THE "ORTHOCENTER" ON OBLIQUE AERIAL PHOTOGRAPHS

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THE writer here proposes the term "orthocenter" as a point in any horizontal plane lying at the orthographic projection of the perspective center of a photo in that plane, and as a point on a photo at which this orthocenter will fall when the horizontal plane containing it is folded down into the photo plane using the intersection of the horizontal plane and the photo as a hinge. When the horizontal plane is that containing the perspective center, the hinge will be the true horizon line, and the orthocenter on the photo will fall at the isocenter. This is the most useful case, which will be illustrated in this article.

In Figure 1, continuous lines all lie in the photo plane, the plane of the paper. This contains the principal line HV, with the points H (on the true horizon), C (photo center), I (isocenter), and V (photo plumb point). The long dashed lines lie in the principal plane, folded down into the photo plane using the principal line as a hinge. This contains the perspective center O, as well as all points on the principal line. The short dashed lines lie in the horizontal projection plane containing the perspective center, hence this contains O and H. When this is folded down into the photo plane using the horizon line as a hinge, the point O falls at O', in coincidence with the isocenter I.

To prove this, we will let I be the isocenter in Figure 1, hence it is required to show that $HO=HI$. The angle HOV must be a right angle since V is on a plumb line through O. It is proven in elementary publications that the line from the perspective center to the isocenter bisects the angle between the axes of the oblique and equivalent vertical photos, making the angles COI and IOV equal in Figure 1. Let these angles be called m, and angle HOC be called n. $n+m+m=$a right angle, but in the triangle OCI the sum of the angles at O and I is also a right angle. This makes the angle at $I=n+m$. But the angle HOI is also equal to $n+m$. Hence $HO=HI$.

It is already well known by all photogrammetrists that all lines radial through the isocenter, as a, b, and c, will make true angles, and equal to those made by the corresponding lines on a map (a, b, and c, of Figure 2), these radial through the map location of the isocenter.

It is the object of the present paper to introduce the far more useful lines radial through the orthocenter, d, e, and f (Figure 1), which correspond to lines falling on the photo at d', e', and f', radial through V, and on the map to d, e, and f (Figure 2), radial through the camera plumb point, or "orthocenter" on the map. The usefulness of these lines arises from (1) the fact that they make true angles at the photo orthocenter (the corresponding photo lines, d', e', and f', do not make true angles), and (2) the simple relation between these two sets of lines (through O' and through V', which is that corresponding lines have a common meeting point on the horizon line.

For a long array of operations of practical usefulness involving the orthocenter, the reader is invited to see a longer paper of the writer's entitled "Field Use of Oblique Aerial Photographs," being currently published by the Geological Society of America. Unfortunately in this paper, the term "orthocenter" had not yet been invented, and the letter I (for isocenter) was used for this point on all diagrams. Many photogrammetrists, reading this paper, may mistakenly take the radial lines through this point as being those with which they are already familiar from photogrammetric literature, namely those radial through the "isocenter." I did point out the fundamental difference at one place,

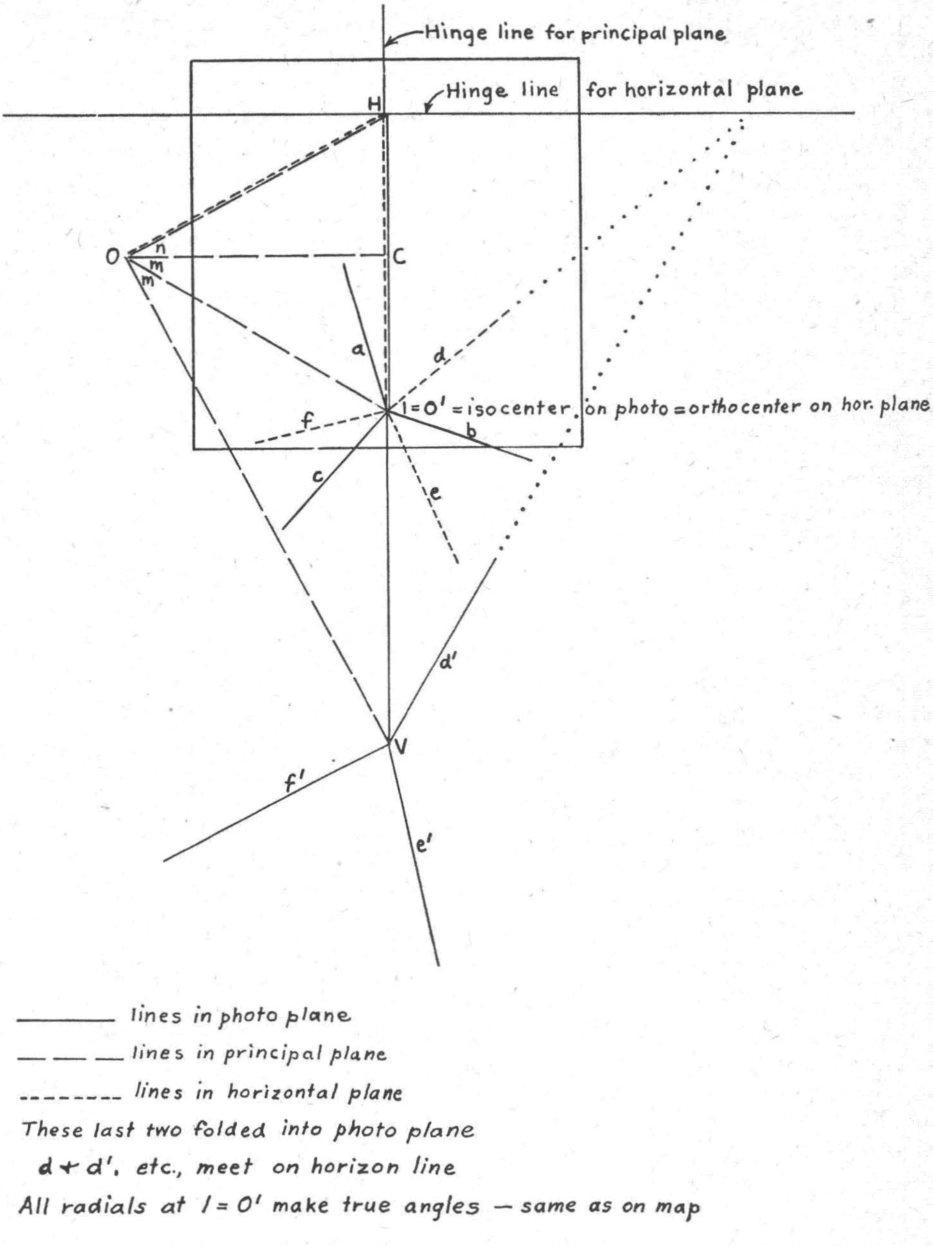

FIGURE 1

in this paper, however, for readers careful enough to catch it. But I wish now I had gone to even greater lengths to avoid this potential confusion. The radial lines at "*I*," so frequently recurring throughout this paper, and so extremely useful, actually represent a system of radial lines unfamiliar to photogrammetrists.[1]

[1] The orthocenter does *not* fall at the isocenter if the horizontal plane containing the radial lines is other than that containing the perspective center. An important use of such a case is found in plotting fully corrected radial lines on tilted vertical photos (ms. in preparation).

The orthocenter (not then named) as a convenient point for getting a simple graphical expression or solution of the "horizontal angle" of any photo point, was first proposed by the writer considerably over a year ago in correspondence with O. M. Miller, according to whom this solution was unknown at that time. In commenting on this in the other article, I forgot to mention

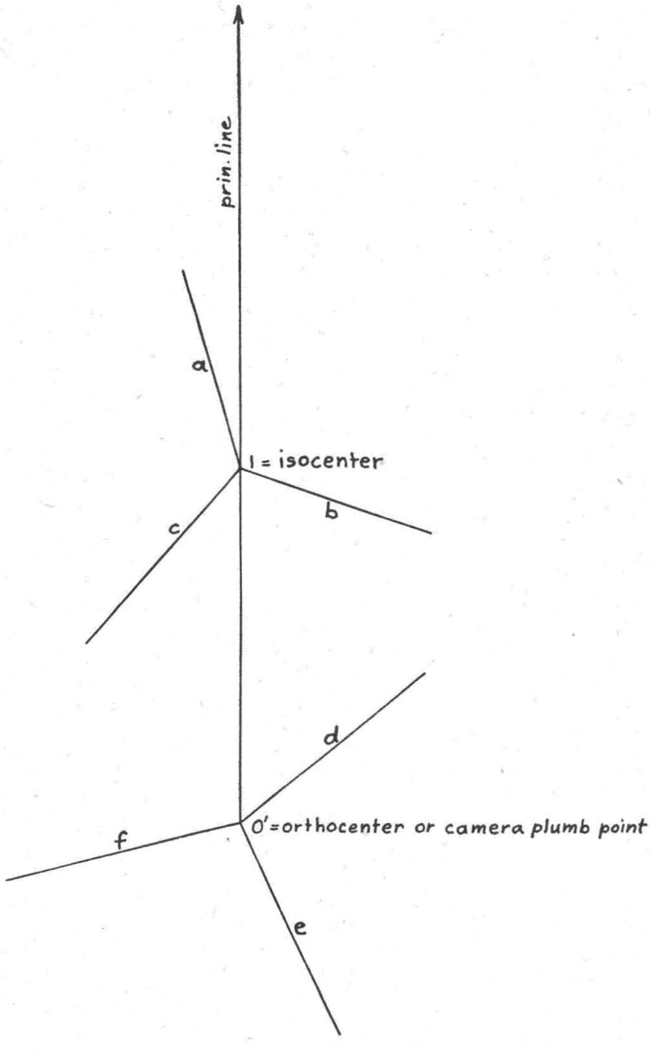

FIGURE 2

an observation of H. T. U. Smith, when first shown this solution. The first step of the solution (Figure 1) was to draw *CO* equal to the focal length, draw *HO*, and then lay off $HO' = HO$. Smith told me: "That's my method of locating the isocenter!" I had previously failed to realize that my O' was at the isocenter, and acknowledge that Smith first called this to my attention. However, at the time Smith had no ready proof, and I arrived at one myself (given above). The balance of the horizontal angle solution was to draw *OV* perpendicular to *HO*, and if any picture point is called *N*, draw *VN* extending to the horizon line from whence draw a line to O', the horizontal angle being made between the last line and the principal line.

The relations between a line on the photo not passing through I or V, and the orthocenter, are also important and useful. Such a line is AB in Figure 3. Extend AB to the horizon at T, and call this line T_1T. The several lines T_1T, T_2T, IT, VT, and T_3T, would be parallel lines on a map, since they converge

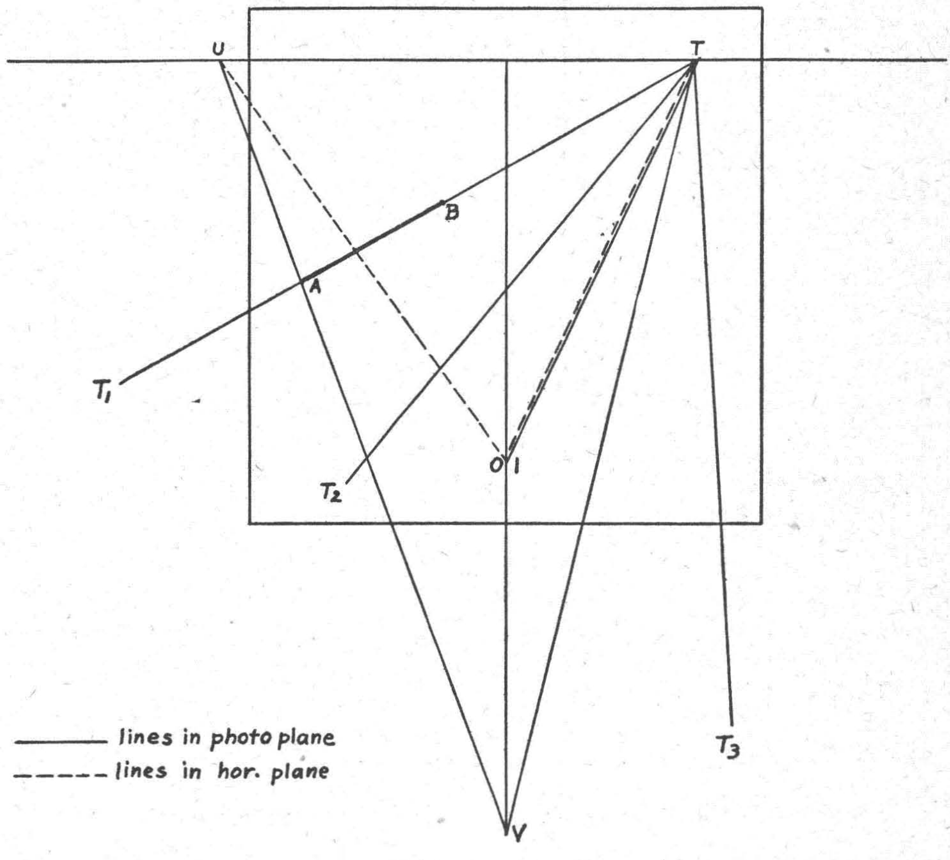

FIGURE 3

on a "vanishing point" on the horizon line. Of these, only IT would pass through the map location of the isocenter, and VT through the camera plumb point (map orthocenter). The line OT (dashed line) would be the same as VT, not the same as IT. OT is the orthographic projection of VT on a horizontal plane through the perspective center, folded into the photo plane. Of all the several lines meeting at T, VT is the only one lying in a vertical plane containing the perspective center, hence is the only one whose orthographic projection on the horizontal plane (or any other horizontal plane) is parallel to (lies on the same line as) its perspective projection on the same plane. The perspective projections of all lines through T on all horizontal planes will be lines parallel to OT when these planes are folded into the photo plane. As a special case, and actually the one here used, the horizontal projection plane contains the projection center (perspective center). The reader is invited to prove that in this case the perspective projections on this plane of all the lines meeting at T are actually the line *OT* in every instance. The reader is invited also to determine (construct) the orthographic projection of *AB* on this horizontal plane (containing the perspective center) folded on the photo, to demonstrate that this projection does not take a direction parallel to *OT*. This exercise should help emphasize the fundamental conception that the horizontal equivalents of all photo lines are their perspective projections, not orthographic projections.

The useful operations to be learned from all this are summarized:

(a) To get the horizontal bearing to any single point, *A* (from the camera plumb point), draw *VA*, extend to the horizon line at *U*, and draw *UO*.

(b) To get the horizontal bearing between any two points as *A* and *B* (or of any photo line as *AB*), extend *AB* to the horizon line at *T*, and draw *TO*.

The reader is invited also to watch for an early forthcoming article, probably in a Canadian publication, on the mathematical and geometric theory and proofs underlying a number of new and useful operations involving oblique photos, including the orthocenter in relation to other new concepts. This will be written by Mr. P. H. Blanchet, who has been the writer's assistant on recent Canadian work, and to whom the writer is indebted for many fundamental concepts regarding obliques.