TREE HEIGHTS FROM SHADOWS*

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INTRODUCTION

THERE are several methods of obtaining the heights of trees from air photographs. The image of the tree may be scaled directly, both in vertical and oblique photographs, or it may be measured stereoscopically. Also the shadow method, as described in reference to Canadian conditions in this paper, may often be applied.

The determination of tree heights from air photographs is affected by varying conditions. For instance detail in dense stands is obscured by the foliage. Furthermore the tree height methods apply with less accuracy to broad-crowned trees than to those with pointed tops, while standing dead timber and bare deciduous trees present special advantages and disadvantages due to increased detail of bole and limbs, offset by loss of distinctness. The shedding of the foliage of deciduous trees in the fall makes possible a greatly augmented view of both trees and shadows which lasts until the coming of the new leaves in the spring. Snow forms a smooth, bright surface upon which the shadows are sharply cast and by which the detail in the depths of the stands is illuminated and accentuated. The slope of the ground and various other sources of error affecting in some cases both the tree and its shadow, are described below with particular reference to the shadow method. When on account of adverse conditions the shadow method becomes of secondary value, the angle of view takes on an increased importance, for if the view is too greatly oblique the tree is obscured by its neighbors and if too nearly vertical the tree's length and shape are not fully visible. Generally, an angle of view of 45 degrees or thereabouts affords the best opportunity for the measurement and identification of the trees. This angle is most closely approached near the edges of a vertical photograph, or in the foreground of an ordinary oblique.

Failing a complete view of the tree or shadow, the portion visible may be measured and the unseen portion may be estimated. The inaccuracies which occur in individual measurements compensate each other to a large extent when the tree heights are averaged for the purposes of volumetric timber estimating. Very often an unobscured and accurately measurable tree may be used, particularly by the aid of the stereoscope, as an index to the height of an adjoining group of trees. Difficulties in obtaining measurements for individual trees are most frequently encountered in dense, even-aged stands. On the other hand, as the average height is fairly constant within these stands, a comparatively small number of actual measurements, such as may sometimes be readily secured on the edges of clearings, burns, blowdowns or cuts, are sufficient.

The measurement of the trees and their shadows in the air photographs is facilitated by the use of magnifying lenses and finely-graduated scales.

The shadow method as employed for volumetric timber estimates was originated by the writer in the course of his duties in the Dominion Forest Service, and has been in use for a number of years. Assistance in developing the method was secured from officials of the Topographical Survey Division and of the Dominion

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Observatory, Department of Mines and Resources. Special photographs taken by the Royal Canadian Air Force were of material assistance.

Sun's Altitude

The calculation of the altitude of the sun is essential to the determination of the relation between the height of the tree and its shadow.

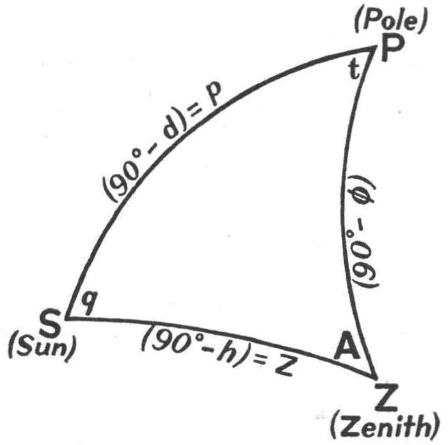

Fig. 1. Astronomical Triangle.[1]

In Fig. 1,

PZ is the co-latitude
PS is the co-declination
ZS is the co-altitude
ϕ is the latitude
d is the sun's declination
h is the sun's altitude
t is the hour angle

From spherical trigonometry,

$$\cos a = b \cos c + \sin b \sin c \cos A.$$

Correspondingly in Fig. 1,

$$\cos (90° - h) = \cos (90° - \phi) \cos (90° - d) + \sin (90° - \phi) \sin (90° - d) \cos t.$$

$$\therefore \sin h = \sin \phi \sin d + \cos \phi \cos d \cos t.$$

The latitude (ϕ) is obtained from the map. The sun's declination (d) is found from tables published by the Surveyor General, or from the Nautical Almanac or similar publications.

The hour angle (t) may be determined by the following steps:

(1) The time of day at which the photograph was taken is secured from the photographer's records.
(2) Standard Time is converted to Local Mean Time.
(3) The Equation of Time is subtracted.
(4) The resultant Local Apparent Time is expressed in terms of the hour angle, which is ordinarily measured from 0 to 24 hours, but which for

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[1] Clark, David, 1931. *Field Astronomy*, p. 14, Constable and Company, Ltd., London.

convenience in these calculations is measured eastwards or westwards from the part of the observer's meridian on which the sun appears at noon.

When the proper values have been substituted for ϕ, d and t, the altitude (h) is found by solving the equation.

Royal Canadian Air Force Photograph

FIG. 2. Tree Shadows in Winter.

Note: Reduced to .75 original size.

TREE SHADOW CALCULATIONS

Region Petawawa Forest Experiment Station
Latitude 46° 00′
Longitude 77° 26′
Date of Photographs March 15th. 1932
d is sun's declination
h is sun's altitude
t is hour angle
φ is latitude

d (with corrections, if desired, for longitude and time of day) (+or–) –2° 13′

Note.--Cos (-d) = +cos d, whereas sin (-d) = -sin d

Sin h = sin φ sin d + cos φ cos d cos t

For noon, Apparent Solar Time

Sin h = sin 46° 00′ sin –2° 13′ + cos 46° 00′ cos –2° 13′ cos 0° . Shadow cast
= .7193 X –.0387 + .6947 X .9993 X 1.00000 . by tree 100 ft.
= –.0278 + .6942 . high
= .6664 . = 100 cot h
h = 41° 47′ . 100 cot 41° 47′
= 112 ft.

Short method for noon, used for verification

h = 180° – φ – (90° – d)
h = 180° – 46° 00′ – (90° + 2° 13′) = 41° 47′

For 11 a.m. and 1 p.m. Apparent Solar Time

Sin h = sin 46° 00′ sin –2° 13′ + cos 46° 00′ cos –2° 13′ cos 15°
= –.0278 + .6942 X .96593 . 100 cot 40° 00′
= –.0278 + .6705 . = 119 ft.
= .6427
h = 40° 00′

For 10 a.m. and 2 p.m. Apparent Solar Time

Sin h = sin 46° 00′ sin –2° 13′ cos 46° 00′ cos –2° 13′ cos 30°
= –.0278 + .6942 X .86603 . 100 cot 34° 59′
= –.0278 + .6012 . = 143 ft.
= .5734
h = 34° 59′

For 9 a.m. and 3 p.m. Apparent Solar Time

Sin h = sin.. .. sin.. .. cos.. .. cos.. .. cos 45°
= .... X .70711 . 100 cot.. ..
= .... =
= ....
h = ...

For 8 a.m. and 4 p.m. Apparent Solar Time

Sin h = sin.. .. sin.. .. cos.. .. cos.. .. cos 60°
= ....X .50000 . 100 cot.. ..
= .... =
= ....
h = ...

To establish relation between Apparent Time and Standard Time

The Standard Time line is moved to the left of the Local Apparent Time line if (1) the region is east of the Standard Time Meridian and (2) the Equation of Time is negative (and vice versa). If Daylight Saving Time has been used a correction of one hour must be made.
Equation of Time (+or–) +9 minutes.
Correction for 2° 26′ ~~East~~, West of Standard Time Meridian 10 minutes.
Standard Time line is 19 minutes of time to Right, ~~Left~~ of Local Apparent Time Line.

Calculations by J. H. G. Seely

FIG. 3

Corrections for refraction and parallax are seldom significant and may usually be ignored.

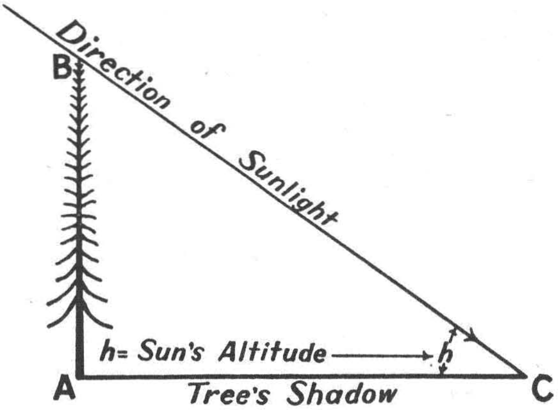

FIG. 4. Length of Shadow and Height of Tree.

In Fig. 4,

$$\tan h = \frac{AB}{AC} \cdot$$

$$\therefore \quad AB = AC \tan h.$$

The first step is to determine the scale of the photograph (see p. 105), after which the value of AC may be found by measuring the shadow in the photograph (see Fig. 2).

The sun's altitude (h) is obtained by the method shown above.

When the proper values have been substituted for AC and h, the height of the tree (AB) may be found by solving the equation.

CURVE OF SHADOW LENGTHS

The foregoing calculations are used when the photographs are dealt with singly but in practice it is often convenient to construct a curve of shadow lengths for use with a whole set of photographs.

Fig. 3 shows a standardized form used to facilitate the compilation of the data required for the construction of the curve. As an example, values have been inserted to indicate the derivation of the curve shown in Fig. 5.

Fig. 5 is a curve of shadow lengths for photographs taken at the Petawawa Forest Experiment Station on March 15, 1932. Local Apparent Time is indicated by a continuous line and Standard Time by dashes.

The hour angle, which is the abscissa of the curve, has been shown in its correct angular proportions, one hour equalling 15 degrees. It would, however, be sound practice to draw the x-axis as a straight line instead of as the arc of a circle.

In constructing the curve, calculations were made for the length of shadow of an imaginary tree of a height of 100 feet. Values were plotted for each hour of Local Apparent Time from 10 A.M. to 2 P.M. The difference between Standard Time and Local Apparent Time is derived in the manner indicated on p. 101. Additional hourly values might have been required if the diurnal period of photography had been longer.

Great economy in the matter of the calculations required is usually made possible by the use of a curve of shadow lengths.

Example.—Refer to the shadow measurement shown in Fig. 2, which is a photograph selected from the set for which the curve shown by Fig. 5 has been prepared. This is a vertical photograph which had been taken at a low altitude for experimental purposes and is accordingly particularly suited for purposes of exemplification.

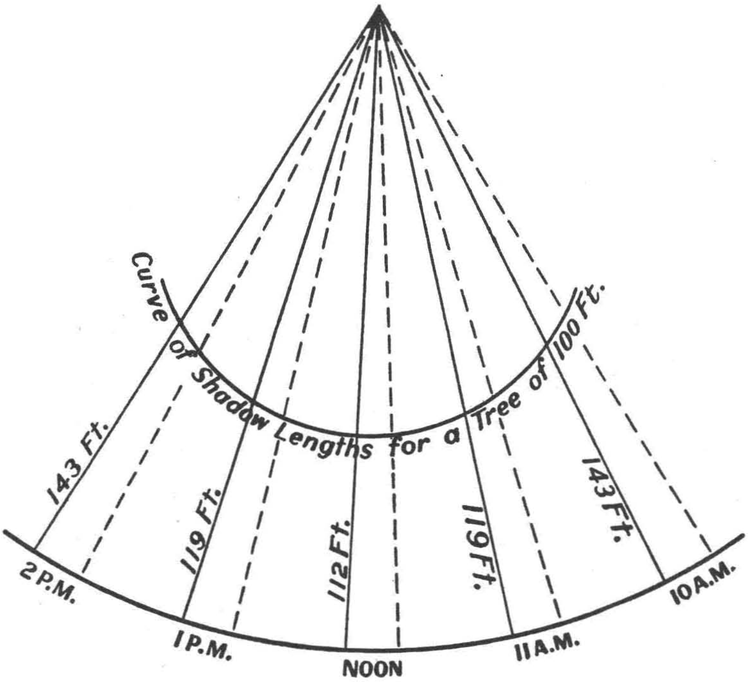

FIG. 5. Curve of Shadow Lengths.

By comparative measurements between points identified on both map and photograph, the scale of the photograph has been found to be 150 feet to 1 inch.

$$\begin{aligned} 1.00 \text{ inch (on photograph)} &= 150 \text{ feet (on ground)} \\ 0.56 \text{ inch (see Fig. 2)} &= \frac{150 \times 0.56}{1.00} \text{ feet.} \\ \text{That is, length of shadow} &= 84 \text{ feet.} \end{aligned}$$

By reference to the data recorded during the flight, it has been found that the photograph was taken at 12:30 P.M., Standard Time.

Interpolating on the curve shown by Fig. 5, it is found that a tree of 100 feet in height has a shadow of 112 feet.

And since tree heights vary in direct proportion to the lengths of their shadows,

$$\begin{aligned} \text{Height of tree} &= \frac{84 \times 100}{112} \text{ feet} \\ &= 75 \text{ feet.} \end{aligned}$$

It may be added that the scale of the photograph is sometimes determined from the relation between the altitude of the aircraft and the focal length of the lens.[2] The stereoscope may be used to estimate local scale variations in hilly country, which are important because the scale at the tree is desired rather than the mean scale of the photograph.

SOURCES OF ERROR

There are a number of factors affecting the accuracy of the shadow method but for the most part the errors are compensating. Thus, while the result for a single tree is often inaccurate, the average for a number of trees may be almost free from error. Furthermore it is often possible to counteract the individual errors. Though many of the sources of error have no practical effect, nevertheless, they are all of considerable interest and are therefore here considered:

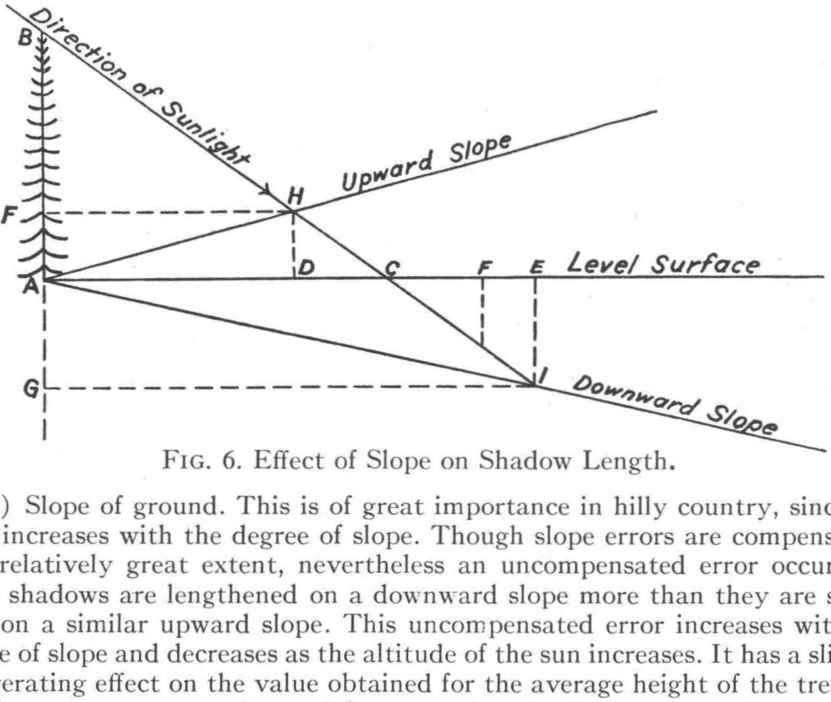

FIG. 6. Effect of Slope on Shadow Length.

(1) Slope of ground. This is of great importance in hilly country, since the error increases with the degree of slope. Though slope errors are compensating to a relatively great extent, nevertheless an uncompensated error occurs because shadows are lengthened on a downward slope more than they are shortened on a similar upward slope. This uncompensated error increases with the degree of slope and decreases as the altitude of the sun increases. It has a slightly exaggerating effect on the value obtained for the average height of the trees.

The stereoscope may be used for correcting the slope errors or for avoiding the difficulty by the preferred practice of utilizing only those shadows that lie along level or nearly level surfaces.

In Fig. 6 the tree AB casts a shadow AC on a level surface. On an upward slope the shadow is shortened by an amount equal to CD and on the equivalent downward slope it is lengthened by an amount equal to CE. CF has been made equal to CD and consequently FE is the uncompensated error.

Fig. 6 also shows that the error in the tree's height, AF or AG, is equal to DH or EI, the vertical distance to the tip of the shadow from a horizontal plane passing through the base of the tree.

(2) Interference by other trees. Nearby trees often interfere with the shadow and prevent an accurate application of the method. The amount of interference depends on forest conditions (see p. 100). This source of error is aggravated when

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[2] The Use of Air Photographs for Mapping, *Topographical Survey Bulletin No. 62*, 1932, pp. 32 and 33, available from Legal Surveys and Map Service, Department of Mines and Resources, 105 George St., Ottawa, Canada.

the sun's altitude is less than 35 degrees, producing unduly long shadows. Accordingly, good results are not usually secured in the months of December and January.

(3) Obscuring of base of tree by tree's own foliage. This prevents the definite locating of the base of the tree image, the point from which the shadow measurement should be made. It may be of some help to visualize a line drawn through the center of the tree image in the direction of the displacement[2] since its junction with the central line of the shadow would mark the base of the tree image.

(4) Depth of snow or low vegetation. An error occurs when the shadow is formed on a layer of snow, grass or underbrush, instead of on the surface of the ground. The error is equal to the depth of the layer and, unless allowance is made, it has the effect of lowering the value of the height of the tree.

(5) Breadth of crown. An exaggerated value is obtained when the crown of the tree is so broad that the shadow is cast from some point other than the tip of the tree. The error increases with the sun's altitude and varies with the slope of the crown but is usually of little importance unless the sun's altitude exceeds 45 degrees. Fortunately the softwood species, especially the spruces, seldom have high, wide crowns.

(6) Leaning trees. Errors from this source are partially compensating and are generally negligible.

(7) Penumbra. The sun's rays do not originate at a single point but on the contrary come from all points of the sun's diameter. The result is that a penumbra appears in all solar shadows. This area of partial shadow increases with the distance from top of tree to tip of shadow, which distance in turn increases with the height of the tree and decreases as the altitude of the sun increases. However, any error that might arise from the consequent lack of a clearly-defined shadow would usually be only a matter of inches.

(8) Miscellaneous. Among other factors may be mentioned the indistinctness caused by slender tree-tops or by shadows cast on dark, uneven surfaces.

TIME FROM SHADOWS

The time at which the photographs are taken is usually recorded during the flight. If, however, the time records are either lacking or inadequate, it is often practicable to resort to a sun-dial method, based on the shadows of the trees as registered in the photograph. Of course it must be kept in mind that the gnomon of the ordinary sun-dial points to the celestial north pole, whereas the tree points to the zenith and must, therefore, be looked upon as a vertical gnomon. Consequently the former marks the hour angle while the latter is directly related to the sun's azimuth.

The following formula provides the link between the hour angle and the sun's azimuth and enables the shadow of the tree to be employed to the same effect as the shadow on an ordinary sun dial.

Referring to Fig. 1,

$$\tan\frac{t}{2} = \frac{\cos\dfrac{\phi - d}{2}}{\sin\dfrac{\phi + d}{2}}\cot\frac{A + q}{2} = \frac{\sin\dfrac{\phi - d}{2}}{\cos\dfrac{\phi + d}{2}}\cot\frac{A - q}{2}$$

$$\text{where } q \text{ is } \sin^{-1}\left(\frac{\cos\phi\sin A}{\cos d}\right).$$

Either of the two forms quoted may be used as convenience dictates.

A line is drawn on the map in the direction of the astronomic north and is transferred to the photograph. A second line is drawn parallel to the direction of the tree shadows. As the position of the sun governs the direction of the shadow, the sun's azimuth (A) may be found from the angles formed by the intersection of the two lines. The formula may then be solved for the hour angle (t), which is reckoned as defined on page 101. Thus it is possible to find the time from the shadows, which provides an alternative when the pilot's time records are missing. If so desired the hour angle (t) may be used to deduce Standard Time (see p. 101).

Of importance from a mapping standpoint is the fact that it is possible to reverse the whole procedure, and determine the orientation of the photograph from the time records.

It may be mentioned that although the direction of the image of the shadow of a vertical object falling on sloping ground may be affected somewhat by displacement,[2] nevertheless the average direction of a number of shadows usually will be sufficiently accurate. In the case of oblique photographs, a transformation of the perspective is necessary, and this may be effected by ordinary plotting methods.

When the time is not demanded for purposes other than the calculation of tree heights or when it is not required for use in a curve of shadow lengths, the calculation of tree heights may be made without reference to the hour angle (t), by employing the following formula:

$$\tan\frac{z}{2} = \frac{\sin\dfrac{A+q}{2}}{\sin\dfrac{A-q}{2}}\tan\frac{\phi-d}{2} = \frac{\cos\dfrac{A+q}{2}}{\cos\dfrac{A-q}{2}}\cot\frac{\phi+d}{2}.$$

In this case, the sun's zenith distance (z), which is, of course, the complement of its altitude (h), is obtained instead of the latter.

The above section on time from shadows is based on information secured from the Dominion Astronomer, Surveys and Engineering Branch, Department of Mines and Resources, Ottawa, Canada.

CONCLUSION

It will be apparent to the reader that the shadows of trees and other vertical objects are of considerable importance in the interpretation of air photographs, not only in determining heights, but also on occasions to indicate either the orientation of the photograph or the time at which it was taken.

While the employment of the shadow method to obtain tree heights for use in volumetric estimating is greatly affected by the character of the forest, seasonal changes, and direction of view, nevertheless the method has proved its value and has been instrumental in securing some surprisingly accurate estimates of timber. The technique of employing the shadow method involves the use of accurately-measurable index trees which may be compared, under the stereoscope, with the adjoining stands.

The shadow method is applied to vertical air photographs almost exclusively because in obliques relatively good opportunities occur for the direct measurement of the tree images, also the shadows generally are obscured by intervening trees.

In certain circumstances the shadow method is the most accurate means of obtaining tree heights from vertical air photographs. The prerequisites are level ground and sharply-defined shadows, such as may be secured in winter when slender-tipped evergreens are found scattered among deciduous trees. Under these conditions the shadow image is usually preferable to the tree image because of its closer approximation to a full side-view of the tree and occasionally because it is affected to a smaller extent by aberration of camera tilt. However, in hilly country or when the view is obstructed by the foliage of dense stands, the measurment of the tree's image is generally more suitable than the shadow method.

To date the main utility of the shadow method has been in relation to trees. However, it may be applied not only to trees but also to ships, buildings, and other vertical objects. When the shadow is cast on a level body of water or on streets, roads, or fields, where sufficient knowledge already exists of the variations in the contour, the slope error (see page 106), does not occur. In these circumstances the height of an object may be obtained more accurately by the shadow method than by measuring the image of the object, either directly or stereoscopically, except when the shadows are unduly short.

AEROPHOTOGRAPHY AND AEROSURVEYING

J. L. Buckmaster, U. S. Geological Survey

COLONEL BAGLEY'S new book *Aerophotography and Aerosurveying* is essentially a textbook and as such it fills a conspicuous gap in our photogrammetric library. Many available books discuss fully limited fields of photogrammetry and others are too technical and too involved for the beginner. Consequently, publication of this volume should be welcomed by that large group of men who are comparatively new in the field of photogrammetry, as they will find it essentially complete and understandable.

It not only contains chapters dealing with the usual photogrammetric matter such as aerial photography and the use of aerial photographs in map preparation, but chapters as well on map projections and the securing of control necessary to map preparation. The latter subjects are frequently omitted by other writers but are none-the-less essential to the student who is interested in the applications of photogrammetry to map making. The reference to the astrolabe is timely, and methods of determining positions with this instrument should be particularly useful to those who are concerned in securing control for maps being made at reconnaissance map scales.

The book should be appreciated by students and others who may have need to inform themselves of those phases of photogrammetry which may fall outside of their field of experience. In this connection it should be said that Colonel Bagley's terminology does not always conform with that employed by the reviewer, which suggests the need for a careful and authentic study of terms peculiar to photogrammetry and the general adoption of the resulting definitions by all photogrammetrists.

The book is descriptive rather than analytical; general rather than exhaustive, but contains as much of mathematics and detailed description as a text of its size and scope will permit. The author is to be congratulated on the preparation of a book which should be found very helpful at this time.