ELEMENTS OF PHOTOGRAMMETRIC OPTICS

Reynold E. Ask
Photogrammetric Engineer
U. S. Coast and Geodetic Survey

EDITOR'S NOTE: This is another chapter for the Manual of Photogrammetry, which the Society is having prepared by outstanding leaders in this specialized field of engineering.

ABSTRACT

In practically every step of map compilation by photogrammetric methods, instruments are used in which optical elements play a very necessary part. Among these are aerial cameras, ratio printers, stereoscopes, projectors, transits, plotting apparatus, etc. Although many books on geometrical optics have been published, it is still difficult for the photogrammetrist to find a single book which presents those phases of the subject which are of chief interest to him. It is the purpose of this chapter to partially fill this need. Necessarily in a single chapter it is impossible to go into each of the many subjects in detail. Numerous references are given, however, to aid those who may be further interested in the subject. Throughout the chapter mention will be made of the practical applications of the various principles presented.

Contents

I. Introduction to Lenses

THE study of optics is usually subdivided into two main fields, physical and geometrical. Physical optics concerns such subjects as wave motion, diffraction, interference, and polarization, and need not be considered in this discussion, although certain aspects of it are important. Practically all of this chapter concerns geometrical optics. Following are the postulates upon which the subject of geometrical optics is based: (1) Light travels in a straight line in a homogeneous medium; (2) when a ray is reflected from a surface, the angle of reflection equals the angle of incidence (the angles are measured from a normal to the surface); (3) when a ray is refracted (on passing from one medium to another medium of different density), the sine of the angle of incidence (i) divided by the sine of the angle of refraction (r) equals a constant called the "index of refraction" (N).

$$\frac{\sin i}{\sin r} = N \qquad \text{eq. (1)}$$

It is this property of refraction which makes lens action possible.

A lens may be defined as an optical medium bounded by spherical surfaces (a plane surface being considered a spherical surface of infinite radius). On entering a lens, light is refracted in such a manner as to form either a real or a virtual image depending on whether a converging (positive) or a diverging (negative) lens is used respectively (see Fig. 1).

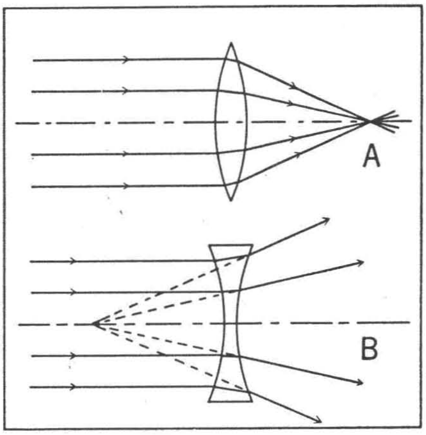

FIG. 1. Lenses. (A) Converging or positive. (B) Diverging or negative.

1. Thin Lenses

The term "thin lens" is used in approximate computations to indicate that the thickness of a lens is ignored, all distances being measured from the lens center. The relationship between the object distance (O), the image distance (I), and the focal length (F) is given by the basic lens equation:.

$$\frac{1}{I} = \frac{1}{O} + \frac{1}{F} \qquad \text{eq. (2)}$$

In the above equation the following sign convention is used: All distances (except focal length) measured from the lens in the direction of the incident light are considered positive; all distances measured against the direction of the incident light are considered negative; the focal length of a converging lens is considered positive; the focal of the diverging lens is considered negative. The distances (O) and (I) are also called conjugate distances. The magnification (M) is defined as a ratio of image to object distance and a minus sign indicates that the image is inverted.

$$M = \frac{I}{O} \qquad \text{eq. (3)}$$

The following examples illustrate the manner in which equations (2) and (3) are used:

Example (1): Given an object distance of 25 inches and a focal length of +10″. Compute the image distance and magnification (see Fig. 2).

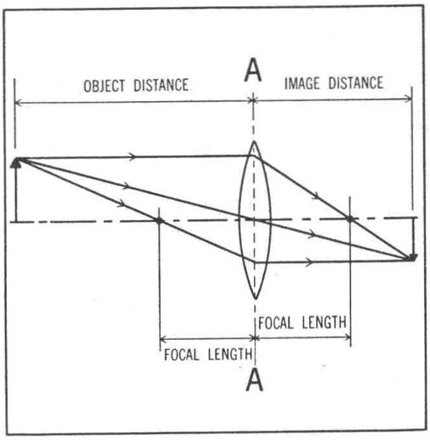

FIG. 2. Image formation positive lens.

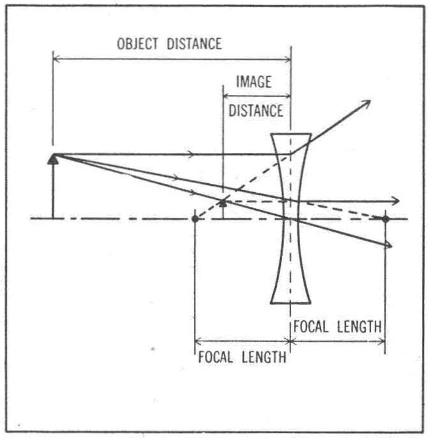

FIG. 3. Image formation negative lens.

$$\frac{1}{I} = -\frac{1}{25} + \frac{1}{10}$$

$$= -0.040 + 0.100 = +0.060$$

$$I = +16.67''$$

$$M = \frac{I}{O} = \frac{+16.67}{-25} = -0.667$$

Thus it has been found that a *real inverted* image was formed which is *smaller* than the object. Note the simple graphical construction used in the figure to obtain a good approximation of the image position.

Example (2): Same conditions as in example (1) except F is $-10''$ (see Fig. 3).

$$\frac{1}{I} = -\frac{1}{25} - \frac{1}{10}$$

$$= -0.040 - 0.100 = -0.140$$

$$I = -7.14''$$

$$M = \frac{I}{O} = \frac{-7.14}{-25} = +0.286$$

Thus in this case we have a *virtual erect* image *smaller* than the object.

If the object distance is infinitely great, it can readily be seen from equation (2) that the image distance equals the focal length. This is essentially the relationship in an aerial camera (see Fig. 1A) where the object distance (flying height) is very large compared to the focal length.

When two thin lenses having focal lengths (f_1 and f_2) are placed in contact, the approximate focal length (F) of the combination is given by the equation

$$F = \frac{f_1 f_2}{f_1 + f_2} \qquad \text{eq. (4)}$$

When working with photographic lenses (or other positive lenses forming a real image) the following equations, in which the magnification (M) is considered, are usually found to be more convenient than equation (2) because no reciprocals are used:

$$\text{Object distance } (O) = F + \frac{F}{M} \qquad \text{eq. (5)}$$

$$\text{Image distance } (I) = \text{F} + FM \qquad \text{eq. (6)}$$

Example (3): The focal length of the lens in a ratio printer is 240 mm. Compute the object and image distances for a magnification of 4.

$$O = 240 + \frac{240}{4} = 300 \text{ mm.}$$

$$I = 240 + 240 \times 4 = 1200 \text{ mm.}$$

Check: $$\frac{1200}{300} = 4$$

In some ratio printers the object the image distances are linked to a cam mechanism which causes these distances to vary as in equations (5) and (6). It is therefore possible to obtain sharp focus at various magnifications or scale ratios merely by turning one handwheel. The term "auto-focus" is given to such an arrangement.

In spectacle optics the dioptry system is used to express the power of a lens. The power in diopters (Δ) being equal to the reciprocal of the focal length (F) in meters:

$$\Delta = \frac{1}{F\,(\text{meters})} \qquad \text{eq. (7)}$$

Thus a 5 diopter lens has a focal length of $\frac{1}{5}$ meter or 20 cm. The advantage of this system is that the power of a series of lenses in contact equals the algebraic sum of the individual powers, thus: $+5\Delta - 1\Delta = +4\Delta$, giving a focal length for the combination of $\frac{1}{4}$ meter or 25 cm. If focal lengths values had been given instead of diopters it would be necessary to use equation (4) which requires a little more computation. Due to their cheapness, spectacle lenses are often used in photogrammetry where great accuracy is not essential, such as in simple lens stereoscopes, magnifiers, and for experimental work.

2. Thick Lenses

The term "thick lens" is used in accurate computations to indicate that the thickness of a lens is considered, the object and image distances being measured from the nodal points N and N' instead of lens center (see Fig. 4). These distances are computed from the same equations as for thin lenses. The nodal points* of most photographic lenses (except the telephoto type—see Section II-1) lie rather close to the lens center and their location is determined from optical bench measurements. It should also be noted that the nodal points

* The nodal points of a lens are sometimes referred to in optical literature as principal points; the two designations can be considered identical in the present discussion.

occasionally overlap, that is, they are reversed in position as shown by the dotted lines in the upper part of the figure. The letter (*S*) will be used in this discussion to indicate the nodal point separation. Referring again to the figure, it is obvious that the total distance from object to image is the sum of: object distance (*O*), nodal point separation (*S*), and image distance (*I*). The graphical ray construction used for a thin lens (Fig. 2) can be modified so that it also will apply to a thick lens. To do this we split the figure along line *A–A* and then separate the two parts a distance equal to the nodal point separation.

In a thick lens it is necessary to distinguish between the following types of focal length (see Fig. 4): The equivalent focal length being the distance from the rear node to the point of best axial imagery; the back focal length being the distance from rear vertex of the lens to the point of best axial imagery; and the front focal length being the distance from the front vertex to the point of best axial imagery, the object distance in all cases being at infinity. For any lens or lens system it is possible to determine by computation the location of the nodal points and values for the various focal lengths. References are given at the end of the chapter to aid those who may be interested in such computations.[1*]

In addition to the above focal lengths, the following two related terms are also used in photogrammetry: (1) *Calibrated focal length* is the perpendicular distance in an aerial camera from the rear node of the lens to the plane of the film emulsion. The calibrated and equivalent focal lengths usually differ in as

* See References at end of chapter.

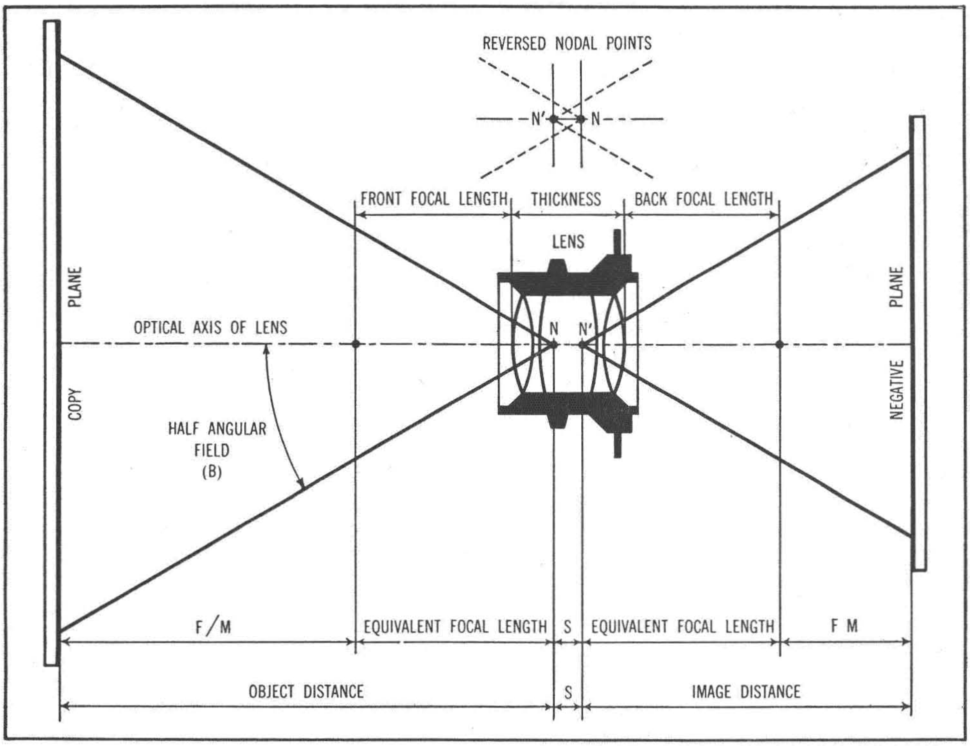

FIG. 4. Important optical dimensions of a photographic objective. The diagram is arranged as if the lens were used in a typical copying camera.

much as the former is selected to give the best average definition over the entire negative while the latter is based on best axial imagery only. A further discussion of this item is given under "Lens Aberrations"; (2) *Principal distance* is a geometrical property of a negative or print and equals the calibrated focal length corrected for both the enlargement or reduction ratio and the film or paper shrinkage. It re-establishes the same angular cone of rays as existed at the taking camera at the moment of exposure.

3. Lens Aberrations

An aberration is a defect in an optical image caused by the fact that essentially no lens system can form a perfect image. A brief description of the aberrations considered in lens design follows.

Spherical aberration (see Fig. 5): Rays from various zones of a lens focus at different places along the axis, this results in an object point being imaged as a blur circle. It is caused by the spherical shape of the lens surfaces. Its magnitude decreases as the lens aperture is reduced.

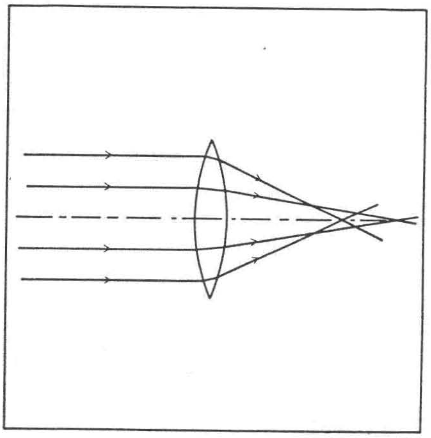

Fig. 5. Spherical aberration.

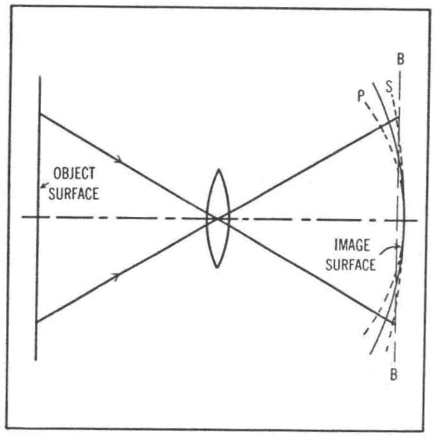

Fig. 6. Astigmatism (dotted curves) and curvature of field (solid curve).

Coma: A comet-shaped blur of light formed around image points off the axis. It is due to spherical aberration of oblique rays.

Astigmatism and curvature of field: Astigmatism is an aberration which causes a point object off the axis to be imaged as two mutually perpendicular short lines located at different distances from the lens. One of these lines is radial and the other tangential with respect to the center of the field. In Figure 6 the tangential image surface (also called primary) is marked (P) and the radial image surface (also called secondary) marked (S). The surface of best definition is located midway between these two (solid curve) and its departure from flatness is termed "curvature of field." A positive value indicates that the field is concave towards the lens. If a lens exhibiting curvature of field is used in a camera, it is obvious that the film cannot be dished to assume the surface of best definition. However, the film plane can be placed in such a position (dashed line B–B in Fig. 6) as to obtain the best average definition over the entire field. In an aerial camera the distance from the rear node of lens to plane of film (as obtained by above method) is called the "calibrated focal length."

Longitudinal chromatic aberration (see Fig. 7): Rays of various wave lengths or colors focus at different distances from the lens (the distances being measured parallel to optical axis). This defect can be practically eliminated in a lens consisting of two or more elements by using glasses of different dispersive powers. The term "achromatic" is applied to such a lens.

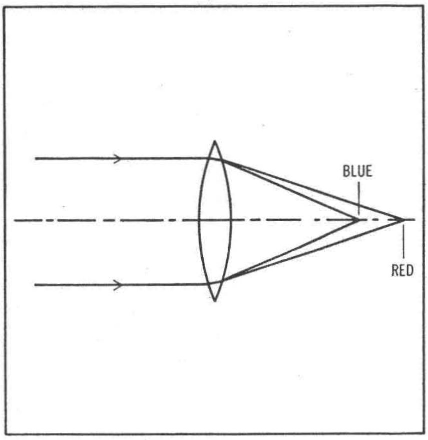

FIG. 7. Longitudinal chromatic aberration.

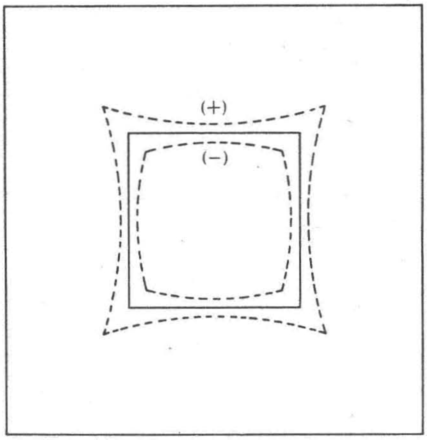

FIG. 8. Effect of distortion on the image of a square object.

Lateral chromatic aberration: A difference in image magnification for various colors caused by chromatic aberration of oblique rays. Due to this defect, images near the outer portion of the field will be fringed with color. In a visual instrument (such as a telescope) the colors are readily apparent to the eye.

Distortion (see Fig. 8): A displacement of an image point radially to or from the center of the field. A positive distortion is considered away from the center. It also may be expressed as a variation in magnification in different parts of the field.

A lens designer is interested in each of the above defects individually. The lens user, however, is mainly interested in distortion and "definition." The latter term is used to express the ability of a lens to record fine detail and really represents the combined effects of all the above mentioned errors except distortion. Definition may be expressed mathematically as "resolving power" which refers to the maximum number of lines per millimeter that can just be resolved (that is, seen as separate lines) in the image. Theoretically the resolving power increases with lens aperture, but practically this is only true when a very small angular field is used (such as in a telescope). As far as photographic lenses are concerned the resolving power is limited by residual aberrations which in most cases become smaller as lens aperture is reduced. However, a reduction in aperture smaller than $f/22$ may reduce the resolving power due to diffraction effects.

II. Photographic Lenses

1. Design

It is not the purpose of this article to go into the highly scientific field of lens design,[2] although a few remarks on this subject appear to be necessary. Photographic lenses are designed for specific types of work. A lens designed for copying maps, for instance, would not be satisfactory for use in a precise aerial mapping camera. In the former case the lens is designed to give good definition and low distortion values for finite object distances (say for scale ratios of 0.25 to 2.00) while in the latter case the lens is designed for an infinite object distance only.

In attempting to obtain a "near perfect" lens, designers have tried hundreds of different combinations. The variables at the disposal of the designer being: the index of refraction of the glass (1.50 to 1.72), the dispersive power of the glass, the curvature of the surfaces, and the number, thickness, and spacing of the elements. The labor involved in designing a new lens may mean months of trigonometric ray tracing. Since it is impossible to design a perfect lens, the best that can be done is to reduce those errors which would impair the work which the lens is intended to perform; in other words a balance is sought between the most desired properties. An example of this balancing is found in the modern aerial camera lens[3] where low distortion, good definition, and a large angular field are secured at the expense of maximum aperture. A cross section of such a lens is shown in Figure 9.

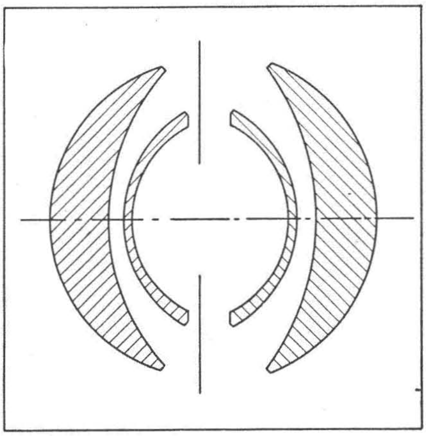

FIG. 9. Section through a Bausch and Lomb wide-angle Metrogon lens (f/6.3 aperture and 93° field).

Although the wide-angle lens is the preferred type to use for mapping work, certain types of aerial photography require lenses of different design. For instance to obtain maximum haze penetration in oblique photography, infra-red film and filters may be used. To secure optimum definition from such a combination it is necessary that the lens be corrected for infra-red light. For military reconnaissance on the other hand, lenses of long focal lengths (up to 50 inches) are necessary to obtain a desirable image size from a great altitude. Such lenses require the use of long cumbersome camera bodies. One method of avoiding this inconvenience is to use a telephoto lens. This lens consists of a positive front element and a negative rear element, the separation of the two being rather large. The result of such a design is to place the rear nodal point of the lens system far out in front of the positive element, thereby obtaining a large equivalent focal length (say 40 inches) with a much smaller overall camera length (say 24 inches). Another special lens for military reconnaissance is the large aperture type used in night photography with flash bombs.

2. MANUFACTURE[4]

The glass used in photographic objectives, as in other optical elements, is specially made for that purpose. It differs from ordinary plate glass both in composition and in the manner in which it is made. The composition of the various types of optical glass differ greatly, but the basic ingredient is usually silica in the form of a fine white sand. Recently a new type of optical glass has been developed in which compounds of the "rare elements" (lanthanum, tantalum, and tungsten) have replaced the silica. This new glass has a higher refractive index for a given dispersive power than was previously obtainable. All the materials that go into a batch of optical glass must be chemically pure, and must be weighed and mixed under rigid scientific control. The mixture is fed into a large clay pot (3 to 5 feet in diameter) which was preheated in a gas furnace. The batch is stirred while the mixture is introduced and also for a considerable period afterward so as to insure uniformity of composition. The pot is then removed from the furnace, covered with an insulating material, and allowed to cool slowly. This is termed "annealing" and usually requires one week. During this cooling period the glass cracks up into irregular chunks which are later inspected for such defects as bubbles, undesirable color, stones, striae and strain.

The next step is to place these irregular chunks in a gas furnace, heating until plastic, and then pressing into a mold of the desired size and shape. The resulting piece of optical glass is now termed a lens blank. These blanks are now fine annealed for about 30 days to remove internal strains.

The lens blank is then ready to be ground. In this operation the blank is cemented to a grinding block and ground with a cast iron tool of the proper curvature, water and emery being used as the abrasive. Finer and finer emery is introduced as the surface approaches the desired shape. When the grinding has been completed the emery is thoroughly washed off, and the grinding tool replaced with a pitch-lined polishing tool. The polishing is accomplished by rouge and water. During this operation the surface is checked periodically by placing a test glass of the opposite curvature on the lens and noting the resulting pattern of interference fringes. This fringe pattern is really a contour map of the air film between the two glasses, the contour interval being one-half a wave length of light (0.00001″). When the polishing of one surface is satisfactory, the lens is turned over and the other side completed.

The next operation is centering, which consists of grinding the edge of the lens element concentric with the optical axis. Small errors in centering of successive elements will cause unsymmetrical distortion in the completed photographic lens. Avoidance of such errors is important in precision photogrammetry.

Most photographic lenses consist of 4 to 8 lens elements, some of which are cemented together. The cementing operation consists in gradually heating the elements on a hot plate, placing a drop of Canada balsam at the center of the desired surface, and pressing the elements firmly together until all air bubbles and excess balsam are squeezed out. The lens elements are now ready to be mounted in their respective metal cells. These cells have been machined very accurately so as to keep the lenses at the designed separation, and also hold them firmly but without strain. The front and rear cells are now mounted in the lens barrel which usually contains the iris diaphragm and a between-the-lens shutter.

3. Testing

Before a given photographic lens is mounted in a camera, it is subjected to a series of suitable tests. These fall into two categories: optical laboratory and camera tests.

In the laboratory an optical bench is used to obtain quantitative measurements of back, front and equivalent focal lengths, nodal point separation, distortion, and definition or resolving power. The optical bench used in these tests may be of the visual or photographic type. In the visual type,[5] the accuracy of the test depends upon the visual ability of the observer to interpret images of suitable test targets. This type is chiefly used for measuring the focal lengths on the axis, nodal point separation, and distortion for various scale ratios. It is very difficult in the visual type to determine the value of the focal length which will give the best average definition over the entire field.

Aerial camera lenses are usually tested in the photographic type of testing equipment.[6] A photographic plate records the images of a series of test charts arranged to cover the angular field of the lens in five degree steps. A chart similar to those used is shown in Figure 10, the numbers opposite each group of lines indicating the number of lines per millimeter that will be imaged on the test plate by a lens of given focal length. Each chart is placed in the focal plane of a collimator (see section V-3) for the purpose of obtaining an infinite object distance. A series of exposures is then made with slightly different lens-plate distances. The resulting photographic plate is then studied with a microscope to determine what value of the focal length will give the best average definition (or resolving power) over the entire field. The value so selected is the "calibrated focal length" and is the distance that is actually set in the aerial camera (this item was previously discussed under "Lens Aberrations"). Also measurements can be made on the plate to determine the distortion for an infinite object distance. The resolving power obtained by the above test may be higher than that obtained under service conditions. The reason being that the emulsion of the test plate is finer grained than that of the panchromatic film used in the aerial camera.

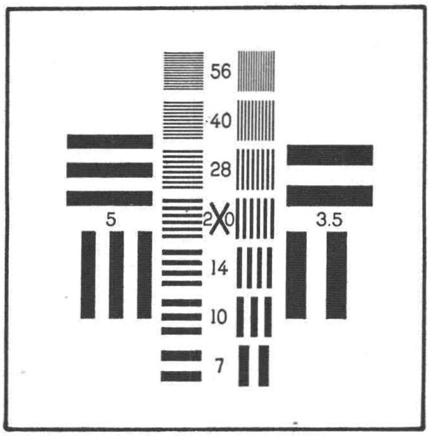

FIG. 10. Chart for testing lens resolving power.

It might be desirable to mention that only a few organizations have available precise optical bench equipment for testing photographic lenses. Among these are large optical manufacturers, the National Bureau of Standards, and a few universities. Lens manufacturers usually conduct tests in their own laboratories and the prospective purchaser can often obtain the results of such tests. In submitting a lens to a testing laboratory it is necessary to specify at what scale ratios and apertures the tests are to be conducted. For instance a lens used in a copying camera or ratio printer should be tested for the scale ratios at which it is to be used, while an aerial camera lens is tested for an infinite object distance.

A typical test report issued by the National Bureau of Standards on a wide angle aerial camera lens is given below:

REPORT

on

One Photographic Objective

Submitted by

Department of Commerce

U. S. Coast and Geodetic Survey

Washington, D. C.

on

August 17, 1942

Lens	FOCAL LENGTHS	
	Back focal length	Equivalent focal length
	mm.	mm.
3244465	119.91	152.18

The values of the focal lengths have been selected to give best average definition across the entire negative and do not necessarily correspond to those values of focal lengths which give best definition on the axis. The probable errors of these determinations of focal length are approximately ±0.10 mm.

Lens	DISTORTION								
	5°	10°	15°	20°	25°	30°	35°	40°	45°
3244465	0.00	0.00	+0.02	+0.04	+0.08	+0.11	+0.12	+0.07	−0.15

The values of the distortion are measured in millimeters and indicate the displacement of the image from its distortion-free position. A positive value indicates a displacement from the center of the plate. The probable error is approximately ±0.02 mm.

Lens	RESOLVING POWER									
	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
3244465										
Tangential	55	55	55	39	28	28	28	28	28	14
Radial	55	55	55	55	39	39	39	39	39	28

The values of the resolving power are given at 5° intervals from the center of the field and are obtained by photographing suitable test charts comprised of patterns of parallel lines. The series of patterns of the test chart are imaged on the negative with the lines spaced 5, 7, 10, 14, 19, 28, 39, 55, and 77 lines to the millimeter. The row marked "tangential" gives the number of lines per millimeter in the image on the negative of the finest pattern of the test chart that is distinctly resolved into separate lines when the lines lie perpendicular to the radius drawn from the center of the field. The row marked "radial" gives similar values for the pattern of test lines lying parallel to the radius.

All measurements were made with parallel light incident on the lens. The effective wave length was approximately 575 millimicrons.

This is a Bausch and Lomb Metrogon lens, nominal focal length 152.4 mm., maximum aperture $f/6.3$ It was tested at maximum aperture. During the test, the lens was mounted in a special test barrel submitted with the lens.

Washington, D. C.
August 26, 1942

The results of the above test show that the distortion error for this particular lens is very small. Also the resolving power is considered good for a lens covering such a wide field.

Sometimes a lens is tested after it is mounted in an aerial camera. Such a test may be performed on the photographic testing equipment previously described. The results of such a test would be a determination of distortion and definition for the focal length actually set in the camera (calibrated focal length). Also the position of the four fiducial marks may be checked to see if lines connecting opposite marks intersect on the optical axis of the lens, thus locating the principal point of the photograph.[7] Another method of obtaining an indication of definition would be by a flight check, in which exposures are made over terrain containing intricate detail.

4. IMAGE BRIGHTNESS

In a camera the amount of light that the sensitized surface receives from an object of given light intensity depends upon several factors, such as: the relative aperture of the lens, the magnification or scale ratio, the angular field, and the light loss in the lens.

The relative aperture of a lens (also called speed or *f*-number) is defined as the ratio of equivalent focal length to aperture, thus a lens of 8 inch focal length and 1 inch aperture would have a relative aperture of *f*/8. The more common *f*-numbers engraved on lenses are as follows: 1—1.4—2—2.8—4—5.6—8—11—16—22—32—45—64. This series is usually followed in American made lenses except in the case of maximum aperture which may fall somewhere between the given values (such as *f*/3.5). Assuming a constant shutter speed, the exposure varies inversely as the square of the *f*-number and the above series is so selected that the exposure is approximately doubled as you go from one number to the next larger, thus the exposure at *f*/11 would be twice that of *f*/8 ($121/64=2$ approx.). This relationship only holds true when the object is essentially at infinity.

In ratio printing or copying work the "effective" aperture is the ratio of image distance to aperture. It can be found by simply multiplying the *f*/number engraved on lens by the quantity $(M+1)$ where (M) is the magnification or scale ratio. If you assume the lens stop to be constant, the exposure will vary as $(M+1)^2$. Example: When $M=2$ the exposure is 10 seconds; what will it be when $M=4$? $(4+1)^2=25$, $(2+1)^2=9$, $25/9\times10=28$ seconds. If an exposure meter is used in copying work it is necessary to multiply the exposure as indicated on the meter by the quantity $(M+1)^2$ to obtain the correct exposure. This must be done because the meter is calibrated for an infinite object distance.

A subject related to lens aperture is depth of definition. Space does not permit a complete discussion of this subject, but references are given to aid those who may be interested in the mathematical relationships.[8] We have seen from equation (2) that for a given object distance we obtain a corresponding image distance at which points are imaged with optimum sharpness. At a greater or lesser distance, points will be imaged as blurred circles, becoming larger the further we depart from the optimum position. In a particular instrument, the maximum circle diameter which will yield satisfactory definition is determined by experimentation. With this diameter fixed, it is possible to compute the usable depth of definition for any given lens aperture. It also can be mathematically proved that this depth increases as the lens aperture is reduced, other conditions being constant. A practical use of the above relationships is found in the multiplex projector. In one model of this equipment the object distance (lens to diapositive) is 30 mm. With a lens of 27.7 mm. focal length, this gives an optimum image distance of 360 mm. By selecting a proper lens aperture (about *f*/22) it is possible to obtain an image depth sufficient to take care of all the relief usually encountered in the stereoscopic model (see section V—Fig. 22). Another practical use of depth of definition is found in that type of ratio printer equipped with a tilting easel. Such equipment is used for approximate rectification of prints for mosaic work (see section III—Fig. 11).

The illumination on the negative in a camera is strongest at the center of the field. Except for the vignetting effect of the lens mount, the illumination varies as $\cos^4 A$ where (A) is half the angular field.[9] This falling off in illumination is rather serious in wide angle aerial cameras. If the half angular field of the camera is 45° the illumination at the edge of the negative will only be 25 per cent of what it is at the center ($\cos^4 45° = 0.71^4 = 0.25$). Subtracting the light loss due to vignetting would probably reduce this to only 15 per cent (the vignetting loss becomes less as lens aperture is reduced). This uneven illumination makes it impossible to obtain a proper exposure over the entire negative in spite of the fact that modern film emulsions have considerable latitude. In actual practice the center is purposely overexposed in order to obtain a printable image on the edges. The resulting negative, however, is rather difficult to print, requiring the center to be exposed much longer than the edges, also the image definition is seriously impaired.

When light passes through glass, part of it is lost by absorption and part by reflection. The amount lost by absorption is very small, about one-half per cent per centimeter of thickness. The amount lost by reflection is considerable, about 5 per cent for each air-glass surface. This reflection loss not only means a longer exposure but also may cause flare spots on the negative due to multiple internal reflections. If a photographic lens contained eight air-glass surfaces then the light transmitted would only be 66 per cent ($0.95^8 = 0.66$). If it were possible to eliminate this reflection loss the effective speed of the lens would be increased considerably.

Within the past few years methods have been developed commercially[10] which reduce the reflection loss by coating the lens surface with a thin transparent film one-quarter wave length thick. To thoroughly understand the theory of this film one would have to study that branch of physical optics which concerns light interference. Briefly, however, it may be said that light on striking the surface of a coated lens, is partially reflected from both surfaces of the film. Since this film is one-quarter wave length thick, the two reflected rays will be out of phase by one-half wave length and therefore interfere with each other and no reflection can take place. A characteristic of the present types of coatings is that the more durable ones eliminate only a part of the reflection loss while the less durable ones are almost 100 per cent effective. A partial solution to this problem is obtained by coating the inner surfaces of the lens with the less durable type, and leaving the outer surface uncoated.

III. Optical Rectification

Optical rectification is the process of projecting the image of a tilted photograph into a horizontal plane. In the case of a multiple lens camera, the projection of the images of the various chambers into a common plane is usually called transformation. The optical and mathematical relationships are similar for both cases, the chief difference being that in rectification we are concerned with small accidental tilts (say under 5°), whereas in transformation we are dealing with large tilts (maybe 40°) whose values are definitely known from the camera calibration data.

Rectifying apparatus is often designed so that a change in scale and tilt removal can be made simultaneously. The resulting prints are used in laying down controlled mosaics, in the measurement of land areas, and in certain types of stereoscopic plotting equipment in which parallax measurements are made directly on the photographs.

1. Approximate Rectification

An approximate rectification, which is satisfactory for mosaic work, may be obtained by simply tilting the easel of a ratio printer (see Fig. 11). Contrasting this method with exact rectification the following items should be noted: (1) Due to mathematical relationships, to be discussed later, the removal of tilt distortion is not theoretically correct; (2) in order to secure sufficient image definition it is necessary to stop the lens down considerably. If in Figure 11 EL and FL are conjugate distances, it can readily be seen that EL and $A'L$ (or EL and $B'L$) are not conjugate distances and therefore points A and B will not be in sharp focus. However, if the easel tilt is not excessive, a sufficiently sharp image is obtained by reducing the lens aperture to about $f/32$ (see depth of definition, section II-4).

The operation procedure for using a ratio printer in the above manner will now be explained. It is first necessary to determine the tilt of the photograph. The necessary data for this determination may be obtained from a previously run radial line plot by measuring the distance from the principal point to several properly distributed radial control points, both on the photograph and on the plotting sheet. An approximate value of the tilt is obtained by analyzing the variations in these distances. For a ratio printer of a given focal length, tables are usually prepared to speed this operation. If the easel of the ratio printer cannot be rotated in its own plane, it will be necessary to resolve the tilt into two components one of which is parallel and the other perpendicular to the length of the film. These two tilt components (corrected for scale ratio changes) are then set on the tilt arcs of the easel and print exposed.

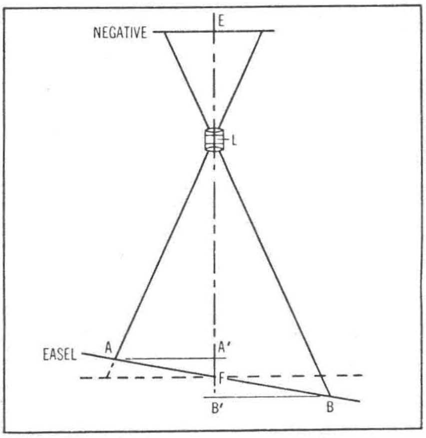

FIG. 11. Approximate rectification obtained by tilting the easel of a ratio printer.

The rectification of photographs for the accurate measurement of land areas is accomplished in a manner similar to that described for mosaic work with the following exceptions: (1) A more accurate determination of tilt is necessary. The Anderson method, which requires the measurement of three ground distances, is often used. (2) The ratio printer must be of precise mechanical construction and the easel settings for any degree of air tilt must be based upon extensive calibration data. Tests on printers of this nature have indicated that for air tilts under 4°, results compare very favorably with those obtained by "exact rectification" which is discussed below.

2. EXACT RECTIFICATION

In order to obtain precise results using plotting instruments of the stereocomparator type, it is necessary that the photographs used in such instruments be tilt-free to a high degree of accuracy and of equal scale. The removal of tilt and change in scale is accomplished in a specially designed rectifying camera, a schematic diagram of which is shown in Figure 12. In order to simplify this figure, the principal line (CN) is assumed to be parallel to edge of tilted negative, a condition which seldom exists in practice. The procedure in operating a rectifying camera is as follows: (1) Determine the tilt of the photograph by any

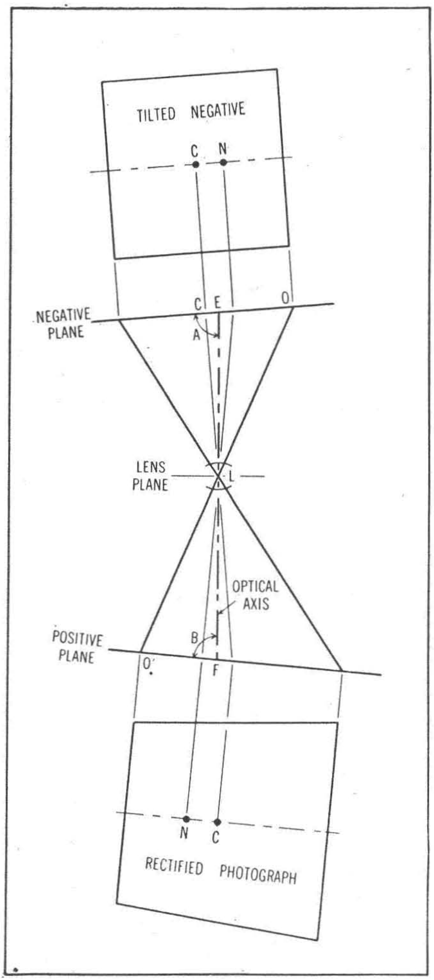

FIG. 12. Schematic diagram of a rectifying camera.

method which gives results within the desired degree of accuracy. The nadir point (N) should be marked on the negative in such a manner that its image will be visible on the rectified print; (2) Compute angles (A) and (B) and lengths EL, FL, and EC according to equations given below; (3) Set these values on the camera scales. The mathematical relationships between these settings are such that any pair of corresponding points (as O and O') when projected to the optical axis, form a pair of conjugate distances. Therefore, it is not necessary to use a small lens aperture to secure good definition, as was the case when only the positive (easel) was tilted. Another point of interest in true rectification is that the negative, lens and positive planes, when extended, intersect in a common line. (4) Place principal point (C) of negative on point (C) in camera. (5) Rotate negative in its own plane until its axis of tilt is parallel to axis of tilt of negative holder. In some types of equipment this rotation cannot be accomplished but the negative holder and easel can be tilted in two directions at right angles to each other. In using such equipment the angles (A) and (B) will have to be resolved into similar components. (6) Expose the print. The only image displacement that now remains in the print is due to relief, which radiates from the nadir point. The image of this point appears on the print because it previously was marked on the negative. A pair of these prints, which have been corrected for both tilt and scale, may now be used in the stereocomparator type of plotting equipment for the accurate determination of elevations.

Equations for computing the items mentioned above are as follows[11]:

$$\cos A = \frac{G \sin T}{MF} \qquad \text{eq. (8)}$$

$$\cos B = \frac{G \sin T}{F} \qquad \text{eq. (9)}$$

$$EL = \frac{G \sin (A + B)}{\sin A \cos B} \qquad \text{eq. (10)}$$

$$FL = \frac{G \sin (A + B)}{\cos A \sin B} \qquad \text{eq. (11)}$$

$$EC = \frac{F}{\sin T}\left(\frac{\sin B}{\sin A} - \cos T\right) \qquad \text{eq. (12)}$$

G = focal length of rectifying camera* (in regards to focal length selection the following should be noted: (1) the shorter the focal length the larger the angular field; (2) the longer the focal length the more acute will be the intersection between the rays and the negative and positive planes).
F = focal length of air camera.
T = tilt of air camera.
M = scale ratio (less than 1 for reduction, more than 1 for enlargement).

In using the above equations, it may be necessary to shift the light source when rectifying various photographs having different degrees of tilt, due to a variation in distance EC. In order to avoid this difficulty, it is possible to base the design of the rectifying camera on another set of equations[11] which refer all measurements to the line of centers (that is, a line connecting principal point of negative to node of rectifying lens). Although the use of these equations allow the light source to be fixed, they complicate the camera design because the lens also must be tilted.

In the actual design of a rectifying camera many refinements must be considered which are not discussed here. Among these are the nodal point separation of the lens, and the errors caused by the glass pressure plate used to hold the film flat. In order to facilitate the operation of a rectifying camera, various mechanical linkages have been designed[12] which reduce the number of necessary settings to a minimum. One of these is an auto-focus mechanism in which a single control wheel moves both negative and positive planes so as to maintain sharp focus for all scale ratios and tilts. Another linkage is designed to manipulate all tilting motions from a single control wheel. Instruments equipped with these two items are given the classification of "automatic."

Sometimes the setting of the various motions of a rectifying camera are accomplished by trial. The negative, containing at least three ground control points, is placed in its holder and a grid or map, with the same control plotted thereon, is placed on the easel. The various tilts and distances are so manipulated that the images of the control on the negative fall on their plotted positions. However, if the ground control points are at various elevations it will be necessary to make repeated corrections for relief displacements until the coincidence of the points are within the desired degree of accuracy. This method is particularly used in machines of the automatic type because of their simple control mechanism.

IV. Errors Caused by Glass Plates

Plane parallel glass plates are often used in the optical path of various photogrammetric instruments. These plates may be in the form of pressure plates for holding film or paper flat during exposure, filters, or reflecting prisms. The latter may be considered as the optical equivalent of a thick plane parallel plate plus one or more plane mirrors depending upon what type of prism is used. To obtain the best possible performance from a photographic lens, it is necessary that filters or prisms used in the system be of a high optical quality. For pressure plates the optical quality need not be so great because the plate is so close to the focal plane that any small irregularities in its surface would affect the image definition very slightly. Carefully selected commercial plate glass is usually satisfactory for pressure plates. The use of the above mentioned plates cause certain errors which may be classed as: displacement of the focal plane; image distortion; and aberrations which impair the lens definition.

* Note special case: If $G=F$ and $M=1$, then $A=B=90-T$. And $EL=FL=2G$. Also the isocenter of negative will fall on optical axis at E.

1. Displacement of the Focal Plane

The insertion of a glass plate between lens and focal plane displaces the focal plane away from the lens, its amount along the axis being about one-third the thickness of the plate. (In Fig. 13 this displacement is the distance 00′.) This axial displacement is

$$\text{displacement} = T - \frac{T}{N} \qquad \text{eq. (13)}$$

in which (T) is the plate thickness and (N) its index of refraction. For most commercial plate glass, filters and prisms this index is about 1.52. The above displacement formula is not absolutely correct for "off-axis" rays in that the plate introduces curvature of field which causes images of points near the edge of the field to have a different displacement than the image of an axial point, thus giving a "dished" image field.

2. Image Distortion

The distortion introduced by the insertion of a glass plate into an optical system results in a radial displacement of an image point toward or away from the center of the field depending on whether the plate is inserted between lens and image or between lens and object respectively (see Fig. 13). The magnitude of this distortion can be found from Table I, which gives the distortion error, expressed as a decimal part of the plate thickness, for rays at various angular distances (in degrees) from the center of the field. The distortion varies with the

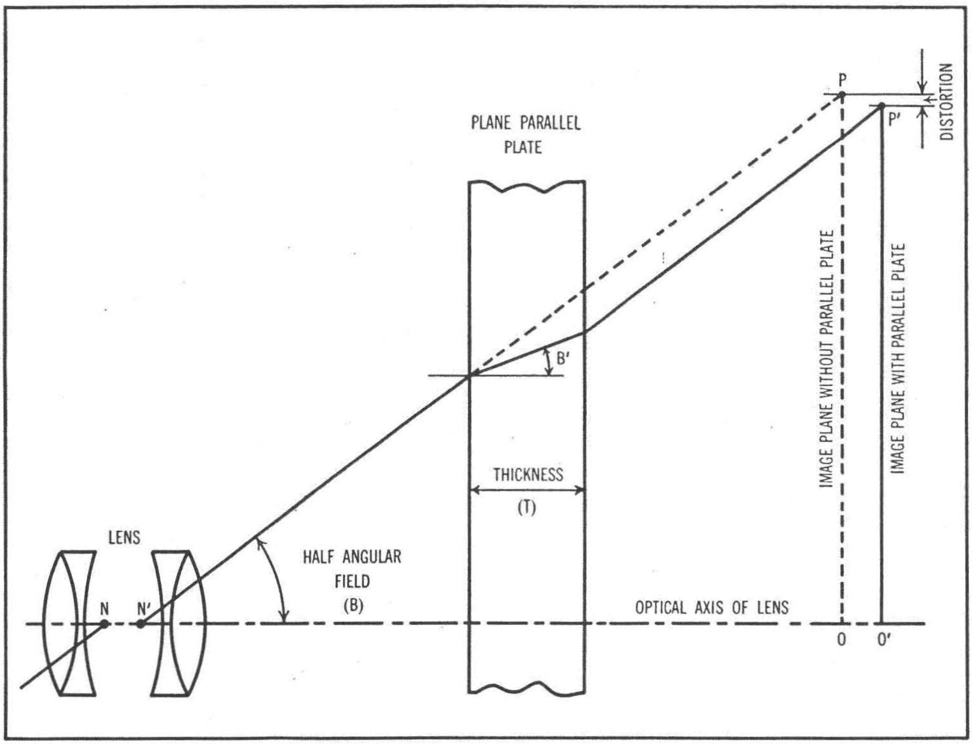

Fig. 13. Image distortion and displacement caused by the insertion of a plane parallel plate.

index of refraction (N) of the glass. The index used in computing the table was 1.52. Since most glass plates, filters, etc., have an index near this value, the results obtained by using Table I will be accurate enough for practical purposes. Computations were made to determine the change in the distortion with change in index. The results indicated that for values of the index between 1.48 and 1.56, the table values are correct to 3 in the 4th place for angles up to 25 degrees, and 6 in the 4th place for angles between 25 and 50 degrees.

TABLE I.—DISTORTION INTRODUCED BY A PLANE PARALLEL PLATE EXPRESSED AS A DECIMAL PART OF THE PLATE THICKNESS

(B) Half angular field in deg.	Distortion $N=1.52$	(B) Half angular field in deg.	Distortion $N=1.52$
1	0.0000	26	0.0197
2	0.0000	27	0.0222
3	0.0000	28	0.0251
4	0.0001	29	0.0281
5	0.0001	30	0.0315
6	0.0002	31	0.0352
7	0.0003	32	0.0391
8	0.0005	33	0.0434
9	0.0007	34	0.0481
10	0.0010	35	0.0532
11	0.0014	36	0.0587
12	0.0018	37	0.0646
13	0.0022	38	0.0710
14	0.0028	39	0.0779
15	0.0035	40	0.0854
16	0.0043	41	0.0934
17	0.0051	42	0.1021
18	0.0061	43	0.1114
19	0.0073	44	0.1215
20	0.0085	45	0.1324
21	0.0099	46	0.1441
22	0.0115	47	0.1566
23	0.0133	48	0.1702
24	0.0152	49	0.1848
25	0.0173	50	0.2006

In computing the distortion error the following two items should be noted: (1) A plate inserted anywhere between lens and object causes a radial displacement of an image point away (sign assumed +) from the center of the field, its magnitude being equal to the value in Table I multiplied by both the plate thickness and the scale ratio. In the case of an aerial camera the scale ratio, that is, image size to object size, is very small (say 1:20,000). For the purpose of distortion computation it may be considered to be zero, hence the image distortion caused by a filter placed over the *front* surface of the camera lens will be zero. Another way of stating this would be to say that if the object is essentially at infinity, the light rays striking the filter are parallel and therefore there will be no image distortion. (2) A plate inserted anywhere between lens and image causes a radial displacement of an image point toward (sign assumed −) the center of the field, its magnitude being equal to the value in Table I multiplied by plate thickness only.

In certain types of photogrammetric equipment it is necessary to compensate for the distortion introduced by a filter or pressure plate. This can be done by inserting a compensation plate, of the proper thickness and index, in the system in such a position as to neutralize the distortion that is present. Sufficient information has been given in the above discussion to enable one to determine the thickness and location of such a plate.

3. Aberrations Affecting Lens Definition

A lens designer, after tedious computations, finally arrives at a design in which he has balanced the aberrations in the best possible manner. Care should be taken lest the insertion of a glass plate into the optical path of this lens upset his elaborate calculations.[13] The introduction of curvature of field has already been mentioned. Fortunately the effect of this error, as well as others, is reduced by stopping the lens down (due to increase in depth of definition). In some cases, however, it may not be possible to do this because of the resulting decrease in illumination. It might be added that the distortion error previously discussed, is not reduced by stopping the lens down.

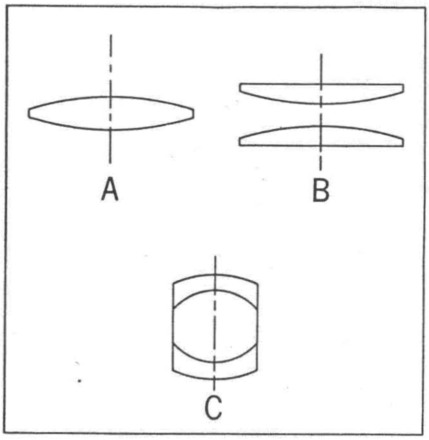

Fig. 14. Types of magnifiers. (A) Simple reading glass (mag. 2 or 3). Definition is good only in center of field. (B) Doublet as used in photo-engraving glasses (mag. up to 5). Gives a large field relatively free from distortion. (C) Cemented triplet (mag. up to 20). Gives a large field free from both distortion and color.

V. Simple Optical Systems

In photogrammetry use is made of a great variety of optical systems. Only the more elementary ones will be described here. A complete study of this subject would bring one into the field of optical instrument design. Those further interested in the subject should consult the references at the end of the chapter.[2]

1. Magnifiers and Eyepieces

A magnifier forms a virtual image of a real object, while an eyepiece forms a virtual image of a real image formed by some other part of the optical system, the image in both cases being erect. In general eyepieces have a shorter focal length and higher optical quality than magnifiers. The image quality obtained by a magnifier depends upon the number and shape of the lens elements. The chief characteristics of several types are shown in Figure 14. The magnifying power (M) of a magnifier or eyepiece does not have a definite value since it depends upon the viewing distance of the observer. An approximate value is given by the equation

$$M = \frac{10}{\text{focal length in inches}} \left(\text{or } \frac{250}{\text{focal length in mm.}} \right) \qquad \text{eq. (14)}$$

This relationship is based on the following assumptions: (1) that the normal viewing distance for most observers is 10 inches and that the purpose of the lens is merely to enable one to accommodate the eye to a shorter viewing distance thereby causing the object to subtend a larger angle at the eye; (2) that the lens is placed at its focal length away from the object, thus forming a virtual image at infinity. Most persons, however, prefer to use an object distance less than the focal length, thereby causing the virtual image to be formed at some finite distance (say about 10 inches). For low power magnifiers the percentage change in magnification with change in object distance is considerable (see problem 11 at end of chapter), but for the shorter focal lengths used in most eyepieces the change is negligible.

Eyepieces (or oculars) are used on such photogrammetric instruments as certain types of lens stereoscopes, transits, levels, and many stereoplotting instruments. The chief function of an eyepiece is to magnify the image. They are also used to erect the image formed by another part of the optical system, to enable cross hairs to be introduced, and to correct for residual aberrations of the system. The two more common types of eyepieces are the Huygenian and the Ramsden, the characteristics of which are discussed below.

The Huygenian type consists of two plano-convex lenses arranged as shown in Figure 15. Both lenses are made from the same type of glass with the focal length of the field lens being about twice that of the eye lens. The separation of the two lenses is made equal to one-half the sum of their focal lengths. A study of the ray path through this eyepiece may be of interest. If the eyepiece was removed, the remainder of the optical system would form a real image at (*D*). This real image is the virtual object for the eyepiece, which in turn forms a virtual image at infinity. If a reticle or cross hair is used it must be placed at (*C*). The use of a reticle in this type of eyepiece is not very satisfactory, since the field is very small and therefore the reticle would have to be very short. Also the eye lens is fixed in reference to the reticle so that it cannot be focused for different observers. In the Huygenian eyepiece the longitudinal chromatic aberration and curvature of field are large, but the distortion and lateral chromatic are small.

The Ramsden eyepiece consists of two plano-convex lenses of equal focal length arranged as shown in Figure 16. The separation of the lenses is usually made equal to two-thirds the focal length of either lens. The rest of the optical system forms a real image at (*H*) and the eyepiece uses this image as an object and forms a virtual image at infinity. When cross hairs are used they are placed at (*H*) in which position it is possible to adjust the eyepiece to accommodate

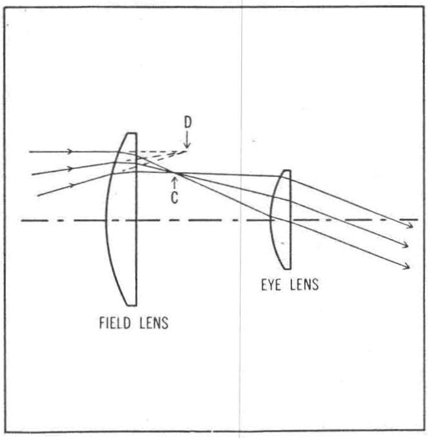

FIG. 15. Huygenian eyepiece.

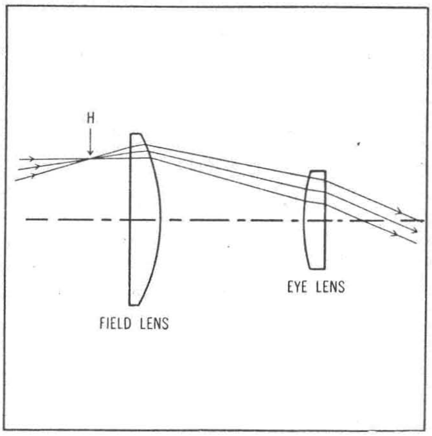

FIG. 16. Ramsden eyepiece.

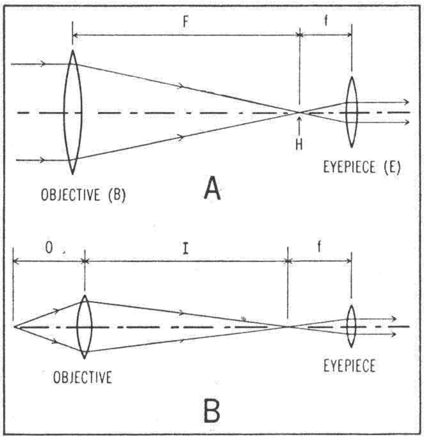

FIG. 17. Optical systems. (A) Telescopic. (B) Microscopic.

different observers. In the Ramsden eyepiece the distortion, curvature of field and longitudinal chromatic aberration are small, but the lateral chromatic is a little large. In order to reduce this latter defect the single eye lens is sometimes replaced by an achromatic doublet, in which case it is known as a Kellner eyepiece.

All of the eyepieces described above form an erect image of the object that they magnify. If the rest of the optical system forms an inverted image, it may be desirable to erect this image. This may be done with an erecting eyepiece using a lens system (see Fig. 18B) or by means of a porro prism system as used in binoculars (see section VII—Fig. 30).

In high power microscopes a "compensating" eyepiece is sometimes used. Such an eyepiece is designed to have aberrations which are opposite in sense to those of the objective, thereby securing a high correction for the entire optical system.

2. TELESCOPIC AND MICROSCOPIC SYSTEMS

There is no definite boundary between these two systems, although the term microscope is usually reserved for cases where the object distance is less than ten inches. The most obvious use of telescopes in photogrammetry is for surveying instruments. Equally important, however, is their use in the viewing systems of stereoscopic apparatus. A simple refracting telescope (Fig. 17A) consists of an objective lens (B) and an eyepiece (E). When the object is at infinity, a real image is formed at H. The eyepiece uses this image as an object and forms an inverted virtual image at infinity. The magnification (M) of such a system is

$$M = \frac{F}{f} \qquad \text{eq. (15)}$$

in which (F) is the focal length of the objective and (f) the focal length of the eyepiece. If the object is relatively close to the objective (as in microscope, Fig. 17B) the magnification is obtained as follows:

$$\text{magnification due to objective} \quad M_B = \frac{I}{O}$$

$$\text{magnification due to eyepiece} \quad M_E = \frac{10}{f\,(\text{inches})}$$

$$\text{magnification due to system} \quad M = M_B \times M_E \qquad \text{eq. (16)}$$

The purpose of Figure 17A was merely to show the metrical relationships between the various elements. In actual practice the optics of a transit telescope[14] may be as shown in Figure 18A, in which an achromatic objective is used with a Ramsden eyepiece. Since this system gives an inverted image it may be

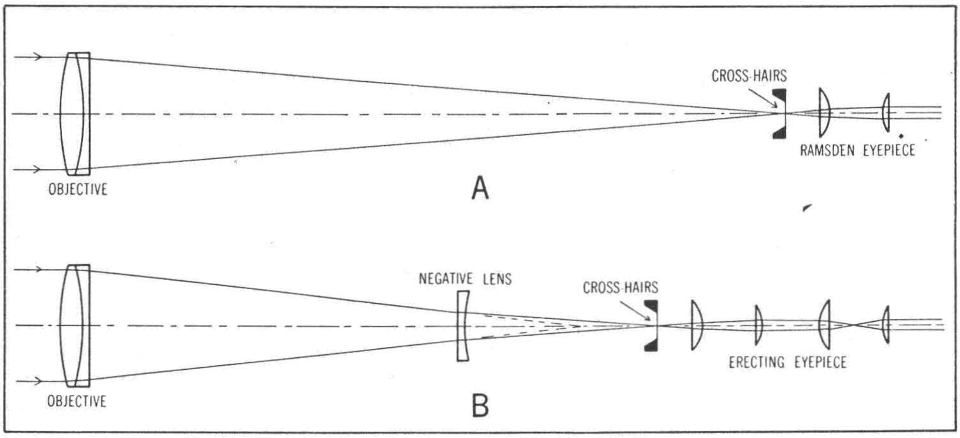

FIG. 18. Optics of a transit telescope. (A) External focusing type with Ramsden eyepiece giving an inverted image. (B) Internal focusing type with erecting eyepiece.

desirable to replace the Ramsden eyepiece with one of the erecting type shown in Figure 18B. A majority of the modern transits are equipped with the "internal focusing" telescope which is also shown in Figure 18B. In this type, focusing is accomplished by moving an internal slide carrying a negative lens. This eliminates the wear caused by dust collecting on an external slide and also eliminates a constant from stadia computations.

The proper way to focus a telescope or microscope is as follows: (1) Focus the eyepiece on the cross hairs; (2) focus the objective on the object until the image appears sharp, in which case the plane of the image and cross hairs should coincide; (3) check the focus by slightly moving the eye from one side to the other. If the cross hairs appear to move over the object the focus is imperfect and parallax is said to exist; (4) refocus the objective to remove this parallax.

3. COLLIMATORS

A collimator is an optical laboratory device which establishes in space a fixed line of sight. It consists of a telescope objective with an illuminated slit or cross hair at its focal point (see Fig. 19). After passing through the lens, rays from each point of the object are made parallel and the object is virtually at infinity. Since the emergent rays are parallel, the established line of sight is in the same direction regardless of what portion of the objective is used.

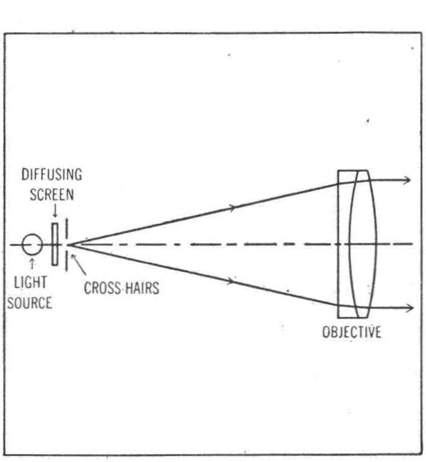

FIG. 19. Optical elements of a typical collimator.

Any telescope can be converted into a collimator merely by removing the eyepiece, inserting a small lamp to illuminate the cross hairs, and then focusing the objective for infinity. The requirements of the telescope thus converted being that it have a higher optical quality and a larger aperture than the instrument to be tested. The focusing operation may be accomplished by the following methods: (1) Replace the eyepiece and then focus on a distant object (such as a star). The objection to this method is that the adjustment may be disturbed when the eyepiece is removed and the illuminating system attached; (2) Focus a theodolite on a distant object, then point the instrument toward the objective of the collimator and change the latter's focus until the image of the cross hairs, as seen in the theodolite, appear sharp; (3) Use an autocollimating or Gauss eyepiece.[15] Other methods of focusing which require the use of two or three collimators are also available.[16]

Collimators are widely used in the laboratory for testing various optical systems such as aerial camera lenses, surveying instrument telescopes, etc.

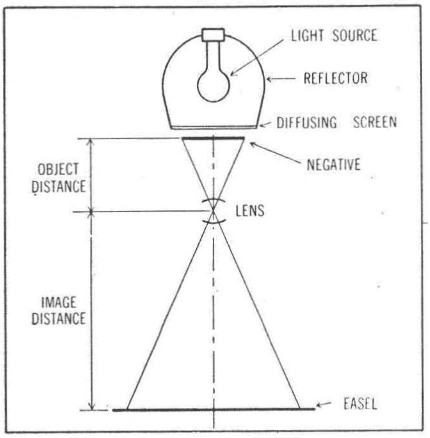

FIG. 20. Ratio printer with diffuse illumination.

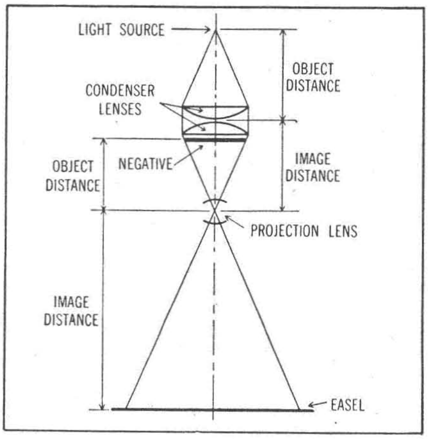

FIG. 21. Ratio printer with condenser illumination.

4. PROJECTION SYSTEMS

Projection systems are used in such equipment as ratio printers, multiplex projectors, reflecting map projectors, etc. The purpose of these projection systems is to form a real image on a receiving screen of sufficient brightness to satisfy the particular work to be done.

Several methods of illumination are used in projectors. The most common type being the diffusion system of a ratio printer shown in Figure 20. The chief parts of such a system are a light source, reflector, and diffusing screen. The light source may be either an incandescent, mercury vapor, or argon lamp. The last two types are preferred because they emit less heat. The light source, mounted in a reflector of suitable design, serves to illuminate an opal or ground glass. Because of its diffusing properties, this glass becomes a secondary source of light which illuminates the entire negative area. Plastic diffusing materials have recently been developed which appear to be superior to glass since they are practically unbreakable and also have a higher light transmission for a given degree of diffusion. The contrast in a print made in a ratio printer equipped with a diffusion system is practically the same as that obtained in a contact print.

Another method of illumination requires the use of a condensing lens. The purpose of such a lens is to concentrate the illumination from a light source upon a limited area, rather than to form an optically perfect image. For this reason high optical quality is usually not necessary. The condensing system often consists of two plano-convex lenses arranged as in Figure 21. The focal length of this system is so selected that a crude image of the light source is formed at the diaphragm of the projection lens. As the scale ratio of the printer is changed it will be necessary to refocus the condenser. The size of the condenser should be slightly larger than the negative diagonal, thus for 9″×9″ aerial negatives, a 14″ diameter would be required. The chief advantages of a condensing system over a diffusing system are that a higher screen illumination is obtained from a given light source, and also a higher image contrast, although negative scratches will be emphasized. These advantages, however, do not offset such items as increased cost, bulk, and weight, and therefore condensers are seldom used in ratio printers (except in the small amateur sizes).

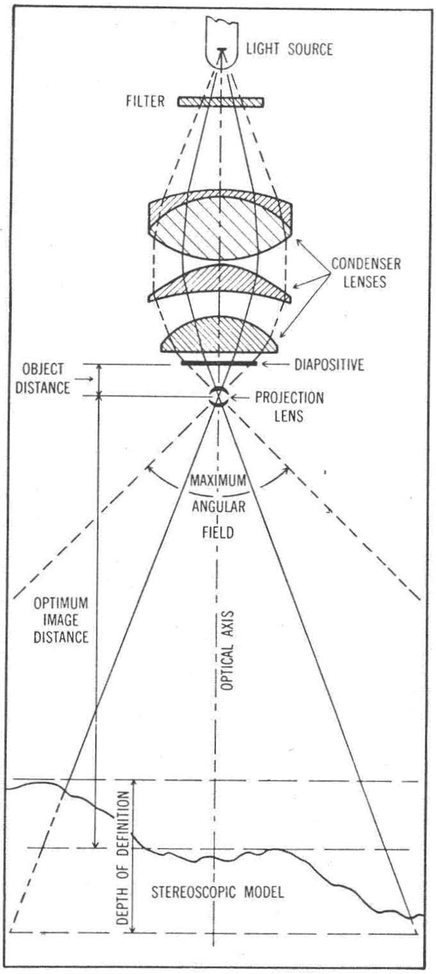

FIG. 22. Optical system of a wide-angle multiplex projector.

The illumination problem in the multiplex projector is much different from that in a ratio printer. In the latter case it is usually possible to compensate for the light lost by the diffuser by merely using a larger lens aperture. As previously mentioned, the aperture of the multiplex projector lens necessarily must be small (about $f/22$) in order to secure sufficient depth of definition to take care of the relief in the spatial model. Also considerable light is absorbed by the red or green anaglyphic filter. For these reasons it is absolutely necessary to use a condensing lens system in order to obtain sufficient screen illumination for stereoscopic observation. Since the light source must be concentrated on an aperture only 1 mm. in diameter, a specially designed system of high optical quality is required (see Fig. 22). The surfaces of these condenser lenses have been coated with a non-reflecting film in order to increase the light transmission.

In the usual map projector the problem is to illuminate an opaque object (a photograph or map). A light source is usually mounted on either side of the object (see Fig. 23). Each of these courses consists of a projection lamp (500 watts) mounted in a spherical glass reflector. Of the three methods of illumination discussed, this is the least effective, as only a small portion of the light striking the matte surface of the photograph or map is reflected toward the projection lens.

VI. Color Filters

A filter may be considered as a colored plate which transmits certain portions of the spectrum and absorbs the remainder. In general it may be said that a filter transmits light of its own color. For photographic purposes it is convenient

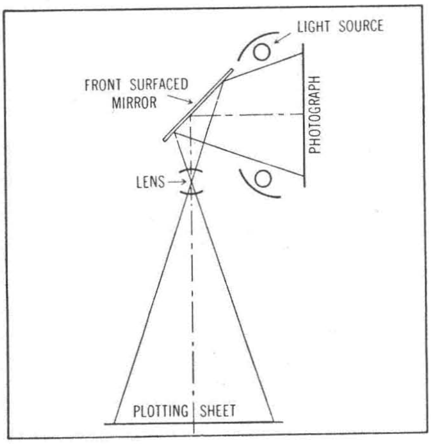

FIG. 23. Reflecting map projector.

to subdivide the spectrum into the following five regions: ultra violet, blue, green, red, and infra red. Figure 24 gives the approximate wave length limits of each region and also indicates the range of sensitivity of various photographic emulsions. By the use of a filter it is possible to select those portions of the spectrum which are best suited for each type of work.

1. CONSTRUCTION

A filter may consist of an unmounted colored gelatin film, a colored gelatin film cemented between two plates of optical glass, or a colored plate of optical glass. Due to its fragility, the first type mentioned is seldom used for other than experimental purposes. The gelatin film mounted between glass has the one disadvantage that the cement deteriorates due to age or heat causing the image definition to be impaired. Filters made of colored glass are considered the best type to use for accurate work. It is very important that a glass filter introduced into the path of a photographic lens have an optical quality equal to that of the lens itself, if critical image definition is to be maintained. By this we mean that the surfaces should be plane and parallel and the glass itself free from defects. Even if a filter is of good optical quality, the items discussed under "Errors Caused by Glass Plates" still apply. Note: It was previously mentioned that in the case of an aerial camera there will be no distortion or displacement of the image if the filter is mounted in *front* of the lens.

2. USES IN AERIAL PHOTOGRAPHY

In aerial photography the purpose of filters is either to eliminate the effects of atmospheric haze[17] (not fog or smoke) or to increase the contrast of certain desired features of the terrain. Atmospheric haze consists of small particles of dust and water vapor which have the property of scattering blue light. That portion which is scattered in the direction of the camera produces a slight fog on the panchromatic film. By placing a blue absorbing filter (yellow in color) over the lens this undesired light is eliminated, the exposure being made by green and red rays reflected from ground objects.

For maximum haze penetration (especially in oblique photography) infrared film and filters are sometimes used. Until recently this type of film was too slow for aerial photography. However, the type II Infra-red Aero Film intro-

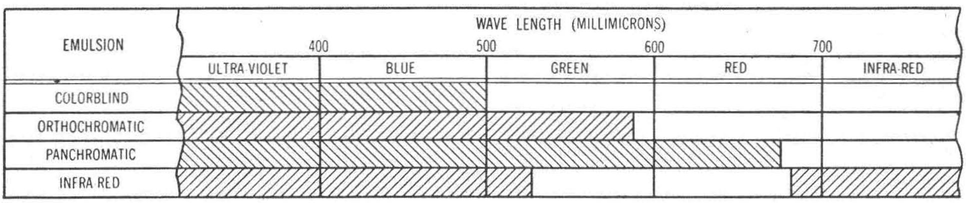

FIG. 24. Approximate sensitivity range of various photographic emulsions.

duced by Eastman Kodak Company has a speed such that satisfactory exposures can be made at 1/150 sec. at *f*/11 (using Wratten filter No. 25 or 89A at an altitude of 5000 feet). This film and filter combination is also useful in military intelligence because it increases the contrast of terrain features, thus aiding in detecting camouflage.

A polarizing screen (which is not a color filter) is occasionally used in aerial photography to eliminate undesirable reflections from bodies of water, thereby aiding in the detection of submerged objects, reefs and shoals. According to the wave theory, ordinary light (unpolarized) is said to vibrate in all planes perpendicular to the direction of propagation, while plane polarized light vibrates in a single plane. Ordinary light on passing through a polarizing screen becomes plane polarized. Such a screen may be compared to a series of parallel slits and only light which vibrates in the plane of these slits is allowed to pass through, thus two such screens placed with their planes of vibration at right angles to each other would allow no light to pass. Sunlight, upon reflection from water, becomes partially plane polarized. If a polarizing screen is placed on the camera (in front of lens) with its plane of vibration at right angles to the plane of vibration of the reflected light, then the latter would be completely cut out.

The type of filter chosen for any particular job depends on several items, such as: flying height, density of the atmospheric haze, type of film used, and character of the terrain. In Table II are listed several filters used in aerial photography. Also given are the filter factors, which indicate how many times the exposure must be increased in order to compensate for the light absorbed by the filter when used with a particular type of film. The filter numbers listed in table below refer to Wratten light filters manufactured by Eastman Kodak Company.[18]

TABLE II.—FILTERS USED IN AERIAL PHOTOGRAPHY

Type	Wratten number	Color	Filter factors*	Conditions when used
Aero 1	3	Light yellow	1.5	Very light haze
Aero 2	5	Yellow	2	Light haze
Minus Blue	12	Deep yellow	2	Medium haze
A	25	Red	4	Heavy haze at high altitudes

* For Eastman Super-XX panchromatic aero film.

3. PROJECTION FILTERS

Another use of filters in photogrammetry is for projection purposes, such as the anaglyphic principle used in the multiplex equipment. This is a method of stereoscopic viewing in which one image of a stereoscopic pair is projected and viewed in one color and the other in a complementary color. This is accomplished by placing a red filter in front of light source on the right projector and also over the right eye of the observer, likewise a blue green filter on the left projector and left eye. Since the filters are complementary each eye sees only one image, this of course being a necessary condition for stereoscopic vision. Often used for this purpose are Wratten gelatin filters Nos. 26 (stereo red) and 55 (stereo green). Other suitable filters of dyed glass are also available.

The polarizing screens previously mentioned in this section may also be employed for stereoscopic projection and viewing.[19] These screens have the advantage over the anaglyphic method in that natural color photographs may be used.

4. Safelights

Filters are also used for safelights in photographic laboratories. The surface quality of such filters can be very poor since they are only used for general illumination. The necessary requirement being that they absorb only that portion of the spectrum which would affect the sensitive material being used. For various types of sensitive material the following Wratten safelights are used.

Contact paper	—series 00 (Yellow)
Projection paper	—series 0 (Orange)
Non-color-sensitized film	—series 1 (Reddish Orange)
Orthochromatic film	—series 2 (Deep Red)
Panchromatic film	—series 3 (Green)
Infra-red film	—series 7 (Green)

Safelights should be used with caution. A check on their "safeness" can readily be determined by exposing a test strip of the sensitive material to be used, and inspecting it for fog.

VII. Prisms

Prisms are used in photogrammetric instruments for several purposes. They may be used to deviate a beam of light through a certain angle, to laterally displace a beam without introducing any deviation, and to invert, revert, or rotate an image in its own plane.

1. Refracting Prisms

Refracting prisms, when used to produce deviations of a few minutes, are called "wedges" (see Fig. 25A). This type cannot be used for large deviations because it disperses white light into its constituent colors (see Fig. 25B). For small deviations, the dispersion of a refracting wedge can be practically eliminated by constructing the wedge from two pieces of glass having different index and dispersion. Such a combination is termed an achromatic wedge. The deviation (D) produced by a wedge is given by the equation

$$D = A(N - 1) \qquad \text{eq. (17)}$$

in which (A) is the refracting angle and (N) the index. Another method of expressing this deviation is by use of prism diopters. A deviation of 1 centimeter

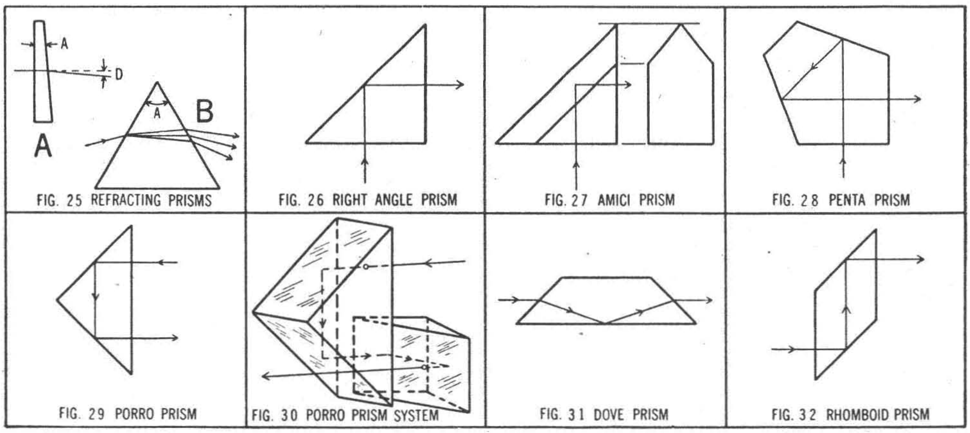

FIG. 25 REFRACTING PRISMS; FIG. 26 RIGHT ANGLE PRISM; FIG. 27 AMICI PRISM; FIG. 28 PENTA PRISM; FIG. 29 PORRO PRISM; FIG. 30 PORRO PRISM SYSTEM; FIG. 31 DOVE PRISM; FIG. 32 RHOMBOID PRISM

Figs. 25 to 32. Typical prisms used in photogrammetric instruments.

in a distance of one meter is one prism diopter. A refracting wedge is sometimes used in a simple lens stereoscope (lens and wedge ground as one piece of glass) in order to use photographs having an air base larger than the observer's eyebase.

2. Reflecting Prisms

Reflecting prisms do not cause the troublesome dispersion mentioned above and therefore can be used for large deviations. Front surfaced plane mirrors could be used for the same purpose but the surface coating deteriorates with age and has to be replaced. Mirrors are generally used in the larger sizes, however, because prisms of this nature are very expensive and difficult to obtain. It must be remembered that the items discussed under "Errors Caused by Glass Plates" applies also to prisms and for this reason mirrors may be preferred. A brief discussion of a few of the more common types of reflecting prisms follows:[20]

(a) The *right angle prism* shown in Figure 26 produces a deviation of 90° and reverts the image. By reversion we mean that the top and bottom of the image will be interchanged without altering the relative position of the right and left sides. This type sometimes replaces the small eye mirrors in a mirror stereoscope.

(b) The *Amici "roof angle" prism* shown in Figure 27 produces a deviation of 90° and inverts the image. It resembles a right angle prism except the hypotenuse face has been replaced by two surfaces inclined at 90° to each other to form a "roof." Entering rays striking the right side of the roof are reflected over to the left side and then out while those striking the left side follow the reverse course. Thus, the final image is formed with rays that have crossed over from one side to the other. If the two reflecting surfaces do not intersect within a few seconds of 90° the beams reflected from them will not match, and a double image will be formed. For this reason the prism is very difficult to manufacture and rather expensive.

(c) The *penta prism* shown in Figure 28 produces a deviation of 90° but does not change the image. The two reflecting surfaces of this prism must be silvered. A particular property of this type is that a 90° deviation is produced even if the beam does not strike the entering face exactly normal.

(d) The *Porro prism* shown in Figure 29 produces a deviation of 180° and inverts the image. It has the same shape as a right angle prism but the ray path is different. Two such prisms arranged as in Fig. 30 form an erecting system often used in optical instruments.

(e) The *Dove prism* shown in Figure 31 does not produce any deviation or displacement but reverts the image. It is also known as a rotating prism, the image rotating 180° when the prism rotates 90°. It is used in the optical system of the aerocartograph to keep the images as seen in the eyepiece, in proper fusion while the floating mark travels over the spacial model.

(f) The *rhomboid prism* shown in Figure 32 produces no deviation or image change but merely displaces the axis parallel to itself. Sometimes used in eyepieces of plotting instruments to obtain a varying interocular distance for different observers.

VIII. Mirrors

Mirrors are used in such photogrammetric instruments as stereoscopes, reflecting map projectors, and in certain types of plotting instruments. The surface quality of the mirror will vary with the type of work to be performed. When used for visual inspection, as in the ordinary mirror stereoscope or reflecting map projector, the surface quality of selected commercial plate glass is sufficient.

Such glass (say for a 12 inch square) will probably be flat to within one or two thousandths of an inch. On the other hand if the mirror is placed into the optical system of a plotting machine, in such a location as to influence its accuracy of measurement, it may be desirable to have its surface flat to within one-fifth of a wavelength of light (0.000004 inch). In order that the surface retain this flatness it is necessary to make its thickness at least one-tenth of its diameter. A few remarks concerning materials used for making mirrors follows.

1. GLASS

The most common mirror of this type is one which has its rear surface silvered and then backed with an opaque enamel to prevent it from tarnishing and to make it durable. The chief disadvantage of such a mirror is that a "ghost" image appears due to a partial reflection from the front surface. This secondary image is noticeable in a stereoscope especially when the prints contain contrasty detail, such as a white concrete road winding through a pine forest. In order to eliminate this double reflection, attempts were made to deposit the silver on the front reflecting surface, and to protect it from tarnishing and abrasion by coating it with a thin layer of transparent lacquer. This coating of lacquer impaired the definition of the reflected image and also reduced the reflectivity from 95 per cent to about 70 per cent. (Note: The reflectivities for the above and following materials are given for light with a wavelength of about 550 millimicrons.)

Within the past few years an excellent process of making front surface mirrors has been developed commercially. It consists of depositing a thin nontarnishing film (about one-quarter wavelength thick) of high reflectivity upon a glass surface. The coating is accomplished by thermal evaporation in a high vacuum. Apparently the most successsful coating is made by first depositing a thin layer of chromium, which exhibits a strong adhesion to glass, followed by a thin layer of aluminum. The resulting composite film has high reflectivity (90 per cent), tenacity and durability.

2. SPECULUM METAL

This is an alloy of 68 per cent copper and 32 per cent tin. It has a reflectivity of about 60 per cent. Its chief disadvantage is that it tarnishes rather readily in the presence of certain atmospheric gases and cases are known where the reflectivity has been reduced to two-thirds of its original value within a few months. It is seldom used in photogrammetric instruments.

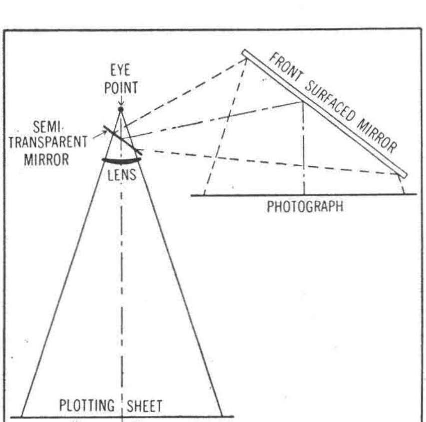

FIG. 33. Schematic diagram of a vertical sketchmaster.

3. STELLITE

This is a trade name for an alloy of chromium, cobalt, and tungsten. It takes a high polish, does not tarnish, is very hard, and has a reflectivity of 60 per cent. However, it is very difficult to machine and must be shaped by grinding.

4. STAINLESS STEEL

This is an alloy of varying composition; one type contains 18 per cent chromium and 8 per cent nickel. This material tkaes a high polish, does not tarnish when polished, and has a reflectivity of about 61 per cent. Some stainless steels may be machined readily.

Metal mirrors should be used instead of glass if there is danger of frequent breakage. It is interesting to know that metal mirrors can have their reflectivities increased to about 90 per cent by the deposition upon their surfaces of an evaporated metallic film as explained above.

5. SEMI-TRANSPARENT

In contrast to the above-described mirrors, in which the greatest possible reflectivity is desired, the semi-transparent type is one which both reflects and transmits the light that falls upon it. The ratio of reflection to transmission can be varied by controlling the density of the evaporated metallic film which is deposited on the surface of the glass. It is often used in instruments of the camera lucida type. An example of such an instrument is the "Vertical Sketchmaster" (see Fig. 33) for transferring detail from vertical aerial photographs to the plotting sheet. The mechanical parts of this instrument are arranged to take care of changes in scale and approximate tilt removal.

IX. PROBLEMS

1. A ray of light strikes a plane polished glass plate at an angle of incidence of 42°12′. Using eq. (1), compute the angle the refracted ray makes with the normal. (Ans: 26°14′)
2. Using equations (2) and (3), compute the image distance and magnification for the following conditions. Also state whether the image is real or virtual, erect or inverted:
   (a) Positive lens of 8″ focal length and object distance of 10″.
   (Ans: $I=+40''$, $M=-4$, real inverted image)
   (b) Positive lens of 8″ focal length and object distance of 6″.
   (Ans: $I=-24''$, $M=+4$, virtual erect image)
   (c) Negative lens of 8″ focal length and object distance of 12″.
   (Ans: $I=-4.8''$, $M=+0.4$, virtual erect image)
   (d) Negative lens of 8″ focal length and object distance of 5″.
   (Ans: $I=-3.08''$, $M=+0.62$, virtual erect image)
3. Two thin lenses, one having a focal length of +10″ and the other a focal length of +5″, are placed in contact. Using eq. (4) compute the approximate focal length of the combination. (Ans: +3.3″)
4. Two thin lenses, one having a power of +5 diopters and the other a power of −2 diopters, are placed in contact. What is the approximate focal length of the combination in cm.? Refer to discussion following eq. (7). (Ans: +33.3 cm.)
5. The focal length of an aerial camera is 6.24″. A negative made with this camera was analysed for tilt and found to have a value of 3°42′. Compute the necessary values for setting a rectifying camera having a focal length of 8.24″. Also assume that the negative is to be enlarged to a scale ratio of 1.080. Use eq. (8) to (12). (Ans: $A=85°28'29''$, $B=85°06'43''$, $EL=15.865''$, $FL=17.143''$, $EC=0.151''$.
6. Given the following data for a multiplex reduction printer; Focal length of lens 50.24 mm., nodal point separation 2.60 mm., thickness of glass plate in front of negative 7.90 mm., index of refraction of glass plate 1.520, scale ratio setting on printer 0.250. Compute the total distance from negative to diapositive using eq. (5), (6), and (13). (Ans: 319.30 mm.)
7. Given a single lens aerial camera with a focal length of 6.24″ and a negative size of 9″×9″.
   (a) Compute the angular field at the extreme corner of the negative. (Ans: 91°04′)
   (b) If the light intensity at the center of the negative is 100% what is it at the extreme corner? Note: except for the vignetting effect of the lens mount, the light intensity varies as $\cos^4 A$, where (A) is half the angular field. (Ans: 24%)
8. Referring to problem (7), assume that a 0.25″ thick glass plate is pressed against the film to hold it flat. Using table I, compute the amount and direction of the image distortion caused by this plate at the corner of the negative. (Ans: −0.035″)
9. The focal length of a ratio printer lens is 10.25″. Using eq. (5) and (6), compute the following:
   (a) Conjugate distances for a ratio of 2.45. (Ans.: 0=14.43″, I=35.36″)
   (b) Conjugate distances for a ratio of 3.68 (Ans.: 0=13.04″, I=47.97″)
   (c) If the correct exposure is 12 seconds for part (a), what will it be for part (b)? Note: The exposure varies as $(M+1)^2$ where (M) is the magnification or scale ratio. (Ans.: 22 Sec.)
10. Referring to problem (9), assume that a 0.375″ thick glass plate is pressed against the film

(9″×9″) to hold it flat. Using table I, compute the amount and direction of the image distortion caused by this plate at the corner of a print made at a scale ratio of 3.68. (Ans.: +0.027″)

11. A magnifying glass used for examining aerial photographs has a focal length of 2.5″. Compute the following:
    (a) The magnifying power, using eq. (14), which assumes that the lens is placed its focal length away from the object thereby forming a virtual image at infinity. (Ans.: 4)
    (b) The magnifying power, using eq. (2) and (3), assuming a virtual image distance of 10″. (Ans.: 5). The purpose of this problem is to show that the magnifying power is not a fixed value but depends upon the object distance used by the individual observer.
12. A transit telescope has an objective lens of 10″ focal length and an eyepiece of 0.50″ focal length. Using eq. (15), compute the magnification. (Ans.: 20)
13. A microscope has an objective lens of 1.00″ focal length and an eyepiece of 0.80″ focal length. Compute the magnification of the system (see eq. 16), assuming an object distance of 1.25″. (Ans.: 50)

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