# CONTOURING AND ELEVATION MEASUREMENT ON VERTICAL AERIAL PHOTOGRAPHS

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A NEW method of contouring and elevation measurement on uncorrected vertical aerial photographs is here presented in brief. The method is simple, accurate, and rapid.

This article will not attempt to explain minor details of application, because these are still in the stage of being changed as improvements are suggested in practice. The reader who wishes to use the method is free to exercise his own inventive talents at many points.

The unique features of the method are mainly the following:

1. A series of visible floating lines are made to rise or fall at will as one observes them under the stereoscope, interesecting the topography in a plane parallel to the natural horizon.

2. The scale attached to the raising and lowering device is calibrated with an elevation scale,—hence permitting the plane of floating lines to be set to the level of each contour in turn.

3. The right-hand photo (or left-hand for a left-handed person) is kept free and unencumbered for ink drawing of contours on the photo under the stereoscope.

4. After the contours are thus drawn on the photographs, the map, including contours and planimetry, is plotted accurately true to scale in all details in one step by an automatic radial-plotting device (published by the writer in a separate paper in PHOTOGRAMMETRIC ENGINEERING, vol. IX, No. 3).

5. The calibration of a parallax (raising and lowering device) scale and the elevation scale is accurate to a degree not heretofore attained by any method using parallax tables, and the use of such tables is entirely avoided. The calibration is quickly prepared as a curved-line graph through a few control points simply derived by slide rule, and based on only exactly determinable factors such as focal length, distance between photo centers on a radial-line plot, and known scale of this plot (avoiding unreliable factors as flying altitude, etc.)

6. A mathematical item of particular mention is that the factor of the photo base becomes an exactly determinable variable in the above-mentioned slide-rule step, whereas in the use of existing methods with parallax tables no provision is made for this variation in the photo base, whose effects are fully equal to those tabulated.

7. The adjustment of the floating lines fully corrects for tilt, lens distortion, etc.

8. The adjustment of the floating lines utilizes ground points of known elevation when available, and one such point per photograph is enough to prevent a gradual departure from sea-level accuracy in the progress of the work.

9. The adjustment of the floating lines fully utilizes all physiographic criteria present, which are usually sufficient to insure accuracy in a local sense (i.e., no appreciable errors in relative elevations in any given area of a few square miles). One good level area found in a stereo-model can entirely control that model (by projection).

10. The adjustment of the floating lines permits the test of visual acuity to be applied at all places. By this is meant the ability of the eyes to judge relative depth of visible objects to a degree of great refinement. The floating lines are

critically tested by eye to lie in a consistent plane, and adjusted until this test is satisfied.

From the above outline, it will be seen that the method is a general one, to be used either with or without furnished elevation control. In the latter case the only error introduced is a gradual one with reference to sea level over a distance of many miles. There is no error introduced in the accuracy of relative elevations in any local part of a region.

Where elevations are furnished, the criteria of physiographic controls and visual acuity are subordinated, though always used in full. Where elevation control is absent, the physiographic control becomes dominant. In this case the procedure greatly resembles a levelling traverse. On each stereo-model in turn one "levels up" on the natural features (ponds, drainage criteria, etc.), "back sights" on elevations on the last overlapping stereo-model, contours the present model which furnishes "fore-sights" for use on the next, and thus indefinitely across the country.

The element of personal skill enters mainly in the recognition of physiographic criteria, which may not be entirely apparent to the unitiated (except the obvious, as ponds, shore lines, etc.). But when trained in their recognition, one will find that in most regions a given photograph possesses a myriad of details related to natural horizontality, such that when used in the aggregate, permits an adjustment of floating lines rigidly fixed to within such extremely narrow limits as to virtually eliminate any appreciable errors in a given stereo-model. This is true because it is the provision of the method that any horizontal plane is *projected* to an indefinite distance surrounding the area of its occurrence. Hence a number of such occurrences of approximate natural levels, when each one is so projected so as to overlap the others, compels an average to be struck of very great accuracy.

The element of personal skill also enters into the acquisition of visual acuity as previously described. This becomes extremely delicate with experience, hence imparts accuracy into the adjustments regardless of what other controls are used. No other method now used for overcoming tilt, etc. (correction graphs) furnished an opportunity for the use of this visual test.

The element of personal skill, of course, enters into the sketching of the contours. This method does not provide a movable floating mark which can *everywhere* be placed on a contour. One sees an *exact* contour location only along each of the several floating lines, where these interesect the topography, and contours are entirely drawn free hand, but kept by eye within the plane marked by the floating lines. Seven lines per stereo-model should be close enough for the beginner. Five are ample for an experienced worker. But to offset any seeming disadvantage of seeing the contours marked only along the lines, is the advantage of seeing them drawn upon the three-dimensional photo, where their intervals can be checked by eye, as well as their conformity to minor topographic details. This advantage is of such great importance that no contouring by conventional methods using drafting arm and floating dot that leaves no visible trace can attain comparable accuracy.

To the extent that personal skills enter into the operation as described in the paragraphs above, the beginner cannot expect either the same accuracy or speed of results as a trained worker.

Another obvious disadvantage of conventional contouring is the inability to correct for relief distortion, which gives contours improper spacing, especially on steep slopes away from the photo centers. The present method, by furnishing the contours on one of the photos, permits them to be copied stereoscopically in true radial-line position throughout by the plotting device described by the writer in the separate article already mentioned.

## THE PARALLAX-ELEVATION GRAPH*

The photos are mounted for use of ordinary lens-type stereoscope. The writer mounts them on a thin board rather than table top, to permit being turned around so that first one side (adjustment of floating lines) and later the other side (drawing of contours) is on the right side. The lines joining center and transferred principal points are aligned (center line).

The floating lines are produced by fusing a series of lines ruled by straightedge on one photo with a series of adjustable lines mounted to a sheet of transparent film resting over the other photo. These lines are oriented approximately at right angles to the center line. The transparent sheet is caused to slide back and forth in a direction parallel to the center line, and the position it occupies at any moment may be read to hundredths of a millimeter on an attached scale. Hence it can also be set at will to the position of any desired reading on this scale. The writer uses a sheet crrying a photo-engraved metric scale graduated to tenths of a millimeter, and sliding by means of two slots through which pins are passed into the board, together with a level for causing slow-motion sliding back and forth. The scale is read under a 7-power lens to hundredths of a millimeter. This device is accurate, efficient, and entirely satisfactory. However, an inventor can doubtless work up the same principles into a better mechanical device, attached to an easy-reading micrometer.

As stated above, a series of adjustable lines are mounted to this film sheet for fusion with the fixed lines drawn on the free photo. These adjustable lines may be prepared on thin narrow strips of film. Theoretically, regardless of tilt in the photos, these lines should be perfectly straight, to attain adjustment that would cause the fused lines to lie in a horizontal plane with reference to the natural criteria of the stereo-model. It is found in practice, however, that perfectly straight lines are incapable of perfect adjustment, the cause being presumably lens-distortion. Uusually the corrected lines are portions of a circle departing very little from a straight line. On all photos taken from the same camera and having a percent of overlap not departing too much from the average, all these lines will have the same curvature, and be convex in the same direction. This can be easily determined, and provides a simple check against the final adjustment of all lines. (A possible variation of the method is to rule the ink lines on the free photo curved, and use inflexible straight lines for the adjustable lines. The

* The writer is aware of confusion in the use of the term "parallax," and has tried to make his meaning clear by the context each time he has used the word. In his use, the "parallax scale" on the sliding sheet is an arbitrarily placed metric scale whose zero end is toward the opposite photo from that over which it is placed, and which is read by reference to any fixed index mark placed beneath it, and the "parallax-bar or equivalent" is a device to measure the true distance between some point on one photo and its corresponding point on the other photo at the relative position the two photos happen to be mounted. Readings on these scales or bars are conveniently called "parallax."

Strictly, of course, "parallax" is an angle, and not a measured linear distance as commonly treated. But if it could be referred to a linear quantity the only one strictly appropriate would be the "variable photo-base" as herein used by the writer. That is, the "parallax" of any ground point appearing on two photos is equal to that value of their photo base referred to that elevation, which can be measured on either photo as the distance from center to transferred principal point, the latter corrected to that same elevation.

The absolute value of parallax has no practical importance, as only parallax differences enter into our work. Hence arbitrary scales are quite sufficient and a new term needed is one that designates any reading on such a scale. The term "parallax" itself has commonly been put to this task.

writer, however, prefers the flexible film lines to become slightly curved in the process of adjustment. In areas decidedly deficient in criteria a physiographic control, however, the other method might have advantages.)

The film strips are mounted to the film sheet by ties of transparent scotch tape at frequent intervals. The adjustments are made by lifting these ties one at a time and resetting slightly to the left or right as demanded. (The writer uses a pointed tool for this operation permitting deftness and rapidity.)

It should be obvious that only on photos entirely free from tilt will the correctly adjusted film lines take the same spacing as those on the free photo, and be respectively parallel to them. A fixed framework of lines cannot be used on tilted photos (compare the Barr and Stroud floating network of lines which are incapable of adjustment for tilt). The adjustments we are describing are the means provided in this method for eliminating tilt, and all associated distortions (serving the purpose of the "correction graph" of other methods).

The first step after mounting the photos and drawing the lines on the free photo is to set up the sliding sheet over the other photo. Two (or more) temporary floating lines selected to pass near the principal points and some point of known elevation are adjusted carefully by eye to float in a single plane approximately level with respect to the topography. This adjustment does not call for the great accuracy as does later adjustments. Its purpose is to calibrate a scale of photo bases with the parallax scale attached to the sliding sheet, also to calibrate this latter scale with an elevation scale. (After these scales are calibrated, the final adjustments are to be made to *exactly* agree with the calibrations adopted. The calibrations are required *before* control points of known elevations can be used to assist the adjustments, hence this preliminary step.)

After these temporary adjustments are made, the lines are raised or lowered (by sliding the sheet) to the level of the selected point of known elevation (which may be taken from the contouring of the last completed photo overlapping this one), and a reading, $P_s$, made of the parallax scale. (This point is referred to as the starting point, $s$, its elevation $e_s$, and its parallax $P_s$.) The next step is the determination of $b_s$, by which is meant the average photo base for the same elevation as $s$. This average is found by the following formula:

$$b_s = \frac{(p_s + b_1 - p_2) + (p_s + b_2 - p_1)}{2}.$$

In this, $b_2$ and $b_1$ are the respective actually measured photo bases of the photos referred to as 1 and 2, and $p_1$ and $p_2$ are the parallax readings when the floating lines are brought to the levels of the indicated photo centers. The photo bases $b_1$ and $b_2$ are measured between center and transferred principal points by metric scale to the nearest tenth millimeter. Hence $b_s$ will be in millimeters.

Next a series of parallax values are taken for each of which the true elevation is computed by slide rule, starting with $b_s$ and its corresponding elevation $e_s$. A tabulation as follows is convenient:

| | $p(mm)$ | $dp(mm)$ | $b(mm)$ | $de(ft)$ | $e(ft)$ |
|---|---|---|---|---|---|
| $s$ | 2.60 | | 63.36 | | 850 |
| $n$ | 1.60 | −1 | 62.36 | −289 | 561 |
| | 3.60 | +1 | 64.36 | +281 | 1131 |
| | 4.60 | 2 | 65.36 | 552 | 1402 |
| | 5.60 | 3 | 66.36 | 815 | 1665 |
| | etc. | | | | |

In this case $b_s$ is 63.36 mm, $p_s$ 2.60 mm, and $e_s$ 850 ft. The several other computed points, referred to as $n$, are selected at whole millimeter intervals above and below $p_s$, their difference from $p_s$ being given in the column headed $dp$. The next column shows the corresponding values of the average photo base, obviously shifted in parallax by the same amounts. Each of these values is referred to as $b_n$.

The values to be filled in the $de$ column ($de$ is the difference of elevation from $s$) are found by slide rule in three steps:

$$(1) \qquad Q = \frac{f(in)\ S_r}{12} = \frac{f(mm)Sr}{304.8}$$

This formula involves only those factors which are constant for an entire series of photos, and hence the determination of $Q$ is performed once and for all. $f$ is the focal length of the taking camera, and $r$ is a radial-line plot of the photo centers, the exact scale of this plot being $1:S_r$.

$$(2) \qquad U = \frac{Q\ b_r(mm)}{b_s(mm)}.$$

This step is performed once only for each stereo-pair. $b_r$ is the measured distance between the two photocenters concerned on the radial-line plot.

Finally, for each position of $n$ in the above tabulation, the following step is taken:

$$(3) \qquad de(ft) = \frac{U\ dp(mm)}{b_n(mm)}.$$

The elevation of each point, $e$, is entered in the last column as the sum of $e_s$ and $de$.

The several points are plotted on cross-section paper ruled 10 lines to the inch, whose two coordinates are elevation (1 in. = 100 ft.) and $p$ (1 in. = 1 mm.) The points are determined by the values found in the columns headed $e$ and $p$. A curved line is drawn through the points on the cross-section paper, giving a graph which will enable the corresponding values of all measures of elevation and parallax to be read.

The three formulas above may be combined into the single formula:

$$de(ft) = \frac{f(ft.)\ S_r\, b_r(mm)\ dp(mm)}{b_s(mm)\ b_n(mm)}.$$

Its derivation will present no difficulties to a reader familiar with mathematical photogrammetry, since the terms are all commonplace except for the separation of the two significant photo bases, $b_s$ and $b_n$, for the upper and lower limits of the interval represented by $de$.

(It is worthy of comment that existing parallax tables utterly fail to assist their user not only in using the true photo base appropriate for a given interval, but for varying this photo base for higher or lower intervals. The result of this, as has been noted, is to ignore a factor whose effects are as large as those given in the table! In other words, such a table goes only half way. It is true that since the table gives parallax for an assumed photo base of 100 mm., it leaves a loophole for a person to take any photo base he pleases as a percentage of a hundred, and hence vary the photo base as he pleases. But the instructions for the use of the table make no mention of varying the photo base, no means of determining the variation are presented even if one wanted to, and if they were the labor would be as great as though no tables were furnished at all. It is strange that none of the many prominent photogrammetric authors who have described the tables have ever commented on this fallacy of their use. At long last one writer, Albert L. Nowicki, in a recent number of PHOTOGRAMMETRIC ENGINEERING (Apr.-June, 1943) at least recognizes that there is a problem here, but unfortunately gets himself involved mathematically in such a way as to fail to see a simple solution. The simple solution is found by rearranging existing formulae—instead of solving for $dp$ on basis of equal increments of $de$, one should solve for $de$ on the basis of equal or definite increments of $dp$. This way permits increments of change of the photo base exactly measureable to be figured in.)

## THE ADJUSTMENT OF THE FLOATING LINES

Having constructed the elevation-parallax graph, the next step is to set individual floating marks at all selected points of known or previously contoured ground elevation, as controls for the final adjustment of the floating lines.

Through each such point on the free photo, rule a short line approximately parallel to the long ruled lines. Then on the sliding film sheet, at each corresponding point, mount a short line (engraved on a short bit of film) to appear to fuse with the other line. This is to be mounted to appear to rest at ground level when the scale of the sliding sheet is set to the parallax measure indicated by the graph as corresponding to the elevation of the ground point in question. (Each of the adjustable short lines should carry a cross line, the intersection of the cross being the exact point to fuse at ground level.)

When all these floating marks are set, it will be seen that they float at a common level, in a plane which is raised or lowered by sliding the sheet. All of the adjustable line strips are now mounted to appear to float at this same uniform level. The central line should have been so placed as to be visible whichever photo overlaps the other.

As previously indicated, the final adjustment must satisfy fully three separate kind of criteria, viz., (a) visual acuity, (b) physiographic control, and (c) agreement with the set control marks. The fullest use of all three is necessary for the greatest accuracy. A proper concept of the role played by all three is gotten by considering that all methods of control aim at establishing a perfectly true and horizontal plane with reference to nature. Visual control is essential to make this plane surface free from local warping and buckling. Physiographic control makes it everywhere lie approximately horizontal. Control marks insure its proper height above sea level at definite points.

The best procedure for making the adjustments is first to bring the lines nearest to the set marks to the level of these, while these are made to float to some level above the ground. Next the strips should be adjusted one by one, and section by section, always sliding the sheet so that the lines in the area being adjusted are at or slightly above ground level. Physiographic features must be reconciled, as well as the consistency of the appearance of the lines. For this last test, raise the lines high enough to bring three adjacent lines clear of the ground.

Physiographic features vary from exact to approximate. The former include ponds, lakes, shore lines, and certain swampy areas. Nearly exact are channels of large streams, and some flood plains. Other criteria are somewhat indirect. The gradient of streams can be estimated very closely with experience. At any rate, if streams are present flowing in several directions, the possible planes satisfying all (that is, keeping *all* streams flowing down hill at once) are not very many, and the adoption of the true horizontal plane is essentially demanded.

Other indirect criteria include a great variety of "downhill" marks, streaks, rills, etc., indicating that horizontal contours must trend at right angles.

After the adjustment has been completed, examination of a straight-edge beside the lines, perhaps with the help of a magnifying glass, should reveal the slight curvature already mentioned attributable to lens-distortion. As stated before, all lines should be convex to the same side and approximately equally curved, and this should be repeated by all other photos from the same series except that the curvature is less if the percentage of overlap is excessive, and vice versa. (Where physiographic and other controls are comparatively lacking, this consistent line curvature can become an aid in securing properly adjusted lines.)

## CONTOURING

After the completion of the adjustment of the lines, contours may be drawn under the stereoscope. The setting of the floating-line sheet for any given contour is taken from the elevation-parallax graph, by finding the parallax value in agreement with the elevation in question, and setting the scale of the sliding sheet at this figure.

An experienced worker at contouring need not set the floating lines to mark the level of every contour, but only those to be drawn heavier than others, and perhaps occasional intermediate contours in areas of gentle slopes or irregular topography. As a general rule he can interpolate intermediate contours by proportioning the vertical interval between those already drawn. The beginner should set the floating lines for at least every other contour, except on very steep uniform slopes.

As previously noted, this method furnishes precise contour location only along the several floating lines. But since two or more such lines are always visible in any local area being contoured, the eye grasping them senses the position of the plane which they define, and can always carry the contour along very accurately in this plane. All things considered, a skilled worker can achieve greater accuracy by this method because of the visibility of the contours. At the same time the large degree of free-hand element permits contouring to proceed far faster and smoother than by any other mechanical method.

## SPOT ELEVATIONS

Spot elevations can be very closely estimated merely by bringing the plane of the floating lines to the ground point in question, and thence reading the parallax scale and referring to the elevation graph. For greater precision of measuring spot elevations, set the floating lines to bring the two nearest to the ground point in question clear of the ground in the immediate area. Thence draw a short line through the ground point on the free photo, and set a control mark (in the manner previously described) first at the level of the floating lines, and again at the level of the ground point. Each time measure the parallax or distance between corresponding marks (by parallax bar) to the nearest hundredth millimeter. The difference between these is subtracted from the reading on the parallax scale, and the result referred to the elevation graph which gives the true elevation of the ground point.

In certain types of geological work, topographic contouring is omitted, while measurement of a large number of spot elevations on outcrops of rock formations is desired, to use in plotting geologic structure. To measure these elevations with greatest precision, a correction graph is prepared after the completion of the adjustment of the floating lines.

To prepare this, lay a sheet of traceofilm over the free photo, fastened on its outer edge only, to hinge. On this photo make marks along any of the ruled lines at selected points distributed to provide well-spaced control of the entire stereo-model. It is well to be liberal with these as the operation is simple and rapid. About twenty such points are desirable.

Before proceeding, the parallax scale of the sliding sheet must be set at zero, and the sheet fastened to preserve this setting during the following step. Then at each of the selected points measure the parallax, that is, the distance between the corresponding lines at the point in question. Care must be taken to use the right point on the line on the opposite photo from the one marked. This will be the point which would fuse with the marked point were the floating lines brought to ground level or above ground. But without fusing the lines it is found simply by first aligning the parallax-measuring device with the marked point seen as a fused point on the ground and then extending this alignment to intersect the other line.

Each of the measures so made is to be entered on the traceofilm beside the tracing of the point to which it pertains. The floating-line sheet may then be removed for all remaining work.

The correction graph is then carefully drawn on the traceofilm by lines expressing equal values in agreement with the figures established in the last step. It is worthy of comment that this correction graph will have an accuracy impossible to approach by any present-used methods.

Measure all spot elevations of selected points on the ground as follows: Measure the ground parallax (parallax bar) at the selected point, and subtract this from the amount taken from the correction graph at that point. This difference, referred to the elevation-parallax graph, will give an immediate reading of true elevation above sea level for the point.

## Tests of the Method

The writer has already contoured hundred square miles in widely separated areas by the method, most of this in areas having no furnished elevation control. In this case the work was carried indefinitely around the country on several rows of photos with exact joining of contours on every picture (using a ten-foot interval in the lowlands to fifty or a hundred feet in the mountains). A rate of some 35 square miles per day is indicated. The most surprising result was the perfect alignment of contours at all levels (maximum relief being 2000 ft.) from one photo to the next. The only appreciable disparities in vertical scale observed were in the case of a few exceptional photos in high mountains at extreme top or bottom edge of a photo. This was obviously the result of tilt, for which the variable photo base value computed for the center line of the photos fell down. The writer sees no practical way of determining the effective photo bases on tilted photos in those portions away from the center lines.** If the end lap of the photos is not too small, these errors are corrected on the next row of photos by tieing in as far toward the central part of the last photo as possible.

## A Less Accurate Method

A quick, less accurate method of measuring spot elevations is here given, using the principle of floating lines, and without the use of the sliding sheet. (This method also provides for contouring entirely free-hand in agreement with

** This statement was made prematurely. See "Supplement" at the end of this article for a means of handling this problem.

as many such spot elevations as it is desired to determine.) The only requirement is a series of a few adjustable lines on strips of film, and a parallax bar or equivalent finely graduated metric scale, also a slide rule, and lens or mirror type stereoscope.

First the photos are mounted with the center lines aligned, and a series of straight lines (at least three) are ruled on one of the photos at approximately right angles to the center line. On the other photo an equal number of film-strip lines are fastened to fuse with the others to give a series of floating lines, adjusted to appear approximately level with reference to the topography, and at a median topographic level. (No effort is made to curve these lines—they may be taped at the ends only.)

A correction graph is then made in the manner described above, but perhaps using fewer control points, and drawing the lines with a very light pencil.

A value of "$K$" (average number of feet elevation for one millimeter parallax) is then determined by any of the following formulas, depending on the available data:

$$K(ft) = \frac{f(in)\ S}{12\ b(mm)} = \frac{f(mm)S}{304.8\ bmm}. \qquad (1)$$

Here the average photo scale is $1:S$, and the photo base, $b$, is the average for the two photos, each corrected an estimated amount by taking the side (transferred) principal point not where actually transferred, but shifted to the estimated position it would have occupied were it at the average elevation (of the floating lines).

$$K(ft) = \frac{A(ft)}{b(mm)}. \qquad (2)$$

Here $A$ is the average altitude of the camera, and $b$ as before.

$$K(ft) = \frac{f(in)\ S_r\ b_r(mm)}{12\ b^2(mm)} = \frac{f(mm)S_r\ b_r(mm)}{304.8\ b^2(mm)}. \qquad (3)$$

Here we have a radial plot of scale $1:S_r$ on which the equivalent photo base is $b_r$, and $b$ again as above.

Having determined $K$, the parallax of some point of known elevation is measured, and by slide rule a "sea-level correction" is computed as follows:

$$\frac{K}{\text{unity}} = \frac{\text{elev. of point } (ft)}{\text{correction } (mm)}.$$

This correction is added to the parallax measured at the point, and this sum marked on the traceofilm at that point in colored pencil. Then colored correction-graph lines are drawn guided by and replacing the light pencil lines, and numbered consistent with the established point.

Spot elevations are everywhere measured as follows: Measure the parallax at the selected ground point, subtract from the value indicated by the correction graph (colored lines), and on the slide rule, constantly set with $K$ over unity, find the elevation:

$$\frac{K}{\text{unity}} = \frac{\text{elevation } (ft)}{\text{dif. from graph } (mm)}.$$

## SUPPLEMENT

Since submitting the foregoing manuscript, the writer has given some thought to the correction for tilt as it affects the parallax-elevation calibration in all parts of the stereo-model, and has arrived at a solution, here given, which seems entirely supported in preliminary tests. In fact, these tests indicate a degree of accuracy beyond our ability to detect any errors, in all parts and at all levels of stereo-models of nearly two-thousand feet relief, with abundant furnished control for checking.

To put the matter in terms familiar to the user of "correction graphs," we now propose to make the "correction graph" three-dimensional instead of two. In the writer's terms, the variable photo base, previously computed for the center of the model only, is now to be replaced by the "effective photo base" for any specified point in the model, varying not only with the elevation, but with horizontal location as well. (The effective photo base for any given point is in reality the true linear parallax of that point, though for reasons already noted it is best not to restrict the term parallax to this application.)

The following procedure need be followed only in cases where tilt appears to be excessive, great relief is present, or where utmost accuracy is desired. The general procedure previously described is followed except for the modifications now to be given.

At the outset of the work, each center point is to be transferred to the opposite photo with the utmost accuracy, and the center lines (joining center and transferred points) drawn also with utmost accuracy, extended the full width of the photos. Then in mounting the photos, utmost accuracy once again is used in aligning these center lines, testing with a magnifying glass.

At the step where temporary floating lines are mounted on the sliding sheet, these should be more carefully adjusted, especially toward their extremities, than was formerly necessary. They should lie in a fairly true plane, horizontal with respect to physiographic and visual criteria.

A sheet of tracing paper is hinged to the free photo in the manner described previously in connection with the preparation of a "correction graph." Such a correction graph is now made, except that the parallax scale on the sliding sheet need not be set at zero reading, but the sheet merely fastened at any fixed position, and only about half as many measured points need be used to control the graph. The lines are to be drawn in light pencil at this stage.

A "starting point" elevation, $e_s$, is selected as previously described, and corresponding parallax reading (sliding-sheet scale), $p_s$, is made. Next the effective photo base, $b_s$, for each center point ($C_1$ and $C_2$) is determined as follows:

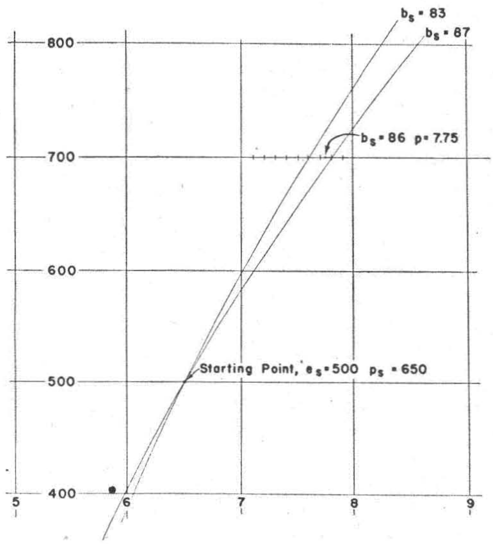
$$b_s \text{ at } C_1 = p_s + b_2 - p_1$$

$$b_s \text{ at } C_2 = p_s + b_1 - p_2 \cdot$$

Each of these values of $b_s$ should be recorded with colored pencil on the tracing paper at the proper point, and then a set of colored graph lines drawn guided by and replacing the light pencil lines, noting, however, that the numerical values of $b_s$ to be assigned to the colored lines increase where the penciled values had decreased, and vice versa.

Two selected round-number values of effective photo base, $b_s$, one near the highest value represented, and one near the lowest, are the only ones that need be used in the tabulation and preparation of two separate elevation-parallax graph curves in the manner previously described. (There is no change in the value of $b_r$ to be used.) These two graph curves will each pass through the starting point as plotted on the graph. Fig. 1 shows a portion of such a pair of graph curves, plotted for values of $b_s$ of 83 and 87, and assuming $e_s = 500$ ft., and $p_s = 6.5$. The double curve is used as follows:

(1) To set control marks for final adjustment of floating lines. Suppose such a control mark is to be set for a ground point of 700 ft. elevation, and located in the stereo-model at a position where the tracing paper indicates a $b_s$ value of 86. The procedure is to select a point on the 700 ft. line falling between the two




graph curves proportionately as 86 falls between 83 and 87. Thence make a parallax reading on the graph, in this case about 7.75, to use in setting the floating mark.

(2) To draw contours. As one draws the 700 ft. contour, in the case illustrated in Fig. 1, he should shift the setting of the sliding sheet scale as he goes from one part of the stereo-model to another, each time getting an average $b_s$ value from the tracing paper and taking a reading on the 700 ft. line proportionate with reference to the two graph curves as before.

This method depends on the theory that the combined displacement components caused by tilt, having produced both the variations in the photo base and the variations in the parallax-bar readings, will have varied these equally, and the same "correction graph" expressing the one will also express the other, provided that this correction graph is freed from effects of improper mounting of the photos. It should be obvious that the floating lines are to be finally adjusted to be truly horizontal at the elevation of the starting point, at which time they are not truly horizontal at any other elevation. This discrepancy could not become apparent to the eye by any other criterion than the physiographic, and will be apparent then only when a considerable magnitude of relief is encountered,—much more than is corrected in the procedure described. It is best to take the starting point at a median topographic elevation, when possible, and most physiographic criteria will retain their optimum value in aiding final adjustments.