TOPOGRAPHICAL MAPPING FROM HIGH OBLIQUE AIR PHOTOGRAPHS

WITH SPECIAL REFERENCE TO THE TECHNIQUES DEVELOPED AT THE AMERICAN GEOGRAPHICAL SOCIETY

O. M. Miller

EDITOR'S NOTE: This is another chapter for the *Manual of Photogrammetry* which the Society is having prepared by outstanding leaders in this specialized field of engineering.

INTRODUCTION

HIGH oblique air photographs are those taken with the camera axis pointing more nearly horizontal than vertical.

Topographical mapping from high obliques, as described in this article, is an offspring of ground photographic surveying of a type first practiced to any great extent by the Canadian Government under the leadership of its famous Surveyor, General E. Deville. Though writers such as B. M. Jones and J. C. Griffiths[1] had discussed the principles involved and outlined possible procedures, little was done in a practical way prior to 1930 to realize the possibilities of high oblique air photographs except for the making of planimetric maps of flat country. Since then, however, various techniques have been proposed and developed and considerable topographical mapping has been accomplished with them.

The techniques and instruments described in this article have been, for the most part, developed at the American Geographical Society and have been in use since 1931. Projects already completed at this institution are maps of Northernmost Labrador[2] and the Sierra Nevada de Santa Marta in Colombia.[3] Work is proceeding on extensive surveys made in the Yukon Territory. Other completed projects include a map of the Queen Maud Mountains in the Antarctic from photographs taken on the First Byrd Expedition[4] and sketch maps made from Leica photographs taken by Lincoln Ellsworth on his flight across the Antarctic Continent.[5]

The use of high obliques is particularly suitable for the mapping of mountainous country on scales smaller than 1:50,000. This is so for a variety of reasons. The flying height at which the photographs are taken need not be nearly so great as in vertical photography in order to cover the ground efficiently for small scale mapping. The taking of high obliques does not necessarily depend on perfect weather conditions. Furthermore straight line flying and a constant flight altitude are not at all necessary. Accurate height control may be obtained from points many miles distant. Finally when topographical features are to be

[1] B. M. Jones and J. C. Griffiths: Aerial Surveying by Rapid Methods, Cambridge, 1925, Chapter VI and Appendix IV.

[2] Alexander Forbes: Northernmost Labrador Mapped from the Air, *American Geographical Society Special Publication No. 22,* New York, 1938. Map of Northernmost Labrador, 6 sheets, in accompanying slip case.

[3] W. A. Wood: Mapping the Sierra Nevada de Santa Marta: The Work of the Cabot Colombian Expedition, *Geographical Review,* Vol. 31, October, 1941, pp. 639–643, accompanied by map, 1:100,000.

[4] Laurence M. Gould: Some Geographical Results of the Byrd Antarctic Expedition, *Geographical Review,* Vol. 21, April, 1931, pp. 177–200, accompanied by map, 1:500,000.

[5] W. L. G. Joerg: The Cartographical Results of Ellsworth's Trans-Antarctic Flight of 1935, *Geographical Review,* Vol.27, July, 1937, pp. 430–444, accompanied by maps.

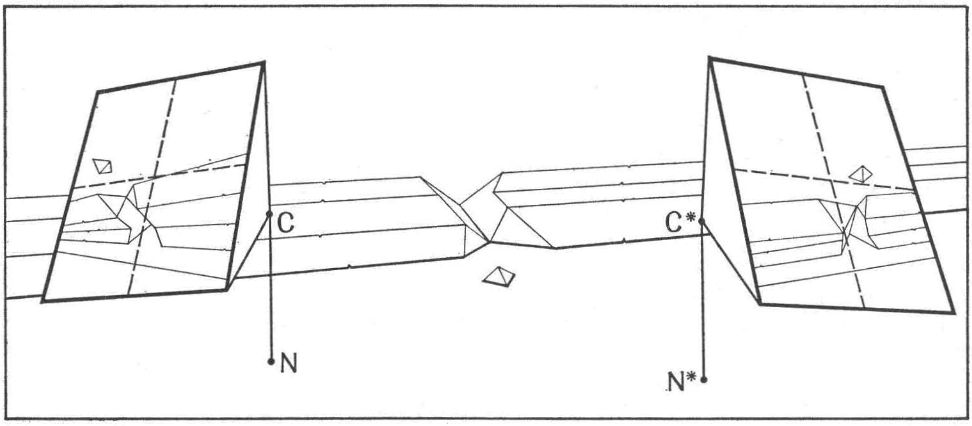

FIG. 1

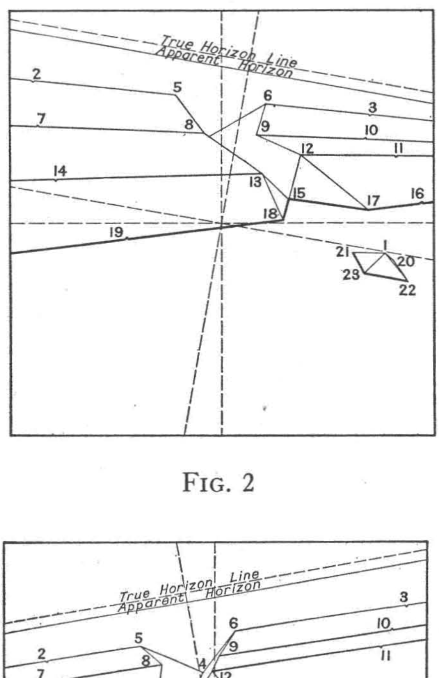

FIG. 2

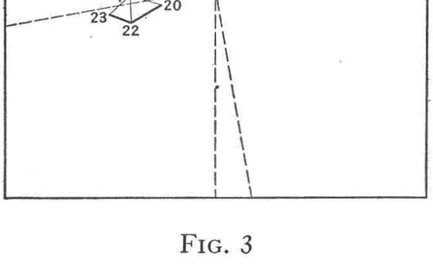

FIG. 3

delineated on small scales by means of sketched contours, oblique views are generally more helpful for this purpose than verticals. One superficial objection to the use of high obliques in mountainous country which has often been made in the past is that there must necessarily be much dead ground not imaged on the photographs. In the vast majority of cases, however, provided the scheme of photography has been carefully planned and executed, what is dead ground from one pair of photographs will not be dead ground from another.

The chief technical problem involved in utilizing high obliques is that of determining the exterior orientations of the camera that took the photographs. When this has been done, it is possible to measure horizontal directions and differences of height from the photographs and the procedure becomes essentially the same as the well-established ground photographic method. The method, therefore, is essentially one of radial line intersections. Contours are sketched as in planetable surveying, but the photographs, instead of field sketch notes are used as guides. Thus each topographical feature can be studied in the comfort of an office from several different aspects. This aids in the truthfulness of the sketching. Stereoscopic examination of the terrain from pairs of photographs is generally possible and is a valuable additional aid.

An outline of the general process of mapping from high oblique photographs is shown schematically in Figures 1 to 6. Figure 1 is a perspective view of two camera stations and the area which is to be mapped from them. Figures 2 and 3 are the photographs. In the area, and imaged on the photographs, are three points, 1, 2, and 3, whose positions and elevations are known. From these three control points the exterior orientations of the two photographs are obtained. The stage in the construction of the map now reached is shown in Figure 4. Shorelines and streams are then plotted and the position and elevations of as many points in the terrain as are considered necessary for constructing the map are determined. The map has now reached the stage shown in Figure 5. From the spot elevations obtained and with the aid of the photographs as guides the contour lines are sketched and thus the final map as shown in Figure 6 is produced.

Most of the work involved in mapping from high obliques is either graphical or instrumental. However, for the analytical processes which required a high order of accuracy such as exterior orientation, computational methods are used advisedly, not only because they are more accurate, but because they provide checks and records of the working and avoid cluttering the plotting sheet with construction lines, and also because in the long run they save time and fatigue. Furthermore, although, in the development of the techniques employed for orientation use is made of spherical trigonometry and elementary differential calculus, in actual practice the computational methods require only the ability to solve linear algebraic equations numerically.

Definitions

The meaning of the symbols used in the article can be ascertained by reference to the figures. A few special definitions are necessary, however, for a ready understanding of the text and these are given below in alphabetical order.

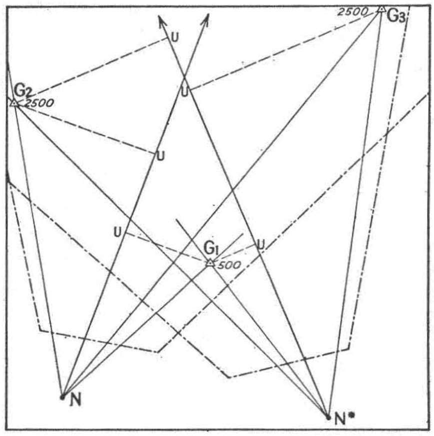

FIG. 4

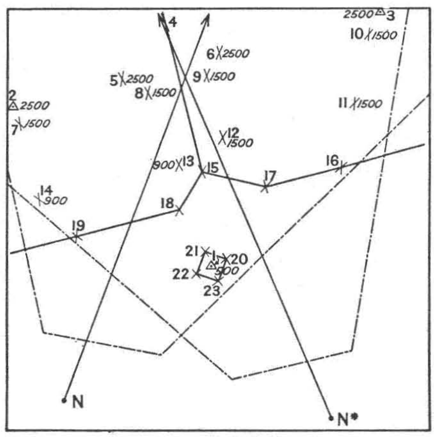

FIG. 5

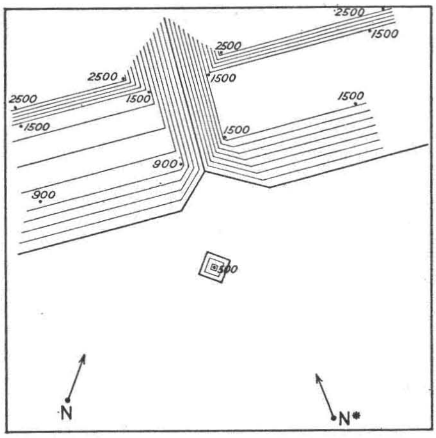

FIG. 6

Air Station (denoted by C) (Fig. 1). The point in space from which a photograph is exposed or the position in space of the *perspective center.*

Apparent Horizon (Figs. 2, 3 and 7). The image on a photograph of the terrestrial horizon.

Exterior Orientation. In this article it denotes the combined process of determining the position of an *air station* relative to the ground and the angular orientation of the photograph.

Horizon Line (True) (Figs. 2, 3 and 7). The imaginary trace on the photograph of the *horizon plane.*

Horizon Plane (Figs. 8, 10 and 11). The horizontal plane containing the *perspective center C.*

Horizontal Angle (Fig. 9). In this article it means specifically the horizontal angle between the *principal plane* and the vertical plane containing the *perspective center* and a ground point G. It is denoted by the symbol O. If G is to the right of the *principal plane* O is positive. If to the left, negative.

Isometric Parallel (Fig. 7). The line on the photograph perpendicular to the *principal line* at an equal angular distance from the *true horizon line* and the *principal parallel.* If j is the intersection of this line with the *principal line* then $pj = f \tan T.2$. (Fig. 8.)

Perspective Center (denoted by C). A single point assumed for mapping purposes where the angular relationships between *perspective rays* to points in the photograph are the same as the angular relationships between rays to corresponding points in the object photographed. Actually the *perspective center* of a photograph corresponds almost exactly to the position, relative to the photograph, of the back nodal point of the camera lens (see also *air station*).

Perspective Ray. The straight line from a *perspective center* to any point object photographed.

Plate Perpendicular (Fig. 8). The perpendicular from the *perspective center* to the photograph. Its distance, commonly called the calibrated focal length or principal distance, is denoted by f.

Principal Line (Fig. 7). The trace on the photograph of the *principal plane.*

Principal Parallel (Fig. 7). The line on the photograph perpendicular to the *principal line* passing through the *principal point.*

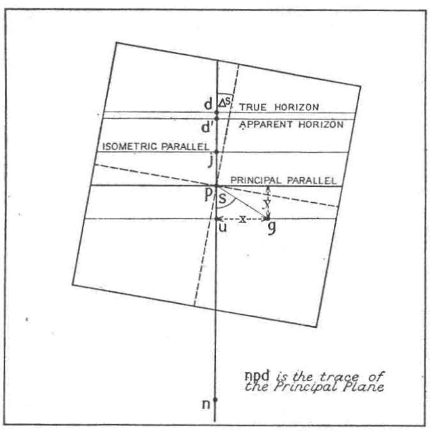

FIG. 7. The paper represents the plane of the photograph.

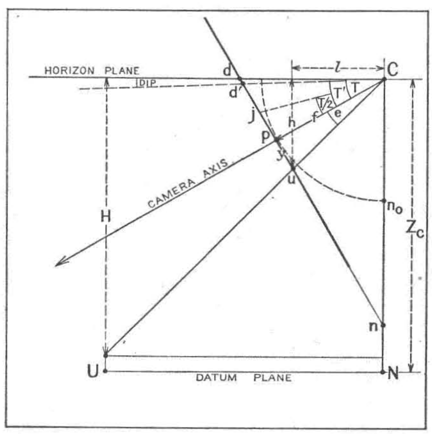

FIG. 8. The paper represents the the principal plane.

Principal Plane. The vertical plane containing the *plate perpendicular.*

Principal Point (Fig. 7). The foot of the *plate perpendicular* on the plane of the photograph. It is denoted by p.

Radial Angle (Fig. 7). The angle subtended at the *principal point* p by the *principal line* and any point image g. It is denoted by S and is measured clockwise on a positive photograph from the *principal line.*

Swing (Fig. 7). The angle of rotation of a photograph in its own plane about its *plate perpendicular.* In this article the term is used to mean specifically the angle at p made by the *principal line* and a line passing through p between the top and bottom collimating marks of the photograph. It is denoted by ΔS and is measured clockwise from the *principal line* on a positive photograph. As it is obvious that a small change in *swing* will cause an equal or corresponding change in any *radial angle*, dS denotes this change in both cases.

Tilt (denoted by T) (Fig. 8). In this article it refers specifically to the angle of depression below the *horizon plane* of the *plate perpendicular* of a positive photograph. Note that in all figures, except 1, 24 and 25, which show negative positions of the photographs, the positive position of the photograph is shown. The positive position of a photograph is assumed to lie in a tilted plane interposed between the *perspective center* and the object space at a perpendicular distance from the *perspective center* equal to f (See *plate perpendicular*).

Tilt Axis. The horizontal line containing the *perspective center* which is perpendicular to the *principal plane.*

Vertical Angle (denoted by V) (Fig. 10). In this article it refers specifically to the angle of depression below the *horizon plane* of a *perspective ray.*

Horizontal and Vertical Angles from Rectangular Coordinate Measurements on a Photograph

In actual practice it would be a laborious procedure to determine the positions of points in the terrain from horizontal and vertical angles derived from rect-

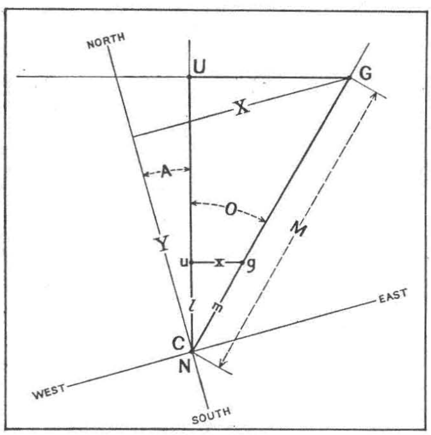

FIG. 9. Horizontal relationships.

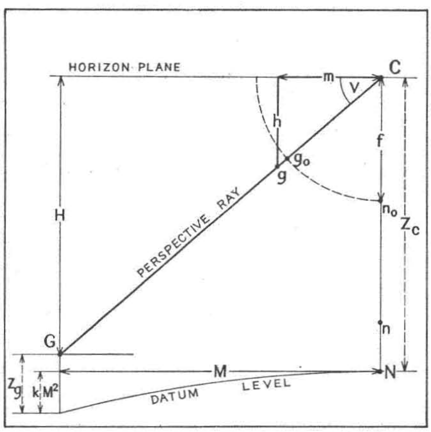

FIG. 10. The paper represents the vertical plane containing a perspective ray.

angular coordinate measurements. Nevertheless, the formulae below constitute the necessary background for the development of the simple graphical methods described later.

From figure 8

$$\tan e = y/f \tag{1}$$

$$l = \text{Cu} \cos (T - e)$$

$$\text{Cu} = f/\cos e\,.$$

From figure 9

$$\tan O = x/l.$$

Therefore by substitution

$$\tan O = x \cos e/f \cos (T - e). \tag{2}$$

From figure 10

$$\tan V = h/m.$$

From figure 8

$$h = l \tan (T - e).$$

From figure 9

$$m = l/\cos O.$$

Therefore

$$\tan V = \tan (T - e) \cos O. \tag{3}$$

Differences of Height Formulae

From Figure 10 the difference of height H between the horizon plane of an air station and a ground point G may be obtained by the usual surveyors' formula

$$H = M \tan V. \tag{4}$$

Substituting for tan V in (3)

$$H = M \tan (T - e) \cos O. \tag{5}$$

Again from Figure 10 the height Z_c of an air station above datum level is

$$Z_c = M \tan V + Z_g - kM^2 \tag{6}$$

where k is a constant of curvature and refraction.

Differentiating Z_c in respect to V

$$dZ_c = M \sec^2 V \cdot dV \text{ when } dV \text{ is in radians}$$

$$= M \sec^2 V \text{ arc } 1' \cdot dV \text{ when } dV \text{ is in minutes of arc}$$

but

$$\sec^2 V = 1 + \tan^2 V$$

and

$$\tan^2 V = H^2/M^2$$

so

$$dZ_c = M(1 + H^2/M^2) \text{ arc } 1' \cdot dV. \tag{7}$$

Formula (7) is of importance in dealing with the problem of exterior orientation.

Differential Relationships Between Angles Measured from a Photograph and Angular Elements of Exterior Orientation

Certain of the fundamental relationships between directions from a perspective center can be determined by means of spherical trigonometry in much the same way as the surveyor or navigator uses the astronomical triangle.

Assume the perspective center of a photograph as the center of a sphere of radius f. The plane of the photograph will then be tangent to the sphere and the principal point p will lie on the surface of the sphere (Fig. 11).

Assume that the horizon plane contains the equator of the sphere and that the direction of the nadir point n is the polar axis. Then in the spherical triangle n_0pg_0 the side n_0p will be the complement of the tilt T, the side n_0g_0 will be the complement of the vertical angle V of any point G and the side pg_0 will be the angle a whose tangent is equal to pg/f. The angle at p will be the radial angle of of the point G and the angle at n_0 will be the horizontal angle O of the point G.

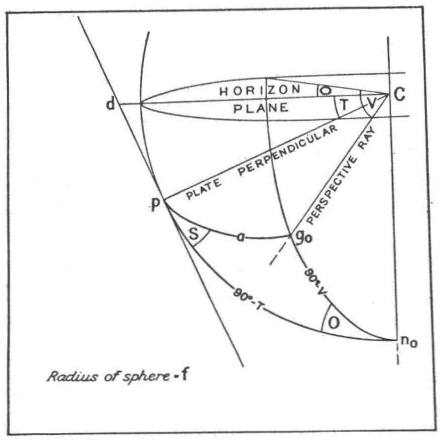

FIG. 11

Using the conventional formulae for solving an oblique spherical triangle

$$\cos(90° - V) = \cos(90° - T)\cos a + \sin(90° - T)\sin a \cos S$$

and

$$\cot O = [\cot a \sin(90° - T) - \cos(90° - T)\cos S]/\sin S$$

or

$$\sin V = \sin T \cos a + \cos T \sin a \cos S \tag{8}$$

and

$$\cot O = [\cot a \cos T - \sin T \cos S]/\sin S. \tag{9}$$

Differentiating V and O in respect to T and S

$$dV = \cos O\, dT + \sin O \cos T dS \tag{10}$$

$$dO = \sin O \tan V dT + (\sin T - \tan V \cos O \cos T)dS. \tag{11}$$

These differential equations are important as they enable the effects on the measurement of vertical and horizontal angles due to small errors in the assumptions of tilt and swing to be readily determined. They will be referred to later in dealing with the problem of exterior orientation.

Differential Variations in the Ground Coordinate System

When great accuracy is required in the determination of position, differential methods are efficient tools for this purpose.

Table of Intersection Errors in Feet
Arguments:—Angle of intersection in degrees and half the sum of the distances in miles.

	10°	20°	30°	40°	50°	60°	70°	80°	90°
50	910	447	301	235	196	173	159	150	146
45	819	402	270	212	176	156	144	135	131
40	728	358	240	188	157	138	128	120	117
35	637	313	210	165	137	121	112	105	102
30	546	268	180	141	118	104	96	90	88
25	455	224	150	118	98	87	80	75	73
20	364	179	120	94	78	69	64	60	58
15	273	134	90	71	59	52	48	45	44
10	182	89	60	47	39	35	32	30	29
5	91	45	30	24	20	17	16	15	15
	170°	160°	150°	140°	130°	120°	110°	100°	90°

Fig. 12

The azimuth of any point G from an air station is the angle $(A+O)$. Therefore from figure 9

$$X_{cg} = Y_{cg} \tan (A + O) \tag{12}$$

$$Y_{cg} = X_{cg} \cot (A + O). \tag{13}$$

Differentiating X and Y in these equations and neglecting the subscripts

$$dX = \tan (A + O)dY + Y \sec^2 (A + O) \text{ arc } 1'd(A + O)$$

$$dY = \cot (A + O)dX - X \operatorname{cosec}^2 (A + O) \text{ arc } 1'd(A + O).$$

Substituting (Fig. 9) linear ratios for the trigonometrical functions and as $d\ (A+O)$ will be the same as dA

$$dX = (X/Y)dY + (M^2/Y) \text{ arc } 1'dA \tag{14}$$

$$dY = (Y/X)dX - (M^2/X) \text{ arc } 1'dA. \tag{15}$$

Standards of Accuracy

A question that is often asked is, "How accurate is mapping from high oblique photographs?" If the question is phrased, "Given a camera and photographic materials which will insure a given standard of accuracy in measurement at what maximum distance from the camera stations can a specified map accuracy be maintained?" then answers for varying conditions may be given. In Fig. 12 is given the maximum error in feet to be expected in the horizontal position of a point intersected from two air stations whose positions are known when the order of accuracy in determining direction is 1 minute of arc. The table has been computed assuming the NG distance to be very much longer than the N^*G distance in the ground triangle NN^*G of Fig. 4. This is the most unfavorable condition. In this connection, however, it is of interest to note that under the most favorable condition (which is when the ground triangle is isoceles) the maximum errors are still very nearly the same. The biggest improvement (as might be expected) is when the angle of intersection is 90°. Even in this case, however, the maximum error from that given in the table is reduced only by about 25 per cent. The table in Fig. 12 can be put to

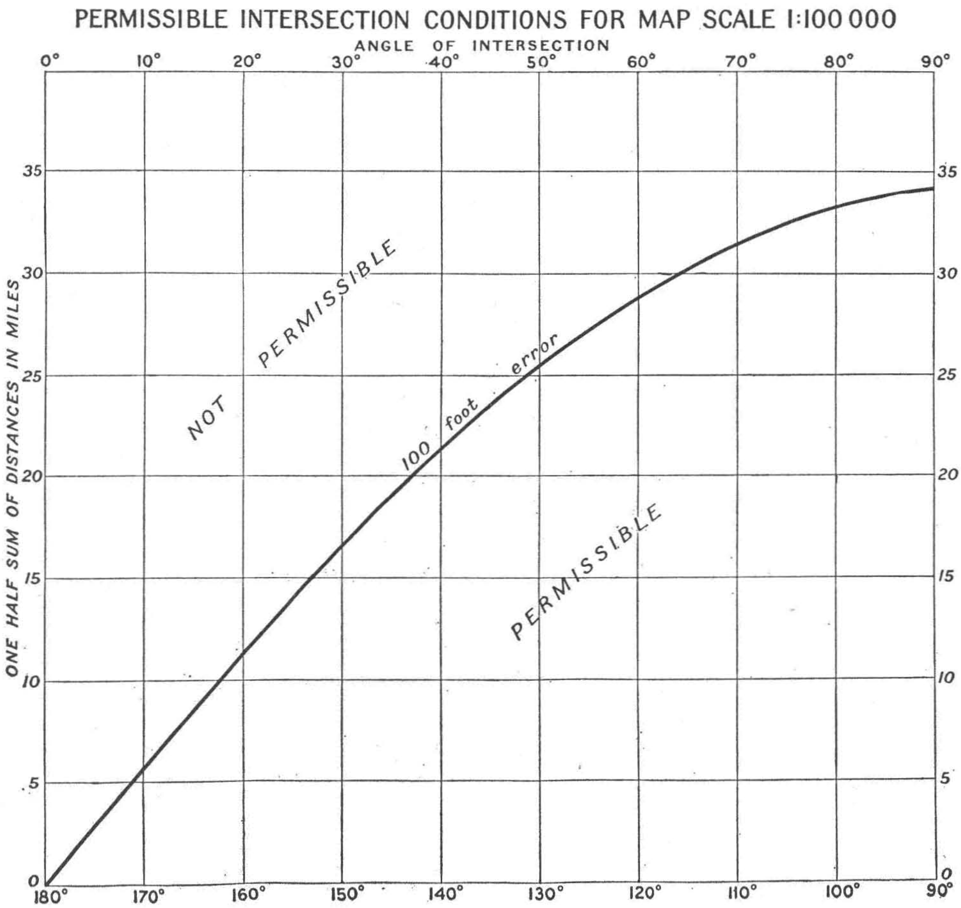

FIG. 13

various practical uses. For instance, if a map is being constructed which will eventually be printed to a scale of 1:100,000 errors of position of less than 100 feet will be unappreciable. The thick line in the body of the table, therefore, is the dividing line for this scale between permissible and non-permissible types of intersections. For any specific scale a graph constructed from the table as in Fig. 13 is more convenient to use. Such a graph is also useful in choosing suitable control points for the resection of air stations and in planning the necessary intervals between consecutive exposures and adjacent flight lines.

When it is assumed that the horizontal distances between the air stations

FIG. 14
POSSIBLE ERRORS IN MEASURING HEIGHTS
from high oblique photographs
Height of Air Station:—10,000 feet.

Distance in miles	05	10	15	20	25	30	35	40	45	50
dZ_v	09	15	23	31	38	46	54	61	69	76
dZ_k	00	01	03	06	09	12	16	21	27	34

and ground points are accurately known, Fig. 14 shows the possible errors (dZ_v) in height determinations introduced by a one minute error in the measured vertical angle when the air station is 10,000 feet above the ground points. For any other set of conditions values of dZ_v may easily be computed from (7).

An independent source of error in height determinations from high obliques is introduced by the fact that in applying a correction to a measured height for curvature and refraction, the precise value of the coefficient of refraction (0.070 is the value normally adopted) is not known. dZ_k in Fig. 14 is the error in height introduced by assuming the coefficient of refraction to be 0.010 in error.

The plotting of detail such as shore lines is often undertaken from single photographs when the difference of height between the air station and the ground level is known. Fig. 15 shows for various values of V and M and a difference of height of 10,000 feet the horizontal displacemements in feet of a plotted point due to one minute errors in the measurement of the V and O angles, dM being in the horizontal direction of the plotted line and the dR being perpendicular to it.

GRAPHICAL METHODS OF MEASUREMENT

In the graphical methods to be described, it is assumed that a positive paper print is used on which to make measurements from the photograph.

TWO METHODS OF EXTRACTING HORIZONTAL ANGLES[6]

Method 1

All points lying in the same vertical plane containing the perspective center of a photograph must obviously have the same horizontal angle O (Figs. 9 and 10). The trace of this vertical plane on the plane of the photograph is a straight line as the intersection of two planes is a straight line. Furthermore, as the vertical plane contains C it must also contain n, the nadir point of C in the plane of the photograph (Fig. 7). It follows that lines of equal horizontal direction on an oblique photograph are straight lines radiating from n. However, the angle formed at n in the plane of the photograph by two lines of equal horizontal direction is not the correct difference in horizontal direction (Fig. 17) except in the case of the truly vertical photograph when T is 90° and n coincides with p (Fig. 16). This statement can be more readily appreciated

[6] It is believed that the first method is here described in print for the first time. Though the author has been aware of the geometrical theory of the method for a number of years it was first suggested by him as a practical procedure in October, 1939, in a letter to Captain Thomas North of the Engineer Board, Fort Belvoir. The second method is the Survey of India method devised by Captain D. R. Crone. See Appendix I. Report of the Air Survey Committee No. 2, 1935. London. His Majesty's Stationery Office.

FIG. 15

HORIZONTAL DISPLACEMENT OF POINTS PLOTTED FROM SINGLE PHOTOGRAPHS

due to one minute errors in the measurement of the horizontal and vertical angles from the perspective center

HEIGHT OF AIR STATION:—10,000 feet

V	80°	70°	60°	50°	40°	30°	20°	10°	05°
M	1763	3640	5774	8391	11917	17320	27475	56712	114300
dM	3.00	3.25	4.00	5.00	7.00	11.50	25.00	97.00	393.00
dR	0.50	1.00	1.75	2.50	3.50	5.00	8.00	16.50	33.00

if one considers the other extreme case when T is zero. Then the nadir line from C will never intersect the plane of the photograph. In other words, in the latter case n will be at an infinite distance from p, and the lines of equal horizontal direction will be parallel to each other and to the principal plane (Fig. 18). The formal proofs of these statements may easily be obtained from formula 2, for when T is 90°

$$\tan O = x \cos e/f \cos (90° - e)$$
$$= x/f \tan e$$
$$= x/y$$

and when T is zero

$$\tan O = x/f.$$

In all other cases when T has some value between 0° and 90°

$$\tan O = x/f \text{ only when } e \text{ equals } T/2.$$

This is demonstrated by inserting this value of e in formula 2.

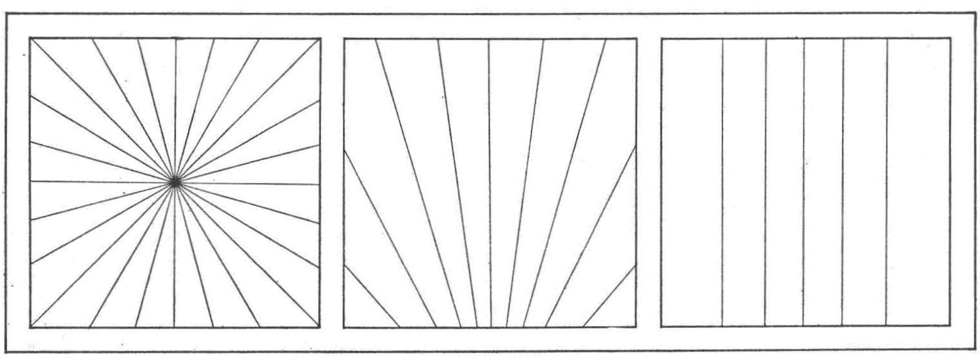

FIG. 16. Lines of equal horizontal direction on a vertical photograph. Angular difference 15°.

FIG. 17. Lines of equal horizontal direction on a high-oblique photograph ($T=30°$). Angular difference 15°.

FIG. 18. Lines of equal horizontal direction on a photograph when T is zero. Angular difference 15°.

Stated in other words

$$\tan O = x/f \text{ when } y = f\cdot\tan T/2$$

or

$$\tan O = x/f \text{ when a point image } g \text{ lies on the isometric parallel.}$$

From the above argument a very simple construction for extracting the horizontal angle O of an image point g may be developed provided the tilt and swing of the photograph are known.

With reference to Fig. 19, lay off a straight line on a piece of paper. Mark a point n at one end of it. Measure a distance np equal to f cot T (Fig. 8).

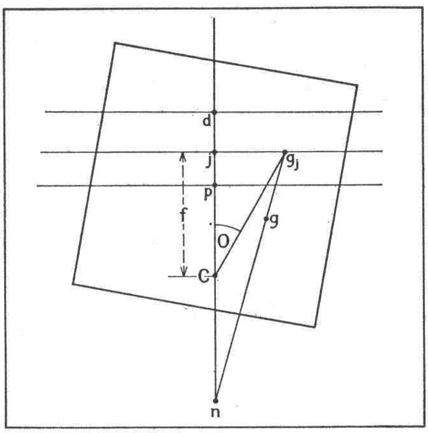

FIG. 19

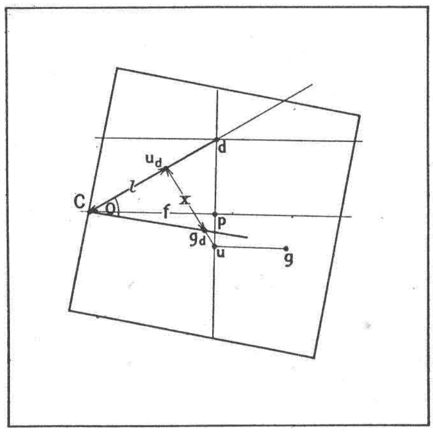

FIG. 20

From p on the same straight line, but in the opposite direction from n, lay off a distance pj equal to f tan $T/2$. Draw lines perpendicular to np through p and j. These lines correspond to the principal and isometric parallels respectively (Fig. 7).

Make marks on the edges of the photograph indicating the principal line and principal parallel and lay the photograph on the paper so that the principal line coincides with the line np and the principal parallel coincides with the perpendicular line through p.

Lay off on a piece of cellulose acetate (or other transparent drawing material) considerably larger than the photograph a straight line. Mark a point C at one end of it. Measure a distance Cj equal to f. Draw a line perpendicular to Cj passing through j. Lay the cellulose acetate over the photograph and drawing paper so that the line Cj coincides with np and the perpendicular at j coincides with the isometric parallel.

The angle O of any image point g on the photograph is then obtained on the cellulose acetate at C in the following manner: Join ng by a straight line and produce this line to cut the isometric parallel at g_j. Join C and g_j. Now because by construction g_j lies on the line of equal horizontal direction passing through g and also on the isometric parallel, O equals the angle jCg_j.

When a large number of O angles are to measured a simple device will greatly expedite this process. Set a thumb tack at n and tie a piece of strong thread to it. Holding the thread taut, line it up with a g point and mark the corresponding g_j point on the cellulose acetate.

This method is not very satisfactory when T approaches zero as then the distance np becomes long. For example, when $f=10$ and T equals 5° np is 114.3. When $f=10$ and T equals 10° np is 56.7. Under ordinary conditions, however, T is usually considerably greater than 10°.

Method 2

When np is too long for convenient handling on a table the following method may be used, though it is not as rapid as Method 1.

With reference to Fig. 20, on a piece of cellulose acetate lay off the right triangle Cpd in Fig. 8. Where Cp equals f, Cd equals f sec T, and pd equals f tan T. Extend Cp and Cd as necessary. Make marks on the sides of the photograph indicating the principal line and principal parallel, and lay the cellulose

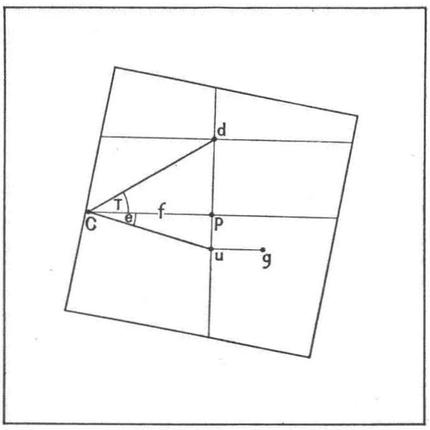

FIG. 21

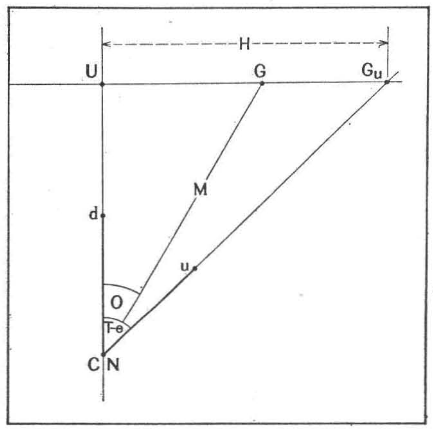

FIG. 22

acetate over the photograph so that Cp coincides with the principal parallel and pd with the principal line.

Drop a perpendicular from g to pd cutting at u. Drop a perpendicular from u to Cd cutting at u_d. From u_d measure a distance $u_d g_d$ equal to ug. Join Cg_d. By comparing Figs. 9 and 20 it will be evident that the angle dCg_d is the required O angle.

A METHOD OF OBTAINING DIFFERENCES OF HEIGHT[7]

Before differences of height can be measured between an air station and a ground point, it is necessary to know the horizontal distance between them.

In this graphical method it is assumed that the horizontal positions are plotted on the map being constructed together with the line NU which is the trace of the principal plane in the datum plane, or in other words the horizontal direction of the axis of the camera taking the photograph.

Referring to Figs. 21 and 22, make marks on the edges of the photograph indicating the principal line. On a piece of cellulose acetate lay off a straight line. Mark a point C at one end of it. Measure a distance Cp equal to f. Draw a line perpendicular to Cp through p and measure the distance pd from p along it (pd equals f tan T). Join Cd.

[7] This is essentially the same as the Survey of India method. See reference in footnote 6.

Place the cellulose acetate over the photograph so that the points p and d lie over the corresponding points on the photograph. Drop a perpendicular from g to pd cutting at u. Mark the u point on the tracing. Join C and u.

Now transfer the tracing to the plotting sheet so that C lies on N and the line Cd lies along the line NU. Draw the perpendicular from the point G to U on the plot cutting the line Cu (produced if necessary) at G_u. Then the length UG_u is the required difference of height H on the scale of plotting.

That this is so is because by construction the angle UNG_u is equal to $(T-e)$ and the length NU is equal to M cos O. Then from (5)

$$UG_u = M \cos O \tan (T - e)$$
$$= H.$$

When a number of elevations have to be determined from one photograph at the same time the method is very fast, especially if set squares or transparent right triangles are used for setting up the perpendiculars from g and G. However, to maintain accuracy great care is required in making the necessary construction.

Horizontal Position of a Point from One Station

If formula 6 is used in order to determine M when Z_c and Z_g are known, it is necessary to solve a quadratic equation. Rather than attempt to do this directly a first approximate value of M is obtained assuming no curvature and refraction. In other words, assuming H to equal $Z_c - Z_g$ determine a value of

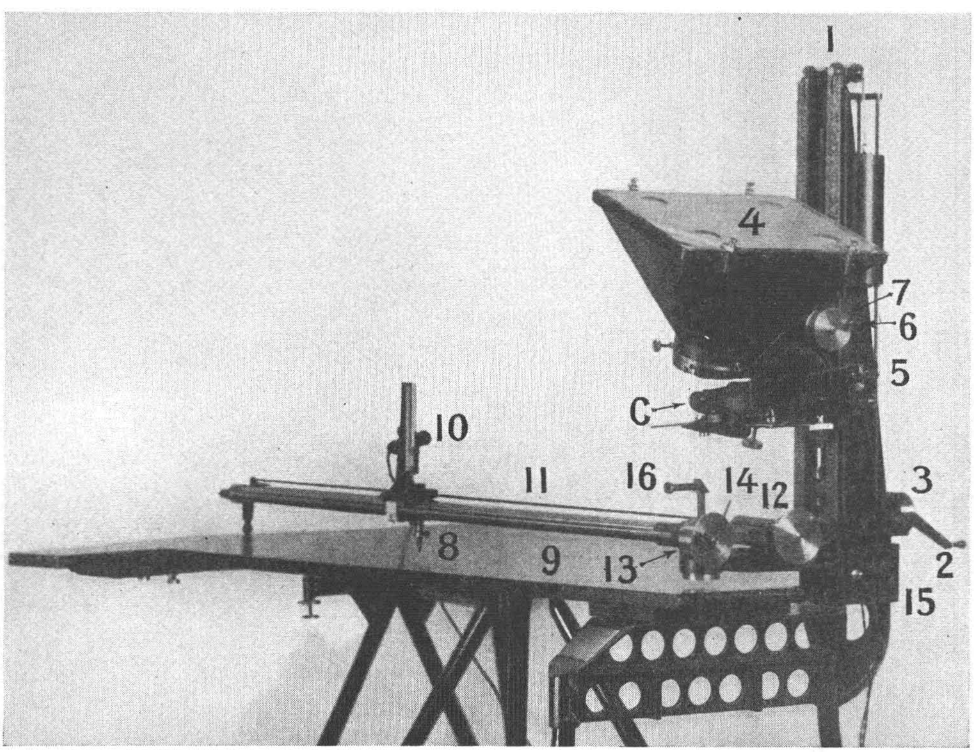

Fig. 23

$'M$ on the plot which will give this value of H. With $'M$ as argument extract the curvature and refraction element from tables and now assume H to equal $Z_c - Z_g + k('M)^2$ and obtain a second value of M, namely $''M$. Repeat the process until the difference between two successive values of M is insignificant. From consideration of Fig. 15, it is obviously poor practice to attempt to fix the horizontal position of a point in this way when M is very large in comparison to H and, consequently, when it is permissible to use the method, never more than two or three approximations will be necessary, as under these conditions the correction kM^2 will always be small.

INSTRUMENTAL METHODS OF MEASUREMENT

THE SINGLE EYEPIECE PLOTTER

The single eyepiece plotter, developed in 1935 by the American Geographical Society, is an instrument with the aid of which a considerable number of things can be done. Its operation may first be described in simple terms. A photograph is placed in the holder shown near the top of the accompanying illustration in Fig. 23 and viewed through the eyepiece, which appears just below and to the right of the uppermost of the three drums. The observer sees a point of light superimposed on the photograph and, by turning the handles on the drums, he can make this point appear to move up or down or to the right or left on the photograph. When the light is so maneuvered, a pencil attached to it is moved on the drawing board and, if the photograph is properly set, shore lines may be plotted, points intersected, and heights determined.

Fig. 24 is a diagrammatic outline of the first model of the oblique plotter. A horizontal axis t–t is mounted on the vertical standard (1) so that it can be moved up and down the latter by turning the crank (2); the height of the axis being read on a drum (3). t–t is the tilt axis about which a "pinhole mirror" C, a photograph holder (4) in which a negative is placed, and a viewing apparatus (5) can be rotated together by turning the crank (6); the tilt being read on the drum (7). The mirror consists of a fully reflecting plane surface in the center of which is a pinhole aperture about 0.4 mm. in diameter. The mirror is adjusted so that the tilt axis lies in its plane and the pinhole is centered on the tilt axis. The photograph holder is adjusted so that the plate perpendicular corresponding to the optical axis of the camera cuts the tilt axis normally at the pinhole. Slow motion movements are provided for swinging the photograph in its own plane about the plate perpendicular and for setting the photograph at its calibrated focal length from the pinhole, and vernier scales are provided for reading the amounts of these movements. The viewing apparatus (5) consists of a telescope the optical axis of whose objective always points to the pinhole.

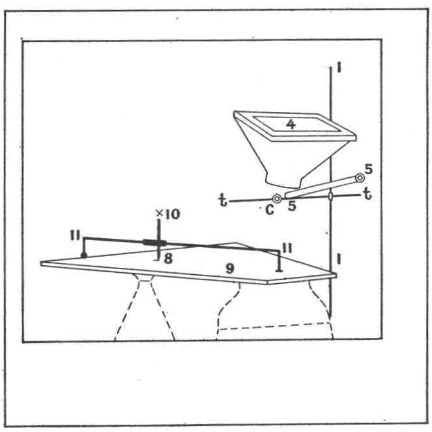

FIG. 24

Attached to and vertically above a plotting pencil (8) which, when operating, touches the surface of the plotting table (9), is an index mark (10) consisting of a small aperture illuminated from behind through a condenser lens. The index mark and pencil are attached to a carriage which is free to move along the horizontal bar (11). The bar (11) is free to swing around the vertical axis passing through the pinhole. By turning the crank (12) the bar is made to swing and a drum is attached which reads horizontal (*O*) angles. By turning the crank (13) the pencil and index mark carriage is made to move along the bar and a drum is attached which reads *M* distances out from the center of rotation. The pencil can be lifted off the plotting table by means of a bar lever operated at (14). The intensity of the illumination of the index mark is controlled by a rheostat situated at (15) and it has been found convenient to make the color of the illumination green or red. The angular scales on the instrument read to the nearest minute and the linear scales all read to the nearest 0.1 of a mm.

It will be apparent from the above description that the pinhole is the perspective center C and that a very thin pencil of rays (virtually a perspective ray) from the index mark on passing through the pinhole is enveloped by a corresponding pencil of rays from a point image in the photograph. The effect to the observer looking through the telescope is to see the image of the index mark superimposed on the photograph. The telescope functions solely as a viewing apparatus whereby the eye is placed virtually at the pinhole.

With the telescope focussed on the photograph the index mark can be seen in sharp definition over a wide range, which on the instrument built is from less than 5 cms. to a meter out from the pinhole.

The operation of the instrument is simple. To plot planimetry once the photograph has been correctly orientated, the operator merely moves the index mark so that its image follows the outline of the feature on the photograph. The scale of the drawing is determined by the ratio between the height of the pinhole above the map and the height of the camera station above the ground. To intersect a point above the map datum level it is necessary to plot first the relative horizontal positions of the two photographs on a plotting sheet and the azimuth lines of their principal planes. The first photograph is taken and correctly oriented, and the sheet is then oriented on the board so that the azimuth line of the photograph's principal plane points at right angles to the tilt axis and the position of the camera station lies directly under the pinhole. To facilitate the latter a centering device (16) has been provided. The image of the index mark is then made to superimpose the image of the point to be intersected, and the pencil is placed in operation and a drawn line drawn on the paper by moving the index mark in towards the operator. The same process is repeated with the second photograph and where the two lines intersect on the paper is the position of the point on the map. Then by placing the pencil at this intersection and lowering the tilt axis until the image of the index mark again covers the photographic image of the point the difference of height (*H*) (Fig. 10) can be read directly on the height scale, the distance (*M*) (Fig. 9) can also be read on the distance scale and the necessary correction to the height for curvature and refraction can be applied at once from tables.

Recently a second model of this instrument has been designed and may possibly be constructed in the near future. It will function in exactly the same way as the first but will incorporate many improvements of a mechanical and operational character. Chief amongst the latter are fast and slow motions for the *M*, *O*, and *H* movements; optical magnification of the image as seen by the operator; an optical linkage system which will enable the whole of a 90° field of view to be scanned over without moving the eyepiece. Ample space (not true of the first model) on the plotting board for orienting a large sheet of paper without interference from supporting structures and more easily read scales are also provided.

Mechanically the new design is more rigid, can be more easily adjusted and can be taken apart for shipment and reassembled without difficulty. It will rest on any strong table rather than having a special stand, as in the first model.

Exterior Orientation

The Method in General

This is the process of determining from measurements on a photograph the position of the air station, the tilt and azimuth of the optical axis of the camera taking the photograph and the swing of the camera at the moment of exposure. The process requires that a minimum of three points whose positions on the ground have already been fixed be imaged on the photograph.

In general the tilt (T), (Fig. 8), of the camera axis and the swing (Δs), (Fig. 7) of the horizon in respect to the sides of the photograph are assumed. With these assumptions horizontal angles to the control points are measured and a horizontal position of the station is determined by the familiar tracing paper method. The height of the air station is then determined independently through each control point and if these heights do not agree the differences are used in solving differential equations which relate the effect of small changes in the tilt and swing to small changes in the height of the air station. Thus, as will later be explained, a consistent solution for the tilt, swing, and height of the air station may be found. As will be gathered from examination of formula 11, small errors in the assumed tilt and swing affect the horizontal angles very little so that only when the assumed values of tilt and swing have large errors or when maximum accuracy is desired for control purposes will the horizontal position have to be redetermined.

Choice of Control Points

If only three control points are available the strongest condition for orientation purposes is when two of the control points are imaged in the background of the photograph on either side with consequent large O angles and small V angles, and the third point is imaged in the center of the photograph near the principal point with a small O angle and a V angle approximately equal to the angle of tilt. There are three reasons for this. In the first place, this arrangement insures against the condition when the horizontal position of the air station lies near the circle containing the horizontal positions of the three control points, for when this happens, as every planetabler knows, no accurate determination of the position of the station is possible. Secondly, errors in the assumed tilt and swing will have the least effect on the resected horizontal position. Again this can be readily appreciated by examination of formula 11. Thirdly, if the expression for dV in (10) is inserted in (7) there results the expression

$$dZ_c = M(1 + H^2/M^2) \cos O \text{ arc } 1'dT + M(1 + H^2/M^2) \sin O \cos T \text{ arc } 1'dS \quad (16)$$

which relates the effects of small errors in tilt and swing on the measured height of the air station. Now in the suggested distribution of the control points it will be observed that the differences between corresponding coefficients in this expression between any two control points will be large. The importance of this will become apparent when the actual process of determining the true tilt and swing from the assumed values is described.

In practice it will very often happen that the control points available are not distributed in the best manner for the resection process. Strong determinations of tilt and swing are always possible, however, provided the horizontal position of the air station does not lie near the circle containing those of the control points and that there is a wide variation in the measured O angles and M distances to the control points.

Assumption of Tilt and Swing

In exposing the camera the endeavor is made to tilt it so that the apparent horizon will appear near the top edge of the photograph. If a clear cut apparent horizon is imaged then a good estimate of tilt and swing may be obtained by determining the so-called dip angle (Fig. 8). This angle in minutes equals

$$0.9878\sqrt{Z_c} \text{ (in feet)}$$

or for purposes of a first approximation

$$\sqrt{Z_c} \text{ (in feet)}$$

Therefore, from Fig. 8

$$pd = \tan (T' + dip)/f$$

where

$$\tan T' = pd'/f.$$

Often in mountainous country or when there is haze the apparent horizon may be only poorly defined or not defined at all. Under these circumstances its position on the photograph must be estimated. It is surprising, however, how accurately this can be done with a little experience. In the Survey of Northernmost Labrador, for instance, the average error in estimating the tilt of a large number of photographs was only 14′ and the greatest single error 1°25′. Only 2 per cent were over 1° in error. In the same test the average error in estimating the swing was 18′ and the greatest single error was 2°56′. Only 5 per cent were over 1° in error.

Having determined the assumed tilt and swing and the tentative position for the point d, it is necessary to indicate these elements on the photograph before measurements can be made. When an instrument is used together with negatives or transparent positives, all that is required to set up the assumed tilt and swing in the instrument is to indicate the position of the points p and d. This should be done very carefully by scratching fine crosses on the emulsion side of the photograph with a needle point. Later when the adjustment is complete small corrections to the assumed tilt and swing may be set on directly by means of the mechanical movements. When graphical methods are used plot the position of p with a fine ink cross and the assumed positions of d and j with similar crosses made with a very soft pencil. Indicate the assumed true horizon line, isometric parallel, and principal parallel with marginal marks on the sides of the photograph also made with a soft pencil.

Tilt and Swing Determinations

With the assumed tilts and swing draw the horizontal direction lines from the perspective center to the three control points and obtain the approximate position of N on the plotting sheet by slipping the drawing under it and orienting them together until the three lines pass through the plotted positions of the control points. Then determine the height of the air station through each control point by the methods already outlined. Now if $'Z_c$ is the height of the air station as determined in this way, then $dZ_c = Z_c - 'Z_c$. Substituting in equation (16)

$$M \cos O(1+H^2/M^2) \text{ arc } 1\, dT + M \sin O \cos T(1+H^2/M^2) \text{ arc } 1\, dS = Z_c - 'Z_c. \quad (17)$$

$M \cos O$ and $M \sin O$ are obviously the distances NU and UG of Figure 4 and are therefore easily derived by direct measurement. Also arc 1, if dT and dS are in minutes, may be given the approximate value 0.0003.

Compute the coefficients of dT and dS for each control point. Then by subtracting one equation from another, Z_c is eliminated and a pair of simultaneous equations will result which enable values of dT and dS to be determined. Z_c can then be determined by solving each of the initial equations and the results should agree. Check the adjustment further by setting the corrected tilt and swing or indicating them on the photograph and then remeasuring the height of the air station through each of the control points. If the results do not check within reasonable limits the whole process should be repeated, using, however, the same coefficients in the equations. A worked out example taken from actual practice is shown in Figure 25.

If graphical methods of measurement are being used, indicate the final positions of the true horizon, isometric and principal parallels with ink lines on the sides of the photograph.

FIG. 25

G	1	2	3
M	19,790	31,610	29,980
UG	8,202	−1,535	−8,623
UN	18,011	31,573	28,713
Z_g	828	1,236	1,082
H	501	110	297
kM^2	−26	−68	−61
$'Z_c$	1,303	1,278	1,318
dZ_t	+79	+140	+128
dZ_s	+31	− 5	− 32
Z_c	1,413	1,413	1,414

Equation 17 for G_1	$+5\cdot 2\, dT - 2\cdot 4\, dS = (1,303)$
Equation 17 for G_2	$+9\cdot 2\, dT + 0\cdot 4\, dS = (1,278)$
Equation 17 for G_3	$+8\cdot 4\, dT + 2\cdot 5\, dS = (1,318)$
(2−1)	$+4\cdot 0\, dT + 2\cdot 8\, dS = -25$
(3−1)	$+3\cdot 2\, dT + 4\cdot 9\, dS = +15$
	$dT = +15'$
	$dS = -13'$

Only slide rule accuracy is necessary for computing the coefficients and solving the equations.

REFINING THE HORIZONTAL POSITION AND THE PROCESS OF GROUPING

If only three control points are used in the orientation process a consistent result will always be obtained, but this may not be the true result because of the impossibility of measuring angles closer than one minute or because of poor distribution of control or personal errors of measurement and identification. Therefore, it is always desirable to use more than three control points if these are available. Let us suppose that horizontal directional rays have been drawn to half a dozen or more control points from a station. It is a familiar experience in making a graphical resection to find that some of the rays give an apparently good result at one point and another selection of rays gives a similarly good result at another point. How is a decision to be made as to which is the best of the points or which the best compromise position?

One obviously practicable way of attacking the problem is to use the most distant control in making the tilt and swing adjustment and in determining the bearing of the optical axis and to make the determination of horizontal position from the near-by control. Another method is the process of grouping. Suppose, for instance, that on one photograph several control points imaged are peaks of a distant mountain range appearing on the right. Others are peaks of a much nearer mountain range appearing on the left, and in the foreground there is a series of fixed points on a river or lake. A graphical resection is made to determine approximately the position of the station and the azimuth of the optical axis. Assuming one of the coordinates of the position—say the X coordinate—and the azimuth of the principal plane, the Y coordinate position is computed by means of formula 13 through each of the control points. If the computed Y coordinates do not agree with each other within prescribed limits compute the coefficients in equation (15) for each point and put $dY = Y_c - {}'Y_c$ where ${}'Y_c$ is the computed Y coordinate of the air station. Then solve for dX and dA in the same way as described for determining dT and dS. Y_c can then be deduced from the initial equations.

As only three sets of initial equations are necessary to get a consistent solution, the control points are placed in three groups according to their locations and the coefficients and values of ${}'Y_c$ of each point within a group are meaned. Thus the finally computed horizontal position is an average position in which the errors within any one group are balanced. If Y is assumed and X computed, formulae (12) and (14) may be used. Y is assumed when the control is more northerly or southerly than easterly or westerly and X when the opposite is the case. The same process of grouping may be employed in the determination of tilt and swing. As only slide rule accuracy is needed for the computations, this process of grouping takes very little longer than if only three points are used, but the resulting increase in accuracy is very marked.

Intersections and the Plotting of Detail

Identification of Points

In making intersections from high oblique photographs, the hardest task and the one that consumes the greatest amount of time is the identification of points on the photographs. The strongest intersections geometrically are, of course, obtained if the optical axes of the two cameras are at right angles to each other. Identification, on the other hand, is difficult because the pictures show features from such entirely different aspects. If the optical axes of the camera taking the photographs were parallel at the moments of exposures, the intersections will be acute but the identification of points will be easy, especially if a hand stereoscope is available so that the terrain imaged can be viewed three-dimensionally. In taking the photographs, therefore, the scheme should be such that both types of pairs of photographs are available, the first for obtaining strong intersections and the second for easy identification of points.

In choosing points experience shows that it is best to choose small surface indications such as edges of rock outcrops, abrupt changes in vegetation, or snow patches, appearing near the topographical point that has been chosen for mapping. Culture, such as roads, houses, etc., are always easy to identify and the same applies to streams. Shorelines, on the other hand, if the pair of photographs have been taken at widely different times of the day should be treated with suspicion as far as identification of points is concerned because if the tidal range in the vicinity is at all large, the shoreline may present completely different outlines from one photograph to another. Similarly, it is always desirable to be extremely skeptical in the identification of features near the surface of the water, such as very small islands, shoals, or sand bars.

Auxiliary Plotting Sheets

When a large number of intersections have to be made from a pair of photographs, it is wise to indicate the points to be intersected on extra prints of the photographs. Auxiliary plotting sheets on which to make the intersections and height determinations are also desirable. If both the auxiliary plotting sheets and the master plot are made of transparent material such as cellulose acetate, it is easy to transfer points and lines from one to the other, and this procedure has the advantage of keeping the master plot clean and free of cluttering construction lines.

Number of Intersecting rays

For purposes of checking the accuracy of the work, it is desirable to make three ray intersections, but when the triangle of error by one three ray intersection is found to be negligible, in order to save time in working over a small area, all other points in the immediate vicinity need thereafter only be intersected from the two stations giving the strongest intersections. However, if a point being intersected is to be used later for extending the control it should be intersected from as many stations as possible. If in this case the intersecting rays do not plot precisely to a point the differential methods already outlined in the horizontal resection processes can be used with advantage and the intersecting lines may be grouped as before in order to get a balanced position. Occasionally points on a shoreline at sea level or on a lake, whose height above sea level has already been determined by elevation determinations to other points on it, may be fixed in horizontal position when only imaged on one photograph by the method of successive approximations already described. This method, however, should only be used when the points are not too far away from the air station (see Fig. 15).

Order of Plotting

In plotting points and planimetrical features it is advisable to do this in two stages. Only when the culture and the drainage have been plotted over a section should the points whose spot elevations are to be used for contour sketching be chosen. The reason for this, of course, is to break down the terrain into comparatively small areas, for by so doing the process of choosing and identifying the points necessary for adequate contour sketching is much simplified.

Tangent Rays

Shore outlines and streams of low gradient can be plotted directly on the single eyepiece plotter or, if an instrument is not available, by the Canadian grid method described elsewhere (not in the present article). Such plottings will usually be distorted to a certain extent because they are made from one photograph (Fig. 15). Consequently a sound practice before drawing such outlines on the master plot is to draw tangent rays from other photographs to headlands and embayments or prominent curves on streams and then to fit the detail to these guiding lines. Streams of high gradient should always have well distributed spot elevations determined along their courses.

SKETCHING TOPOGRAPHICAL FEATURES

A knowledge of planetabling and physiography is of great help in the selection of points to be used as control. As in all other mapping processes, experience improves the quality of sketching from photographs. Before sketching a topographical feature by means of contour lines it should be studied from as many photographs as are available, stereoscopically if possible. Again it is convenient to do the sketching on the auxiliary plotting sheets and only to draw the finished lines on the master compilation.

FIELD WORK

TYPE OF PLANE AND CHOICE OF CAMERA

In mapping projects in which high obliques are going to be used circumstances beyond the surveyor's control generally dictate the type of plane and camera that he will use. This, incidentally, brings out one of the advantages of high oblique surveying, which is that this process of mapping is not completely dependent on special types of planes or precise mountings for cameras. The cameras themselves, therefore, though they should be precise survey cameras, may be hand cameras. When this is so it is obvious that lightness is advantageous, so that cameras having short focal lengths are to be preferred. The airplane used should, if possible, have either a clear range of view on both sides of the line of flight or there should be clear views both forward and to the rear. If neither of these conditions exist and the fields of view from the plane are very limited, the amount of flying necessary to cover the ground adequately with photographs is, of course, increased. If the airplane is sufficiently well adapted it would be an efficient plan to use two cameras and two operators, one working on either side or at either end of the aircraft. The ideal camera equipment would be a multiple lens camera so mounted in the plane that the coverage at one exposure extends around the whole horizon.

PROGRAM OF PHOTOGRAPHY

Experience shows that the height of flight should, if possible, be at least as high above the highest point in the terrain as the latter is above the lowest point. On the other hand, it is impossible to give any general rule as to the frequency of exposures or the separation of the flight lines in high oblique surveying. The tables in Figures 12 to 15 will be of aid, but nevertheless each mapping project has its own individual problems and in planning flight lines consideration must be given to the intended scale on which the map is to be published, to the type of country being photographed, its ruggedness, etc., the meteorological conditions in the area, the time of day when the photographs should or must be taken, and the types of airplane and camera available. Even then, when all the contributing factors have been studied, it is largely a matter of individual judgment as to what will be the best scheme.

As has already been remarked, in order to obtain strong intersections but easy identification of points the scheme of photography should be planned as far as possible in such a way that 50 per cent of the photographs are taken at right angles to each other and 50 per cent with parallel axes. One special point should be emphasized. The photographer taking the photographs should know and understand how they are to be used later. Furthermore, he should be en-

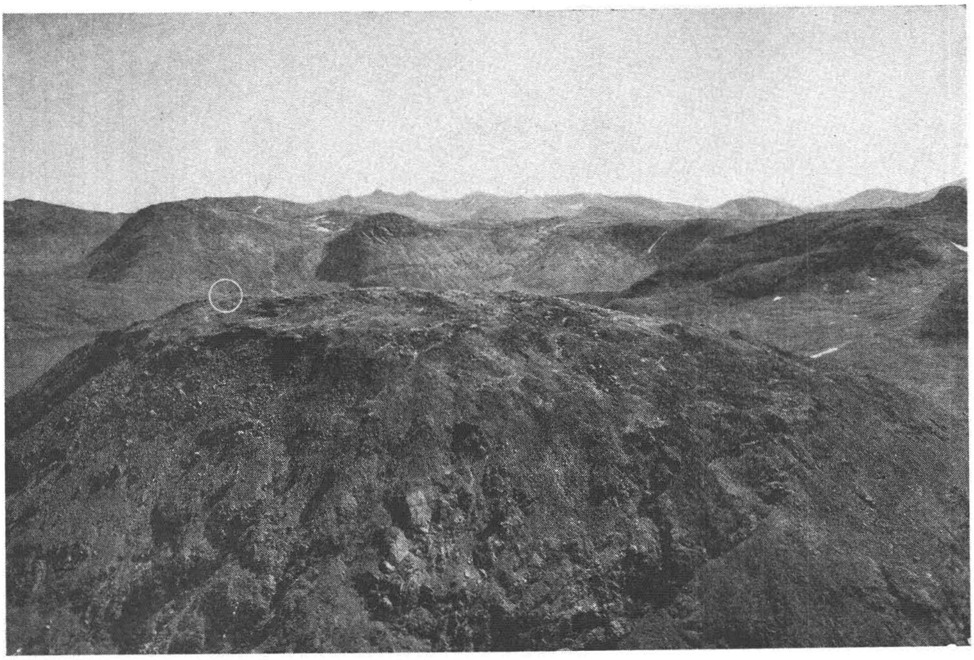

FIG. 26 (top) and FIG. 27 (bottom) are much reduced reproductions of two air photographs, the first taken at a low altitude and the second from about 7500 feet. They both show the same hill on the top of which was a survey flag 7 feet high. In the original photograph of Fig. 26 the flag is just discernible within the white circle.

couraged to use his own initiative and take additional photographs beyond those planned in order to get especially good views or when he sees that if they are not taken certain features will be hidden.

Initial Ground Control

Under ideal conditions where secondary or tertiary triangulation systems cover the terrain being mapped, it is still necessary to put in additional ground control. This is because small survey beacons are generally not-identifiable from high oblique photographs. This is clearly brought out in Figures 26 and 27.

The additional control would consist of points intersected from the triangulation stations which can be readily identified in the photographs. It is highly desirable, therefore, to have this field work done after the photographs have been examined. Unfortunately, this will rarely be possible in practice.

Local Triangulation System

The more usual circumstances are that no triangulation system exists and that a ground field party must put in whatever control they can within the limited period during which the reconnaissance and photographic flights are being undertaken. Under these circumstances the most efficient ground survey work will consist of small local triangulation nets extended from small measured base lines fixed geographically by astronomical methods. From the extended local triangulation stations prominent features in the landscape will then be intersected.

Auxiliary Ground Photographs

As it is very difficult for the surveyor on the ground to know what points are likely to be conspicuous from the air and to safeguard against his intersecting nothing but useless points, it is highly desirable that he be provided with a surveying camera. Then at the local triangulation stations, instead of making sketches for identification purposes and taking rounds of angles with a transit, he will cover the whole circumference of the horizon with photographs. This procedure saves a great deal of time in the field, especially as the necessary and desirable intersection of points for control purposes may be made later from the ground photographs in the home office using similar techniques to those already described.

Extending Control

In reconnaissance surveying from high obliques it sometimes happens that there is no time to put in even these local triangulations. When this is the case the photographs can still be used to make a map by extending the control through the photographs by alternately intersecting and resecting, but the scale and orientation of the map will then only be approximate and will depend to a large extent for this on previous work in the area by others, such as independently determined positions in latitude and longitude and small local surveys.

General Office Procedure

If a plotting instrument is available one set of paper prints in addition to the negatives or transparent positives should be made. If graphical methods are used two sets of prints should be made, one for measurement purposes and one on which to mark the identification of points.

Before the field work has been commenced a map from all available sources should have been compiled for planning purposes. As soon as the photography is completed and the photographs have been received in the home office together with navigation and flight data, the approximate courses of the flight lines and the positions of the exposure stations should be plotted on this preliminary map. When this has been done the individuals who are actually going to construct the map should then learn the country from the photographs. It saves time in the long run to spend plenty of time on this. In fact, those who are undertaking the work should not attempt to start mapping until they feel that they know the country almost as if they had lived in it. For instance, any photograph taken at random should be immediately identifiable in position on the preliminary plot without reference to its serial number. As familiarity with the country increases, uncontrolled sketch maps of the general topographical relationships should be made. It helps also to give names to prominent features (not for permanent use, but merely for easy reference and identification).

As a further preliminary to actual measurement all available ground control points should be plotted on the master plotting sheet and also identified and indicated on the auxiliary prints of the photographs on which they are imaged.

The general order of procedure thereafter is first to complete the exterior orientation of all necessary photographs, then to plot the planimetry such as shore lines, drainage, and culture, then to choose and intersect the points required for contour sketching, and finally to sketch the contours.

In order to carry out the first of these steps efficiently it is essential as a further preliminary measure to make out a program outlining each successive step, as for instance the order in which the photographs are to be oriented and the order in which the major points are to be intersected for purposes of extending the control by alternate intersection and resection.