USE OF THE PRISMATIC ASTROLABE FOR ASTRONOMIC POSITIONS BY THE HYDROGRAPHIC OFFICE*

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THE Prismatic Astrolabe with a 60-degree prism was developed by French engineers in 1903. Later on British engineers brought out a similar instrument with a 45-degree prism, for which certain advantages are claimed. The principles of both types are analogous in determining astronomic latitude and longitude by equal altitudes of heavenly bodies when the time of a prime meridian, such as that of Greenwich, is known at the place of observation. This method of equal altitudes originated with Gauss, the noted astronomer, who in 1808 demonstrated that if the right ascensions and declinations of three or more stars be known, and equal altitudes of these stars be observed at times recorded, then from these times there may be computed: the observed altitude, the error of the chronometer on local time and the latitude of the observer. Nowadays, with radio time signals and an approximate value of the latitude, both latitude and longitude may be deduced precisely by this process of equal altitudes.

The first determination of astronomic position with the astrolabe by the Hydrographic Office was made in Venezuela in 1938. Previously we used the meridian transit—zenith telescope. From actual observing with each I am convinced of many worth-while advantages of the astrolabe over the transit-zenith telescope, without sacrifice of precision. There is nothing at all complicated about the observations, computations, or adjustments of the astrolabe.

The French "Geodetic Model" of Claude and Driencourt with a 60-degree prism, which we use, weighs 30 pounds, for the instrument, with case and accessories about 45 pounds, and can be carried by one man. Whenever possible we build a pier for observing and for a permanent marker, but a tripod support is part of the equipment furnished by the manufacturer. It serves its purpose well and weighs only 11 pounds. The instrument is set up and adjusted without much difficulty or exertion, and the observer can be seated. No more care is required for leveling the astrolabe than for an ordinary surveyor's theodolite. Orientation in the meridian closely enough for a start is done with an attached magnetic compass, applying the local magnetic declination. Collimation, which the makers term "auto-collimation," is effected with two adjusting screws on the exterior of the alidade and requires but a moment's work. The instrument having been leveled, oriented, and collimated, there remains but to pour a small amount of mercury in the pan for an artificial horizon, and the observations are in order.

We, of the Hydrographic Office, in appreciation of our success with the 60-degree astrolabe, feel greatly indebted to Colonel Bagley, U. S. Army, to Mr. Shea of the Massachusetts Institute of Technology, and to Major Weld Arnold, U. S. Army, formerly of the Institute of Geographical Exploration, for the help they have all given us in astrolabe work, and also for Major Arnold's compilation of a very comprehensive list of stars for each degree of latitude from 60 degrees south to 60 degrees north, without which our astrolabe observations could hardly have been attempted. On occasions when no program of stars had been prepared in advance, we have read them off directly from Major Arnold's cata-

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Revisions were made by the author for this publication.

logue, taking them in sequence as tabulated for local sidereal times and corresponding azimuths. A pre-selected list of stars to observe is, however, desirable, so as to get the proper distribution of azimuths in each quadrant, but no special preparation of observing programs, such as required for the transit-zenith telescope, is necessary in astrolabe procedure. No reference to old catalogues, such as the Boss, need be made for the astrolabe, as current star publications suffice without having to work up star positions to any date.

As to the dependability of the Astrolabe, we had an excellent opportunity a year or so ago to verify this at the Naval Observatory by accepting for standard of comparison the astronomic position of the pier we used as determined by Naval Observatory astronomers. In latitude two of our student observers obtained results with the astrolabe that differed respectively six-tenths of a second of arc and zero seconds of arc from the established value; in longitude each came within one and one-tenth of a second of arc. An experienced observer in the same test checked to two-tenths of a second of arc in latitude and five-tenths, also arc, in longitude. On one occasion two of our engineers received instruction in the astrolabe during the winter under adverse weather conditions permitting but three clear nights. Only seven days in all could be spared for the period of instruction, but nevertheless their subsequent determinations have been very gratifying, with probable errors of observations not more than two-tenths of a second of arc for both latitude and longitude. No test for personal equations of the observers has been possible as yet, but from an inspection it appears that the results of our observers at the Naval Observatory, as mentioned, would have been even closer with a small correction to the longitude on the assumption of a slight lag in the general or usual personal reaction. The makers of the astrolabe have developed a machine for testing personal equation which is reputed to be very efficient, but owing to war conditions we have been unable to get any of them.

Procedure in observing with the 60-degree astrolabe is simple and not difficult to learn. Our junior hydrographic engineers, without previous assignment to astronomical work, have received a fair understanding of the observing and computations in a few days when pressed for time. Our experience with the astrolabe has proven the field operations to be economical and expedient. An astronomical expedition consisting of two officer observers completed 24 stations for latitude and longitude in less than six weeks. This time included ship transportation between stations, averaging about 60 miles apart, and time for landing gear through surf, making and breaking camp, and for small surveys to connect the piers to lighthouses and nearby portions of the shore line and topography.

Optics

The optical arrangement of the 60-degree astrolabe is illustrated on Figure 1.

R and *R′* are rays from a star. *R* passes normally through the upper face of the prism and is reflected from the lower face through the object glass to the focal plane of the objective at *f*. *R′* is first reflected from the mercury surface to the upper face, thence to *f*. The diagram is drawn to show the convergence of *R* and *R′* at the instant when the two star images coincide at the altitude governed by the angle of the prism. The three figures at the bottom indicate the images on entering the field, at passage, and on leaving the field. Of the two eyepieces shown, the upper one is the finder eyepiece, of 30 diameters, having a field of 1 degree 36 minutes, for training the alidade on the stars. A small prism actuated at will by a lever transmits the star images from *f* to this finder eyepiece, and after centering the images therein the observer throws back the lever and the images are then set to appear for observing in the lower eyepiece of 80 diameters and 36-minute field.

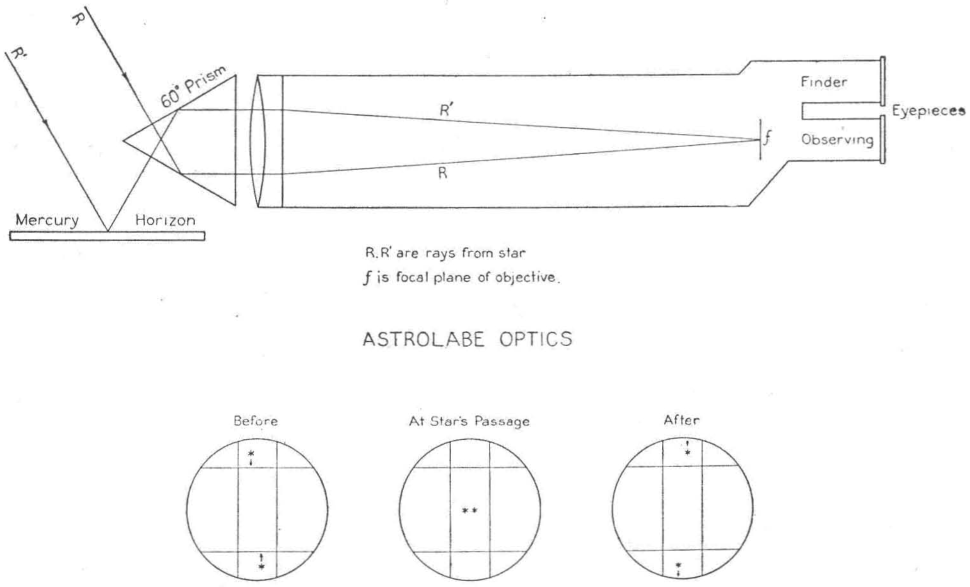

R
R'
60° Prism
R'
R
f
Finder
Observing
Eyepieces
Mercury
Horizon
R, R' are rays from star
f is focal plane of objective.
ASTROLABE OPTICS
Before
At Star's Passage
After

FIG. 1

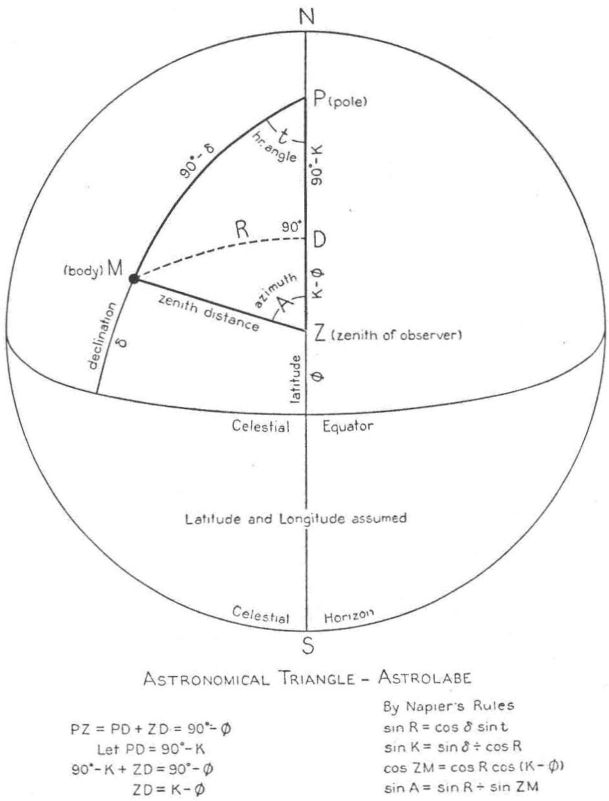

N
P (pole)
90°-δ
t
hr. angle
90°-K
R
90°
D
(body) M
azimuth
A
K-φ
zenith distance
Z (zenith of observer)
declination
δ
latitude
φ
Celestial
Equator
Latitude and Longitude assumed
Celestial
Horizon
S
ASTRONOMICAL TRIANGLE - ASTROLABE
PZ = PD + ZD = 90°- φ
Let PD = 90°- K
90°- K + ZD = 90°- φ
ZD = K - φ
By Napier's Rules
sin R = cos δ sin t
sin K = sin δ ÷ cos R
cos ZM = cos R cos (K - φ)
sin A = sin R ÷ sin ZM

FIG. 2

Astronomical Triangle

The astronomical triangle concerned in the computation, shown on Figure 2, is projected on the plane of the observer's horizon extended to the celestial. *NS* is the observer's meridian through his zenith point *Z* and *P* the elevated pole. *M* is the body observed and delta (δ) its declination. *R* by construction is a perpendicular from *M* to the meridian at *D*.

The latitude and longitude are assumed. The Greenwich sidereal time and the body's right ascension are known, from which the body's hour angle (*t*) is deduced. The declination of the body is known. The two spherical right triangles *PMD* and *DMZ* are successively solved, from which the values of *ZM* the zenith distance, and *A* the azimuth of the body, are computed in relation to the assumed position.

Computations

The form of Figure 3, filled out to illustrate its use, is arranged for computation of stars observed with the Astrolabe. Practically all of the stars in Major Arnold's "60° Star Lists," previously mentioned, are tabulated in the British catalogue, *Apparent Places of Fundamental Stars*, which now is being published each year. Referring to Figure 3, the chronometer times of the star passages at 60° altitude are carefully scaled from the chronograph sheets. The right ascensions and declinations are interpolated from the British catalogue. The identity of each star used may be checked by its right ascension as approximated in the "60° Star Lists."

ZENITH DISTANCES AND AZIMUTHS

Greenwich Sidereal Time Of Observation Known

	Star	#214 β Aql.	#218 41 Cyg.	#221 ε Aqr.	#234 ι Cet.	#236 ε And	#246 η Psc
	Chro. time	1 53 36.74	2 06 11.49	2 14 10.01	2 46 52.75	2 54 44.21	3 27 28.20
	Chro. Corr.	+ 11 57.65	+ 11 57.65	+ 11 57.65	+ 11 57.65	+ 11 57.65	+ 11 57.65
	G. S. T.	2 05 34.39	2 18 09.14	2 26 07.66	2 58 50.40	3 06 41.86	3 39 25.85
	λ	4 13 00.00	4 13 00.00	4 13 00.00	4 13 00.00	4 13 00.00	4 13 00.00
	L. S. T.	21 52 34.39	22 05 09.14	22 13 07.66	22 45 50.40	22 53 41.86	23 26 25.85
	R. A. Star	19 52 27.84	20 27 02.07	20 44 31.99	0 16 27.34	0 35 28.41	1 28 21.30
	(Time) t	2 00 06.55	1 38 07.07	1 28 35.67	1 30 36.94	1 41 46.55	2 01 55.45
	(Arc) t	30 01 38.25	24 31 46.05	22 08 55.05	22 39 14.10	25 26 38.25	30 28 51.75
	Star's δ	+ 6 15 46.12	+30 10 29.78	- 9 42 27.67	- 9 08 45.97	+28 59 33.52	+15 02 37.21
	Log sin t	9. 699 3281	9. 618 2166	9. 576 3534	9. 585 6456	9. 633 0926	9. 705 2248
(add)	Log cos δ	9. 997 4004	9. 936 7624	9. 993 7363	9. 994 4431	9. 941 8502	9. 984 8550
	Log sin R	9. 696 7285	9. 554 9790	9. 570 0897	9. 580 0887	9. 574 9428	9. 690 0798
	R	Note: Convert log sin R to log cos R and enter second line down.					
	Log sin δ	9. 037 7815	9. 701 2587	9. 226 9132	9. 201 2674	9. 685 4707	9. 414 2297
(subtract)	Log cos R	9. 938 2727	9. 970 0558	9. 967 7296	9. 966 0839	9. 966 9416	9. 940 4149
	Log sin K	9. 099 5088	9. 731 2029	9. 259 1836	9. 235 1835	9. 718 5291	9. 473 8148
	K	+ 7 13 26.70	+32 34 58.94	-10 27 52.63	- 9 53 46.25	+31 32 09.31	+17 19 15.57
	Ø	+10 40 15.00	+10 40 15.00	+10 40 15.00	+10 40 15.00	+10 40 15.00	+10 40 15.00
	K - Ø	3 26 48.30	21 54 43.94	21 08 07.63	20 34 01.25	20 51 54.31	6 39 00.57
	Log cos (K - Ø)	9. 999 2137	9. 967 4341	9. 969 7562	9. 971 3974	9. 970 5430	9. 997 0681
(add)	Log cos R	9. 938 2727	9. 970 0558	9. 967 7296	9. 966 0839	9. 966 9416	9. 940 4149
	Log cos Z	9. 937 4864	9. 937 4899	9. 937 4858	9. 937 4813	9. 937 4846	9. 937 4830
	Computed Z	30° 00′ 36.″38	33.″50	36.″85	40.″58	37.″83	39.″17
	Assumed Z	30 00 10.00	10.00	10.00	10.00	10.00	10.00
	Correction = ΔZ	26.″38	23.″50	26.″85	30.″58	27.″83	29.″17
	Log sin R	9. 696 7285	9. 554 9790	9. 570 0897	9. 580 0887	9. 574 9428	9. 690 0798
(subtract)	Log sin Z	9. 699 1027	9. 699 0922	9. 699 1043	9. 699 1179	9. 699 1079	9. 699 1128
	Log sin A	9. 997 6258	9. 855 8868	9. 870 9854	9. 880 9708	9. 875 8349	9. 990 9670
	Computed A	264° 01′	314° 09′	227° 59′	130° 31′	48° 42′	78° 21′
	Star List A	263¼°	313¼°	227¼°	131¾°	49¾°	79°

Ø = Assumed Latitude
t = Hour Angle
A = Azimuth
λ = Assumed Longitude

δ = Declination
Z = Zenith Distance
R & K are auxiliary angles

$\text{Sin R} = \text{Sin t Cos } \delta$

$\text{Sin K} = \frac{\text{Sin } \delta}{\text{Cos R}}$

$\text{Cos Z} = \text{Cos (K - Ø) Cos R}$

$\text{Sin A} = \frac{\text{Sin R}}{\text{Sin Z}}$

North Latitude and Declination are regarded (+), South as (-). If Latitude and Declination are both (+) or both (-) then K-Ø = the numerical difference of K and Ø. If Latitude and Declination are of different names, K-Ø = the numerical sum of K and Ø. The plotting requires a constant "Assumed Z" to be less than the smallest "Computed Z".

Fig. 3

As already mentioned in the solution of the astronomical triangle, the zenith distances and azimuths of the stars are computed from assumed co-ordinates. The position sought is then plotted graphically, using the various zenith distance corrections and azimuths in somewhat similar manner as in plotting lines of position in navigation, except that for the astrolabe the differences between observed and computed zenith distances are plotted instead of the altitude differences.

One "wrinkle" in the computations is concerned with the "Assumed Z," which appears at the lower part of Figure 3. In this connection it should be noted that the absolute value of the prism angle does not enter into the computations. As a rule the variation from 60° is only a few seconds. The observations being based upon a constant observed zenith distance, any value for a constant assumed zenith distance ("Assumed Z") may be arbitrarily chosen for convenience in plotting. For the computations of Figure 3 the constant of 30°00′10″ was selected so that the plotting of zenith distance corrections (ΔZ) would permit a scale of two divisions to one second of arc in the plotting of Figure 6, such a scale being sufficiently large for accurate graphic determination of this particular computed position. Another consideration in selecting the value of "Assumed Z" is to choose a value less than the smallest "Computed Z" so that all star position lines may be plotted from the assumed position in direction *toward* the stars. Increasing or decreasing the assumed zenith distance affects plotting of the computed circle of position only to the extent of lengthening or shortening the radius without displacing the center.

ASTROLABE COMPUTATIONS NHO-750

LATITUDE AND LONGITUDE

GREENWICH SIDEREAL TIME OF OBSERVATION KNOWN SET No. 6

PLACE Carupano, Venezuela ASSUMED LAT. 10°40′15.″00 N. ASSUMED LONG. 63°15′00.″00 W. ASSUMED Z.D. 30°00′10.″00
SHIP OR EXPEDITION U.S.S. Sumner DATE, 1941 MEAN EPOCH OF OBSERVATIONS July 21.3
SIGNALS FROM Arlington FREQUENCY 113 K.C. OBSERVER W. S. Davis COMPUTER W. S. Davis

LATITUDE AND LONGITUDE OBSERVATIONS
Carupano, Venezuela
July 1941

The latitude and longitude of a concrete pier near E. Guayacan Point at Carupano, Venezuela, was determined with a 60° astrolabe using a sidereal chronometer. The rate of the chronometer was checked with radio time signals from Arlington. Both clock ticks and radio signals were recorded by means of a chronograph. Six sets of from 15 to 20 stars each were observed and plotted. The scaled values of these 6 sets are as follows:

	Longitude	Latitude
Set No. 1	63-14 56.6	10-40 13.1
2	63-14 57.1	10-40 12.8
3	63-14 58.0	10-40 12.0
4	63-14 56.8	10-40 13.0
5	63-14 58.2	10-40 12.6
6	63-14 57.9	10-40 12.8

See Fig. 3 for computations of six stars of set No. 6

The means of these 6 sets were taken as the observed position of this pier:

Longitude	Latitude
63-14 57.4 W.	10-40 12.7 N.

The probable error was found to be :
± 0.2" in longitude ± 0.1" in latitude

The probable error was computed from the formula:

$$P.E. = \pm 0.6745 \sqrt{\frac{\sum v^2}{n(n-1)}}$$

CLOCK CORRECTIONS

Greenwich Date, July 21, 1941	h m s	h m s	h m s
G.C.T. of Signal	6 00 00.00	7 00 00.00	8 00 00.00
G.S.T. of Greenwich 0^h	19 53 28.25	19 53 28.25	19 53 28.25
Corr. for Int. since 0^h	+ 59.14	+ 1 9.00	+ 1 18.85
G.S.T. of Signal	1 54 27.39	2 54 37.25	3 54 47.10
Clock Time of Signal	1 42 29.70	2 42 39.63	3 42 49.50
Clock Corr. on G.S.T. at Signal (F−, S+)	+ 11 57.69	+ 11 57.62	+ 11 57.60

Average Hourly Rate between Signals		Interval between Signal and Mean Epoch		Change in Error in Interval	
Gaining Losing	s 0.045	×	Hours 1.0	=	s 0.045 Gaining Losing

Chronometer Correction for Set = + $0^h\ 11^m\ 57.^s65$

LATITUDE DIFFERENCE	Divisions	° ′ ″	
Assumed Latitude, ϕ_a		10 40 15.00	
Y-coordinate (~~N~~, S)	2.2 =	− 2.20	
Observed Latitude		10 40 12.80	

LONGITUDE DIFFERENCE	Divisions	° ′ ″	Log
X-coordinate (~~W~~, E)	2.1		0.322 2193
Log Cos Lat. (subtract) or Nat. Sec Lat. (multiply)			9.992 4250 0.329 7943
Corr. to assumed Long.		2.10	
Assumed Longitude		63 15 00.00	
Observed Longitude		63 14 57.90	

COMPUTER'S NOTES:

FIG. 4

Figure 4 shows a form and illustrative computation such as is used for recording data in general concerning the observations of one set of stars, and includes the results of the entire observations at one station.

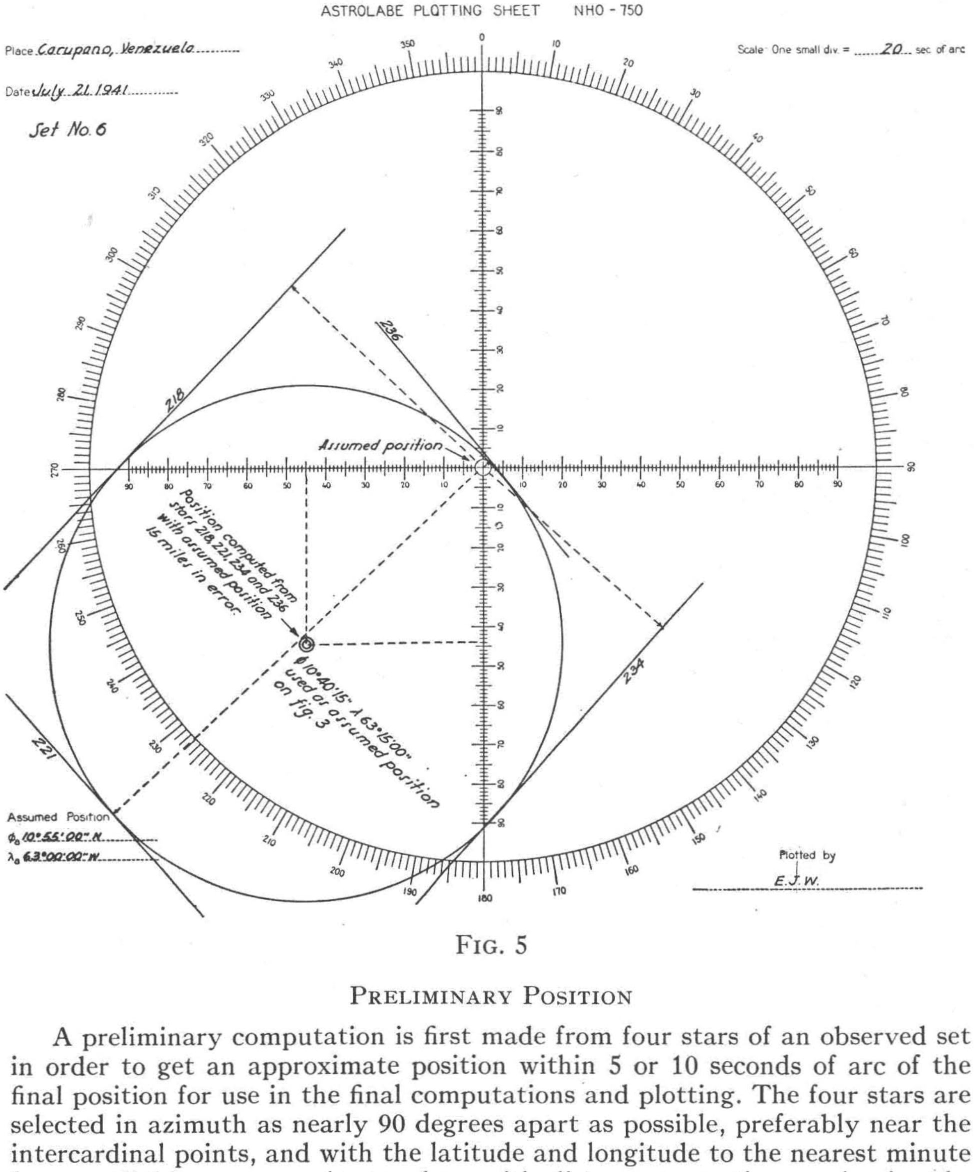


FIG. 5

PRELIMINARY POSITION

A preliminary computation is first made from four stars of an observed set in order to get an approximate position within 5 or 10 seconds of arc of the final position for use in the final computations and plotting. The four stars are selected in azimuth as nearly 90 degrees apart as possible, preferably near the intercardinal points, and with the latitude and longitude to the nearest minute from available maps or charts, the zenith distance corrections and azimuths are computed in the same manner and form as the final position. Illustrating how the approximate position is derived, Figure 5 was prepared assuming a latitude and longitude purposely grossly in error about 15 miles. This position is represented at the center of the diagram. The four dotted lines are the plotted azimuths of the four selected stars. The normals to these dotted lines are drawn at their respective zenith distance corrections from the diagram center, and by construction are tangents to a circle the center of which is the observed point.

The scale of this figure is 3 divisions of Figure 5 to one minute of latitude, and the derived position proved to be within 5 seconds in arc of the final position. A close value of the longitude may be found directly if any star of the set is observed on both sides of the meridian. In such a case the longitude is the difference between the mean of the Greenwich sidereal times of the star's passages and the star's right ascension. The computation procedure of the four stars, for an approximate assumed position, is identical with the computation for the final position as on Figure 3.

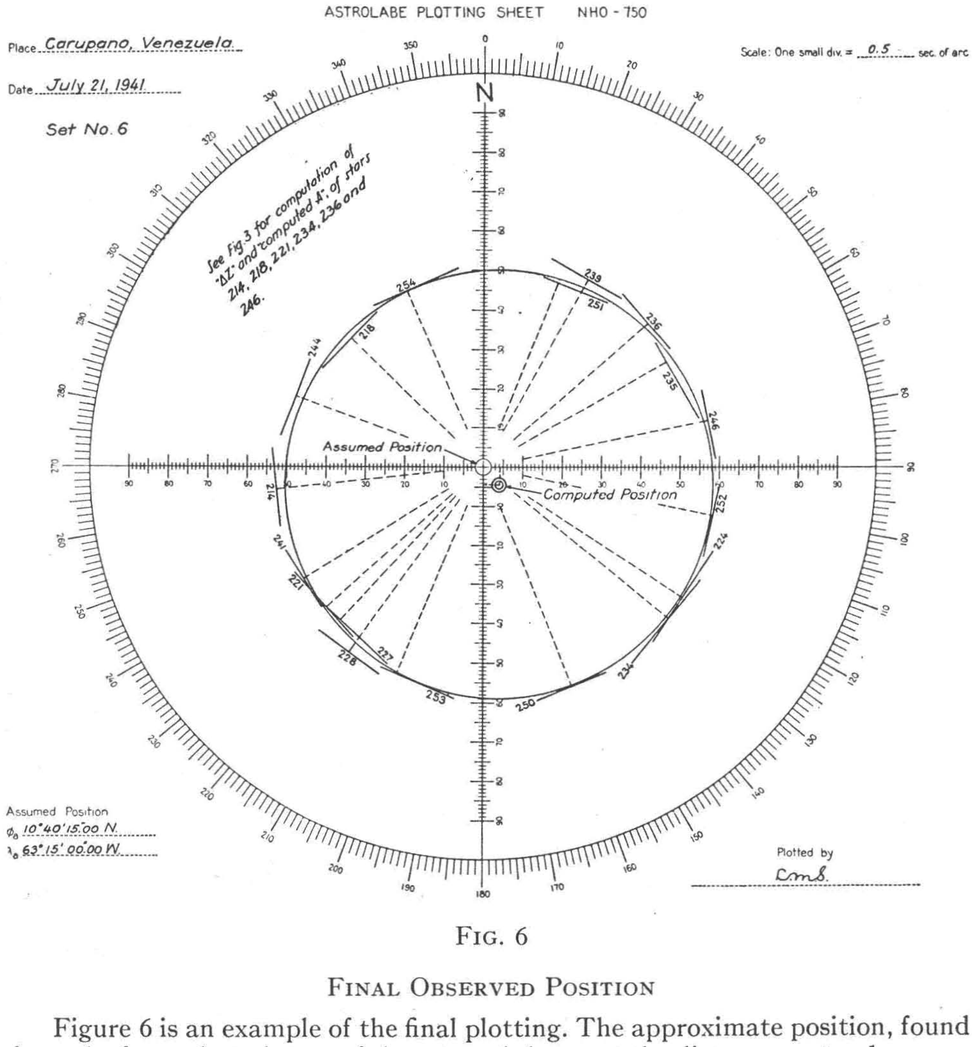


FIG. 6

FINAL OBSERVED POSITION

Figure 6 is an example of the final plotting. The approximate position, found from the four selected stars of the set, and shown at the diagram center, becomes the assumed position for the final computations and plotting. The azimuth lines of the various stars of the set, including the four selected stars recomputed for the new assumed position, are shown with their corresponding normals, these normals being tangents to a circle the center of which is the final observed position and indicated by the two small concentric circles. The value of this final observed position is referred to the assumed position as derived from the preliminary computation, the latitude correction in arc being scaled direct, the correction for longitude arc being the scaled difference times the secant of the latitude.

Opinions differ as to the number of stars which should be observed for each set. Our system is, whenever possible, to control the set with three radio time signals, an hour apart, and within this two-hour period to observe as many stars as practicable, consistent with careful work. Generally a greater number of stars are observed than are utilized in the computations, thus permitting a selection for distributing the position-circle tangents to advantage in the quadrants, and also providing additional stars for checking if needed. We prefer not less than five stars to each quadrant. As a rule, stars closer than 10 degrees to the meridian are avoided as they change too slowly in altitude for accurate timing, and time is the deciding element for precision in astrolabe operations.

A few minutes before commencing the observations it is advisable to take out several stars from the catalogue and point on them to check the compass orientation of the azimuth circle, and also to adjust the focus of the observing eyepiece. This focus must not be altered during any set, as so doing will materially vary the constant measure of the altitude.

A unique feature in observing with the Prismatic Astrolabe is that the two images of the star, as seen in the telescope, approach from opposite directions, and therefore at twice the apparent motion, so if there be any displacement in the observer's estimate of the coincidence of the images his error is halved.

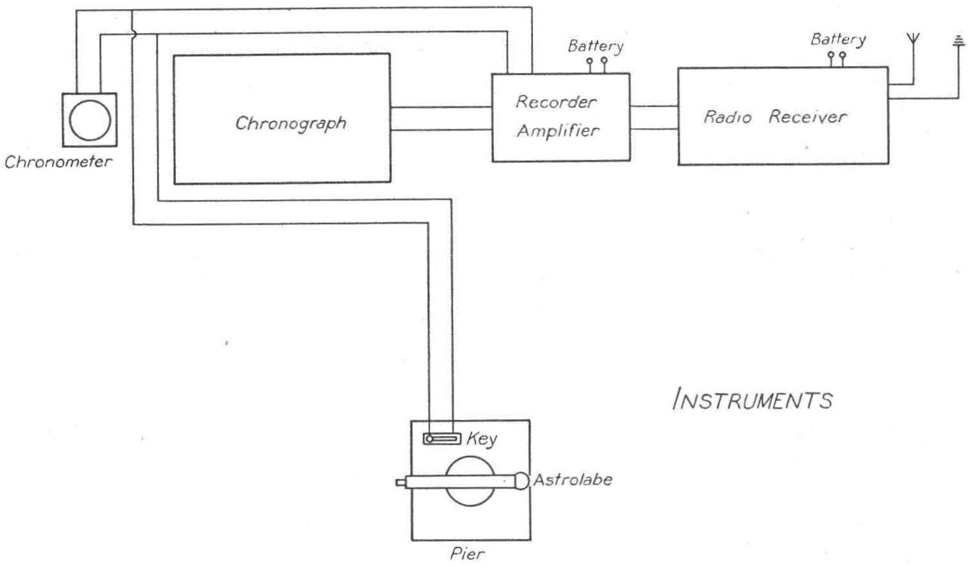
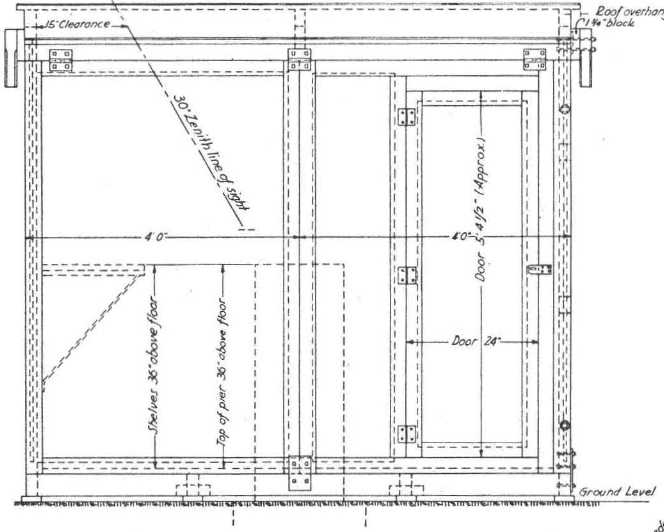


FIG. 7

INSTRUMENTS

Figure 7 shows our usual arrangement of instruments for astrolabe observations. At the bottom of the figure is the pier with astrolabe and observer's break-circuit key. In the upper part of the diagram at the left is the break-circuit sidereal chronometer, and thence towards the right are the chronograph, the radio recorder-amplifier and the radio receiver. Current for electrical needs is supplied from flashlight cells and a 6-volt storage battery.



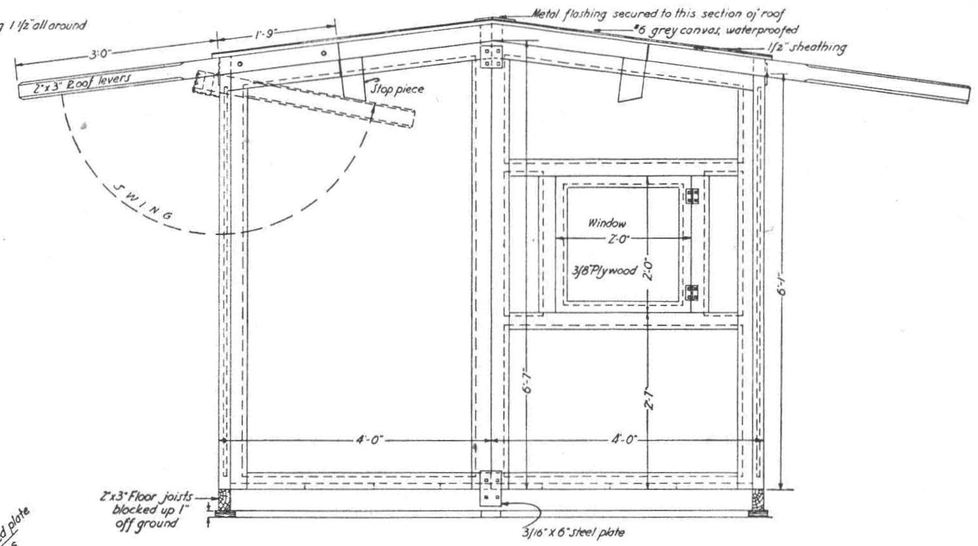
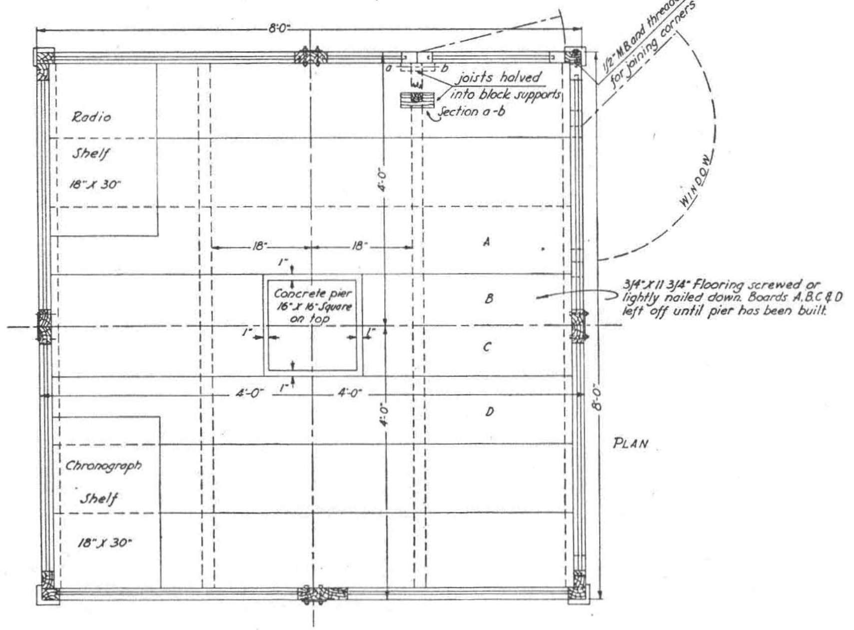


FIG. 8



PORTABLE SHELTER FOR ASTROLABE

1 FT. 1/2 0 1 FT.

Floor joists, and framing for sides and roof 1 3/4" X 2 3/4" dressed 4 sides.
The roof, sides and floor each is built on sections secured in place by plates, bolts and hinges as shown.
Sufficient clearances around pier are allowed so that floor may be placed in position while pier is being built.
Top of pier should be 36" above floor.

All lumber except paneling to be of white pine free of knots.

Frame joints to be halved and screwed or mortised, bound with waterproof glue. Plywood paneling, 3/8", to be glued in rabbetted framing, waterproof glue.

All sections and parts to be given 3 coats of outside white paint.

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" 11-4-42 C.m.S.

Astrolabe Shelter

Figure 8 shows the portable type of shelter we use for the astrolabe. It is 8 feet square, $6\frac{1}{2}$ feet high, and designed so that it may be set up at the same time the pier is being built. The sides and roof are each in two sections, 4 feet wide and 6 to 8 feet long, and in the erection are bolted together in place. The paneling is $\frac{3}{8}$-inch plywood rabbetted into 2-inch by 3-inch white pine framing. The roof is in two sections as mentioned. Each section is hinged at the side and can be thrown back with two levers to clear the ceiling for observing. All sections of the structure are thoroughly waterproofed and painted, and the whole may be towed as a raft in the water for landing if desired. There is sufficient room inside the shelter for a couple of cots and also for the observer's living and computing quarters.

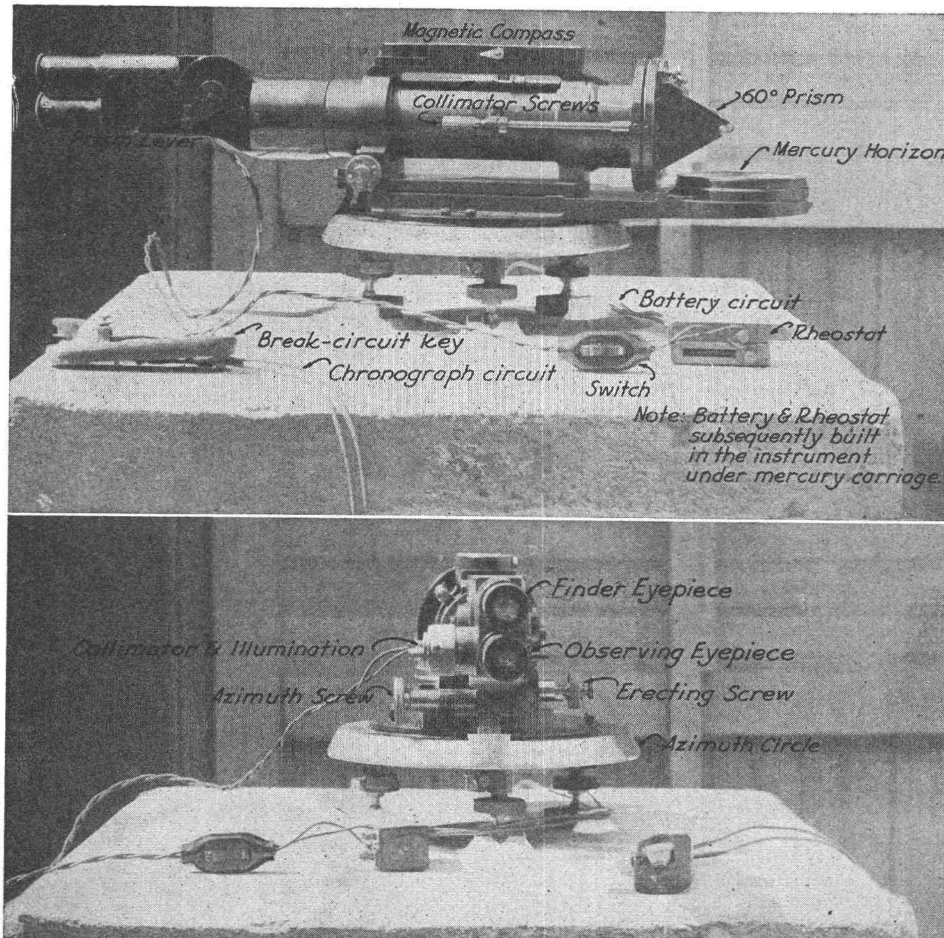

FIG. 9

Astrolabe Photographs

Two photographic views of the Claude and Driencourt "Geodetic Model" astrolabe are shown on Figure 9. The pictures were taken of the instrument set up for observations at the Naval Observatory.

The Hydrographic Office has adopted the Astrolabe for astronomical determinations in connection with beginning or checking surveys. This is in keeping with a progressive policy to constantly improve our methods in field surveys and likewise in the compilation, construction and reproduction of surface and air charts. A keen interest is always manifested in new equipment and procedures, whether developed elsewhere or by our own personnel.