A FEW PRINCIPLES REGARDING THE MULTIPLEX PROJECTOR

Earl Church, Syracuse University

THIS is merely the brief statement of a few fundamental principles concerning the relation between the horizontal and the vertical scale of a stereoscopic model formed by overlapping aerial photographs in a Multiplex projector.

In these statements, let f_c be used to designate the perpendicular distance in the aerial camera from the emergent node of the camera lens to the plane of the photograph, a constant value built into the camera, f_p the fixed value in the projectors of the perpendicular distance from the incident node of the projector lens to the plane of the emulsion of the reduced plate print, and f_1 and f_2 the fixed built-in image and object distances respectively in the reducing printer, or the perpendicular distances from the incident node of the lens of the reducing camera to the plane of the film being reduced and from the emergent node to the plane of the reduced print in the reducing camera; and let m_1 be used to designate the value of the ratio f_1/f_2 and m_2 that of the ratio f_c/f_p.

1. The horizontal and vertical scales of the stereoscopic model formed in the Multiplex will be equal when, and only when, m_1 and m_2 are exactly equal. This equality must be attained by the proper design of the reducing camera.

2. If the reducing printer is properly designed in relation to the aerial camera and the projectors, so that m_1 exactly equals m_2, then the horizontal and vertical scales in the stereoscopic model will be exactly equal regardless of the existence of any tilts in the original aerial photographs.

3. If the reducing camera is so constructed that f_1/f_2 or m_1 is not exactly equal to f_c/f_p or m_2, when the original photographs are absolutely free from tilt, the stereoscopic model will have a vertical scale differing from the horizontal scale by a constant ratio.

For example, if $f_c=184.03$ mm., and $f_p=45.49$ mm., giving f_c/f_p equal to 4.056, and if f_1/f_2 equals 3.902, then $m_2/m_1=4.056/3.902=1.035$. If the horizontal scale of the stereoscopic model is 1:7,500, then the vertical scale will be 1:(7,500)(1.035) or 1:7,765.

This corrected vertical scale remains constant regardless of the positions of points on the stereoscopic model and regardless of the variations in topographic relief.

Thus, with truly vertical aerial photographs, if the vertical scale on the plotting table of the Multiplex is calibrated in accordance with the corrected vertical scale of the model, the incorrect ratio of f_1 to f_2 in the reducing printer will introduce no errors whatever in either the planimetry or the topography of the plotted map.

4. Inasmuch as f_c, f_p, and f_1/f_2, can all be determined by actual measurements, the ratio m_2/m_1 for correcting the vertical scale of the model can be readily found for any Multiplex apparatus. It must be remembered, however, that this correction factor holds exactly only in the case of vertical photographs which are entirely free from tilts.

5. If the reducing printer is so constructed that f_1/f_2 or m_1 is not exactly equal to f_c/f_p or m_2, when the original aerial photographs have tilts there is no constant ratio between the horizontal and vertical scales of the stereoscopic model. In this case the correction of the vertical scale by the factor m_2/m_1 as before will approximately eliminate the errors, but not exactly.

Let it be supposed that a pair of aerial photographs taken from an altitude

of 10,000 feet, have three-degree tilts, that they are taken with the aerial camera mentioned above for which f_c is 184.03 mm., that they are reduced with the reducing printer in which f_1/f_2 is 3.902 and then oriented in the Multiplex having projectors with f_p equal to 45.49 mm., in which case $m_2/m_1 = 1.035$ as mentioned above. With the photographs properly oriented and with the vertical scale of the model corrected by this factor 1.035, there will still remain errors in elevations determined in the Multiplex which may be as great as 20′.

Of course, an error exactly analogous to that mentioned above might be introduced by attempting to use in a Multiplex projector aerial photographs taken with a camera other than that for which the reducing printer was designed. In this case again, if the original aerial photographs were entirely free from tilts, all errors in the Multiplex plotting can be eliminated by the above-described calibration of the vertical scale on the plotting table; but if the original photographs have tilts, even with the correction of the vertical scale of the model, errors in elevation will still remain. Again an analogous error could be introduced if the focal lengths of the projectors in the Multiplex were not uniform.

Although this discussion is based upon a somewhat exaggerated error in the ratio of image to object distances for any precise construction of a reducing printer, it is thought that the above statements may be of some intrest

This entire subject is at present being investigated by Mr. G. C. Tewinkel, a graduate student at Syracuse University, and his thesis showing the results of his work will be available later in the college year.

EDITOR'S NOTE: The members of the American Society of Photogrammetry are indebted to Syracuse University and particularly to Professor Church for the valuable publications issued by the University which have been furnished, free of charge, to the entire membership.