ONE FOR EACH SUCCESSIVE BEAM OR FLASH. THE MOTOR ROTATES THE SENSITIZED PAPER SO THAT EACH FLASHED IMAGE IS DISPLACED FROM THE PRECEDING ONE. THE MIRROR SPEED IS KNOWN AND SINCE THERE ARE 60 MIRRORS, THE FLASHING RATE IN FLASHES PER SECOND IS EXACTLY THE SAME AS THE MIRROR SPEED IN REVOLUTIONS PER MINUTE. FLASHING RATES AS HIGH AS 2400 PER SECOND ARE THUS EASILY OBTAINED. THE SHUTTER SPEED IS THEN FOUND DIRECTLY BY COUNTING THE NUMBER OF IMAGES ON THE TEST. THE EFFICIENCY IS FOUND AS FOLLOWS: THE AREA OF EACH IMAGE IS MEASURED WITH A PLANIMETER AND PLOTTED AGAINST THE CORRESPONDING TIME. THE AREA ENCLOSED BY THE CURVE SO FORMED AND THE BASE LINE IS MEASURED AND THE AREA OF THE RECTANGLE FORMED BY THE BASE, THE FULL-OPEN LINE AND THE VERTICALS AT THE EXTREMITIES ARE ALSO MEASURED. THE EFFICIENCY IS THE CURVED AREA DIVIDED BY THE RECTANGULAR AREA. A TEST OF A FAIRCHILD SHUTTER IS ILLUSTRATED IN FIGURE 1. AREA A E F D REPRESENTS THE ACTUAL LIGHT ADMITTED. AREA A B C D REPRESENTS WHAT WOULD HAVE BEEN ADMITTED HAD THE SHUTTER OPENED AND CLOSED AT INFINITE VELOCITY. THE EFFICIENCY IS AREA A E F D DIVIDED BY AREA A B C D.

FOR TESTING MULTI-LENS SYNCHRONIZED ELECTRIC SHUTTERS, A SPECIAL MODIFICATION OF THE ABOVE MACHINE WAS BUILT. FIVE INCANDESCENT LAMPS ARE USED AND FIVE MIRROR DRUMS MOUNTED ON A SINGLE SHAFT ROTATED BY A SYNCHRONOUS MOTOR. THE SHUTTERS ARE MOUNTED IN A BOX IN THEIR ACTUAL POSITION (HORIZONTALLY). FIVE LENSES BRING THE IMAGES TO FOCUS ON A REVOLVING DRUM CARRYING THE SENSITIZED PAPER SO THAT AS THE SHUTTERS ARE TRIPPED, THE FIVE TESTS APPEAR SIDE BY SIDE. ANY LACK OF SYNCHRONISM IS THEREFORE IMMEDIATELY SHOWN UP. ALL FIVE SHUTTERS ACTUALLY OPERATE TOGETHER WITHIN 1/1000 OF A SECOND. IN THIS MACHINE, THE SHUTTER BOX IS SO CONSTRUCTED AND INSULATED THAT THE INSIDE MAY BE BROUGHT DOWN TO EXTREMELY LOW TEMPERATURES AND THE SHUTTERS THEN TESTED, THE LIGHT BEAMS PASSING THROUGH PLATE GLASS WINDOWS IN THE BOX.

RECTANGULAR COORDINATES AND STANDARD HORIZONTAL DATUM

BY

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PLANE RECTANGULAR COORDINATE SYSTEMS TO REPRESENT STANDARD DATUM, WHICH MAY BE EXTENDED OVER LARGE AREAS WITHOUT EXCESSIVE LOSS OF ACCURACY, HAVE ONLY RECENTLY BEEN DEVISED FOR USE IN THE UNITED STATES. IN ORDER THAT INDIVIDUAL SYSTEMS MAY BE STATE-WIDE WITHOUT INTRODUCING OBJECTIONABLE DISTORTION THEY MUST BE PLANNED CAREFULLY AND MUST HAVE THEIR FOUNDATIONS LAID IN EXACT MATHEMATICS. THE UNITED STATES COAST AND GEODETIC SURVEY, IN CERTAIN OF ITS PUBLICATIONS SETS FORTH THE THEORY OF THESE PLANE COORDINATE SYSTEMS AND BY NUMERICAL EXAMPLES ILLUSTRATES THE DIFFERENT COMPUTATIONS INVOLVED. IT IS NOT INTENDED HERE TO OUTLINE THE THEORY AGAIN, NOR TO PROVIDE A DETAILED GUIDE TO THE USE OF THE SYSTEMS. ONE URGENT PURPOSE IT IS HOPED THAT THEY WILL SERVE IS TO WELD TOGETHER UPON A COMMON HORIZONTAL DATUM MANY OF THE VARIOUS INDIVIDUAL LOCAL SURVEYS CONDUCTED BY ENGINEERS IN ALL BRANCHES OF THE PROFESSION. WITH THIS PURPOSE IN MIND, IT IS INTENDED IN THE FOLLOWING PARAGRAPHS TO DISCUSS CERTAIN FEATURES OF THE SUBJECT, PERHAPS FROM A NEW POINT OF VIEW, WITHOUT THE DISTRACTION OF MATHEMATICAL DERIVATIONS.

THE STANDARD PLANE COORDINATE SYSTEMS MAY BE REGARDED AS PARTS OF THE FUNDAMENTAL WORLD-WIDE GEODETIC SYSTEM THAT HAVE BEEN TRANSLATED INTO A FORM MORE CONVENIENT FOR EVERYDAY USE. IN THE GEODETIC SYSTEM DISTANCES MEASURED ON THE GROUND ARE REDUCED TO SEA LEVEL IN ORDER THAT ALL PARTS OF A SURVEY MAY BE CONSIDERED AS LYING ON THE SURFACE OF THE GEODETIC REFERENCE SPHEROID. THIS ASSUMPTION, TAKEN FOR GRANTED IN THE FOLLOWING DISCUSSION, IMPLIES THAT THE PLANE COORDINATE SYSTEMS ARE ARRANGED ALSO SO AS TO RELATE DIRECTLY TO SEA LEVEL DISTANCES RATHER THAN TO DISTANCES MEASURED AT GROUND LEVEL.

NOW A TRUE RECTANGULAR COORDINATE SYSTEM MUST BE CONTAINED IN A PLANE: IT IS NOT POSSIBLE TO ARRANGE SUCH A SYSTEM SO THAT IT WILL BE FORM-FITTED TO THE CURVATURE OF THE EARTH OR OF THE REFERENCE SPHEROID. THE PROBLEM HERE, THEREFORE, AS IN MAPPING, IS TO TAKE THE DETAILS OF A SURVEY FROM THE

CURVED SURFACE OF THE SPHEROID AND SHOW THEM AS PROJECTED INTO A PLANE OR FLAT SURFACE. A GREAT MANY DIFFERENT KINDS OF MAP PROJECTIONS ARE IN USE, EACH HAVING BEEN DEVISED OR CHOSEN TO MEET SOME PARTICULAR REQUIREMENT. FORTUNATELY, THE MATHEMATICS DEVELOPED IN THE STUDY OF PROJECTIONS FOR MAPS, ALREADY AVAILABLE, APPLIES AS WELL TO PROJECTIONS FOR RECTANGULAR COORDINATE SYSTEMS. FOR SYSTEMS DESIGNED TO BE USEFUL TO THE SURVEYOR, ONE OF THE FUNDAMENTAL REQUIREMENTS IS THAT ANGLES SHOWN BY THE PROJECTION IN THE PLANE COORDINATE SYSTEM SHALL BE AS NEARLY EQUAL AS POSSIBLE TO THE CORRESPONDING ANGLES MEASURED ON THE GROUND. CONFORMAL PROJECTIONS ARE DESIGNED TO MEET THIS REQUIREMENT: THEIR DERIVATION PROVIDES THAT ANGLES BETWEEN LINES AT ANY GEODETIC POINT SHALL NOT BE CHANGED IN THE PROJECTION AND THAT SCALE CHANGES AT ANY PROJECTED POINT MUST BE EQUAL IN ALL DIRECTIONS ABOUT THAT POINT. THUS INFINITESIMALLY SMALL AREAS ARE PROJECTED WITHOUT CHANGE IN FORM, ALTHOUGH THEY MAY BE CHANGED IN SIZE. ANOTHER REQUIREMENT IS THAT THE PROJECTION CHOSEN SHALL REMAIN AS TRUE TO SCALE AS POSSIBLE OVER LARGE AREAS. THE LAMBERT CONFORMAL AND THE TRANSVERSE MERCATOR PROJECTIONS APPEAR MOST NEARLY TO MEET THESE REQUIREMENTS.

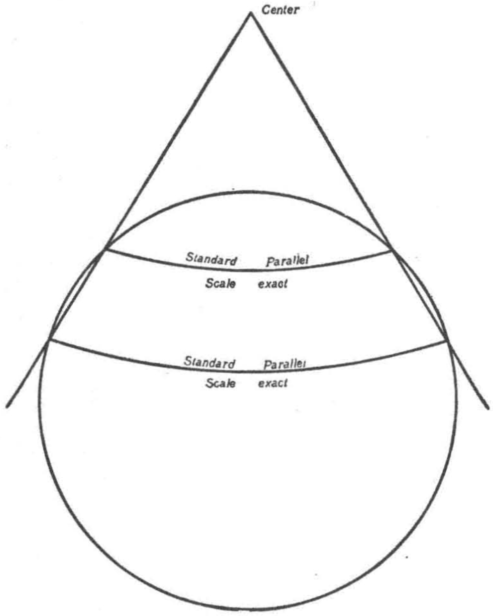

FIGURE 1
SPHERE WITH INTERSECTING CONE
FOR LAMBERT PROJECTION

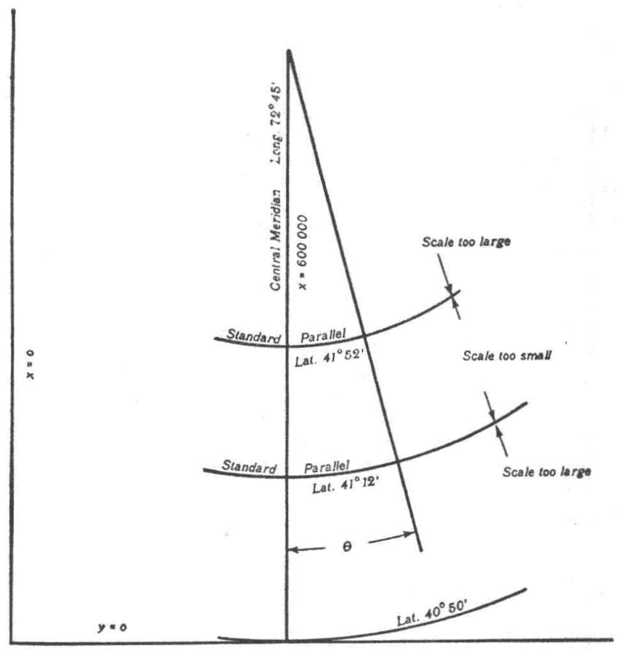

FIGURE 2
DIAGRAM OF THE LAMBERT PROJECTION
SHOWN ON THE PLANE

IN ARRIVING AT THE RELATION BETWEEN RECTANGULAR AND GEODETIC COORDINATES THERE ARE THEREFORE TWO DISTINCT SURFACES INVOLVED--THE SURFACE OF THE SPHEROID, UPON WHICH ARE SITUATED THE POINTS USED IN GEODETIC COMPUTATIONS, AND THE PLANE SURFACE OF THE PROJECTION, ON WHICH LIE THE CORRESPONDING POINTS THAT ARE USED IN THE RECTANGULAR COORDINATE SYSTEM. THEREFORE THE CHANGE FROM GEODETIC TO PLANE RECTANGULAR COORDINATES, OR THE REVERSE, IS NOT MERELY A TRANSFORMATION OF COORDINATES FROM ONE KIND TO THE OTHER FOR AN IDENTICAL POINT TO WHICH BOTH KINDS OF COORDINATES REFER. INSTEAD, THE CHANGE IS EFFECTED BY COMPUTING RECTANGULAR COORDINATES FOR A SEPARATE POINT, LYING IN THE PLANE, WHICH IS TO REPRESENT THE POINT GIVEN ON THE SPHEROID FOR WHICH THE GEODETIC COORDINATES ARE KNOWN. OF COURSE THIS PROCEDURE IS REVERSED IF IT IS THE RECTANGULAR COORDINATES THAT ARE KNOWN. EVEN THOUGH THE PLANE IS IMAGINED AS BEING PLACED INTO THE CLOSEST POSSIBLE JUXTAPOSITION WITH THE SURFACE OF THE SPHEROID, THE TWO CORRESPONDING POINTS MAY STILL BE SEPARATED VERY WIDELY IN SPACE BECAUSE OF THE DIVERGENCE BETWEEN THE DIFFERENT SURFACES UPON WHICH THEY LIE. THEIR POSITIONS ARE REALLY RELATED TO ONE ANOTHER ONLY THROUGH THE MATHEMATICS AND THE CONSTANTS OF THE PROJECTION THAT IS BEING

USED. SOME CONFUSION MAY BE AVOIDED BY REMEMBERING THESE TWO SEPARATE SURFACES, AND BY VISUALIZING TWO SETS OF POINTS FOR ANY SURVEY BEING CONSIDERED --THE POINTS ON THE SPHEROID AND THE POINTS REPRESENTING THEM IN THE PLANE OF THE PROJECTION.

NOW CONSIDER WHAT HAPPENS WHEN A SIMPLE GEODETIC TRIANGLE ON THE SPHEROID IS PROJECTED TO A PLANE. THE TWO TYPES OF PROJECTION USED FOR THE STANDARD SYSTEMS UNDER DISCUSSION ARE BOTH CONFORMAL. THEREFORE, IN USING EITHER ONE, THE THREE ANGLES OF THE GEODETIC TRIANGLE WILL BE PROJECTED TO THE PLANE WITHOUT ANY CHANGE IN SIZE. BUT THE SIDES OF THE GEODETIC TRIANGLE WILL NOT BE PROJECTED TO THE PLANE AS STRAIGHT LINES. SO BEFORE USING THE PROJECTED TRIANGLE IN PLANE COMPUTATIONS ITS SIDES MUST BE STRAIGHTENED. WHEN THIS IS DONE IT IS EVIDENT THAT THE TRUE ANGLES AS PROJECTED MUST BE CHANGED SLIGHTLY, AND THEREFORE THE QUALITY OF EXACT CONFORMALITY CANNOT BE MAINTAINED IN CONSIDERING AS A WHOLE A TRIANGLE OR OTHER GEODETIC FIGURE COVERING AN APPRECIABLE AREA. MOREOVER, THE LENGTH OF THE PROJECTED SIDES WILL NOT BE THE SAME NOR EVEN EXACTLY IN THE SAME PROPORTION AS THE LENGTH OF THE SIDES OF THE GEODETIC TRIANGLE. BUT THE PROJECTED AND STRAIGHTENED TRIANGLE IS READY TO USE IN ANY KIND OF PLANE COMPUTATION IN THE RECTANGULAR COORDINATE SYSTEM. THE SAME GENERAL CONSIDERATIONS APPLY TO THE ELEMENTS OF A TRAVERSE LINE ALSO.

THE MATHEMATICS OF THE PROJECTIONS PROVIDES FORMULAS FOR OBTAINING THE PLANE COORDINATES OF PROJECTED POINTS FROM GIVEN GEODETIC COORDINATES OF POINTS ON THE SPHEROID, OR VICE VERSA. FORMULAS AND SCALE FACTORS ARE PROVIDED ALSO TO SHOW THE DIFFERENCE BETWEEN LENGTHS ON THE SPHEROID AND THE CORRESPONDING LENGTHS AS PROJECTED IN THE PLANE. FORMULAS ARE PROVIDED TO DETERMINE THE SLIGHT CHANGES IN DIRECTION AT THE ENDS OF THE CURVED PROJECTIONS OF THE GEODETIC LINES WHEN THESE ARE STRAIGHTENED IN THE PLANE.

THE CORRECTIONS JUST DESCRIBED PROVIDE THE MEANS TO PREPARE THE OBSERVATIONS OF ORDINARY SURVEYS FOR COMPUTATION DIRECTLY IN THE PLANE OF RECTANGULAR COORDINATES, THUS AVOIDING THE COMPLICATIONS OF PASSING THROUGH THE GEODETIC PHASE. BY THE APPLICATION OF THESE CORRECTIONS TO THE OBSERVED ANGLES AND DISTANCES (ASSUMED AS REDUCED TO SEA LEVEL FIRST), CORRESPONDING ANGLES AND DISTANCES BETWEEN PROJECTED POINTS IN THE PLANE ARE OBTAINED JUST AS IF THE MEASUREMENTS HAD BEEN MADE THERE. THE OBSERVED VALUES CONTAIN THE ACCIDENTAL ERRORS THAT ALWAYS OCCUR IN ANY SURVEY; THE SAME ACCIDENTAL ERRORS ARE STILL CONTAINED IN THE ANGLES AND DISTANCES AS PREPARED FOR PLANE COMPUTATION, SO THAT ADJUSTMENTS BY LEAST SQUARES OR OTHER METHODS SHOULD BE MADE AS IN ANY PLANE SURVEY. SIMPLE PLANE COMPUTATIONS SUFFICE TO CONTINUE THE WORK.

BY THIS TIME THE READER MAY HAVE COME TO THE CONCLUSION THAT THESE COORDINATE SYSTEMS, BASED ON MANY CORRECTIONS, DISTORTIONS, AND PROJECTIONS BETWEEN SURFACES, WILL SO SERIOUSLY JUGGLE THE RESULTS OF A SURVEY AS TO DAMAGE ITS USEFULNESS. SO PERHAPS THE MAGNITUDE OF THE QUANTITIES INTRODUCED BY THE JUGGLING SHOULD BE INDICATED IN ORDER TO SHOW THAT PLANE COORDINATE POSITIONS ARE NOT SO FAR-FETCHED AFTER ALL. ANY GEODETIC TRIANGLE CONTAINS SPHERICAL EXCESS AT THE RATE OF ONE SECOND FOR EACH 75 SQUARE MILES OF ITS AREA, APPROXIMATELY. IT MAY BE FLATTENED TO A PLANE BY REDUCING THE SUM OF ITS ANGLES BY THE AMOUNT OF ITS SPHERICAL EXCESS. ANY OTHER FIGURE OR SURVEY NEEDS TO ABSORB CORRECTIONS TO ITS OBSERVED ANGLES ONLY AT THE SAME SMALL RATE TO ELIMINATE ITS SPHERICAL EXCESS AND SO FLATTEN IT TO A PLANE. CORRECTIONS TO LENGTHS DUE TO THE VARYING SCALE OF THE PROJECTIONS RARELY EXCEED ONE PART IN TEN THOUSAND. THE STATE SYSTEMS (WITH ONE EXCEPTION) HAVE BEEN CHOSEN AND ARRANGED TO REMAIN WITHIN THIS LIMIT OF SCALE DIFFERENCE. SCALE CORRECTIONS USUALLY APPLY IN UNEQUAL RATIOS TO DIFFERENT LINES OF THE SAME SURVEY, THEREBY PRODUCING DISTORTION. BUT THIS DISTORTION CAN BE NO GREATER THAN IS IMPLIED BY THE CHANGE IN THE RATIO OF LENGTH CORRECTIONS, WHICH ARE AT THE MOST ONLY 1:10,000 IN AMOUNT AND WHICH CHANGE ONLY GRADUALLY OVER LARGE AREAS. NOTE THAT IT IS NOT THE AMOUNT OF THE SCALE CORRECTION RATIO ITSELF BUT RATHER THE CHANGE OF THAT RATIO THAT PRODUCES THE DISTORTION.

AN ENGINEER MAY ASK: "WHY BOTHER WITH A STATE-WIDE SYSTEM THAT SEEMS COMPLICATED, WHEN BY MERELY ASSUMING CONVENIENT COORDINATES FOR SOME INITIAL POINT AND AN INITIAL DIRECTION NEAR NORTH I CAN PROCEED INTO A SIMPLE PLANE SURVEY THAT WILL SERVE MY IMMEDIATE NEEDS?"

Consider first the "bother" that is involved. In simple plane surveying a certain degree of tolerance has always been invoked to allow the plane computing of measurements taken on the curved surface of the earth; that is, spherical excess and slight local distortions customarily have been ignored in such work. If an engineer is content to continue with the same degree of tolerance, he may still ignore these same details in using the standard systems so long as he operates in a compact area of only a few square miles. Corrections due to spherical excess and distortion should be considered in connection with surveys over extended areas, or where long triangulation lines are involved. In the great majority of local surveys, however, these corrections may be neglected entirely.

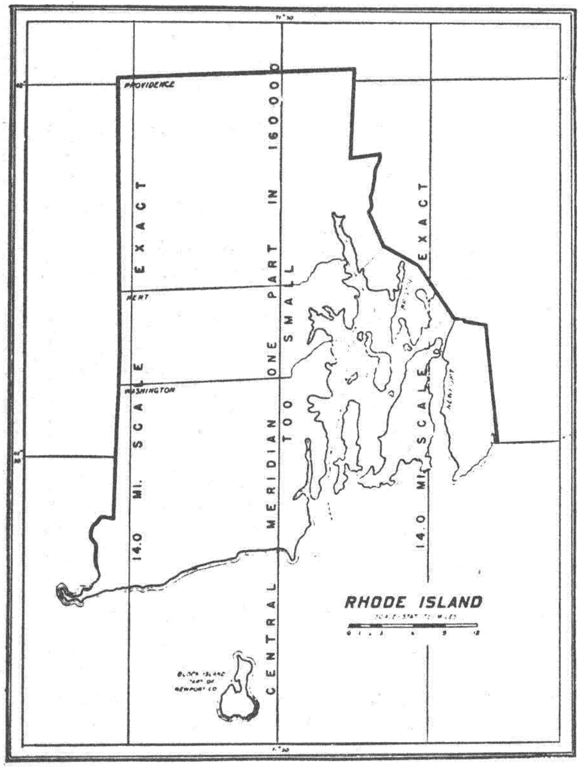

Fig. 3, Rhode Island
with grid system outline

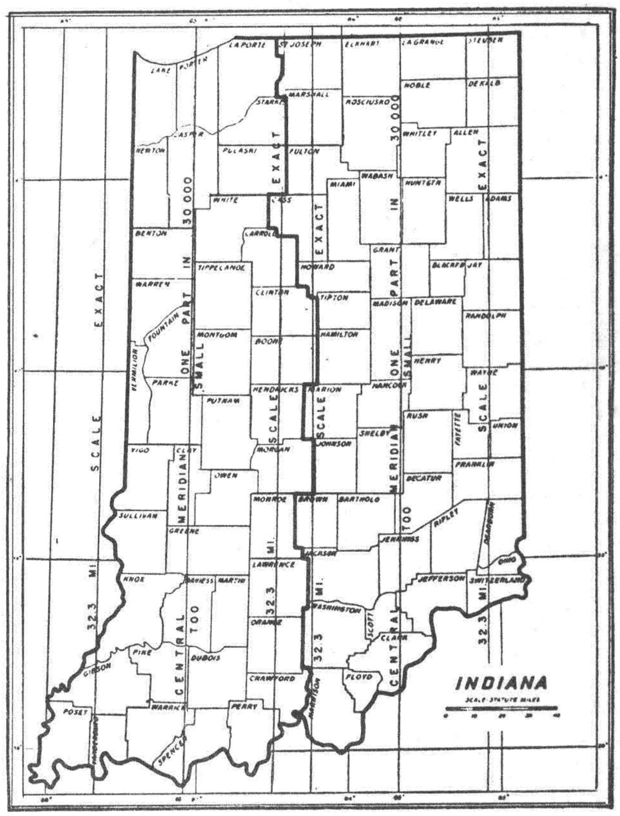

Fig. 4, Indiana
with grid system outline

The scale corrections are of greater magnitude and importance. In the established practice of plane surveying, measured horizontal distances are not usually reduced to sea level. But this should usually be done in preparing to use the standard systems, and the further correction due to the scale of the projection should also be applied. In many local surveys conducted in relatively flat areas it will be found practicable to combine the two factors into a single constant ratio. It is but little bother to apply this proportional correction to measured distances. Then, except for the probably insignificant changes of these factors within a limited area, other dependent unmeasured lengths computed within a local survey also will be shown to the desired scale. Thus the local survey may be adjusted in overall size so that it will fit accurately and consistently into its allotted place on the plane of the projection.

The feature that may bother the engineer most is the necessity of including in his survey at least one station (preferably two or more) of the fundamental control net of the country. One such station, with its given rectangular coordinates referred to the standard system, provides starting values of x and y to introduce into the local survey. If other basic stations can be included they will provide the means to check and adjust the local survey more safely and rigidly into place. Tying to only one basic station may make it difficult to obtain an initial direction which will orient the local survey harmoniously with the standard coordinate system. Many of the more recently established triangulation stations of the first-order net have been provided with auxiliary azimuth marks from which a reference direction in terms of grid azimuth may be obtained. Lacking such an azimuth mark, an initial direction may be obtained by reducing to grid azimuth the azimuth obtained from an observation on Polaris. The reduction is not difficult, but the observation itself may be inconvenient. If two or more basic stations are included in the survey, the grid azimuths between such stations provide the best initial directions, and they are easily computed by the ordinary formulas of analytic geometry from the given rectangular coordinates of the stations.

The "bother" confronting an engineer in using a State system as the reference datum for his survey has now been outlined in its four parts. These relate to shape, scale, place and orientation, all four being adjustments of his survey to fit others upon the plane of the projection. After attending to these preliminary steps, any ordinary survey may be conducted in both field and office by the usual methods of simple plane surveying, and it will be based at the same time upon standard datum.

In conclusion, and to answer the engineer who asks "Why bother---?", these next remarks are submitted. It would be a very desirable coordination of information if the results of all surveys -- private, State, and Federal -- could be expressed in common terms. Small surveys would expand into harmonious contact with others adjacent to them. The results of one survey made for some specific purpose would be supplemented perhaps by information already available from preceding surveys made for other purposes. Successive surveys of the same area would not need to repeat the previous work, or possibly a new survey might be wholly unnecessary because of the accumulation of information already available for that area, referred in an orderly manner to a datum common to all surveys.

In contrast to this ideal of coordination and cooperation in effort, the actual situation is often such as to bring no credit to the engineering profession. Does it not frequently happen that different surveyors at various times work over and over again in the same area, each for some separate purpose, all using different kinds of reference systems with different origins and orientations? Plainly, in the results of our surveys we do not always speak to one another in the same language. We should take care lest we build another Tower of Babel.

DISCUSSION
BY
HUGH C. MITCHELL

Rectangular coordinate systems such as are described in the accompanying paper have, in the past few years, become realities, and are steadily securing more and more favor from engineers through use, and endorsement of State officials through legislative action. Mr. Wilson has given a clear picture of the elements involved in the use of stations of the national triangulation for the control of local surveys, and a good analysis of how the problems ordinarily presented in such use have been satisfactorily solved by the U. S. Coast and Geodetic Survey through the establishment of systems of plane coordinates for the various states. It should be emphasized, however, that no