# PRACTICAL CONSIDERATIONS IN MAPPING FROM OBLIQUE AIR PHOTOGRAPHS BY NON-STEREOSCOPIC METHODS

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THE American Geographical Society's experimental survey of Northernmost Labrador from high oblique photographs is now virtually completed. More than 2,000 square miles of mountainous country have been mapped on a scale of 1:50,000 with 50 meter contours and the maps have been reproduced on a scale of 1:100,000. In addition, at least another 2,000 square miles of mapping on a scale of 1:100,000 are completed and have been reproduced on a scale of 1:300,000 with 250 meter form lines.[1]

A description has already been given to the American Society of Photogrammetry on the methods and instruments used at the American Geographical Society.[2] The latter consist of a quick reading photogoniometer for measuring horizontal and vertical angles and a monocular plotting instrument which incorporates the pinhole mirror optical device invented by me.

The purpose of this paper is to give in the light of my own experience and that of my associates, Messrs. Walter A. Wood and Charles B. Hitchcock, certain statistics concerning what we consider the accuracy and speed of mapping from high obliques by non-stereoscopic methods.

In the first place, the instruments have been designed to read and plot directions to the nearest minute of arc. They are certainly sensitive to changes of direction of this magnitude and we feel confident that when they are in adjustment our measurements are correct as far as the photographs will allow. This is so because we find that independent measurements of the same items on both instruments consistently agree.

The question as to whether the photographs themselves can be relied upon to give an accuracy in the extracted angles of 1′ is another matter of course, but from the evidence in recent papers on the subject as far as optical distortion and dimensional changes in film and paper are concerned, it appears that, provided the original films are used and not paper prints for making measurements, a one-minute accuracy may reasonably be expected. On the other hand, to expect consistent measurements to a much greater degree of accuracy than 1′ is perhaps asking too much of the material available at the present time.

In discussing the accuracy and efficiency of our technique it is necessary to recognize that four distinct processes are involved in constructing a map from oblique photographs. These are (1) the resection and orientation of camera stations from known ground control, (2) the extension of control by air triangulation, (3) the intersection of minor control, and (4) the plotting of detail.

## Plotting of Flat Surface Features

Supposing we assume that an air station has been perfectly fixed in position and oriented sufficiently precisely for tilt, swing, and the bearing of the optical axis to insure our being able to measure directions from a photograph to the nearest minute of arc. Also let us assume that the points which we wish to plot from the photograph lie at sea level or at some constant height above sea level.

[1] Alexander Forbes: Northernmost Labrador Mapped from the Air, *American Geographical Society's Special Publication No. 22*, New York, 1938.

[2] *News Notes of the American Society of Photogrammetry*, Vol. 1, No. 5, May–June, 1935, pp. 25–34.

The first question to be answered is how accurately can the horizontal position of such points be plotted from a single photograph.

Fig. 1.

HORIZONTAL DISPLACEMENT OF POINTS PLOTTED FROM SINGLE PHOTOGRAPHS

due to one minute errors in the measurement of the horizontal and vertical angles from the perspective center

HEIGHT OF AIR STATION:—10,000 feet

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| A. | 80° | 70° | 60° | 50° | 40° | 30° | 20° | 10° | 05° |
| B. | 1763 | 3640 | 5774 | 8391 | 11917 | 17320 | 27475 | 56712 | 114300 |
| C. | 3.00 | 3.25 | 4.00 | 5.00 | 7.00 | 11.50 | 25.00 | 97.00 | 393.00 |
| D. | 0.50 | 1.00 | 1.75 | 2.50 | 3.50 | 5.00 | 8.00 | 16.50 | 33.00 |

A. is the angle of depression in degrees.
B. is the horizontal distance in feet.
C. is the horizontal displacement in direction of ray in feet.
D. is the horizontal displacement perpendicular to ray in feet.

In Fig. 1 the answer is given for an altitude of the air station of 10,000 feet. Row 1 of this table shows the angle of depression of the point below the air station; in Row 2 the distance from the air station in feet. In Rows 3 and 4 are given the horizontal displacements in feet in the direction of the perspective ray and at right angles to it respectively, assuming an error of 1′ in the measurement of the horizontal and vertical angles.

When the horizontal distance exceeds 3 miles or so or the angle of depression is less than 30°, the possible errors in the positions of points plotted by single perspective rays increase very rapidly. Now in high oblique photography when a single-lens camera is used having an average angular field of view, as for instance a camera with a 10 inch focal length producing a photograph 7×9 inches, in order that the trace of the apparent horizon will appear near the top of the photograph, it is necessary to tilt the camera axis not more than 15 to 20 degrees below the horizontal. This means that the bulk of the detail appearing on the photographs will have angles of depression of less than 30° from the camera stations.

In plotting coastline detail and drainage patterns or any other flat surface features from single high obliques we must be content, then, to use only a small portion on the lower half of each photograph or else plot only to very small scales. This is unless we take further precautions. As Fig. 1 shows, the possible horizontal displacement of a point due to errors in measuring directions from a photograph is always much less in a direction perpendicular to the direction of the perspective ray than along it. Consequently tangent rays from air stations whose field of view is across that of the photograph from which the plotting has been made will tend to increase the accuracy of the plotting when the latter is adjusted to them.

Consequently the general procedure in plotting flat detail from high obliques is obviously to build up a skeleton framework of tangent rays from all available photographs and adjust the plotted detail to it. In the actual plotting of outlines from a single high oblique photograph one cannot trace directly from the photograph as in vertical work, and if one sketches freehand between control, even with the aid of perspective grids, there is a strong inclination to exaggerate the curves and angularity of shore lines and other line features when looking down them and to minimize them when looking across. We have found that the most accurate and generally satisfactory method of plotting such detail is by means of a perspective plotter, such as has been developed at the American Geographical Society. With this type of instrument the compiler is not asked to make a judgment of distance or direction. He merely guides a mark over the feature to be plotted as he would a pencil when making line plots from a rectified vertical photograph.

## INTERSECTION OF POINTS

In high oblique surveying, as in vertical radial line plotting, when relief features are to be shown on the map and stereoscopic plotting instruments are not available, it is necessary to intersect a thick network of critically chosen points and determine their heights so that contours may be sketched in. This, of course, is similar to small-scale plane-tabling but has distinct advantages over the latter, in that the photographs supply a permanent record of what can be seen from each survey station and the same feature may be viewed from various aspects at the same time in the process of sketching the contours.

With what accuracy can points be intersected and their heights determined from high obliques?

Fig. 2.

TABLE OF INTERSECTION ERRORS IN FEET

Arguments:—Angle of intersection in degrees and half the sum of the distances in miles.

| | 10° | 20° | 30° | 40° | 50° | 60° | 70° | 80° | 90° |
|---|---|---|---|---|---|---|---|---|---|
| 50 | 910 | 447 | 301 | 235 | 196 | 173 | 159 | 150 | 146 |
| 45 | 819 | 402 | 270 | 212 | 176 | 156 | 144 | 135 | 131 |
| 40 | 728 | 358 | 240 | 188 | 157 | 138 | 128 | 120 | 117 |
| 35 | 637 | 313 | 210 | 165 | 137 | 121 | 112 | 105 | 102 |
| 30 | 546 | 268 | 180 | 141 | 118 | 104 | 96 | 90 | 88 |
| 25 | 455 | 224 | 150 | 118 | 98 | 87 | 80 | 75 | 73 |
| 20 | 364 | 179 | 120 | 94 | 78 | 69 | 64 | 60 | 58 |
| 15 | 273 | 134 | 90 | 71 | 59 | 52 | 48 | 45 | 44 |
| 10 | 182 | 89 | 60 | 47 | 39 | 35 | 32 | 30 | 29 |
| 5 | 91 | 45 | 30 | 24 | 20 | 17 | 16 | 15 | 15 |
| | 170° | 160° | 150° | 140° | 130° | 120° | 110° | 100° | 90° |

Now every surveyor knows that the accuracy of an intersection decreases as the angle of intersection departs from a right angle and as the sum of the distances from the two stations to the intersected point increases. So instead of, "How accurately can points be intersected?," the question to be put is, "What are the limits of distance and angle beyond which a prescribed accuracy cannot be maintained?" Figure 2 gives this information, assuming perfect positioning and orientation of two photographs and an accuracy of 1′ of arc in determining directions from them. The arguments are the $\frac{1}{2}$ sums of the distances from the air stations to the intersected point in miles and the angles of intersection ranging on either side of 90°. The body of the table, under these conditions, gives the maximum error in feet to be expected in the horizontal position of an intersected point. Supposing a map is being constructed which will eventually be printed to a scale of 1:100,000. In this case, errors of position of less than 100 feet need not be considered. The thick black line on Fig. 2 is the dividing line between permissible and non-permissible types of intersections for this scale. For any specific scale a graph such as Fig. 3 is more convenient to use and can be constructed easily from the table in Fig. 2. Such a graph has other uses as well, as for example, in the choice of suitable control points for the resection of air stations and in estimating exposure and flight line intervals.

After points in the landscape have been intersected, the next question is the determination of their heights. Here a comparison is useful. In vertical photography with single-lens cameras it must often be necessary to measure the difference of height of a point from an air station when the angle of depres-

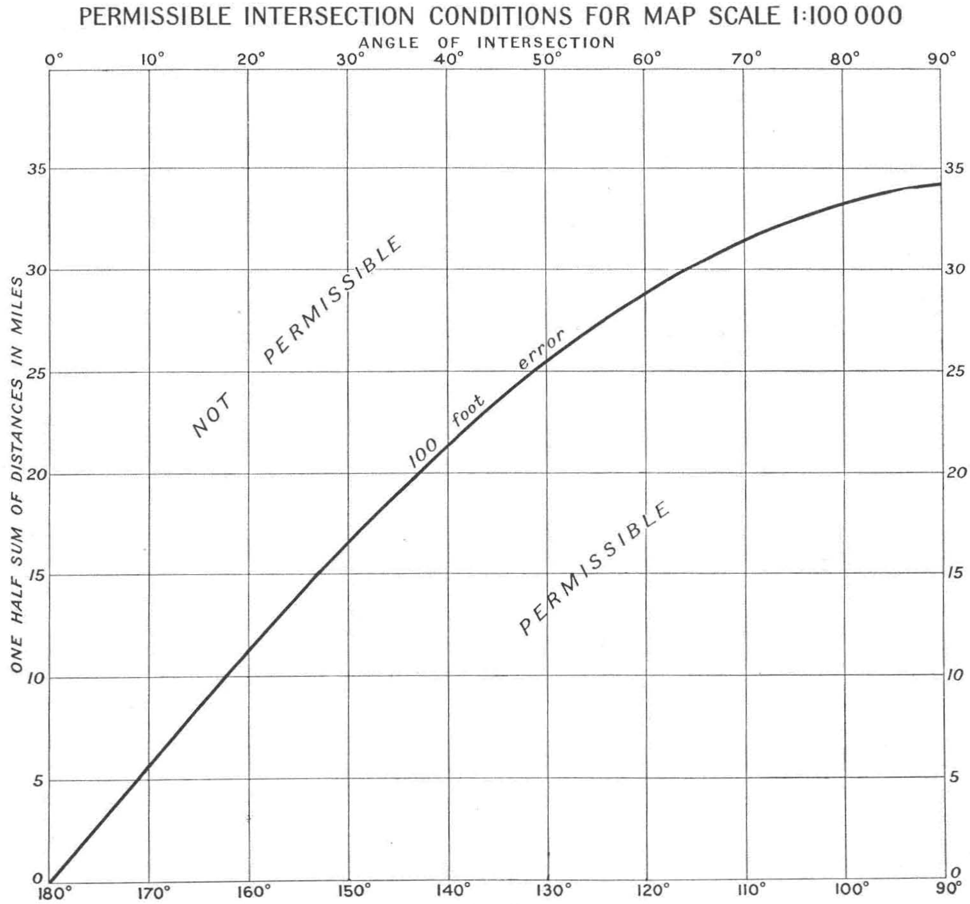


Fig. 3

sion from the horizontal to the point is as great as 80° or more. Assuming a difference of height between station and point of 10,000 feet and that the horizontal distance is known accurately, an error of 1′ in the determination of an 80° depression angle will give an error in the difference of height of 17 feet.

In Fig. 4 the top row shows various distances in miles and the second row the corresponding error in the determination of the height of a point from an air station 10,000 feet above it due to a 1′ error in the measurement of the vertical angle.

Note that the height of a point ten miles distant can be determined more accurately than the difference in height in the case given for the vertical photograph, and that when the distance is greater than 10 miles or so, the increase in the error is practically directly proportional to the distance.

An independent source of error in the determination of heights from high obliques is introduced by the fact that in applying a correction to a measured height for curvature and refraction, we can never determine the precise value of the coefficient of refraction under the conditions imposed during a flight. It is usual to adopt a value of 0.070 for this coefficient but the true value, for all we know, may vary from this by 25% of the total. In the third row of Fig. 4 are given the amounts of errors introduced by assuming the coefficient of refraction to be 0.010 more or less from its true value. It will be observed that up to 20 miles the error is insignificant compared to that in the second row, and that even at 50 miles out, the error due to refraction is less than half that introduced by a one-minute error in the vertical angle.

Fig. 4.

POSSIBLE ERRORS IN MEASURING HEIGHTS

from high oblique photographs
Height of Air Station:—10,000 feet.

| Distance in miles | 05 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|---|---|---|---|
| A. | 09 | 15 | 23 | 31 | 38 | 46 | 54 | 61 | 69 | 76 |
| B. | 00 | 01 | 03 | 06 | 09 | 12 | 16 | 21 | 27 | 34 |
| A. & B. | 09 | 16 | 26 | 37 | 47 | 58 | 70 | 82 | 96 | 110 |

A. is the error in height in feet due to a one-minute error in the measured vertical angle.
B. is the error in height in feet due to an error of 0.010 in the coefficient of refraction used.

## Resection of Air Stations

So far we have assumed that the air stations have been perfectly positioned and oriented.

The next question to be answered is what conditions are necessary to obtain sufficient accuracy in the resection process so that no appreciable errors will be apparent on the scale of reproduction.

As in all horizontal resection problems, three control points are a necessary minimum. The process,[3] briefly is as follows. A tilt and a swing for the photograph are assumed. On these assumptions, differences of horizontal direction to the control points are measured and a horizontal position for the station is determined usually by the familiar tracing-paper method used in plane-tabling. The height of the air station is then determined independently through each control point. If these heights do not agree within the prescribed limits, the differences are used in differential equations which relate the effect of small changes in the tilt and swing to small changes in the height of the air station. Thus the true tilt and swing and the true height of the air station are found.

Small errors in the assumed tilt and swing only slightly affect the extraction of horizontal angles from high oblique photographs, so that in most cases the original horizontal position can be accepted. However, it is always good practice to check this after the adjustment for swing and tilt has been made.

If only three controlled points are used, an apparently consistent result will always be obtained, but this may not be the true result because of the impossibility of measuring angles closer than 1′, or because of poor distribution of control or of personal errors of measurement and identification. Therefore it is always necessary to take further precautions. The first essential precaution is

[3] Same as Note 2.

to insure that at least two of the control points are so situated in respect to the air station and one another that the probable error in the horizontal position of the air station is not greater than permissible for the scale of mapping. A table or graph showing the permissible types of intersection as in Figs. 2 and 3 is very helpful.

The second essential precaution is to make sure that the positions of the selected control points are sufficiently widely separated both in bearing and distance from the air station to give a wide variation in their differential coefficients of tilt and swing.

The third precaution, of course, is to use more than three control points if these are available. This last introduces various interesting problems of analysis. Supposing directional rays have been drawn to half a dozen or more control points from a station. It is a familiar experience in making a graphical resection to find that some of the rays give an apparently good result at one point and another selection of rays gives a similarly good result at another point. How is a decision to be made as to which is the best of the points or which the best compromise position?

It would be possible to work out least square methods for adjusting positions when more than the minimum control is available, but the means and the end in view would hardly justify such involved procedure.

However, one practical way of attacking the problem—a way which in many cases is sufficient—is to use the most distant control in making the tilt and swing adjustment and in determining the bearing of the optical axis, and, having done this, to make the final determination of horizontal position from the nearby control. By this procedure an additional series of checks on the horizontal position become available, because if the position is correct then the heights of the air station as computed independently through each of the control points that are not used in the tilt and swing adjustment should agree.

Another method which is especially useful when there are many control points to choose from is what we call the process of grouping. Suppose, for instance, that on one photograph several of the control points imaged are peaks of a distant mountain range appearing on the right. Others are peaks of a much nearer mountain range appearing on the left, and in the foreground there is a series of fixed points on a river or lake. A graphical resection is made to determine approximately the position of the station and the bearing of the optical axis. With one of the co-ordinates of the assumed position—say the $x$ co-ordinate—and the assumed bearing of the optical axis, the $y$ co-ordinate position is computed through each of the control points. If the computed $y$ co-ordinates do not agree with each other within prescribed limits, very simple differential equations can be introduced relating small changes in the computed $y$ co-ordinates with small changes in the $x$ co-ordinate and the bearing of the optical axis. Only three sets of equations are necessary to get a solution, so when a large number of control points is available, we group them—that is to say, having computed the differential coefficients for every point, we take the mean value of a group. The result is that the finally accepted horizontal position is an average position in which the errors within any group are balanced. This process of grouping takes very little longer than if only three points are used. It may also be used in the determination of swing and tilt.

We constantly use this grouping process, and by so doing are convinced that the results are much closer to the truth than would be the case if a selection was made of what appears to be the best combination of three points for the resection and the evidence from the others was neglected.

In an area that has been triangulated and where there are plenty of well-defined points such as road intersections, buildings, and the like, no further forms of analysis are necessary in handling the resection process. However, in undeveloped country, whether it be mountainous or flat, it often happens that the only control appearing on a photograph is very distant and not too well defined.

Fully to appreciate the problem that one is now up against, an example should be considered.

Supposing two photographs approximately 10 miles apart in position are both resected and oriented for tilt and swing from ground control, 30 to 50 miles from the air stations. Also suppose that the maximum difference in horizontal bearings from each of the air stations to the control is 30°, then Figs. 2 and 4 tell us that each of the air stations may be as much as 240 feet out in position and 80 feet out in height. If we now intersect a point in the foreground of each photograph we shall probably find that the results of computing the height of the point independently from each air station do not agree.

For instance, suppose the point to be approximately 7 miles distant from each station and 10,000 feet below them; then, if the initial errors of resection do not cancel each other, we may find the two computed heights of the point to differ from one another by as much as 300 feet.

Such uncertainty in the height of the point is not tolerable. Must we be content to leave the matter there or can we further refine the process? Now the distant control will have given us accurate determinations of tilt and swing and the bearing of the optical axes. Therefore, if we intersect three points in the foreground of both photographs we can take the differences in their computed heights and introduce them into differential equations which relate these differences to small changes in the relative position of the two stations. By considering one of the stations as fixed we can find the relative position of the other by a solution of these equations. In this way, though we may not improve the absolute position of the stations, we do get a constant relation between them so that the height determinations of the points from both stations agree when the intersections are plotted. The process is, of course, much simplified if one or more of the foreground points is known to be at sea level.

## Extension of Control

Differential methods of analysis are also extremely useful in extending control through the air stations themselves in cases where the ground control is not well distributed. The usual procedure in extending control is to resect and orient stations from ground points that have been intersected previously from other stations, the process of resection and intersection being worked alternately. However, in many cases this is not possible, and in any case it is unsatisfactory in that errors tend to accumulate very rapidly. To offset this, two additional forms of analysis have been worked out. In the first case, if one photograph has been correctly resected and oriented and five points imaged on it are also imaged on two other photographs, these two photographs can be oriented horizontally in respect to the first photograph without more than an initial first approximation for tilt, swing, and position and without our having to bring in any height relationships. We assume a position and orientation for one of the unknown stations and make intersections to the five points, using the already resected station as the second station. From these intersections we then resect the third station. If the rays from the five points do not meet in a point then we know that one or more of the four variables of our first assumption is wrong. These four variables are the bearings of the optical axes of the three stations and the relative position of the third station. By differential analyses similar in principle to those already outlined, a correct solution can be found. In this case, the absolute scale of the setup will not be known. But final adjustments for tilt and swing on the two unknown stations together with the evidence of the flight records will generally give sufficient clues to determine this.

In the second case, we can adjust a single air station to one other station already adjusted if five unfixed foreground points are common to both by taking differences of height into consideration. In this case, the position of the unadjusted station is assumed and the height of this air station is computed independently through each of the unknown points after they have been intersected. A similar series of differential equations can then be formed which relate the differences of height of the computed air station to the errors in the first assumptions of position and orientation.

Such adjustments as the last two should not be undertaken lightly because they each involve the solution of four simultaneous equations and this is quite heavy computing. However, a sense of proportion here is desirable. In ordinary routine work no such heavy computations are necessary. The tilt and swing adjustments and other simple forms of differential analysis involve merely the solving of pairs of simultaneous equations. In every case the differential coefficients are extremely simple to extract. Only in the absence of an adequate ground control will the heaviest computations be necessary, and if we take into consideration the fact that by extending control through the photographs we can eliminate much of the field work on the ground, the extra computing work involved does not seem out of proportion to the results obtained.

## Density of Air Stations and Control

Once the type of camera that is to be used has been chosen, it is theoretically possible from the figures in Fig. 2 to choose the optimum spacing of flight lines and exposure intervals. This is true at any rate for flat country. However, to try to standardize schemes of this sort seems unwise, because in any project the nature of the topography both in its geographical distribution and its size must be taken into account. In addition, the type of airplane used in the photography will probably be a deciding factor in choosing the directions to point the camera in respect to the line of flight. The proportion between the height of flight and the maximum range of height in the landscape must also be taken into consideration, and it is not always practical to fly at the height that in theory would appear to give the most economical results. For instance, over the St. Elias range in the Yukon Territory, where the elevation of points varies between 2,500 feet and 17,000 feet, Mr. Walter A. Wood and Mr. John D. Kay[4] had to fly at a height not exceeding 14,000 feet because of the limited ceiling of their plane. In theory it would certainly have been better had they flown at 20,000 feet.

The experience gained in making the Labrador map, however, gives a rough clue as to the density of air stations required under average conditions. When a single-lens camera having a field of view approximately 50 by 40 degrees is used in flat country, one air station should certainly be sufficient to cover 16 square inches of plotting paper. In mountainous country, provided the plane can be flown at twice the height of the greatest height difference in the landscape, the figure should be roughly half this or one station to 8 square inches of plotting

[4] Walter A. Wood: The Wood Yukon Expedition of 1935, *Geographic Review*, Vol. 26, No. 2, April, 1936, pp. 228–246.

DENSITY OF CONTROL

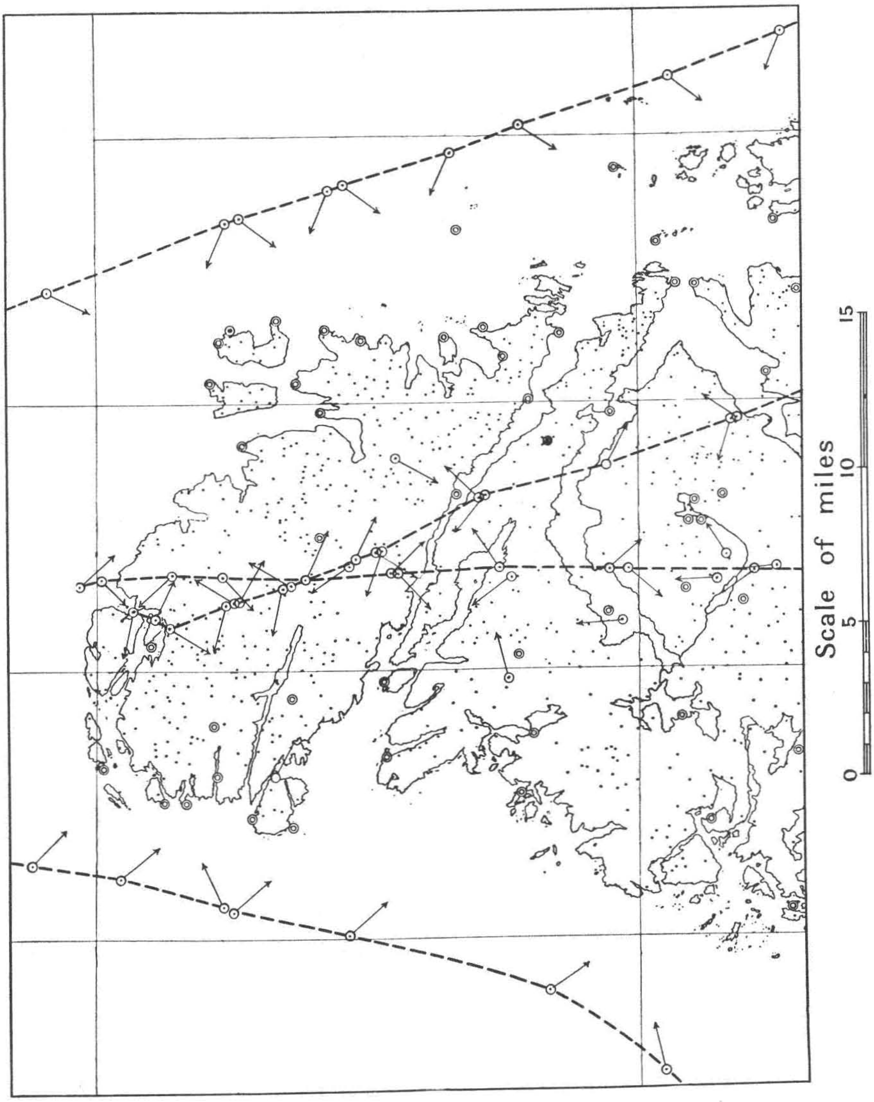


Fig 5

DENSITY OF CONTROL

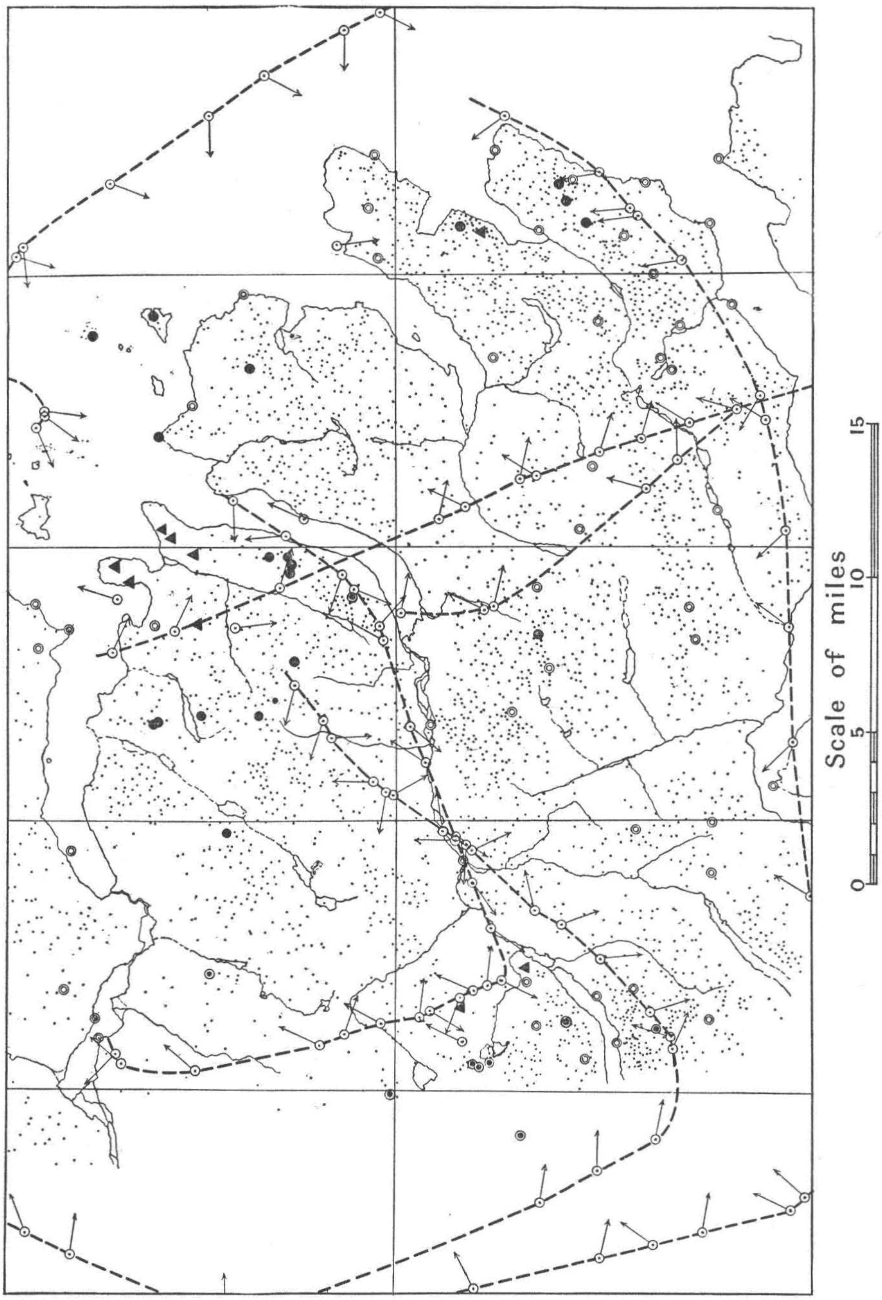


Fig 6

paper. If the height is lower than twice that of the range of height in the landscape, then the number of air stations necessary to cover the landscape adequately must be larger.

In our Labrador work the height range of the landscape was about 5,000 feet. Consequently, if the photographs had been taken at 10,000 feet for the reproduction scale of 1:100,000 the density of air stations should have been about 1 to every 20 square miles of territory, according to the above reasoning. Actually the density of stations resected worked out at about 1 to every 10 square miles. This was mainly because the flying height averaged about 6,000 feet instead of 10,000 feet and the tilt of the camera axis was generally considerably less than 10°, which meant that the field of view of the photographs was not fully utilized. As for the density of minor control, Figs. 5 and 6 which show this for two of the Labrador sheets give a picture better perhaps than that given by numerical statistics, because the density of points fixed varies with the type of topography. In fact we have found this variation to be as much as 1 to 10 per square mile.

In the building up of major control either on the ground or by air triangulation, the density of the control points, provided they are well distributed, should be about the same as the density of the air stations.

## Speed and Efficiency of Method

Finally we come to the difficult problem of estimating the speed and efficiency of mapping from high obliques.

The first point to emphasize is that the plotting of coast line features is extremely rapid on the pinhole plotter developed at the American Geographical Society. For instance, on a plotting scale of 1:50,000 one can make the plotting pencil, while still maintaining the requisite accuracy, move as fast over the paper, relatively speaking, as an express train moves over the actual ground. In plotting intersections and sketching contours we estimate that one man can cover only about 10 square inches of paper in one day. This, however, includes the identification of points on pairs of photographs, the actual making of intersections, the actual determination of heights and the sketching of contours. The use of our plotting instrument makes the determination of heights very rapid because differences of height are read off directly on the machine instead of having to be computed by the regular tangent formula. As far as the resection process is concerned, we find that resections can be worked through from start to finish in less than two hours provided the control is good and adequate. However, time is always consumed in the identification of points and because of this I would consider three resections a day by one man an excellent average. Where the control is distant or poorly distributed or badly defined, then one must have resource to the more involved methods of analysis. In these cases, one is doing well to complete one resection in a day.

We have tried from time to time to adjust a photograph for tilt and swing by trial and error methods in our plotting instrument. Though by doing this one avoids computation, we have never found that actual time is saved or that the resulting adjustment is consistently satisfactory. This is particularly so when one wishes to adjust the air station to more than three control points. However, we should welcome any systematic procedure that would do this surely and accurately with less computation than we now use if we could be certain that accuracy was not being sacrificed.

One feature of the Geographical Society's plotter which we consider quite essential is that all setting and measuring movements on the instrument have scales. This makes the use of the various differential methods extremely simple, as the elements required for building up the differential coefficients are in all cases simple functions of tilt, distance, height, and horizontal direction. Just as important as this is the fact that the scales permit complete records of the resection and orientation data. Thus all the work of resection and orientation for a given area can be completed first. In this way, accumulations of errors become apparent before the detail plotting is attempted and the necessary adjustments can be made before it is too late.

Fig. 7.

ESTIMATES OF THE SPEED OF MAPPING FROM HIGH OBLIQUE PHOTOGRAPHS

Note: The figures give the area in square inches covered on the scale of plotting by one man in one day.

A. is for flat country when no contours are required.
B. is for mountainous country when contours are sketched.

| | A. | B. |
|---|---|---|
| Work on minor intersections, tangent rays, plotting & sketching | 20 | 10 |
| Resection of air stations when full ground control is available | 48 | 24 |
| Resection of air stations when minimum ground control is available | 16 | 08 |
| Complete process when full ground control is available | 14 | 07 |
| Complete process when minimum ground control is available | 09 | 04.5 |

Finally in Fig. 7 an attempt has been made to give a summary of the time required to construct a map from high oblique photographs under varying conditions. In compiling this, it has been assumed that the camera to be used would have a horizontal field of view of about 50° and a vertical field of view of about 40°. Obviously much time would be saved by using a camera having a wider field of view. The data in Fig. 7 can easily be translated into terms of square miles per unit time. For instance, if we take the scale 1:100,000 and consider a working year of 230 days, then we can say that one man should be able to work the whole process of mapping and complete 1,300 square miles a year by this method, if the ground is flat and the ground control adequate. If the country is extremely mountainous and the ground control extremely sparse, as was the case in the Labrador mapping, then not more than 400 square miles a year should be expected of one man. We estimate that the Labrador topographical sheets were completed at a speed of about 333 square miles per man a year. Considering that we were all along experimenting with method, this is fair evidence that with experienced personnel and good material to work upon, the figure of 400 square miles per man per year is a conservative estimate to say the least.