A GRAPHIC INVESTIGATION OF THE EFFECT OF LENS DISTORTION ON STEREOSCOPIC MODELS

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IN VIEW of the interest and activity in the problem of compensating for the effect of aerial camera lens distortion on stereoscopic models, it seems advisable to add a graphic determination of this effect to the list of papers dealing with the subject. The following discussion relates to the effect of aerial camera lens distortion on the models used in the Multiplex aero-projector mapping instruments, but it could be adapted to the determination of distortions for any similar case. The principal reason for preparing this paper lies in the fact that a graphic solution of this type may be clearer to some students of photogrammetry than the analytical determination. The author also feels that this method will afford a clearer picture of the variation in the "Y" parallax in the corners of the models due to air base and flight height variations.

The problem resolves itself into the determination of the amount of distortion in a stereoscopic model as a result of distortion in the lens of the original aerial camera. The effect of variation in flight height and of the air base between the camera stations on this distortion will also be considered. It may be noted that the effect of relief of the terrain photographed may be considered as a variation in the flight height.

This demonstration presupposes the use of the distortion curve of a particular Zeiss 10 centimeter Topogon lens such as is used in the Zeiss precision mapping camera. The distortion curve that is employed is the curve determined for the calibrated focal length of the lens and not the equivalent focal length, although the same numerical result would be secured if the latter were used. The ideal condition of aerial photographs, exposed with the lens axis truly vertical, and with the negative plane normal to the lens axis, is assumed and a corresponding disposition of the axes of the vertical projectors and the diapositive plates. The further assumption is made that no distortions have been introduced into the model from any source except that of the taking camera lens.

The plane of best definition for the Zeiss 22 millimeter wide-angle Multiplex projectors, such as are used by the U. S. Geological Survey, is found at a distance of 360 millimeters below the projectors; so the effect of distortion in that plane will be investigated first. The hyperfocal distance for these projectors extends from 280 millimeters to 440 millimeters below the projectors and marks the limits of the field in which satisfactory measurements can be made. This discussion will include an investigation of the distortion effect at the 300 millimeter and the 420 millimeter planes as well as the plane of best definition.

Figure 1 is an elevation and plan of two projectors, *A* and *B*. In this sketch the lines *AC* and *BD* represent the lens axes, while the points *A* and *B* represent the perspective centers of the projectors *A* and *B* situated at a height of 360 millimeters. This sketch also contains a random point *E*, located in the 360 millimeter plane, with rays drawn to it from projectors *A* and *B*. In preparing this drawing for actual use, it should be constructed to full scale, that is, the distance *AC* should be drawn as 360 millimeters, while the distance *AB* should be the actual separation of the projectors on the bar for the ideal case. This distance *AB* should equal the net gain on any picture, say 40%, brought to the scale of the model. It should be noted that this discusion provides a method of determining the amount of error in a stereoscopic model; if one is interested in converting these figures into errors of elevation in nature, then it will be neces-

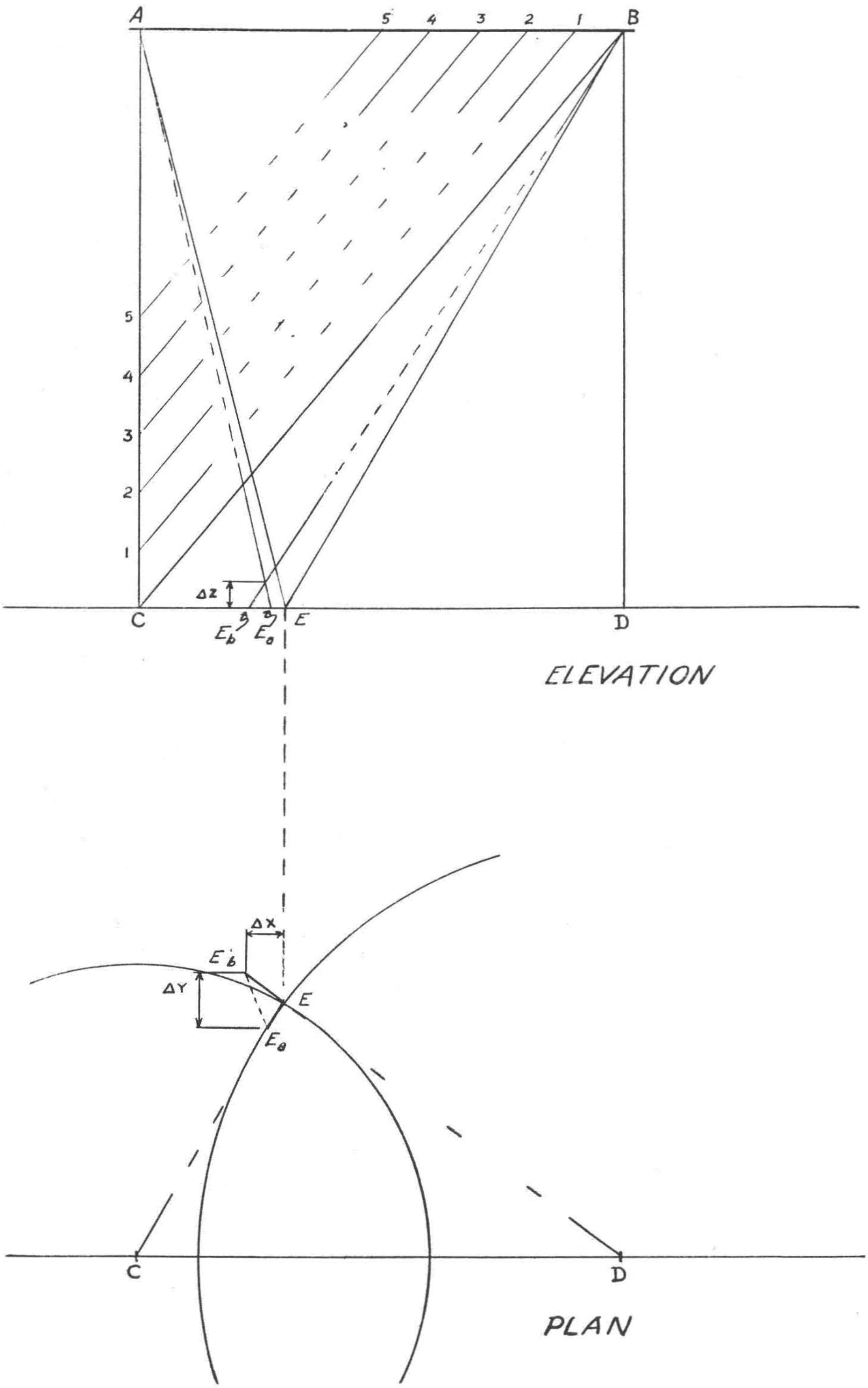

FIG. 1.

sary to multiply the quantities determined for the model by the inverse of the map scale fraction.

In the lower part of Figure 1 will be found the representation of the two intersecting rays shown in plan view. The common objective is represented by by the point E, where the rays from projectors A and B would pass through the 360 millimeter plane were they not displaced by the effect of camera lens distortion. The following formula permits one to determine the displacement that occurs for any ray:

$$\frac{\text{Calibrated focal length}}{360} = \frac{\text{Lens distortion}}{\text{Distortion at the 360 mm. plane}}$$

Or it might be said that we convert the distortions from the scale of the photograph to the scale of the model.

The lens distortion in the above formula should be determined from the curve showing the lens distortion for the calibrated focal length for the particular angle of the ray in question. Referring to the plan view, the distortion for the ray BE as shown as EE_b, and for the ray AE as EE_a, the distance EE_b representing a positive distortion, and the distance EE_a a negative distortion of the aerial camera lens.

Due to the fact that we are dealing with radial distortion of the aerial camera lens, the distorted ray BE_b will fall in the plane EBD. The line ED is the trace of this plane in the 360 millimeter horizontal plane; therefore, the distorted position of the point E will fall in the trace ED or its extension. The same reasoning will apply to the AE ray, so that the ray from A will fall in the trace CE. For this reason, the horizontal angle C-E-D is the true angle for these two ray planes. Consequently, one may plot the amount of distortion along the legs of the triangle C-D-E to any degree of magnification that we may choose. In this case, for ease in plotting, the effect of distortion will be magnified 100 times.

Consideration of Figure 1 will show that the two distorted rays to the point E will fail to intersect in the 360 millimeter plane by the distance E_bE_a. By resolving this distance into an x-component, parallel to the flight line CD, and a y-component normal to the flight line, it will be possible to consider the effect of the distortion in the same manner that the Multiplex operator must consider it. The x-component can then be considered as a measure of the error in elevation of the point E, due to the distortion; while the y-component, in the common parlance of the Multiplex operator, will be the "parallax." While the two distorted rays can never form a true intersection in space, they will *appear* to intersect above the 360 millimeter plane when the point E_b falls to the left of E_a, or below the plane, should the points be in the reverse position, providing the parallax is not too great.

Consideration of the figure will show that the x-component (Δx) is to the vertical error (Δz), as the air base of the camera stations is to the flight height. As the air base and the flight height are a constant for any given case, equal x-components must result in equal vertical errors no matter where the point may fall in the model. To determine the actual value of the vertical error, lay off 10 millimeter intervals along the line CA beginning at the point C. These intervals will be equivalent to 0.1 millimeter in the model in order to magnify the distortion 100 times. To determine the vertical error for the point E, one may lay off the x-component of the line E_aE_b parallel to the line CD in such a position that it will form the third side of a small triangle similar to triangle CAB. The amount of the distortion, or vertical error, can then be read directly from the magnified scale previously laid out on the line AC. 1/100th of the y-component will be the actual value of the "parallax."

A quicker method of solving the x-component may be desirable where we have a great number of points to solve. Draw lines parallel to CB, from thepoints 1, 2, 3, etc., on the line CA, to an intersection with the line AB. A scale will be formed on the line AB with which one may measure the vertical error directly from the x-component, since this scale will be a solution of the equation $\Delta z = \Delta x\ AC/AB$.

The above described method will enable one to determine the veritcal error as well as the "parallax" for any given point in the model. To obtain a systematic series of points covering the model as a whole, it will be necessary to draw a series of circles about the points C and D, the radius of each circle will represent each 10 millimeters abscissa of the distortion curve. The intersection of these circles will provide a series of points each of which must be solved in the manner illustrated for the point E. By interpolation, and connecting the points of equal error, one can obtain what is known as a distortion chart. This will be a diagram at a one to one scale showing vertical errors throughout the model. One can also indicate in the same manner the distribution of the "parallax" components over the model and thus determine the lines where the "parallax" will be zero. This information is of particular importance where one must

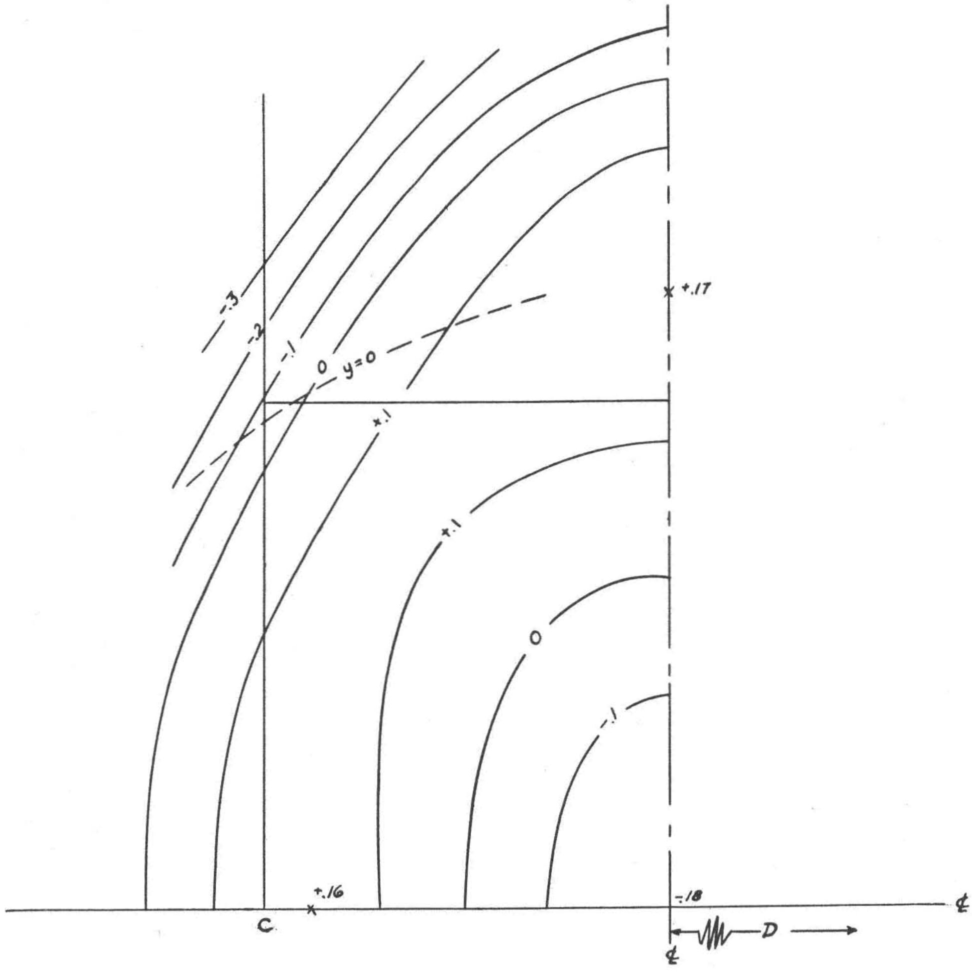

FIG. 2. Height = 360 mm.

take into account the variations of relief, or flight height, in setting up a model for Multiplex drawing.

As has been stated, the above discussions apply only to the ideal cases of flight height and air base. In practice this ideal condition is seldom realized and

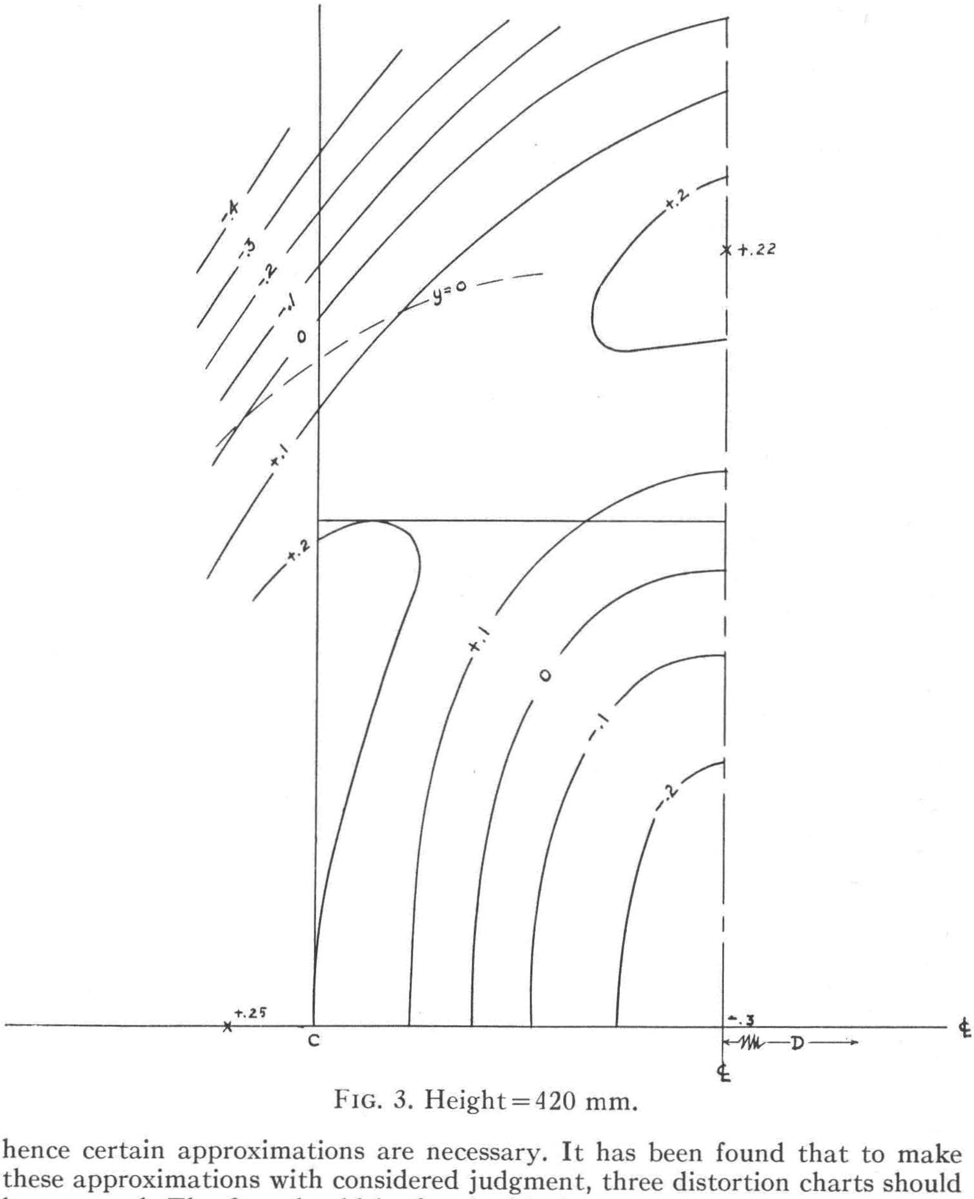

FIG. 3. Height = 420 mm.

hence certain approximations are necessary. It has been found that to make these approximations with considered judgment, three distortion charts should be prepared. The first should be for the ideal case; the second, where the air base is less than normal; and the third, where the air base is greater than normal. These are all compiled with an ideal flight height. In addition, diagrams should be drawn at the 300 millimeter plane and the 420 millimeter plane for each of the above mentioned bases. This may sound complex and burdensome, but the operator would prefer to select a diagram that will fit his particular case, rather than to make too many assumptions from a limited number of diagrams. These nine diagrams will cover practically all of the cases ordinarily encountered by the Multiplex operators.

The author has prepared such diagrams for the 300, 360, and 420 millimeter planes of reference, utilizing the air base of normal length. Reference to Figures 2, 3, and 4 will show these diagrams in one quadrant of a model. The errors affecting the balance of the model can readily be found by placing the diagram in

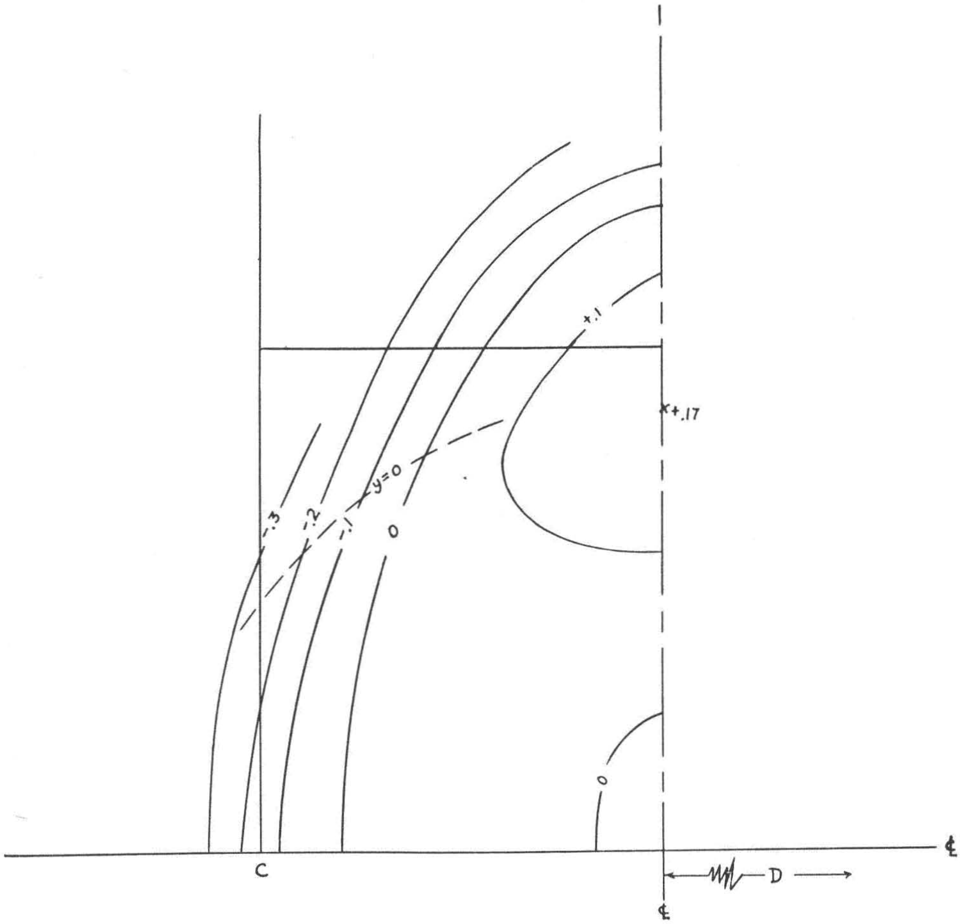

FIG. 4. Height = 300 mm.

appropriate positions as the distortion is symmetrical about the two center lines of the model.

It is evident that no one diagram, or even a series of nine diagrams, can be expected to fit exactly the very large number of cases of variation in base length and flight height that are encountered in practice. For this reason, approximations must be applied in using the diagrams. Usually these approximations can be confined in application to the effect due to changes in air base, or flight height, from that indicated by the diagram. Consideration of the complete diagram as prepared for the Topogon lens previously mentioned will show that the distortion diagrams take the general shape of an elongated doughnut. In other words, the model will be depressed in the center, will rise to a maximum near the points *C* and *D*, and then fall off sharply to limits of the model (or stereoscopic area).

In the use of the charts, it is customary to transfer these lines of equal distortion to the Multiplex map sheet prior to drawing the contours in order that the operator can readily compensate for the distortion effect. To do this, one can center the left projector point of the diagram (point C in Figure 1) under the left projector of the Multiplex instrument and then swing the diagram about the point C until the line CD, which represents the line of flight, passes through the axis of the right projector. One can then transfer the *right* half of the diagram, including the line of no parallax, to the map sheet. Similarly, one can then move the diagram until the point D is centered under the right projector in a corresponding position and then trace off the *left* half of the diagram. The explanation for this method of transferring the diagram to the map sheet can best be illustrated by the use of two diagrams of equal flight height but different air base. By superimposing these two diagrams over each other with the position for the left projector of each diagram coincident, it will be seen that the ridge formed by the maximum distortion on the right hand side of the two diagrams will have about the same position with respect to the left projector in both cases. It will also be noted that the line of zero parallax on the right side will coincide closely. The numerical values taken from any two diagrams having different base-height ratios, will show the variation of the errors and enable one to make the necessary approximations within the reading ability of the Multiplex operator.

To illustrate the application of these diagrams to provide corrections for differences in flight height, use one diagram under, and one over, the elevation in question. An approximation should then be made for the error in base length as described above, and then an interpolation between the two diagrams for the elevation in question. Great care must be taken in applying these approximations when the point in question lies on the one side of the maximum ridge in one diagram and on the other side of the ridge in the second diagram. The variation in this maximum ridge should be borne in mind as the flight height changes. The line of no "parallax" can be interpolated between the two diagrams to conform with the elevation in that part of the model. The general flight height should be determined as soon as the model has been brought to the approximate scale of the map to be drawn.

The applications of these approximations for the purpose of correcting the distortions that may exist in the models of the Multiplex instrument, may be the cause of much anguish to those whose minds may function mathematically. They may not be satisfied with other than definite determinations of the amount of these errors. It is a generally accepted fact that the precision to which various operators can measure elevations in a stereoscopic model is of the order of 0.1 millimeter. Consequently, it is believed that the method of determining corrections that are described above will be satisfactory in practice. The experience of the author has included several cases where there was considerable variation in the elevation of the terrain in a model. The above method of approximation was used in determining the distortion corrections for these errors with the result that the resulting diagram agreed well with the distortion throughout the model, as revealed by the super-abundance of field control.

The author desires to extend his thanks to those who assisted in the preparation of this paper. Their discussion and criticism has been greatly appreciated.