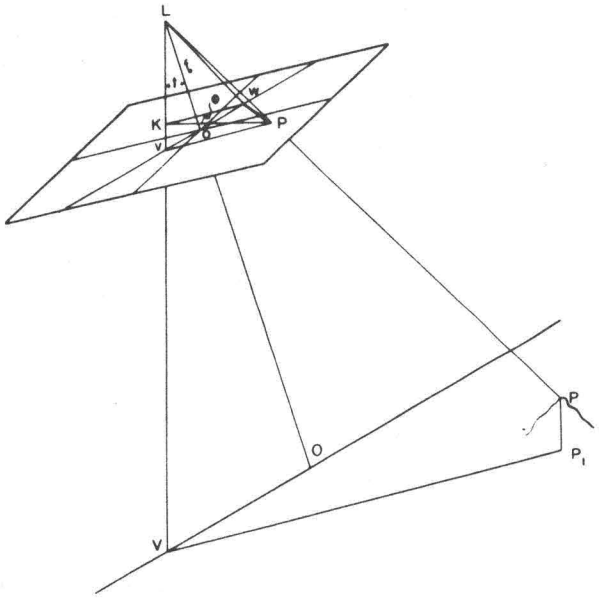

FIGURE 8
TILTED PHOTOGRAPH

BY INSPECTION:

angle wkp = angle OVP_1

wp = x

kw = y cos t

AND $\Phi = \tan^{-1} \frac{x}{y \cos t}$, WHICH IS THE TRUE VALUE OF THE GROUND HORIZONTAL ANGLE OVP; AND THE INTERSECTIONS AND RESECTIONS BASED UPON THESE Φ's FOR DIRECTIONS OF NADIR POINT RAYS ARE ABSOLUTELY CORRECT; AS WILL BE THE RESULTING RADIAL PLOTTING.

WHEN ATTEMPTS HAVE BEEN MADE TO DETERMINE THE TILTS EXISTING IN AERIAL PHOTOGRAPHS IN ORDER TO FIND THE ISOCENTER FOR USE IN RADIAL PLOTTING A STEP ONLY HAS BEEN MADE IN THE RIGHT DIRECTION. IF, INSTEAD OF ISOCENTER RAYS, NADIR POINT RAYS SHOULD BE USED WITH SOME SCHEME DEVISED FOR FORESHORTENING ALL ORDINATES BY THE FACTOR cos t, THEN THE PLOTTING WOULD BE ABSOLUTELY CORRECT, AND ENTIRELY FREE FROM ALL ERRORS OF BOTH TILT AND TOPOGRAPHIC RELIEF.

SHOULD ANY READER BE INTERESTED IN THE INVENTION AND DESIGN OF PHOTOGRAMMETRIC INSTRUMENTS, IT MIGHT BE SUGGESTED THAT AN INSTRUMENT FOR THE MECHANICAL EXECUTION OF RADIAL PLOTTING BASED UPON THESE TRUE HORIZONTAL ANGLES (Φ's) OBTAINED MECHANICALLY, WOULD BE SIMPLE AND CHEAP; AND SUCH A PLOTTER FOR PLANIMETRIC MAPS WOULD COMPLETELY ELIMINATE ALL ERRORS WHICH HAVE HERETOFORE CAUSED MUCH DIFFICULTY.

ANNUAL MEETING OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

THE ANNUAL MEETING OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE WILL BE HELD IN INDIANAPOLIS DECEMBER 27, 1937 TO JANUARY 1, 1938. IN CONNECTION WITH THE MEETING THERE WILL BE HELD AN EXHIBITION IN THE MURAT THEATER. AMONG THE IMPORTANT EXHIBITS WILL BE A DISPLAY OF THE SCIENTIFIC RESULTS AND NEW TYPES OF EQUIPMENT OF THE NATIONAL GEOGRAPHIC - U. S. NAVY ECLIPSE EXPEDITION TO CANTON ISLAND TO PHOTOGRAPH THE TOTAL ECLIPSE OF JUNE 8, 1937.

THE 14-FOOT CAMERA AND SPECIAL LENS DESIGNED BY DR. I. C. GARDNER WILL BE A PART OF THE EXHIBIT TOGETHER WITH A "POLAROID CAMERA" DESIGNED BY DR. F. K. RICHTMYER. OTHER EXHIBITS INCLUDE ISOTOPES OF WATER AND NITROGEN, THE LATEST COSMIC RAY COUNTING APPARATUS, AND A POPULAR EXHIBIT BY THE MUSEUM OF SCIENCE AND INDUSTRY IN CHICAGO.