# PLOTTING STEREOSCOPIC TOPOGRAPHIC MAPS

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RECENTLY, during the course of our work at Syracuse University, the question arose as to whether it would not be desirable to construct stereoscopic mates for topographic maps plotted with the multiplex-projector to enable a person to examine and study topographic maps stereoscopically, in the same manner that we study stereoscopic pairs of aerial photographs. After reviewing the possible values of such maps, we next considered whether an instrument could be designed which would plot a stereoscopic mate at the same time that the correct map was being plotted without destroying the accuracy of the latter. The following deals with our findings on this question.

*Suggested Design for Attachment to Multiplex-Projector Plotting Table*

The most economical way to make stereoscopic mates for the regular maps plotted with the multiplex-projector would be to plot both maps at the same time. This suggests a double plotting instrument or the use of some type of carbon-copy device. The former was regarded more practical.

In order that a pair of topographic maps may be viewed stereoscopically, parallax must be present. Since it is desirable to have a correct map at all times, the parallax must be introduced in the stereoscopic mate for the correctly plotted map.

The amount of parallax for any point appearing on both maps would be equal to the parallax for that same point on the photographs multiplied by the approximate ratio of the map scale to the scale of the photographs ($P_m = P_{ph} \times$ map scale/photo scale, where $P$ denotes the parallax). The logical conclusion, here, would be to construct a second plotting instrument to be connected to the multiplex plotting table by means of a parallax screw device which would transform the vertical ($Z$) movement of the plotting table into a horizontal ($X$) movement on the second plotting instrument in the $x$-direction. The latter movement would be equal to the change of parallax corresponding to the change in elevation of the plotting table. But, the multiplex plotting table does not operate on the parallax principle; changes in the vertical position of the floating mark represent differences of elevation reduced to a constant vertical scale. Therefore, the parallax micrometer principle would be difficult to introduce into such an instrument.

However, it is not necessary to have exactly the correct parallaxes in the stereoscopic map, for in all probability there will be no need to make measurements on the stereoscopic model formed by a pair of maps. The correct map is always available for horizontal measurements and the contours will determine the vertical positions of all points. The stereoscopic map would be used principally for inspection. A model that is approximately correct would serve almost as well as a perfect one. Assuming these conditions, quite satisfactory results could be obtained if the $Z$-movement ($\Delta Z$) of the plotting table could be changed, according to a fixed ratio, to an $X$-movement ($\Delta X$) on the second plotting instrument, which would be approximately equal to the correct parallax.

The second plotting instrument must be connected to the plotting table so that no $y$-parallax can be introduced in plotting. This means that the plotting pencils of both instruments must retain a fixed relationship in the $y$-direction, and that the relative movement of the second instrument in the $x$-direction must always be parallel to the line of flight. Furthermore, both instruments must be connected to each other and to a universal drafting machine so that they always retain a fixed orientation, moving parallel to each other and parallel to themselves. For practical reasons, the plotting instruments should be designed so that the plotting table could be readily attached and detached from the universal drafting machine and the second plotting instrument.

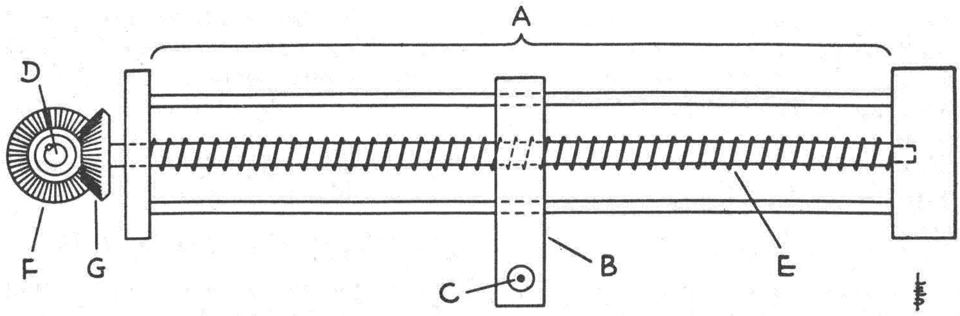


FIG. 1. Proposed design for attachment to multiplex-projector plotting table. (A) frame of second plotting instrument and guide for plotting table B; (B) plotting table of second plotting instrument; (C) plotting pencil attached to B; (D) screw of multiplex-projector plotting table; (E) screw of second plotting instrument; (F) horizontal beveled gear, attached to foot of screw D; (G) vertical beveled gear.

Figure 1 shows one manner in which the $Z$-movement of the plotting table could be made to produce the correct proportional $X$-movement in the second plotting instrument. The desired ratio between $\Delta Z$ and $\Delta X$ could be determined by the gears used in the design or by the ratio between the pitch of the screws of the plotting table and the second plotting instrument. In either case, the horizontal gear, $f$, would be attached to the foot of the plotting table screw which raises and lowers the table.

Because the ratio, $\Delta X/\Delta Z$, will vary for aerial cameras of different focal lengths, the proposed plotting instrument would have to be constructed for individual cameras. Although this is a disadvantage, it is the same disadvantage found in the multiplex equipment, which likewise must be constructed for individual cameras.

### *Determination of* $\Delta X/\Delta Z$

The term $\Delta X/\Delta Z$ represents the ratio of the desired $X$-movement (the correct parallax) of the second plotting instrument to the $Z$-movement of the plotting table. Table I shows the computation of $\Delta X/\Delta Z$ for the aerial camera used at Syracuse University. This camera, hereafter referred to as camera A, has a focal length of 184.03 mm. and takes photographs 184 mm. by 130 mm. The method of computing $\Delta X/\Delta Z$ can easily be followed from the supplementary notes included in the table. The same computations were made for an aerial camera, hereafter called camera B, with a focal length of 99.23 mm. which takes photographs 177.8 mm. by 177.8 mm. The results of the latter are tabulated in the third column of Table III.

TABLE I. COMPUTATION OF $\Delta X/\Delta Z$ FOR CAMERA *A*

| $H$ (Feet) | $Bxf$ (a) | $P_0$ $h=0$ | $P_{100}$ $h=100$ | $\Delta X_1$ $P_{100}$ $-P_0$ | $P_{1000}$ $h=1000$ | $P_{1100}$ $h=1100$ | $\Delta X_2$ $P_{1000}$ $-P_{1100}$ | $\Delta X_A$ $\frac{\Delta X_1+\Delta X_2}{2}$ | $\Delta Z_A$ (b) | $\frac{\Delta X_A}{\Delta Z_A}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 5000 | 368060 | 73.612 | 75.114 | 1.502 | 92.015 | 94.374 | 2.359 | 1.930 | 3.681 | .524 |
| 7500 | 552090 | 73.612 | 74.607 | 0.995 | 84.937 | 86.264 | 1.327 | 1.161 | 2.454 | .473 |
| 10000 | 736120 | 73.612 | 74.355 | 0.743 | 81.790 | 82.710 | 0.920 | 0.832 | 1.840 | .452 |
| 12500 | 920150 | 73.612 | 74.206 | 0.594 | 80.013 | 80.715 | 0.702 | 0.648 | 1.472 | .440 |
| 15000 | 1104180 | 73.612 | 74.106 | 0.494 | 78.870 | 79.437 | 0.567 | 0.530 | 1.227 | .432 |
| 17500 | 1288210 | 73.612 | 74.035 | 0.423 | 78.073 | 78.549 | 0.476 | 0.450 | 1.052 | .428 |
| 20000 | 1472240 | 73.612 | 73.982 | 0.370 | 77.486 | 77.896 | 0.410 | 0.390 | 0.920 | .424 |

(a) $B$ = Air Base = $.40S/f \times H = .40H$; $f = 184.03$ mm.; $S = 184$ mm.
(b) $\Delta Z = f/H \times 100$ = mm/100 ft.

The ratio of $\Delta X/\Delta Z$ is always larger than $.40S/f$, where $f$ is the focal length of the aerial camera; $S$, the dimension of the photograph along the line of flight, and where $.40S$ represents the length of the air base (60% overlap) on the photograph. The value of $\Delta X/\Delta Z$ approaches $.40S/f$ as the vertical distance between the exposure station and datum increases beyond 10,000 feet. However, the rate of change in $\Delta X/\Delta Z$ decreases as this vertical distance increases.

In the $\Delta X/\Delta Z$ computations for cameras *A* and *B*, parallaxes were determined for the photograph and $\Delta Z$ (mm/100 ) was based on the datum scale of the photograph. The values of $\Delta X/\Delta Z$ thus determined will always be correct for a given set of conditions and a given camera regardless of the amount of enlargement of the map scale over the photographic scale. The value of $\Delta Z$ at the datum scale of the photograph and the parallax of the photograph are multiplied by the same amount, the ratio of the map scale to the photographic scale, in plotting a map to any given scale.

For convenience, an assumed datum of 0 was used in computing $\Delta X/\Delta Z$. The same results would be obtained if some other datum were used, providing the new datum was taken into account wherever $H$ was used.

Table II shows values of $\Delta X/\Delta Z$ computed for camera *A* at different flying heights and for various ranges in topography. The exposure stations of aerial photographs taken with camera *A* are usually about 10,000 feet above datum, and the range in topography is seldom greater than 500 feet. For these conditions, the ratio .452 would be used to convert the vertical movement of the plotting table to a horizontal movement in the $x$-direction on the second plotting instrument in constructing the latter for camera *A*. This ratio, we believe, will

TABLE II. VALUES OF $\Delta X/\Delta Z$ FOR CAMERA *A*

| Range of $h$ | $H$=Feet 5000 | 10000 | 15000 | 20000 |
|---|---|---|---|---|
| 0–100 | .408 | .404 | .403 | .402 |
| 1000–1100 | .641 | .500 | .462 | .448 |
| 2000–2100 | 1.149 | .633 | .532 | .497 |
| 0–1000 | .524 | .452 | .432 | .424 |
| 1000–2000 | .895 | .566 | .499 | .471 |
| 0–2000 | .779 | .518 | .461 | .450 |

$f$=184.03 mm. Overlap=60%
$S$=184 mm.

plot stereoscopic mates which, used with the correct maps, will produce stereoscopic models that will most closely approach the models formed by the pairs of photographs from which the maps were plotted and for the conditions under which the photographs were taken. Furthermore, this ratio will serve almost as well in making stereoscopic mates where the values of $H-h$ exceed 10,000 feet, for there is only a slight change in the ratio as the height of the exposure station increases to 20,000 feet.

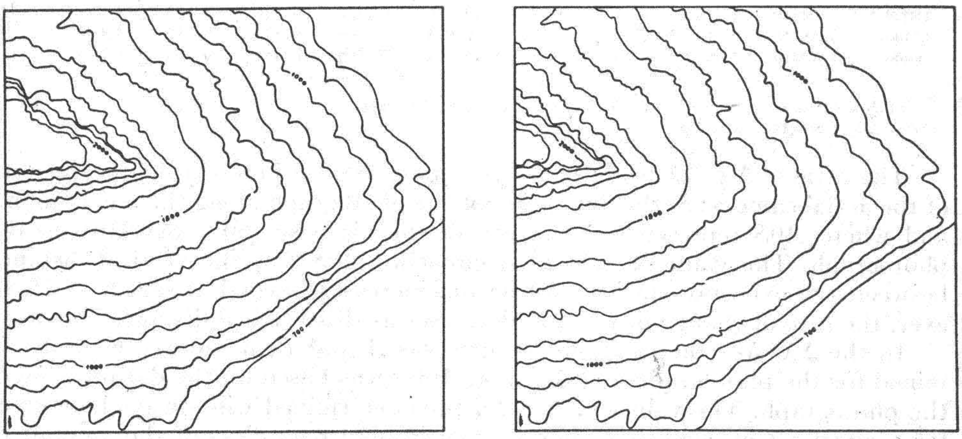

FIG. 2. A pair of stereoscopic maps. The left map is the correct one and the one on the right is the stereoscopic mate.

Figure 2, showing contours only, indicates how a pair of stereoscopic maps would appear when plotted with the proposed instrument. Since no plotting table with the attachment described has been constructed, these maps were actually made by tracing the original map and displacing the contours on the stereoscopic mate along the line of flight to obtain exactly the same parallaxes as would be produced automatically by the proposed instrument. This attachment would also automatically plot planimetric details such as streams, roads and trails, always maintaining the correct relationship between the two plotting pencils so that the planimetric details would appear in correct stereoscopic relief.

It can be proved that the ratio between the values of $\Delta X/\Delta Z$ for any two cameras is always equal to the ratio between the values of $S/f$ for those same cameras. In the case of cameras $A$ and $B$

$$\frac{S_A/f_A}{S_B/f_B} = \frac{184/184.03}{177.8/99.23} = 0.5583,$$

which is the same as the $\Delta X/\Delta Z$ relationships for these cameras shown in column 4 of Table III.

The values of $\Delta X/\Delta Z$ for camera $B$ or any other camera can be obtained by multiplying the $\Delta X/\Delta Z$ of camera $A$ by $S_B/f_B/S_A/f_A$, where $S_B/f_B$ is the ratio of the dimension of the photograph along the line of flight to the focal length of camera $B$ or any other camera for which $\Delta X/\Delta Z$ is desired. If $S$ and $f$ of a camera whose $\Delta X/\Delta Z$ is known are equal, as they are for camera $A$, the $\Delta X/\Delta Z$ of any other camera may be determined merely by multiplying $\Delta X/\Delta Z$ of the former by $S/f$ of the camera whose $\Delta X/\Delta Z$ is desired. Thus the values of

TABLE III. $\Delta X/\Delta Z$ FOR CAMERAS $A$ AND $B$

| $H$ (a) | $\frac{\Delta X_A}{\Delta Z_A}$ | $\frac{\Delta X_B}{\Delta Z_B}$ | $\frac{\Delta X_A}{\Delta Z_A} \Big/ \frac{\Delta X_B}{\Delta Z_B}$ | $\frac{\Delta X_B}{\Delta Z_B}$ (b) |
|---|---|---|---|---|
| 5000 | .524 | .940 | .558 | .940 |
| 7500 | .473 | .848 | .558 | .848 |
| 10000 | .452 | .810 | .558 | .810 |
| 12500 | .440 | .788 | .558 | .788 |
| 15000 | .432 | .775 | .558 | .776 |
| 17500 | .428 | .766 | .559 | .767 |
| 20000 | .424 | .759 | .559 | .765 |

(a) Range of $h$ is 0–1000 feet

(b) $\frac{\Delta X_B}{\Delta Z_B} = \frac{\Delta X_A}{\Delta Z_A} \times \frac{S_B/f_B}{S_A/f_A}$

$\Delta X/\Delta Z$ for camera $A$, listed in column 2 of Table III, can be used as constants to find the corresponding values of $\Delta X/\Delta Z$ for any other camera.

*Summary*

There is no doubt that stereoscopic topographic maps would have a practical value in reconnaissance work, planning and teaching. Although the instrument proposed herein might not find an extensive use, it would be an interesting device to have available for use with the multiplex-projector; and as suggested above, it could be designed to be readily attached and detached from the plotting table so as not to interfere with the normal use of the plotting table.