REVIEW OF MR. RALPH O. ANDERSON'S RECENT PUBLICATION ENTITLED "TILT OF THE AERIAL PHOTOGRAPH BY GRAPHICAL RESECTION"*

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MR. RALPH O. ANDERSON'S recent fifty-page pamphlet on *The Tilt of the Aerial Photography by Graphical Resection*, presents a new method of determining the scale, the tilt, and the photographic direction of the tilt of an aerial photograph. It is in reality a supplement to his earlier book entitled *Applied Photogrammetry*, and a reading of the pamphlet must necessarily follow the reading of the original publication.

In the original book Mr. Anderson presented a method for determining the tilt and scale of an aerial photograph covering cases of moderate tilts and limited topographic relief. This initial method was a very simple, practical, useful, and interesting application of photogrammetry based upon Mr. Anderson's original conception of the "scale point". The method had some recognized limitations however. There was no check upon the work, and the method needed amplification to render it applicable to unpredictable cases of large tilts or to extreme cases of topographic relief. These necessary amplifications explained in *Applied Photogrammetry* assumed considerable complexity.

The new pamphlet presents an entirely different graphical method which is intended to solve the same calibration problem for the cases where the tilts are large and where the topographic relief is excessive. This pamphlet therefore not only supplements the original book but it actually supersedes much of it.

It is generally understood that the scale of an aerial photograph is a variable quantity depending upon the position of the point on the photograph at which the scale is under consideration, and upon the elevation of the terrain appearing at this point. Mr. Anderson has introduced the term "datum scale" at any particular point on the photograph to signify the scale at this point corresponding to ground at zero elevation.

The new method, like the original one, is based fundamentally upon the following principles: (1) The photographic scale found by dividing the ground distance between two points by the photographic distance between the corresponding images, being of course a scale for *some* point for *some* ground elevation, is actually the scale corresponding to an elevation which Mr. Anderson calls the "equivalent elevation" and for which he gives a specific formula, and this scale exists at a point on the photograph called the "scale point", for whose position Mr. Anderson also gives a definite criterion. (2) The formula for the equivalent elevation is $h_e = h_2 \pm (X/P)(h_1 - h_2)$, with certain corrections for certain cases, where h_1 and h_2 are the ground elevations of the terminals of the control line, P is the photographic length between images, and X is the distance on the photograph from the image corresponding to the point of higher elevation to the foot of the perpendicular let fall to the control line on the photograph from the *nadir point.* (3) The scale point on the photograph where this scale is true for this equivalent elevation, is situated on the line on the photograph joining the images for the two control points after they have been corrected for topographic relief displacement, at a point as far from the mid-point of this line on the opposite side as is the foot of the perpendicular let fall to this line from the *iso-center*. As a matter of fact, unless I am mistaken, none of Mr. Anderson's

* Copies can be obtained from the author whose address is P.O. Box 882, Chattanooga, Tennessee for $1.00 each.

publications presents a very explicit, rigid, and coherent proof of all of these principles.

The current pamphlet extends these principles to include the consideration of the change in datum scales between the iso-center and any other point in the photograph, together with the consideration of the changes in the computed equivalent elevation caused by variations in the position of the nadir point. The graphical method presented for determining the position of the nadir point is indeed a very ingenious one, and for this contribution Mr. Anderson deserves much credit. Both the merits and the complete originality of the method are unquestionable.

There is one difficulty which Mr. Anderson encounters. His method, as he states, is one of approximation in that more than one solution is needed to arrive at the final results for the position of the nadir point and the datum scale at the iso-center. This in itself is not particularly objectionable, but the real difficulty lies in the fact that successive solutions do not lead convergently to the desired values. There is a definite reason for this. The correction called $S_{di}(E \sin t/f)$ to reduce an iso-center datum scale to the datum scale for the scale-point, as it is computed using the position of the iso-center from the *previous solution*, is not sufficiently close to the correct value to cause the new determination of the nadir point to be correct or to approach the correct value very rapidly. Herein, I believe, lies the only flaw in the method, one which I fear will cause serious difficulty.

Had Mr. Anderson been able to devise a graphical solution which did not involve this approximation for the correction to an iso-center datum scale to find the corresponding datum scale at the scale point, but which solved for this reduction, itself a direct function of the position of the nadir point, simultaneously with the solution for the desired position of the nadir point, the difficulty mentioned would have been circumvented. Inasmuch as such a solution appears impracticable, he has devised what is probably the best solution utilizing his method of attack. I doubt seriously that the method as it stands can be used without considerable difficulty, inasmuch as the results in successive solutions will not converge, or approach the desired values as limits. Only experience will determine the practicability of the method.

There are several statements in Mr. Anderson's pamphlet, more or less unrelated to the topic presented, to which I would take exception. The statement that other solutions of the tilt and calibration problem are likewise methods of approximation in that several solutions are required to obtain the desired results, only partially applies to our exact space geometry method. In the space geometry method the simultaneous equations set up for solving the problem are *exact*, and as far as the theory is concerned no approximations are required. It is of course true that algebraic processes are not powerful enough to solve the equations themselves without sometimes resorting to successive solutions. However two solutions are invariably sufficient and frequently one is sufficient. I cannot understand how the theory underlying the space geometry solution of the calibration problem can be regarded as difficult. The method is based exclusively upon the method of finding direction cosines of lines in space between pairs of points whose space co-ordinates are known, that for finding the angle between two lines in space when their direction cosines are known, the differential calculus method of solving higher degree equations, and a few of the simplest principles regarding planes and lines. The theory is so simple that the entire space geometry method for determining the complete orientation of an aerial photograph can easily be presented to a class in two or three ordinary lecture periods. The space geometry computation of the complete space orientation of an aerial photograph requires an hour or an hour and a half and covers one and one-half pages. It is very exact, is not limited by the amount of the tilt or of the topographic relief, and *is absolutely self-checking*. It provides complete space orientation data for the aerial photograph instead of merely the tilt, swing and exposure altitude as found by the graphical method, thereby supplying data for subsequent absolute determinations of ground positions instead of merely the relative positions and lengths as in the graphical method. In other words, the space geometry method is a complete solution of the space resection problem.

Mr. Anderson suggests the use of his publications as texts by students of photogrammetry. As a matter of fact, a text book, to satisfy the demands of an adequate course in photogrammetry, should include many other topics which must be taught, such as radial plotting, proof of the geometric properties of various ray centers, stereoscopic principles and problems, rectification methods, stereocomparator problems, orientation of stereoscopic pairs of photographs in the multiplex projector, and so on. At Syracuse we do use Mr. Anderson's publications with excellent results in teaching the scale determination problem, one of many topics which constitute a semester's work.

Aside from my own personal exceptions to what I have termed the irrelevant material in this pamphlet, the graphical solution of the calibration problem as presented by Mr. Anderson is a splendid piece of work. It furnishes a very original application of Mr. Anderson's own original idea of the "scale point." The pamphlet should be carefully read by everyone interested in photogrammetry. We are indebted to Mr. Anderson for another splendid contribution to photogrammetry.

DISCUSSION OF PROFESSOR CHURCH'S REVIEW ON "TILT OF THE AERIAL PHOTOGRAPH BY GRAPHICAL RESECTION"

R. O. Anderson

THE controversial statements are: "*the fact that successive solutions do not lead convergently to the desired values*" (*12th and 13th lines, page 180*), and (2) "*successive solutions will not converge, or approach the desired values as limits*" (*28th and 29th lines, page 180*).

These statements are definitely erroneous as successive values converge to the correct values. Oscillatory convergence is encountered in some cases (rules are clearly stated in pamphlet). In every case successive differences (computed value minus correct value) vanish at the correct values which means that the method will serve to compute tilt. It is also applicable when the three scale points fall upon a straight line. This method is only needed in cases of extremely excessive relief; the efficiency increases as the relief predominates the tilt. The successive positions of the principal line may be likened to the amplitudes of a swinging pendulum, the point of rest being the correct position of the principal line.

The mathematical proofs of the basic tilt formulae vanish into Brook Taylor's Perspective (1715), the connecting link being a geometric progression. Oblique calibrations, of importance to the Military, was overlooked in this review.

Attention should be called to the fact that the contents of *Applied Photogrammetry* covers a complete planimetric solution, embracing analytical and graphical expansion of control.