AN EXPERIENCE WITH AERIAL TRIANGULATION

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The following paper is a digest of a communication from the Geodetic Institute at Delft, which was prepared by Dr. Schermerhorn (President of the International Society of Photogrammetry), for presentation at Washington. Dr. Schermerhorn returned to the Netherlands on the outbreak of the war before presenting his paper. It was read, however, by Mr. B. G. Jones at the last annual meeting. The following digest has been prepared to describe the general procedure and the results obtained without going into the detailed description of the adjustments given by Dr. Schermerhorn. A copy of the original paper can be loaned to members particularly interested.

THERE are many territories for which the value of air survey would be nil, if it were necessary to measure sufficient ground control points to fix each stereoscopic pair of photographs. It is not astonishing, therefore, that a number of methods of aerial triangulation have been developed.

The accuracy of all the different methods of aerial triangulation varies with the amount of ground control. If only one or two pairs of photographs must be bridged, many methods may give the limit of accuracy which can be used on the map. When the distance between ground control is increased, the differences between the methods become more evident. The exact proportion of accuracy of the different methods, in my opinion, has, by no means, been fixed definitely as yet. In his report on the Rome Congress, Lieutenant Commander Reading records that the accuracy claimed for a number of methods he mentions is apparently about 1:2,000 to 1:3,000 in each case.

Nothing has yet been published on important aerial triangulation work on which to test such claims. In some cases indications are given of the accuracies obtained, which are obviously meant as propaganda by interested parties. In other cases, no more than a single strip has been measured. In still other cases, important work has indeed been carried out, but this has not yet been published or else there has been no terrestrial survey to check the work. Until aerial triangulation surveys have been accomplished over a larger area than one single strip, I consider a definite opinion on the comparison of the various methods premature.

In order to further the comparison of methods, I shall try to explain by means of the following practical example, not only the result but also the method used at Delft, so as to make clearer the possibilities of this method of aerial triangulation. It should be pointed out that the example chosen is typical of many. Progress made since it was accomplished shows that what is here published need not be looked upon as the ultimate limit that can be obtained.

The example selected concerns an island about 600 square miles (1600 km^2) in area. Fig. 1 shows the scheme of flight strips and the ground control points. Only three ground control points were available. The error of these points as regards situation with respect to each other plus the error in identification on the photographs, is of the order of 10 meters. For points *T 2* and *T 3* (distant 16 miles (25 km.) from each other) identification was certain; for *T 1* about $2\frac{1}{2}$ miles (4 km.) from *T 2* there was an uncertainty estimated at 20 meters. Point *T 1*, therefore, was not used in the final computation of co-ordinates. For further terrestrial control to check the survey, the original sheets of a 1911 hydrographic survey of the island were used. From these sheets the co-ordinates of 20 "sea chart" points distributed along the coast were taken, of which 15 could be satisfactorily identified.

Three stereoscopic machines were used in the aerial triangulation, a radial triangulator, the Zeiss Stereoplanigraph, and the Wild Autograph A5. The details of these machines will not be discussed in this article, but in order that the computation of the results may be better understood, a short explanation of the triangulation in space will be given.

After setting the first pair of photographs in the Stereoplanigraph or Autograph the X, Y, and Z co-ordinates of the points selected for the aerial triangulation are read. The transfer of scale to the next pair of photographs is carried out by measuring in the first overlap the height of a clearly visible point in the area which is situated near the plumb point. This height is determined in the

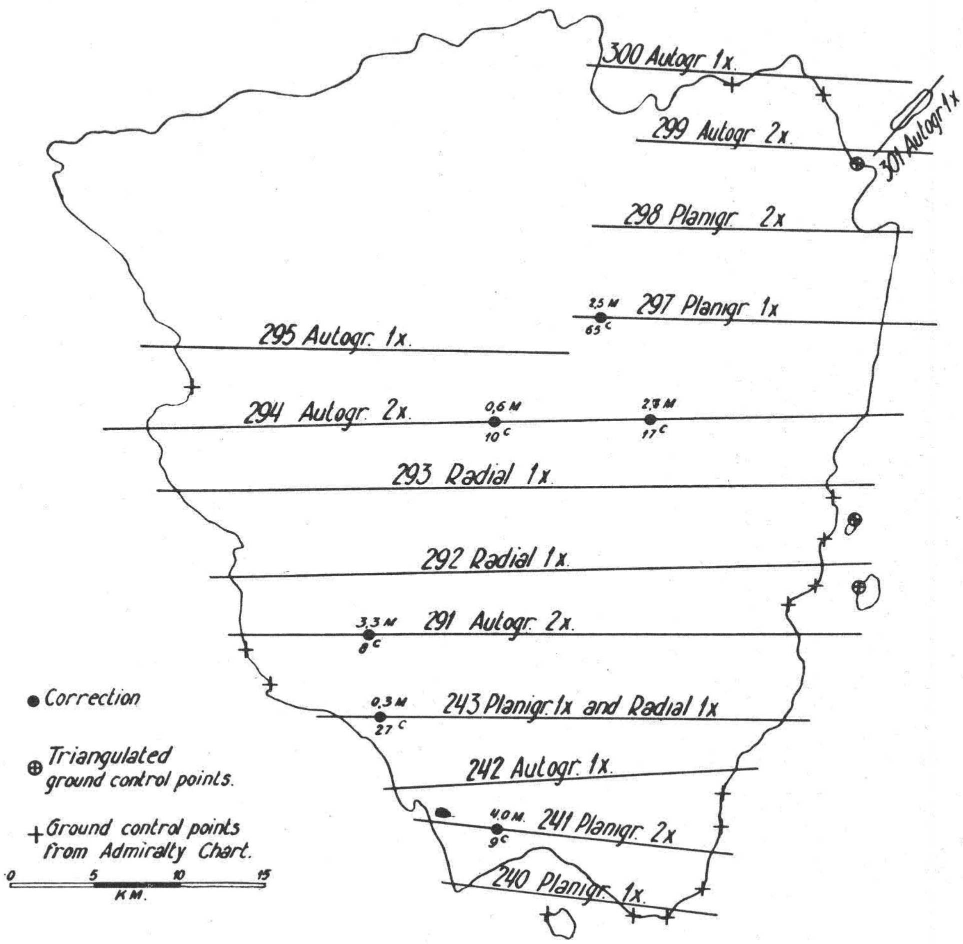

FIG. 1. TRIANG SCHEME.

first pair of photographs, and the length of the air base in the second pair is altered until the same height is read on the co-ordinate scale. In the second pair of photographs, the co-ordinates of the points to be triangulated also are read, and so on through the strip.

As already known, there are a number of errors in this method.

1. Systematic errors occur in the scale and consequently also in the height of points in the area.

2. Errors of considerable magnitude also occur in the transfer of scale and azimuth from overlap to overlap, and these errors exceed the measuring accuracy of the instrument.

Systematic errors in the transfer of the scale cause an error in the length of each strip which is proportional to the square of the distance from the starting point. Besides this, a practically circular deviation occurs perpendicular to the flying direction. If these errors were absolutely regular, it would be possible by retriangulation of a strip in the reverse direction to eliminate the influence of the systematic errors from the mean of the two results. Fig. 2 shows the difference in X between the two triangulations counted from the end point, and Fig. 3 shows the same for the deviation in the Y direction. The parabolic and circular character of these deviations clearly appear in these figures.

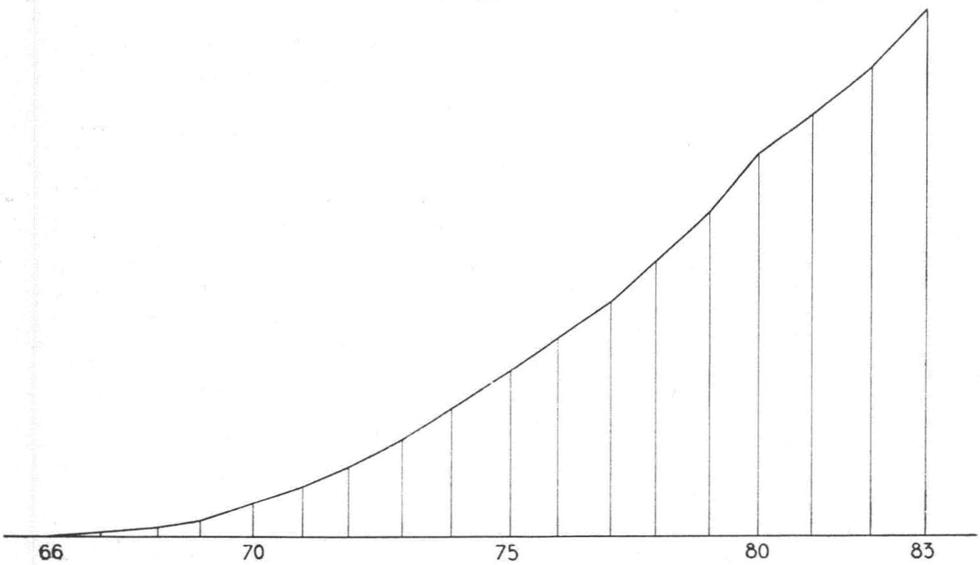

FIG. 2. DIFFERENCES IN X. A PARABOLA.

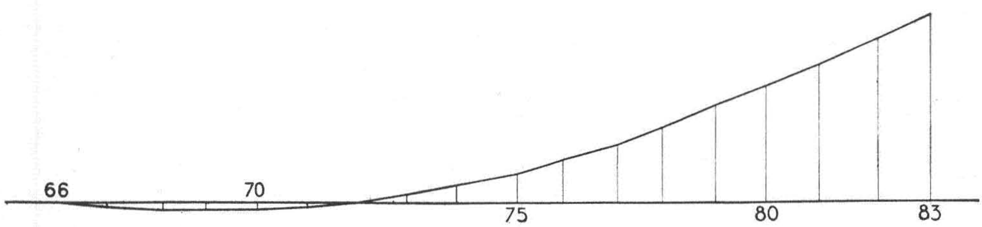

FIG. 3. DIFFERENCES IN Y. A CIRCLE.

The causes of these errors have not been sufficiently discussed in modern literature—they are probably due not only to the influence of the instrument, but also to the operator, and probably also to the photographs. When the Stereoplanigraph was used for the first time in Delft in 1936, it was found that corrections derived from the forward and backward triangulation comparisons were not always the same, either for various strips or even for triangulation of the same strip. We are now carrying out a test of the utmost importance for this method of aerial triangulation, because if a constant correction cannot be found, it cannot be supposed that the influence of systematic errors has been eliminated from the mean of both forward and backward observations.

Some results of the studies in the Delft Institute on this subject are published in Volume 4 of *Photogrammetria*, 1939.

Bad setting of pairs of photographs in the machine, or ordinary reading mistakes may cause substantial errors.

If the so-called systematic or "C" correction be subtracted from the values of the X and Y co-ordinates, and a graph or diagram be made of the values so derived, an indication of the occurrence of such errors will be obtained. By repeating the triangulation at the points where such differences occur it can be determined whether the error was made in the forward or backward triangulation. Fig. 4 shows an example of such errors.

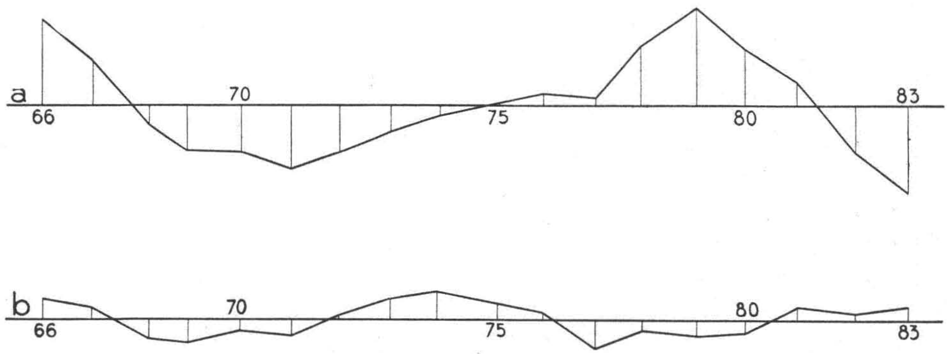

FIG. 4. a. DIVERGENCES OF THE MEASURED PARABOLA AS COMPARED WITH THE COMPUTED ONE. b. AS ABOVE BUT NOW CORRECTED FOR ERRORS IN SCALE.

There are also considerable errors having the same value in both backward and forward measurements which only appear by lateral comparison of adjacent strips. These errors cause the main difficulties in aerial triangulation. It is only when lateral comparisons between strips check within acceptable limits that there can be certainty in the results of strip aerial triangulation.

After this explanation, we give below a brief report regarding the computation of the triangulation of Fig. 1. The X-Y co-ordinates and the heights will be discussed separately.

Strips 292 and 293 were triangulated in the Stereoplanigraph; the others as indicated in Fig. 1. All strips had common points above and below their principal points throughout their length. An effort was made to reconcile the differences in the co-ordinates of the common points by means of graphs showing the effects of probable scale and azimuth corrections, but without acceptable results.

Strips, 292, 293, and 243 (flat territory and low hills) were retriangulated in the radial triangulator. Strip 293 showed excellent agreement with strip 294 without a systematic difference. Strip 292 was, therefore, adjusted to 293, Fig. 5, diagrams 1 and 2.

Strip 292, so adjusted, was then compared with strip 291 which indicated either a scale jump or a large correction. The double triangulation comparison indicated a scale jump in X values; therefore, 291was adjusted to 292 by dividing the adjustment into two parts at the jump. When so adjusted, 291 gives reasonably good agreement with the values given by the double triangulation of strip 243, with only small acceptable differences which were reconciled. This adjustment of the above strips gave an agreement of 1.05 mm/km (1/1,050) with the "sea chart" (hydrographic survey) points. The sea chart points have a mean error of 40 meters in consequence of the way in which they were determined and mainly because of the identification error in the photographs.

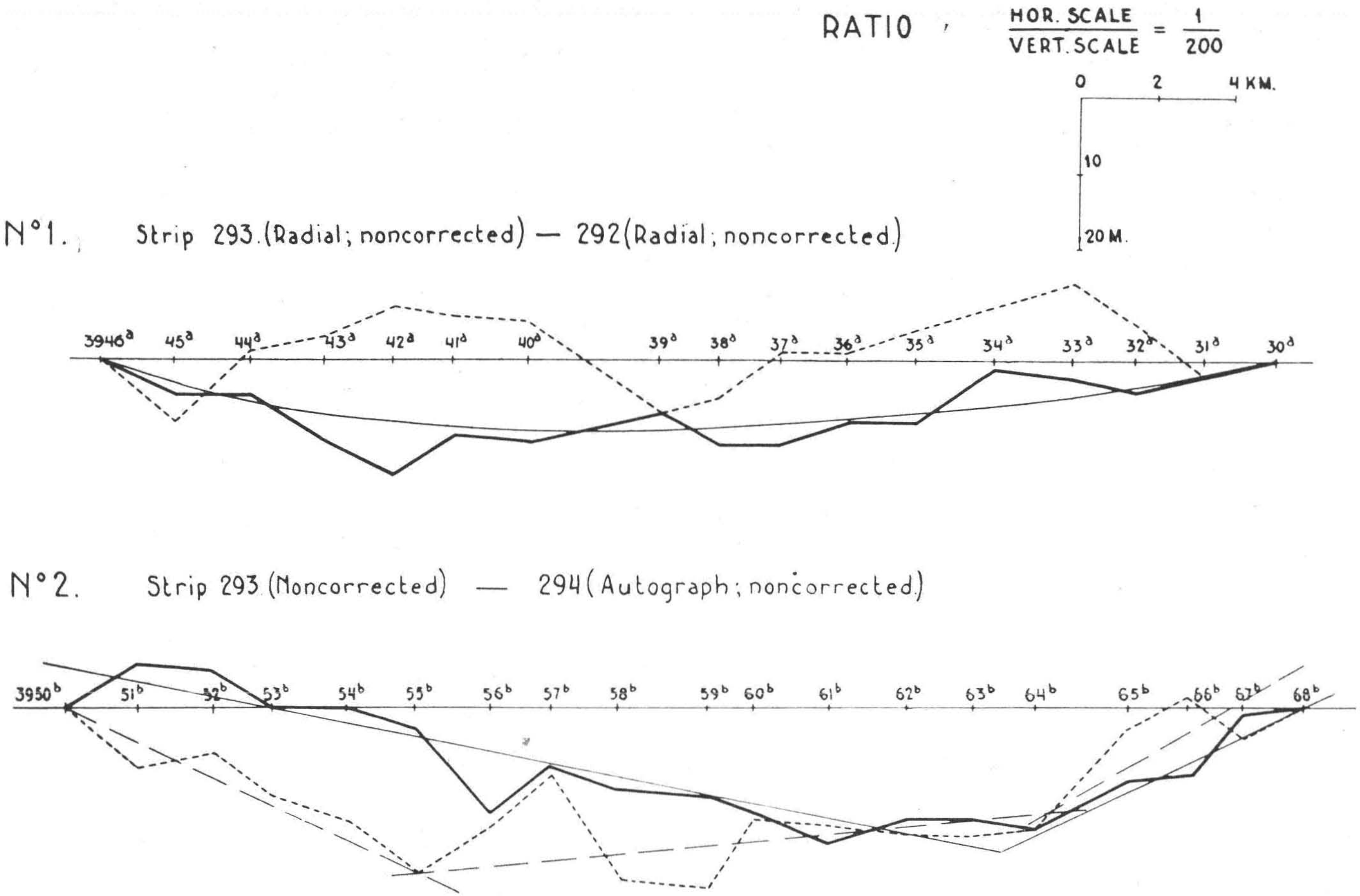

FIG. 5. LATERAL COMPARISONS.

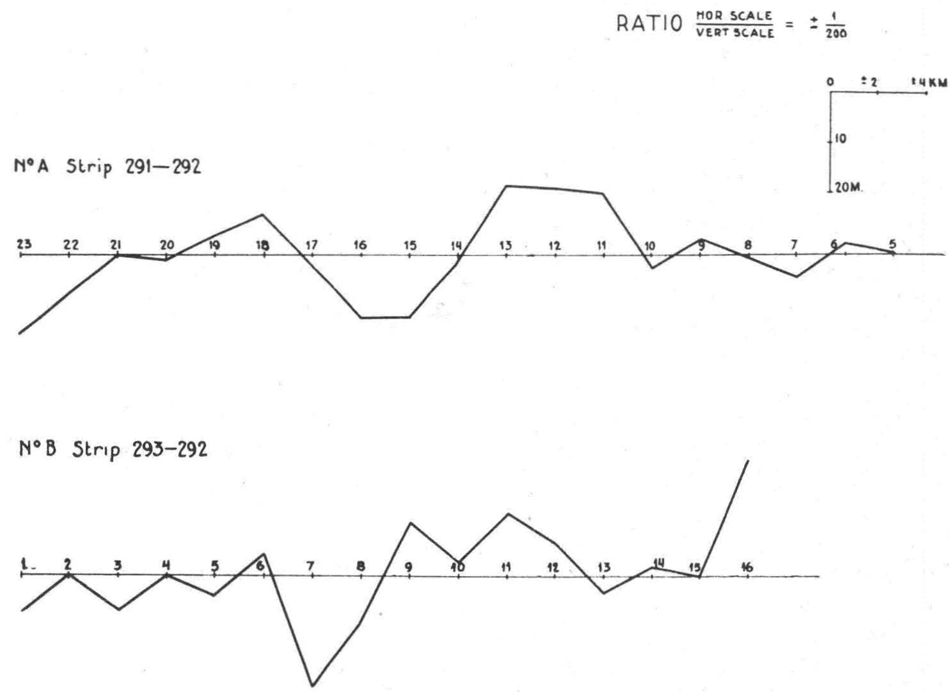

FIG. 6. LATERAL COMPARISONS OF HEIGHT.

The remaining strips were corrected in a similar manner. Connection to the two control points *T 2* and *T 3* which could be positively identified, gave a factor for final adjustment of all co-ordinates. The final adjustment gave a correction of 0.46 mm/km when compared to the 15 reliable sea chart points. This amount is rather smaller than the accuracy to be expected from the sea chart. The mean error of the remaining differences in the 141 common points after adjustment by strips, as outlined above, was 3.7 meters in X and 4.0 meters in Y direction.

The heights were not needed in the triangulation of the area, however, four of the longest strips, 22 miles (35 km), were selected for experimental height determinations. Neither statiscope nor horizon photographs were available, both of which would now be considered essential for first-grade work.

Strip 291 had a difference of 41 meters between the height of the water at its eastern and western ends; strip 292, 61 meters between water at its western end, and the esternmost common point of strip 291; Diagram (A) Fig. 6, shows the differences in heights remaining after adjustment. Strip 293 was adjusted to 292 by common points at both ends by proportionally distributing a discrepancy of 46 meters. The differences remaining are shown by Diagram (B), Fig. 6. When so adjusted two points on the water at its western end were +23 and +14 meters; 3 points on the east side, probably on the water, were +30, +24, and −3 meters.