STEREOSCOPY

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EDITOR'S NOTE: This is another chapter for the Manual of Photogrammetry which the Society is having prepared by outstanding leaders in this specialized field of engineering.

CLOSELY allied to the field of photogrammetry, which may be aptly defined as the science of the measurement of photographs, is the field of stereoscopy—the viewing of objects in three dimensions. Its application to photogrammetry is the observation of photographs with optical instruments for the purpose of measuring relative heights of objects thus shown, and also to define the shape and positions of such objects.

Stereoscopic instruments may be of the mirror (reflecting) type, the prism type, or the lens (refracting) type, or a combination of all. In any case, definite optical characteristics are present and must be accurately determined if the instrument is to be used to its best advantage. Hence, it would appear that at least an elementary knowledge of the development and formulation of the principles of optics would be essential to a thorough understanding of the science of stereoscopy.

HISTORICAL

Literature and historical data concerning the early pioneers in optics are rather vague. History reveals, however, that the Chaldeans in 5000 B. C. were the first to discover and use glass. No further fundamental developments appear to have been made in the field of optics until a few centuries before Christ. At that time the noted philosophers, mathematicians, and physicians gave to man the first theories of light and vision. Such men as Plato, Sophocles, Aristotle, Hipparchus, and others, all made valuable contributions along this line.

Several centuries later, a number of scientists, namely, Euclid, Galen, and Ptolemy made discoveries and expounded theories which appear to mark that era as the main landmark in the history of optics. So important were their contributions that they were to have an effect on the world for over 1200 years. Even with the poor instruments and equipment available, they were able to formulate and prove the following optical principles: that angles of incidence and reflection are equal; that the rays of each are in the same plane; that the angles of the two rays are not the same when light is passed from one medium to another medium of greater density; and that the same rules hold true for plane surfaces as well as curved surfaces.

Even though the above principles in the field of optics were established, practical optical instruments other than those using plane mirrors were not developed until 1100 years later. One of the first of the latter-day scientists to illustrate the applications of optics was Roger Bacon who recommended the use of "glass lenses for those of poor eyesight." Others who later contributed greatly to the field were Tacharius Jansen, who developed the Dutch compound telescope-microscope in 1590; Galileo, who developed the compound microscope in 1610; Newton, who formulated the spectrum theory in 1666; Thomas Young, who made glasses corrected for astigmatism in 1800; Fresnel, who promoted polarization theories in 1814; and G. G. Airy, who developed the theory of distribution of light through lenses in 1834.

Previous to the Nineteenth Century, only the simplest lenses made of poor grades of glass were available to the optical instrument makers. Lenses were either limited to crown glass, which has a low refractive power, or flint glass,

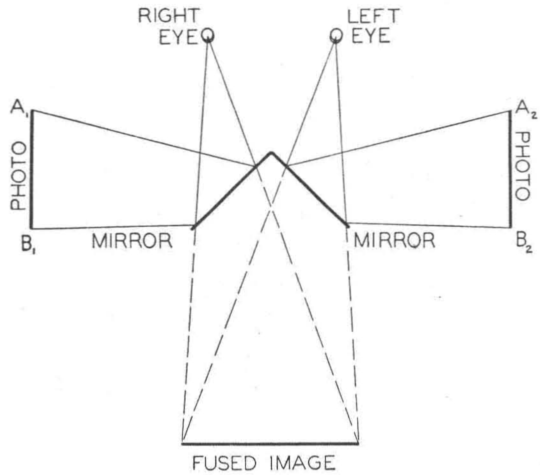

FIG. 1. Wheatstone Mirror Stereoscope.

which has a high dispersive power. A few of the men who helped to develop glass lenses with new characteristics, which, in turn, directly affected the development of precise and delicate optical instruments, were Charles Spencer and Robert Towles in the United States, Ernst Abbe, Ziess, and Schott in Germany, Amici in Italy, and David Brewster in England. As a result of their studies, glass lenses are now made of materials which may include borate, barium, lead, and phosphate compounds, as well as the original materials of quartz, soda, and lime.

In 1838, the first recorded optical instrument incorporating the principles of stereoscopy was developed by Robert Wheatstone (1802–1875). This instrument consisted of two mirrors which reflected the images from a pair of stereoscopic pictures directly to the eyes. Figure 1 shows the Wheatstone mirror stereoscope, while Figure 2 shows the Helmholz (1857) four-mirror stereoscope, which is similar to those in use at present. See Figure 3. A few years after Wheatstone developed the reflecting stereoscope, Sir David Brewster (1849) developed a lens-stereoscope which consisted of two convex lenses separated about $\frac{3}{8}$ inches farther apart than the interpupillary distance of the observer's eyes. The characteristics of the lens-stereoscope are also shown in Figure 2. Also see Figure 4.

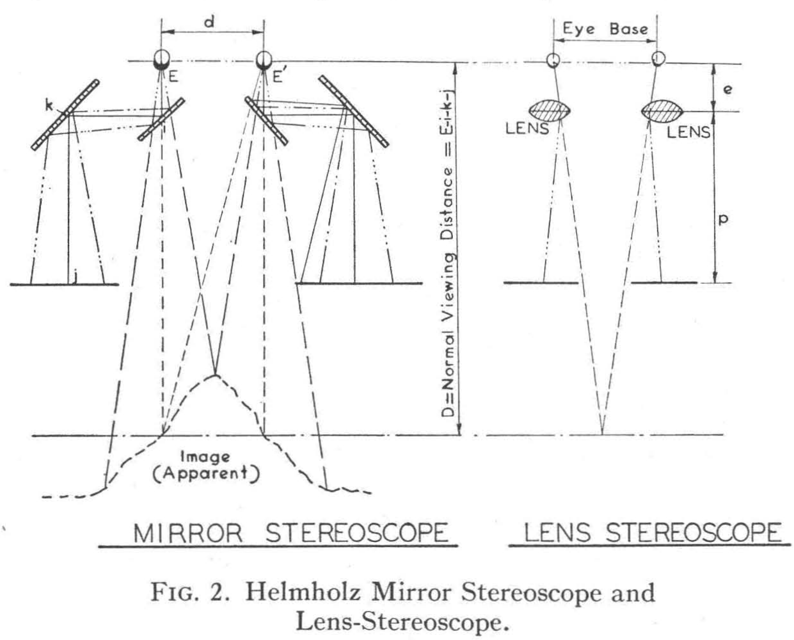

FIG. 2. Helmholz Mirror Stereoscope and Lens-Stereoscope.

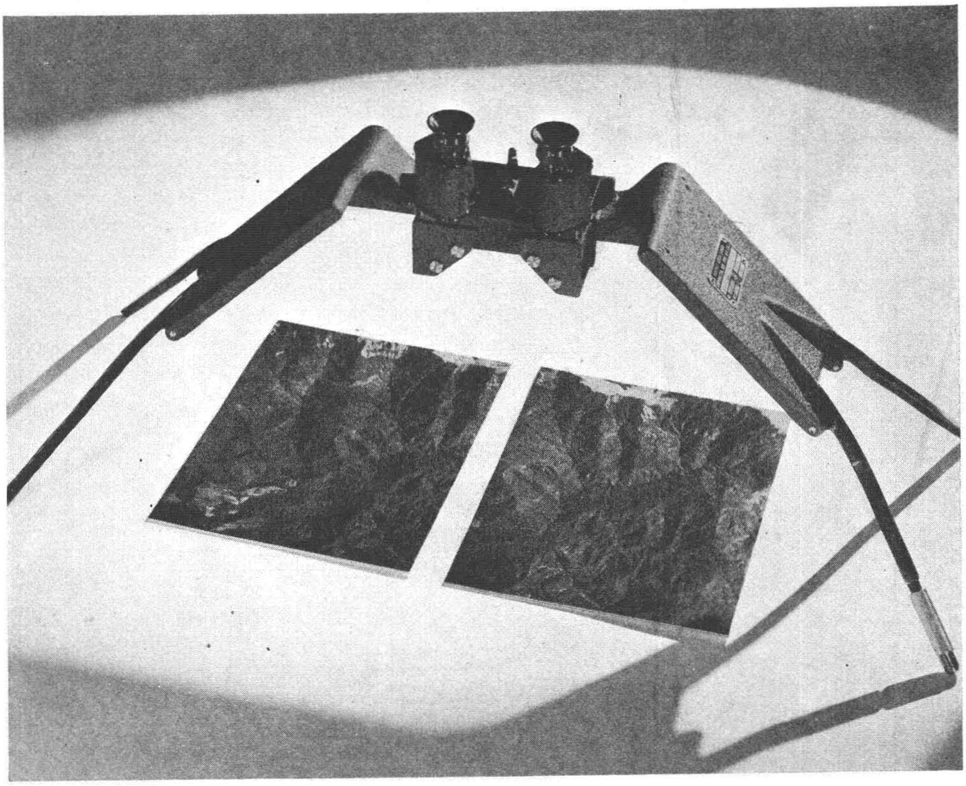

FIG. 3. Fairchild F-71 Stereoscope with Binoculars.

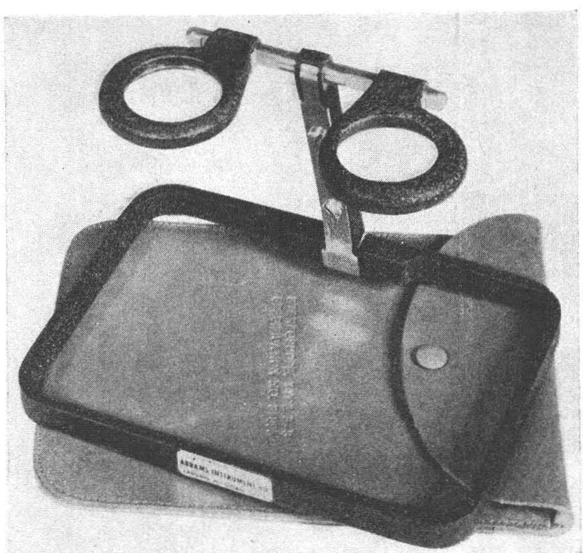

FIG. 4. Abrams Folding Type
Lens-Stereoscope.

Principles of Vision

The principles and mechanics of stereoscopic perception are relatively simple and should be studied by every one who is to work in the field of photogrammetry. Some of the principles can readily be indicated by diagrams and simple formulae, but certain phases of these phenomena must be considered from a physiological standpoint inasmuch as the workings of the human eye and mind also enter into the process.

The faculty of vision is so natural and customary that we seldom pause to appraise it or are in the least bit conscious of the intricate processes involved. In the process of vision, either monocular or binocular, three important elements appear to be linked together, namely, the eyeball, the optic nerve, and the visual centers of the brain. The eyeball is globular in form and contains the dioptric apparatus and nervous mechanism which is sensitive to stimulus by luminous radiation (light) from without. Each sensation of light is then conveyed by means of the optic nerve to the brain where it comes to consciousness and gives rise to the impressions, or perceptions, of vision. Figure 5 shows a perspective view of the globular eye, and Figure 6 shows its internal arrangement.

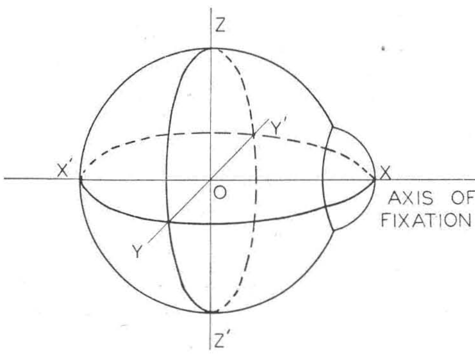

FIG. 5. Perspective View of the Globular Eye.

The retina, which constitutes the beginning element of visual perception, is perhaps the most important of all the eye components. A transverse section of the retina would show that it is made up of about seven million "cones" and

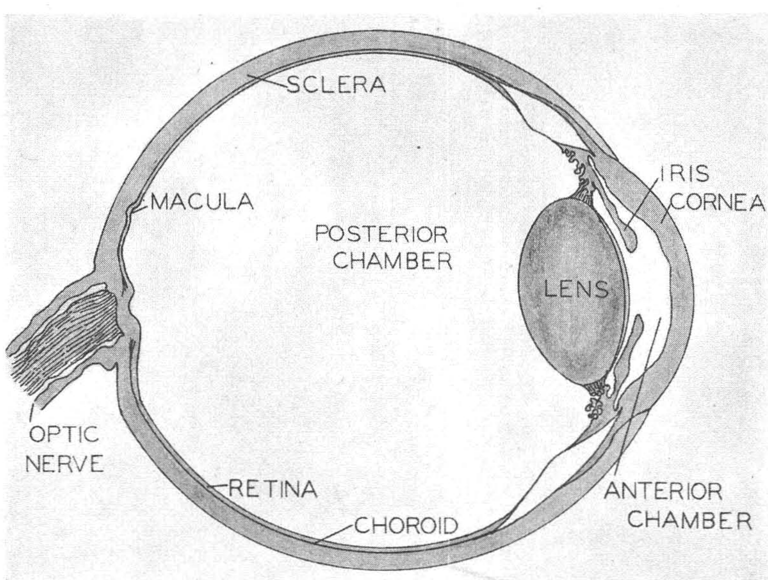

FIG. 6. Internal Arrangement of the Human Eye.

about one hundred million "rods" in addition to miscellaneous nerve fibres and cells. As light falls upon the rods, a chemical change occurs within them which, in turn, stimulates the optic nerve and sends a message to the brain. Each rod may be likened to the sensitive coating on a photographic film with the primary difference being that it has the power to regenerate itself. This latter process is continuously being carried out both in daylight and in darkness at a rate of about five hundred times per minute.

Whenever the eye fixes its attention on an object, the image is sharply focused on the element of the eye called the "macula." A high concentration of cones is located at this spot and tends to enhance the perception of detail and acute vision. As a general rule, it may be stated that the cones make possible the abilty to see objects sharply over a small central field of view, while the rods dominate the viewing of movements and orientation of gross objects in the remainder of the outer portion of the field of view. Nerve fibres leading from the retina to the brain carry the numerous stimulations that are thus set up and "develop" them by a mental process into a composite picture.

Normally, the mobile human eye is capable of covering a horizontal field of view of about 45 degrees inward and 135 degrees outward and a vertical range of approximately 50 degrees upward and 70 degrees downward. In those cases where the eye is kept perfectly motionless (referred to as "instantaneous fixation"), the horizontal range is limited to about 160 degrees (45° plus 115°). Figure 7 shows the range of monocular and binocular vision.

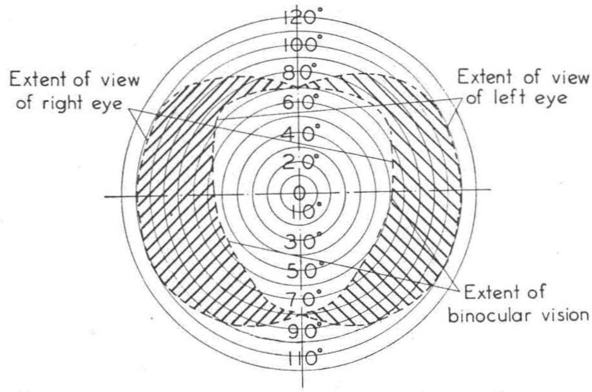

FIG. 7. Range of Monocular and Binocular Vision.

Although the single human eye (monocular vision) affords a wide range of view in a horizontal and vertical direction, it is very limited in its ability to form accurate conceptions of depth. Relative directions of objects fixed in space can readily be determined, but the process of being able to determine accurately (except by inference or association with other objects) whether one object is nearer or farther from another is impossible. An orthographically projected view is all that can be ordinarily obtained.

BINOCULAR VISION

Fortunately, man is blessed with two eyes instead of one; thereby his faculty of vision is greatly enlarged and reinforced. Each eye is capable of executing its own movements, but constant training and use in the interest of distinct binocular vision has linked the units together to function as a "double eye." Reactions and movements are invariably made in unison. It will be found that the eyes will work together either as parallel lines of fixation (i.e., in viewing a star which may be considered an infinite distance away), or as a duplex organ of sight in converging or diverging operations. In the process of convergence, the two eyes tend to work in unison whenever a change is made in the position of fixation. Such unified change in the position of fixation may occur outward or inward along the same line of vision or as a unified movement to another line of vision.

An optical characteristic which is often encountered in connection with binocular vision is that of a "double image." For example, when the eyes of the observer are focused for a certain distance, any object lying nearer or farther away will be seen as a double image. The following simple experiment may be used to substantiate this phenomenon: If an object on the opposite side of the room, such as a small picture hanging on the wall, is observed momentarily with both eyes, and then the right eye is closed, the object apparently will shift its position to the right with respect to the wall. On the other hand, if the right eye is quickly opened and the left eye simultaneously closed, the object will appear to shift its position to the left. Or, if the same object is observed with both eyes, while holding one finger up about ten inches in front of the eyes in line with the object, it will be noted that the finger will appear to be doubled, i.e., two images will appear. Conversely, if the eyes are concentrated on the finger, the object on the wall will appear to be doubled.

In normal binocular vision, double images will not ordinarily be noticeable, for, as a rule, they are seen only when the viewer's attention is drawn to them by concentration. On the other hand, persons with defective vision, such as squint, heterotropia, or cross-eye, may see all objects doubly, although one of the images may be suppressed in consciousness. Detailed studies of the human eyes have shown that there are corresponding places on the retinas of the two eyes which receive identical impressions and, conversely, that if the retinal images of one and the same object do not correspond, a double image will be seen.

Additional factors which affect vision are intensity of light, differences in brightness between adjacent areas of an object, distance of an object away from the observer, and sharpness of boundary between adjacent areas. For instance, a brightly colored dot can easily be seen on black background but can hardly be seen on a background having a color which contrasts slightly from the color of the dot. An elementary experiment which will bring out clearly the basic factors involved in seeing is illustrated. Upon a sheet of white paper place a dark colored, irregular-shaped spot of a size just large enough to be visible at a convenient distance, i.e., fifteen feet. The sheet of paper should then be held so that the spot is viewed alongside of a relatively faint star in the sky (considered to be at an infinite distance from the observer). Under these conditions, both the star and the spot will appear to be the same size. However, as the observer approaches the sheet of paper, the spot will appear to get larger and more easily identifiable, whereas the star will appear to remain constant in size (since it actually is an infinite distance away).

Definite conclusions may be gathered from the above experiment, namely, that vision depends upon the ratio of actual size to viewing distance (i.e., angular size rather than actual size of the object), and that all objects appear about equal in size at the limit of visibility.

Radius of Stereoscopic perception

In monocular vision, as previously indicated, all that can be determined about an object is its relative direction in the field of view. Binocular vision,

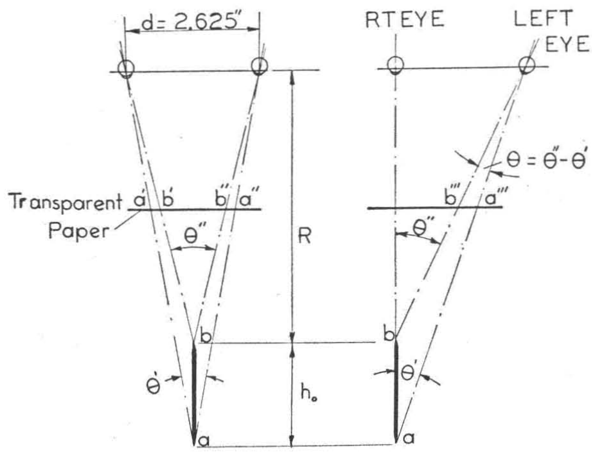

FIG. 8. Minimum Angle of Depth-Perception.

on the other hand, affords some estimate of distance and depth perception provided the image is not too far away as compared with the interpupillary distance (average value is about 2.625 inches) of the observer; see Figure 8. Even with normal binocular vision, it has been found that it is impossible to distinguish between objects if the difference of the angle of convergence ($\theta = \theta'' - \theta'$) is less than about 20 seconds of arc (0.000965 radians). Carl Pulfrich (1901) found, however, that for some extraordinary individuals this minimum angle of depth-perception was as low as 10 seconds.

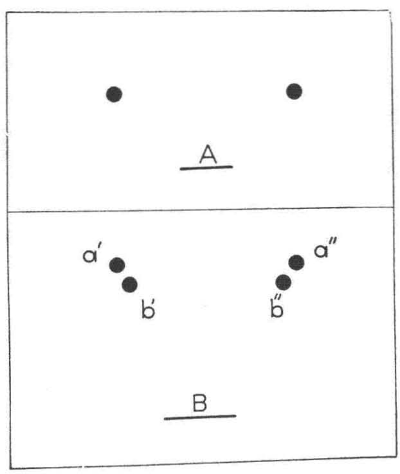

FIG. 9. Simple Stereogram.

Thus, for a range of interpupillary distance (d) of 1.97 inches to 2.85 inches (the general range for different persons) and angle (θ) of 20 seconds, the distance from an observer (R) than an object would still appear to have depth would be from 1700 feet to 2450 feet, respectively. Beyond that distance the naked eyes, alone, cannot discriminate differences of distance or depths of objects, since beyond that point all objects appear to be projected on the infinite background of space. The distance (R) has often been referred to as the "radius of stereoscopic perception."

When the distance (R) is relatively large the following relation can be used for approximate results:

$$R = \frac{d}{\theta} = \frac{d}{0.000097} = 10{,}315(d)$$

where both (R) and (d) are in the same linear units and (θ) is expressed in radians. This equation was based upon the relation that the arc distance (d) is equal to the radius (R) times the subtended angle (θ).

In aerial mapping operations involving the use of stereoscopic instruments, it will be found that visual acuity is dependent not only upon the inherent limitations of the instruments used and upon the physical nature of light but also upon the physiological state of the individual. Such factors as stimulants, fatigue, mental depression, distracting noises, unsatisfactory illumination, uncomfortable viewing position, and the improper humidity and ventilation of workshop —all tend to interfere seriously with results. Because of the number of complicated biological factors thus involved, it is very difficult to establish or confirm limits of visual acuity.

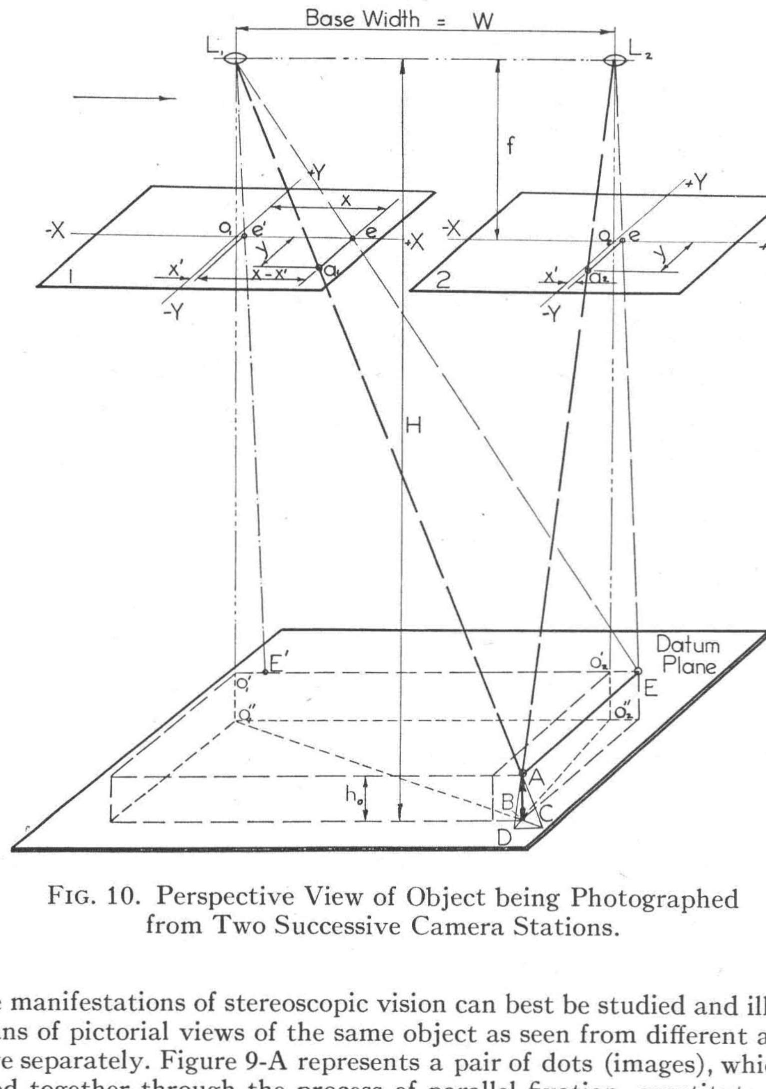

FIG. 10. Perspective View of Object being Photographed from Two Successive Camera Stations.

The manifestations of stereoscopic vision can best be studied and illustrated by means of pictorial views of the same object as seen from different angles by each eye separately. Figure 9-A represents a pair of dots (images), which, when observed together through the process of parallel fixation, constitute a simple stereogram (spatial model).

Figure 9-B shows a simple stereogram consisting of two parallel rows of two dots each, with the lower set of dots spaced a little closer together than the upper set. By staring at the two left-hand dots with the left eye and the two right-hand dots with the right eye, it will be found that the dots (a') and (a'')

will fuse and the dots (b') and (b'') will also appear to fuse, but above the other fused pair. Figure 8 explains how this phenomenon of the "floating dot" is possible. Such a result is obtained because the angle of convergence of the inner (upper) row of dots is greater than the outer (lower) row. Hence, it is seen that such distances as ($a'b'$) and also ($a''b''$) may be used as a direct measurement of the relative heights of the objects they represent.

Absolute Parallax

Further reference to Figure 8 will show that the difference of the angles of convergence (i.e., the angles of parallax θ'' and θ') may be used as a direct

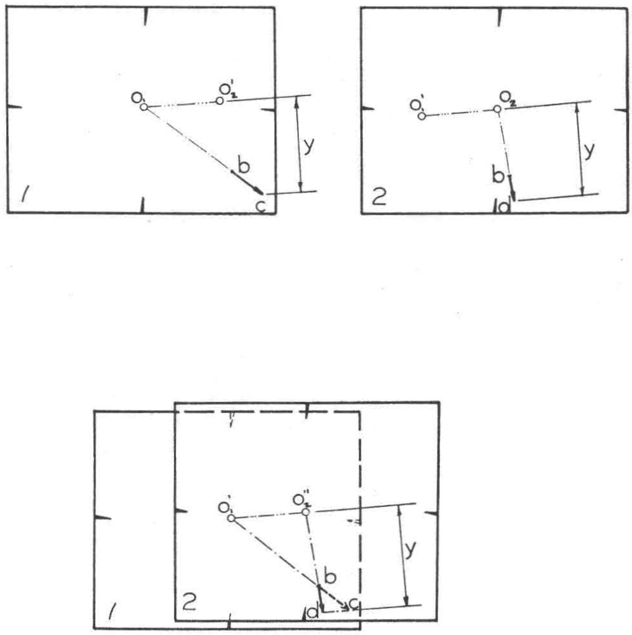

FIG. 11. Method of Obtaining "Differential Parallax Displacement."

measurement of the height of the object (h_0). However, since it is quite difficult to measure the angles of parallax in the case of an actual pair of stereo photographs, it is much simpler to resort to linear measurements on the photos, such as the distances ($a'b'$) and ($b''a''$). Certain geometric relations involving such distances on the photo and the corresponding height of the abject can be readily set up.

Figure 10 illustrates the perspective view of the two successive camera station positions (L_1 and L_2). Photographs 1 and 2 show the object point (A) as falling at the points (a_1) and (a_2). respectively. Point (a_1) has the coordinate value of (x, y) and the point (a_2) the coordinate value of (x', y). It is to be noted that in the case of two photos in which the scale is the same (i.e., when photographed at the same height above the datum, with the same focal length camera, and in which no tilt exists) the successive position points, (a_1) and (a_2), will both lie at the same distance (y) from the line connecting the two centers of the photographs. This latter line is the flight line represented by the points (O_1) and (O_2). This feature is more clearly shown on Figure 11. In this case the object

(AB) of Figure 10 is shown on photo 1 as the displacement (bc) and on photo 2 as (bd). These two distances are referred to as "relief displacement" distances. When the two photographs are superimposed one upon the other so that the two flight lines and the two photo centers coincide, it will be found that the line connecting (d) and (c) will be parallel to the flight line (O_1'' and O_2''). By geometric relations it will be found that the line (dc) can also be used as a direct measurement of the height of the object (AB).

If, in Figure 10, the line (L_1E') is geometrically constructed parallel to (L_2E), the distance (O_1e') will then be equal to (O_2e). For convenience, both these

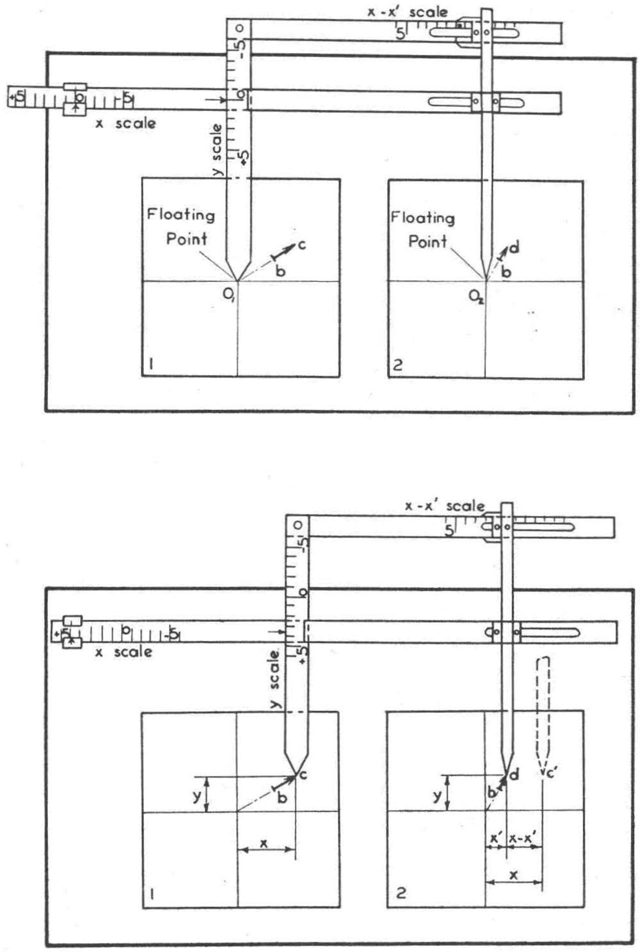

FIG. 12. Principles of Stereocomparator and Contour Finding Machines.

latter distances are labelled as (x'). The distance ($x-x'$) is known as the "absolute parallax" of the point (A). The value of (x) and (x') must be added algebraically, however, if proper relations are to hold. The value of (x) and (x') are positive if they lie on the forward (as measured in the direction of flight) side of the photo and negative if they lie to the rear of the photo center. In the illustration, the values of (x) and (x') are both positive, as is the value of (y).

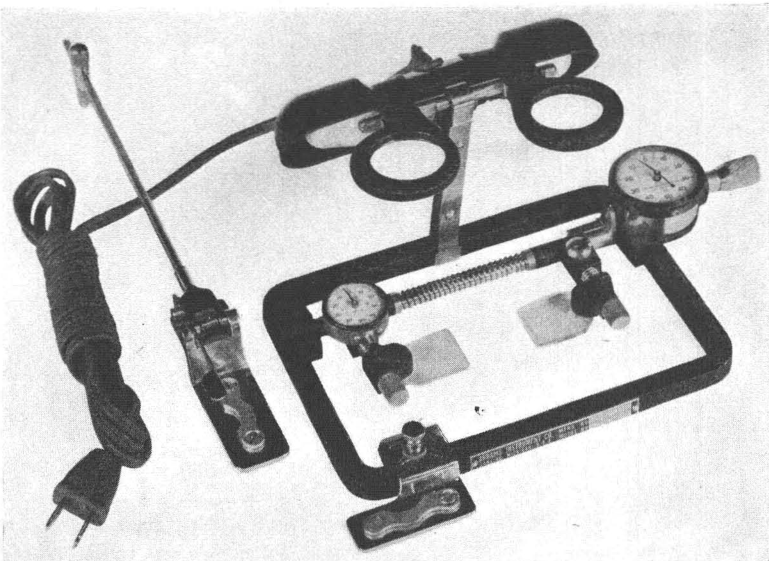

FIG. 13. Abrams Contour-Finder.

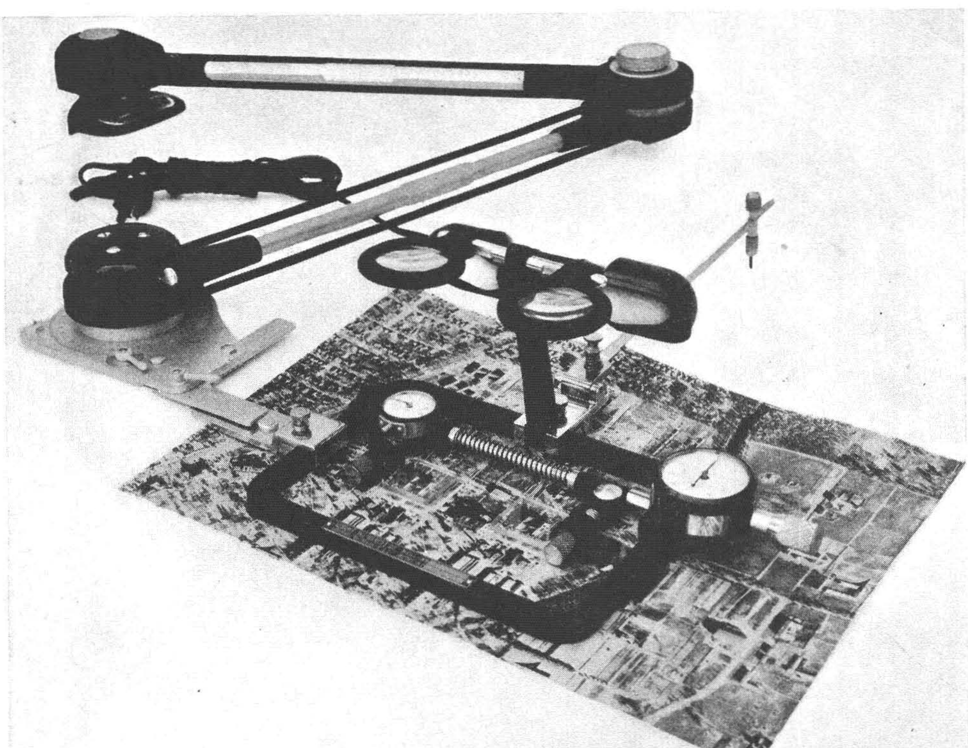

FIG. 14. Abrams Contour-Finder ready for operation.

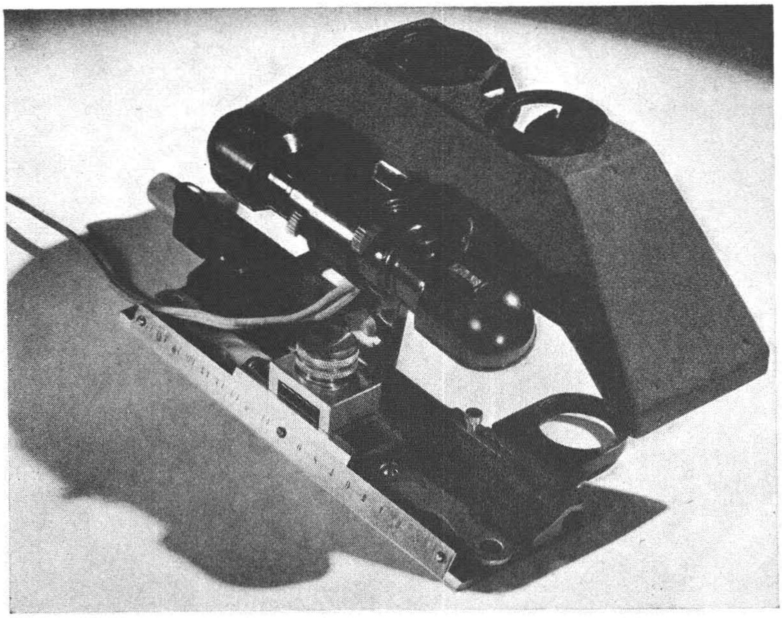

FIG. 15. Fairchild Stereo-Comparagraph.

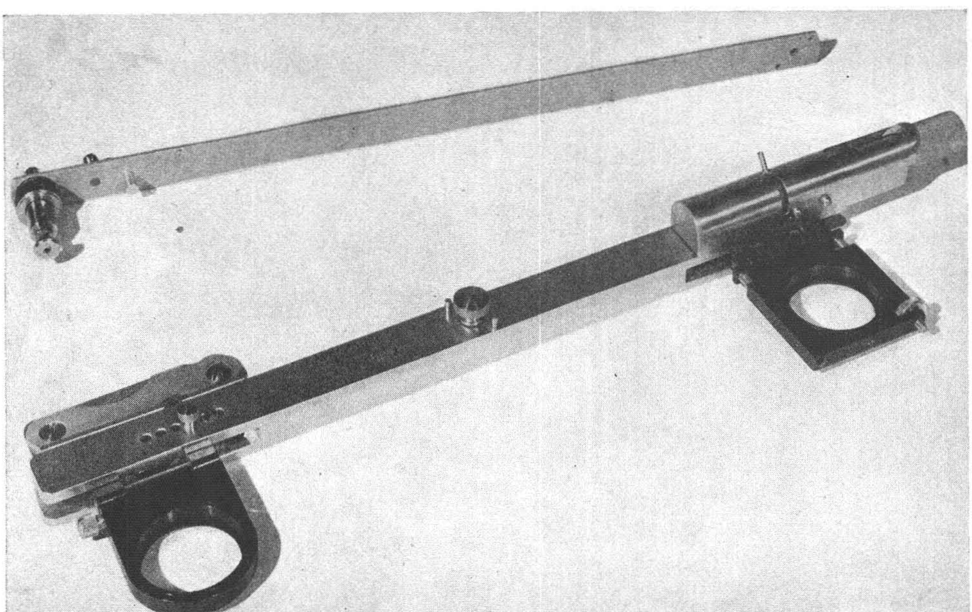

FIG. 16. Fairchild Parallax Bar.

PARALLAX EQUATIONS

Figure 12 shows the principle of the many commercial stereoscopic devices which are available for measuring differences of relief. Instruments of this type are the Abram's Contour Finder, Figures 13 and 14 and the Fairchild Stereocomparator and Parallax Bar, Figures 15 and 16. Figures 10, 11, and 12 will be used to derive the "parallax equations" which are the basis of all contour plotting machines and machines for determining the relative heights of objects.

From the similarity of the various triangles in Figure 10, it is seen that:

$$\frac{EO_1'}{eO_1} = \frac{EE'}{ee'} \cdot \quad \text{Let:} \quad X = EO_1$$

$$x = eO_1$$

$$x' = ee'$$

$$EE' = L_1L_2 = \text{Air Base width} = W.$$

Then:

$$\frac{X}{x} = \frac{L_1L_2}{x - x'} = \frac{W}{x - x'} \cdot$$

Likewise:

$$\frac{AE}{a_1e} = \frac{EE'}{ee'} \cdot \quad \text{Let: } Y = AE.$$

Then:

$$Y = \frac{x \cdot W}{x - x'} \cdot$$

Also:

$$\frac{L_1O_1'}{f} = \frac{EE'}{ee'}; \quad \text{Let: } Z = L_1O_1' = H - h_0.$$

Then:

$$Z = H - h_0 = \frac{f \cdot W}{x - x'} \cdot$$

Or:

$$x - x' = \frac{f \cdot W}{H - h_0} = \text{"parallax distance."}$$

And:

$$h_0 = H - \frac{f \cdot W}{x - x'} \cdot$$

Differential Parallax Displacement

Reference to Figure 11 will show that the distance (dc) is equal to the value of $(x-x')$ minus the distance (O_1O_2'). To provide for a simpler expression let $(D_p) = (x-x') - (O_1O_2')$, which may be defined as the "differential parallax displacement" of the point (A).

Then:

$$\frac{O_1O_2'}{W} = \frac{f}{H} \cdot$$

Therefore:

$$O_1O_2' = \frac{f \cdot W}{H} \cdot \quad \text{And: } x - x' = \frac{f \cdot W}{H - h_0} \cdot$$

Hence:

$$D_p = \frac{f \cdot W}{H - h_0} - \frac{f \cdot W}{H} = \frac{f \cdot W \cdot h_0}{H(H - h_0)} \cdot$$

Or:

$$h_0 = \frac{D_p \cdot H}{D_p \cdot H + f \cdot W}$$

in which

D_p = differential parallax displacement for point (A), in feet.
h_0 = elevation of point (A) above the datum plane, in feet.
W = distance between successive exposure stations, in feet.
H = elevation of camera lens above datum, in feet.
f = focal length of camera, in inches.

Since:

$$\frac{\frac{f}{12}}{H} = \text{R.F. scale} = S; \quad \text{Or: } \frac{f}{H} = 12 \cdot S.$$

Then:

$$D_p = \frac{12 \cdot S \cdot h_0 \cdot W}{(H - h_0)}; \qquad \text{Or: } h_0 = \frac{H \cdot D_p}{(D_p + 12 \cdot S \cdot W)} \cdot$$

It is evident from the above equations that all points which have the same differential parallax displacement, regardless of their positions on the photographs, will have the same elevation above datum. Or, conversely, all points at the same height above a given datum plane will have the same parallactic displacement.

Stereocomparator and Contour Machines

The commercial comparator machines are very precise instruments in which the (x), (y), and ($x - x'$) distances are measured by means of micrometer screw adjustments which may be read to the nearest 0.01 mm. Each photo of a stereo pair is attached to a drafting table with the center of each in direct alignment and in a vertical plane containing the line joining the two pointers, as shown in Figure 12. When the pointers are placed in this position the (x), (y), and ($x - x'$) scales should all read zero.

Instead of a pointer, the actual contour machines have two "floating dots," or "grids," placed on transparent glass plates, located in the plane of the photographs. The dots are called floating dots because when set apart at the proper distance they will fuse into a single dot which will appear to be "floating" beneath the eyepieces of the stereoscope. If the dots are moved further apart, by means of the ($x - x'$) vernier attachment, the fused dot will appear to be lowered vertically through space away from the observer, and if the dots are brought together, the floating dot will appear to rise, relative to the landscape. Hence, by varying the spacing between the dots, the fused image can be made to approach, recede from, or actually touch a given portion of the ground.

A "Universal Drafting Attachment" is connected to the stereocomparator machine to insure that any movement of the entire unit (i.e., the stereoscope and the floating dots) will occur only in a direction parallel to the X-axis (flight line). In moving the unit from the original zero position of the dots to another position for which the new (x), (y), and $(x-x')$ values are to be found, the following procedure is carried out: The left-hand floating dot is moved to the new object (point (c) in Figure 13) whose coordinates are shown as (x, y). The right-hand dot will then be at the position marked (c'). At this point the two dots will appear separately to the observer. The right dot is moved parallel to the X-axis by means of the $(x-x')$ adjusting screw to the position of (d) at which point it will be fused with the left-hand dot. In this position the fused dot will appear to be just touching the object whose elevation and coordinates are being determined. The scale reading on the $(x-x')$ scale may then be substituted in the formula:

$$h_0 = H - \frac{f \cdot W}{x - x'} \cdot$$

If it is desired that contours be plotted with the stereocomparator machines, the $(x-x')$ scale reading can be set for the proper contour level and retained at that value as the floating mark is moved from point to point. Under these conditions the $(x-x')$ scale value will remain unchanged but the (x) and (y) coordinate values will be continuously changing as the instrument is moved about. Contours may then be traced by moving the unit so that the floating dot will always be kept in apparent contact with the ground as it thus moves in a horizontal plane. The path of the floating dot is traced upon a map sheet by means of a pencil that is located a suitable distance from the stereoscope and which is held in place by an extension arm leading from the stereoscope. If a new contour line is to be plotted, then the $(x-x')$ scale value must be calculated and properly set in place.

In actual contour mapping operations resort is made to "parallax distance" charts which are computed by measuring the parallax of three points which appear on the overlapping photographs and whose elevations are known. If the elevations above a given datum plane are plotted as ordinate values and the parallax distances as abscissa values, a curve will be obtained which deviates slightly from a straight line. It will be convex toward the ordinate axis. However, where only slight variations in elevation occur on a pair of photographs, this graph can be assumed as a straight line.

Usually the horizontal plane at the elevation of the lowest ground control point is chosen as the datum plane and the graph plotted from that point upwards. With such a graph available for a pair of photographs the parallax distance reading can be found for any desired level, and, if such levels are taken so as to coincide with contours, a satisfactory topographic map can be compiled.

In order to determine the value of the air base width (W) in the formula:

$$(x - x') = \frac{f \cdot W}{H - h_0}$$

the $(x-x')$ values of the three ground control points can be found by means of he stereocomparator machine and then substituted separately in the above equation. By solving any two of the three equations simultaneously the value of (W) can be obtained. This constant value can then be used for finding the elevation (h_0) of any other point falling on the overlapping portions of the two photographs.

Limitations of Stereo Perception

Simple and obvious as the above geometric relations appear, it cannot be inferred that stereoscopic perception can always be actually verified by one's own eyes. It has been shown that there is a definite limit (20 seconds of arc) to the difference between the two converging angles of fixation upon an object. Such limit would be equivalent to a minimum measurement of differences of elevation of two objects of approximately 0.004 inches (i.e., without magnification). At a scale of 1/20,000, differences of elevation of seven feet or more could be determined by the above process, but a difference of less than seven feet could be determined only by means of some type of magnifying apparatus.

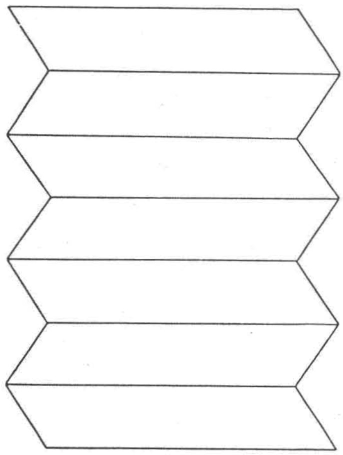

FIG. 17. Optical Illusion of Depth.

From the formula for determining the radius of stereoscopic perception, it was found that a value of about 2200 feet would result if an average interpupillary distance of 2.55 inches was chosen. It can further be shown that differences in distances can be determined stereoscopically up to the square of the distance to the object divided by the above figure of 2200 feet. Thus, if an object is 1000 feet away from the observer, other objects up to $(1000)^2/2200=455$ feet beyond the first object cannot be perceived stereoscopically because up to that point, the difference in the angles of convergence is 20 seconds or less.

Oftentimes, certain geometric figures present themselves as illusions which are purely mental. That such illusions have nothing to do with binocular vision may be proved by the fact that they are more obvious when regarded only with one eye. Monocular conception of depth appears, therefore, to be more of a mental than optical process as differentiated from binocular vision which is strictly optical. Figure 17 illustrates such an illusion.

Stereo-Power of Lenses

Artificial enhancement of the power of stereoscopic vision can be obtained by increasing the virtual base-line (i.e., interpupillary distance) or by the introduction of a magnifying optical instrument (which directly tends to lower the effective value of the angle (θ)). The "stereo-power" of a binocular instrument is found by multipying the magnifying power of the lens, designated by the letter "M," by the ratio of increased optical base to interpupillary distance. The latter ratio can be designated by the letter "c." Thus, if a binocular instrument has $M=3$ and $c=2$, its stereo-power value would be 3×2, or, 6. An illustration of the above type of instrument would be the prism field glass or the prism stereoscope. The above instrument would result in an increase in the power of depth perception of an object six times the distance at which stereoscopic vision could be obtained without the use of the instrument.

In using a magnifying lens of a stereoscope, the eye should be kept as close to the lens as possible (i.e., position (E) in Figure 18 rather than at (E')) and the

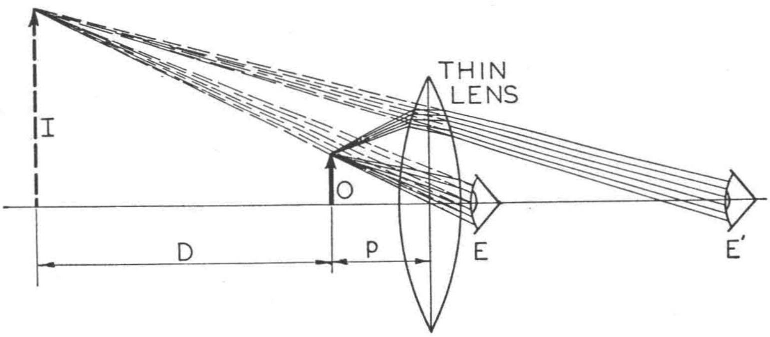

FIG. 18. Relationship Between Size of Object and "virtual" Image.

lens in turn should be brought up to the object (photograph) until the latter is seen as distinctly as possible. This results in a condition in which the rays from all parts of the photo object come to the eye through the central part of the lens, thus reducing the possibilities of spherical and chromatic aberration. On the other hand, if the eye is placed at position (E') the rays from the object (O) would be refracted in the outer portion of the lens, resulting in some distortion.

GENERAL LENS FORMULAS

Magnification may be produced through the use of a convergent stereoscopic lens only when the object (the photo) is placed slightly nearer to the lens than

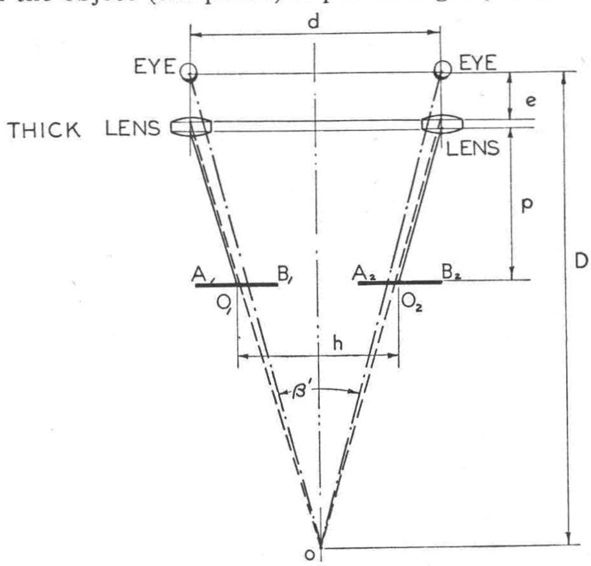

FIG. 19. Correct Geometric Relations in "Orthostereoscopic" Perception.

its principal focal distance. In that case a virtual, and enlarged, image will appear to the eye at the position (I) of Figure 18. The ratio of the length of the image (I), formed at the distance of distinct vision (D) (usually taken as 10″ for normal eyes), to the length of the object (O), is called the "magnifying power" (M) of the lens. The magnifying power also may be expressed as $1+D/f$, where

(f) is the focal length of the lens. In deriving this last expression, the following steps were utilized:

The general lens formula for "thin" lenses states that:

$$\frac{1}{p} + \frac{1}{D} = \frac{1}{f}$$

where (f) is always positive for convergent lenses, and (p) and (D) may be either positive or negative values. In the case where the position of the object (O) and the image (I) are on the same side of the lens, as would be the case in the observation through a stereoscope, the value of (p) is positive while that of (D) is negative. In this case the image (I) is a "virtual" image rather than a "real" one; hence, the negative sign. The image becomes a virtual one whenever the distance (p) is made to be less than the principal focal distance (f) of the lens. The resulting formula then becomes:

$$\frac{1}{p} - \frac{1}{D} = \frac{1}{f}.$$

Multiplying through each term by (D):

$$\frac{D}{p} - 1 = \frac{D}{f};$$

or:

$$\frac{D}{p} = 1 + \frac{D}{f}.$$

But by geometric relation:

$$\frac{D}{p} = \frac{I}{O}; \quad \text{therefore} \quad \frac{I}{O} = 1 + \frac{D}{f} = M$$

which is the formula for determining the magnifying power of a lens.

Other relations which can be derived from the general lens formula and from the geometric relations shown in Figure 19 are as follows:

$$\frac{D}{p} = M$$

$$f = \frac{p \cdot D}{D - p} = \frac{D}{M - 1}$$

$$M = \frac{f + D}{f}$$

$$D = \frac{p \cdot f}{f - p}$$

$$p = \frac{f \cdot D}{f + D}$$

$$h = \frac{D \cdot d}{f + D} = d - \frac{d \cdot p}{D} = d - \frac{d}{M}$$

where (h) is the distance between an object on photo (I) and the same object on photo (II) (i.e., photo spread). It can be concluded, therefore, that, everything else being the same, the smaller the focal length of the lens, the larger the distance between the centers of the two stereoscopic photos. Stating it in another way, an increase in the spread between pictures can be accomplished by choosing lenses with large magnifying powers (i.e., small focal lengths).

Magnifying Power of Lenses

The distance (D) is usually established at 250 mm. (10 inches) for average individuals, and the magnifying power of lenses are determined on that basis. Therefore the magnifying power is normally equal to $1+250/f$, if (f) is expressed in mm., or $1+10/f$, if (f) is expressed in inches. Thus a 4″-focal length lens would be equivalent to $1+10/4$, or $3\frac{1}{2}$-magnifying power. As a rule, stereoscopic lenses are limited to a magnifying power of from 2 to 5, thereby restricting the focal lengths from 10″ to $2\frac{1}{2}$″. It may be stated, generally, that as the magnification is increased the field of view is decreased, and the lens will have to be placed closer to the photograph. Another restriction to excessive magnification (above 5-power) is that emulsion grain particles may become enlarged to such an extent as to obliterate small cultural features and to confuse the stereoscopic impression.

The distance (p), for ordinary conditions (when $D=10''$) can be expressed as:

$$p = \frac{D \cdot f}{D+f} = \frac{10 \cdot f}{10+f}.$$

Thus for:

$$f = 10''; \quad p = 5''; \quad M = 2$$
$$f = 5''; \quad p = 3\tfrac{1}{3}''; \quad M = 3$$
$$f = 2\tfrac{1}{2}''; \quad p = 2''; \quad M = 5.$$

It is seen from the above formula that the distance (p) will vary for different individuals because the distance (D) will not be the same for all persons. Although for most persons a value of 10″ may be considered for (D), it may be possible that for some this value may be as high as 17″ or more. This is especially true of those who wear glasses. In the latter case, it is to be noted that persons wearing bifocals may have some difficulty in making consistent readings through a stereoscope. The difference in the magnifying power of the two component parts of the spectacle lens may result in two different readings. A difference in the distance (h) will also be noted for different persons because of the variation in the distance (D).

Proper Stereographic Observation

Judging from the practical requirements of a lens stereoscope, it would appear that the lens should be held at least $3\frac{1}{3}$″ above the photo so as to allow for the tracing of topographic details, or for the pricking of objects on the photo while using the steroscope. A focal length lens larger than $3\frac{1}{3}$″ would thus have to be used for this purpose (i.e., 5″) which would result in a magnification of 3.

Figure 20 shows the result of observing objects through different stereoscopic systems in which the angles of convergence of emerging bundles of rays are not the same as the angles of convergence in the original taking cameras. In the diagram, it is assumed that the $8\frac{1}{4}$″-focal length is the effective focal length which results in the proper reproduction of the model whereas the 4″- and the 12″-focal lengths are not.

Only when the observations are made with the proper stereoscopic system will the spatial model have the same vertical and horizontal scale and, hence, be a true reproduction of the original model. The principles of stereoscopic reconstruction just outlined are necessary whether such reconstruction is accomplished by direct stereoscopic observation, by projection as in the case of "colour" and "polaroid" anaglyphs, or by projection with the various stereocomparator or contour plotting machines.

For proper stereoscopic observation, then, it may be stated that each person should set up his stereoscope according to his own individual requirements.

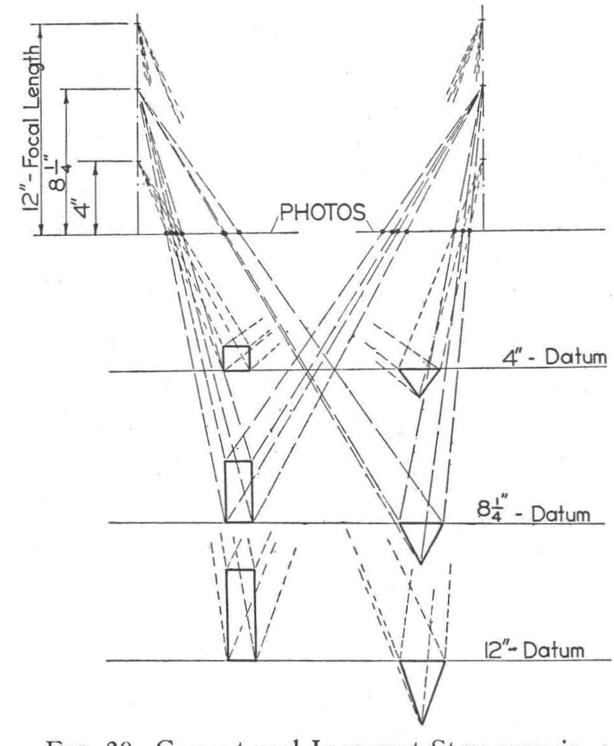

FIG. 20. Correct and Incorrect Stereoscopic Systems for Observation of Spatial Models.

Each person should check his own interpupillary distance (d) and the distance (D), as well as the value (p), (h), (M), and (f) for each stereoscope that is used. Such care in the observation of stereo images will result in a harmony between convergence and accommodation which in turn will provide views with sharp definition, undistorted perspective, correct "depth" relations, and complete ease in fusion. It is only when all the factors are correctly applied that a true, or "orthostereoscopic," image will result.

In order to be able to determine the magnification of a stereoscope lens when the focal length is not known, the following simple procedure may be followed:

Set a scale, ruled to decimals of an inch, on a support below the lens. The distance (p), which is the distance between the lens and the scale, should be such as to result in a clear image. Set up the lens at a distance (D) of 10″ above the table top and place upon it a piece of white paper at the base of the support, and on the side facing the light. Keeping both eyes open, and the head very still, focus your attention on the scale. The image of the scale will then appear as if on the paper below. With a pair of dividers, measure the distance between any two lines of the image the points "O" and "6" of Figure 21). The distance between the points of the dividers (A') will give the size of the virtual image, and as the size of the scale object (A) is known, the magnification can be found directly by dividing the scaled size of the image by the size of the object. In this case, the magnification equals A'/A.

If the distance (p) is also measured, the focal length (f) can be found from the relationship $1/p - 1/D = 1/f$ or from the relationship where magnification power (M) equals $1 + D/f$.

The position of the principal focus of a lens can also be obtained by allowing light rays to emanate from a source at an infinite distance from the lens, i.e., rays from the sun, and noting at what point on the opposite side of the lens, the rays converge. Placing a piece of paper on the side of the lens opposite the sun and slowly drawing it away from the lens will cause the image to diminish in size. The focus will be found at the point at which the image is a minimum (i.e., burning glass principle).

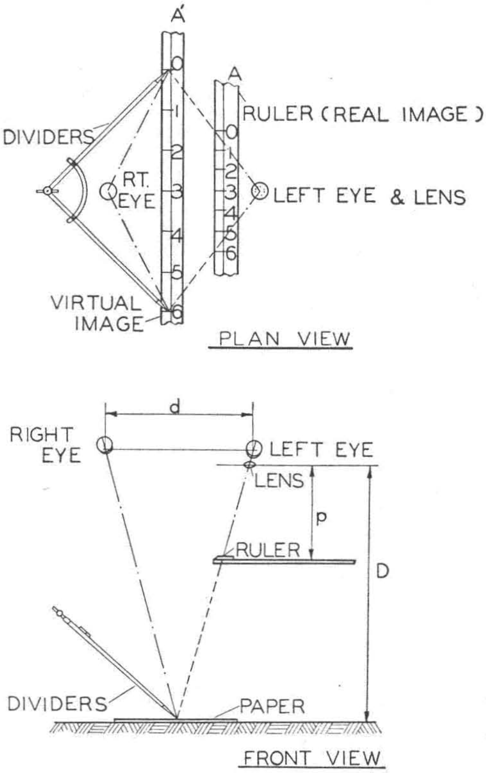

FIG. 21. Method of Obtaining the Magnifying Power of a Lens.

ANAGLYPHS

Another singular effect which may be produced with stereoscopic pairs of photos is that of the "anaglyph." In this case, two separate pictures of a stereo pair are printed in complementary colors and then superposed upon a single sheet of paper. The image that is to be observed by the right eye is printed in blue-green and that for the left eye in red. Binocular fixation of corresponding points is then obtained by observing the dichromatic over-print with a pair of goggles with a blue-green glass filter or celluloid film, in front of the left eye and a red one in front of the right eye. Thus, the blue-green image will be seen by the right eye alone and the red one by the left eye alone. The resulting effect will be a spatial model, in black and white, that had been formed mentally by observation of the two different optical impressions. In general, it may be stated that the ordinary anaglyph serves no practical purpose except as an instructional aid in attaining stereoscopic fusion.

Anaglyphs are printed by the half-tone process in order to insure satisfactory results. The half-tone process consists of photographing each print through two glass plates upon which have been engraved at least 120 fine lines per inch. The plates are placed so that the lines are at right angles to each other, resulting in a checkered pattern of fine dots. In order to prevent the dots of the blue-green print falling on those of the red print, the half-tone plate of one is rotated about 30 degrees with that of the other. If the images in the foreground of the two half-tones are matched one on top of the other, the resulting spatial model appears to be behind the plane of the paper. This affords a clearer mental impression of the anaglyph than if the objects in the background were thus matched.

Similar depth-impression effects may be obtained by projecting upon a single screen two stereoscopic pictures that have been illuminated by light of two different colours. Spectators are then able to observe the effect of relief by the use of suitable goggles as indicated above. It is this same principle which is followed in the observance of spatial-models in connection with the Multiplex Plotting Machine.

White light, which has been polarized in two planes at right angles to each other, may also be used to illuminate two stereo pictures instead of the two colours (i.e., red and blue). It is necessary, however, that "polaroid" spectacles be used in the observance of the model which is formed by the illumination of each picture with a different beam of polarized light. In this method of relief visualization, the right eye will receive one kind of polarized light while the left eye will receive the opposite kind.

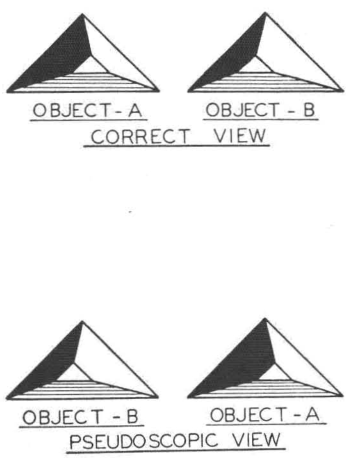

FIG. 22. Stereograms Showing Correct and "Pseudoscopic" Views.

PSEUDOSCOPIC VIEWS

In observing an anaglyph or a stereogram, care must be taken to assure that a reversal of relief is not obtained. Such an effect is known as a "pseudoscopic illusion." A reversal of relief is obtained if the photo originally intended to be observed by the left eye is placed at the right-hand side of the stereoscope (or observed through the red-colored glass in the case of the anaglyph), and if the photo designated to be seen by the right eye is placed at the left-hand of the stereoscope (or observed through the blue-colored glass). The same result can be obtained with the anaglyph by merely rotating it through 180° (i.e., the top of the picture is placed at the bottom) and observing it with the spectacles in their original position. Figure 22 shows a simple geometric stereogram in correct (orthoscopic) position and also in reversed (pseudoscopic) position.

Pseudoscopic illusions can be obtained much more easily through the use of simple geometric figures than with complicated objects. In the first case, the converse figure is as easy to visualize as the original one because it is one of frequent occurrence. On the other hand, objects such as buildings, hills, and valleys, are difficult to observe conversely because no previous observation in nature has been encountered.