RATIO AND APPROXIMATE TILT DETERMINATION FOR PHOTO MOSAICS*

THE Ratiograph is expected to be of vital interest to photo mapping units employing the radial triangulation method of establishing supplemental control. Use of this device eliminates laborious scaling of photo and map distances and the subsequent computations. Operation does not require photogrammetric experience or mathematical knowledge.

General Requirements for Controlled Mosaics

The accuracy of a mosaic may be considered as ranging from a general picture of the terrain involved to a near precise photo map, depending upon the intensity of control, and the amount of pyramiding in laying the component parts, which is in turn governed by the amount of laboratory work expended. It is assumed that resolution at the reproduction scale will be adequate and that uniformity of tone, in so far as possible, will be secured in the laboratory. In the precisely controlled mosaic it is expected that mismatches will not be evident.

Governing Limitations

Construction of controlled photomaps of general areas is usually handicapped by the impracticability of securing control in inaccessible areas and the fact that appropriate equipment may not be available. Further, the photography available may not be theoretically suitable to the work at hand. Since a precision mosaic contemplates converting the images of photographs of varying scale to an orthographic projection, long focal length vertical photography is ideal. Photography of enemy controlled territory is customarily secured for a wide variety of uses, and so, is seldom the type of photography that would be chosen for mosaic construction. Photomaps can be prepared from the so-called "split camera" photography or from short focal length wide angle cameras, but, in such cases, and when extremely rugged terrain is involved, loss of position accuracy must be expected.

This outline presupposes that rectification equipment is available and that supplemental control is to be secured by either the slotted templet or metal templet method of radial triangulation based upon identified ground control positions or selected map control.

The Ratio as Applied to Aerial Photos Used in Mosaics

Since the successive contact prints of any flight vary in scale with relation to each other due to change in flying height, and since the scale of any individual photograph varies throughout due to relief and mechanical or air tilt, the first problem is the determination of the average scale of the photograph in order that it may be converted to the common scale of the mosaic assembly by laboratory methods.

The term "ratio" as applied to mosaic photography indicates the relation of the average scale of the photograph to the desired, or assembly scale. Customarily, this is secured by determining the mathematical average of eight or more selected points on each exposure, having considered such ratios as may be unduly affected by excessive relief.

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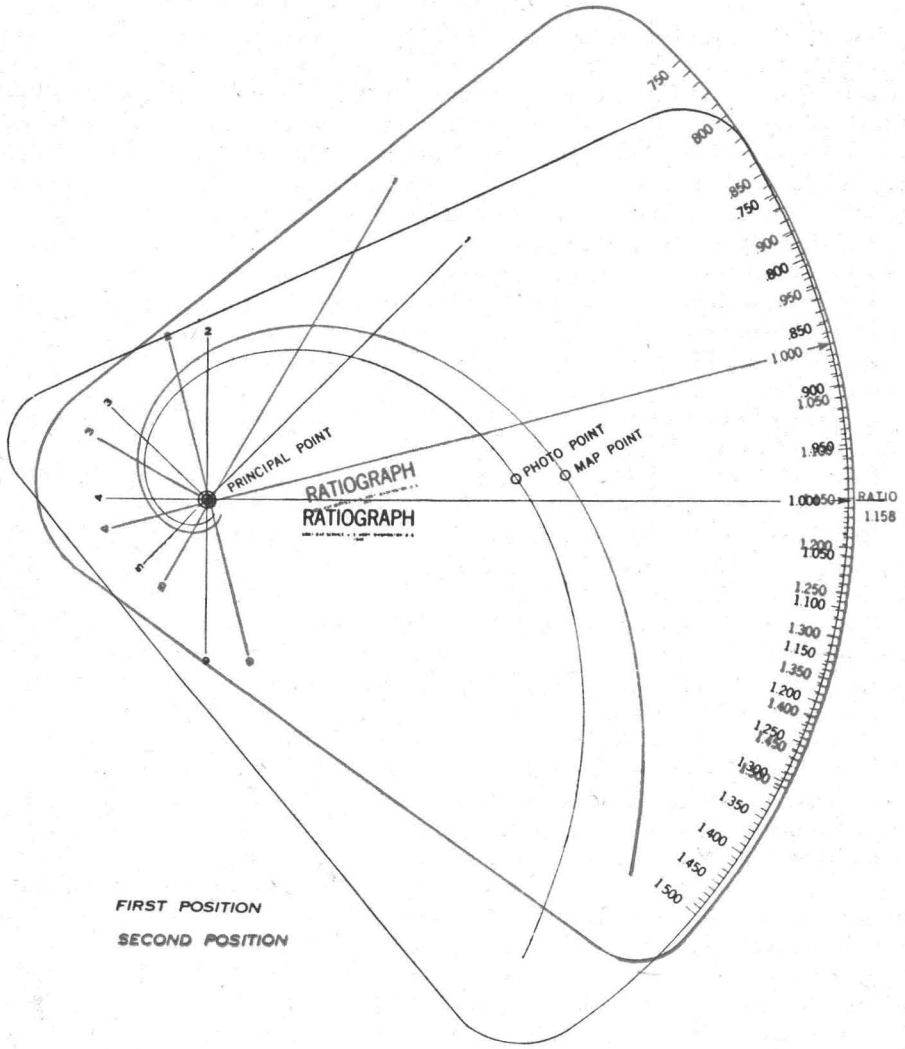

FIG. 1

Since the displacement due to relief occurs radially, the determination of the ratio at any point on the photograph becomes simply the distance from the photo center on the map base to the map position of the point involved, divided by the distances from the principal point of the photograph to the photo position of the same point. In other words, the map distance divided by the photo distance equals the ratio at any given point.

The usual method for determining the ratios of the radial control points on each photograph depends upon scaling the photo distance and the map base distance, dividing the map distance by the photo distance and recording the ratio at or near the point in question. Log templets, graphs, and various devices have been conceived to reduce the time involved in making these determinations. The Ratiograph recently developed provides an accurate, rapid means of determining point ratios on aerial photographs with relation to the same positions on the assembly base.

RATIOGRAPH

The Ratiograph is a logarithmic spiral plotted on any convenient scale in its correct relation to a calibrated arc of any convenient radius having as its center the principal point of the device, which, in turn, is referred to the principal point of the photograph and its corresponding position on the map base. (See illustrations.) The device is printed on a transparent medium of sufficient weight to insure rigidity and durability. To eliminate parallax the features of the device are printed in reverse on the under side of the transparent medium.

This device has been mathematically evolved, is theoretically correct, and due to the simplicity of operation, is of extremely high accuracy.

For the purpose of describing the operation, the various portions of the Ratiograph will be referred to by the following terms:

Ratio Scale: The calibration on the right hand edge of the device ranging from 1.00 down to .75 and from 1.00 up to 1.5.

Principal Point: The star near the left hand edge of the device which is also the center of the circle of which the Ratio Scale is an arc.

Principal Curve: The logarithmic spiral extending from a point near the Principal Point to the lower right edge of the device.

Reference Lines: The numbered lines radiating from the Principal Point which serve as Reference Lines to obviate the necessity of extreme rotation of the device in operation.

Customarily, the control established from the radial triangulation is traced on acetate to permit transfer from the templet assembly to the base to be used for the photomap. In this operation the photo centers are transferred also and the print center designated.

By this method, the device is used in the following manner:

Each individual print is successively laid on a table, face up. To prevent movement, scotch tape is fastened to the table with the adhesive side up which holds the print firmly enough for the period required. The acetate on which the control has been traced is laid over the picture in such a way as to bring the respective print center on the acetate in coincidence with the principal point of the print involved and so that the rays through the control points on the print pass through the same control points on the acetate overlay.

The Ratiograph is then laid over the print with the needle point through the star on the principal point of the photograph. The device is then rotated until the control point in question falls on the principal curve. While in this position, a light pencil mark is made at the arrow point or 1.00 on the ratio scale. By again rotating until the corresponding map control point on the acetate falls on the principal curve the ratio of the photo distance to the map distance may be read directly on the ratio scale at the point indicated by the pencil mark previously made (Figure 1).

The ratio of each of the remaining photo control points is similarly determined from which the average ratio of the print may be quickly computed. A simple form facilitates recording the respective ratios and permits determination of the average ratio at any time by an independent operator.

When it is desired to work from the photograph to the map base without use of the acetate overlay, the ratio of the photo distance to the arbitrary 1.00 on the device may be obtained and the ratio of the map distance to the photo distance may then be determined by placing the device on the map base, properly centered in such a manner that the desired point falls on the 1.00 line. Then, marking the point on the ratio scale as determined from the photograph, the device is rotated until the desired map point falls on the principal curve in which

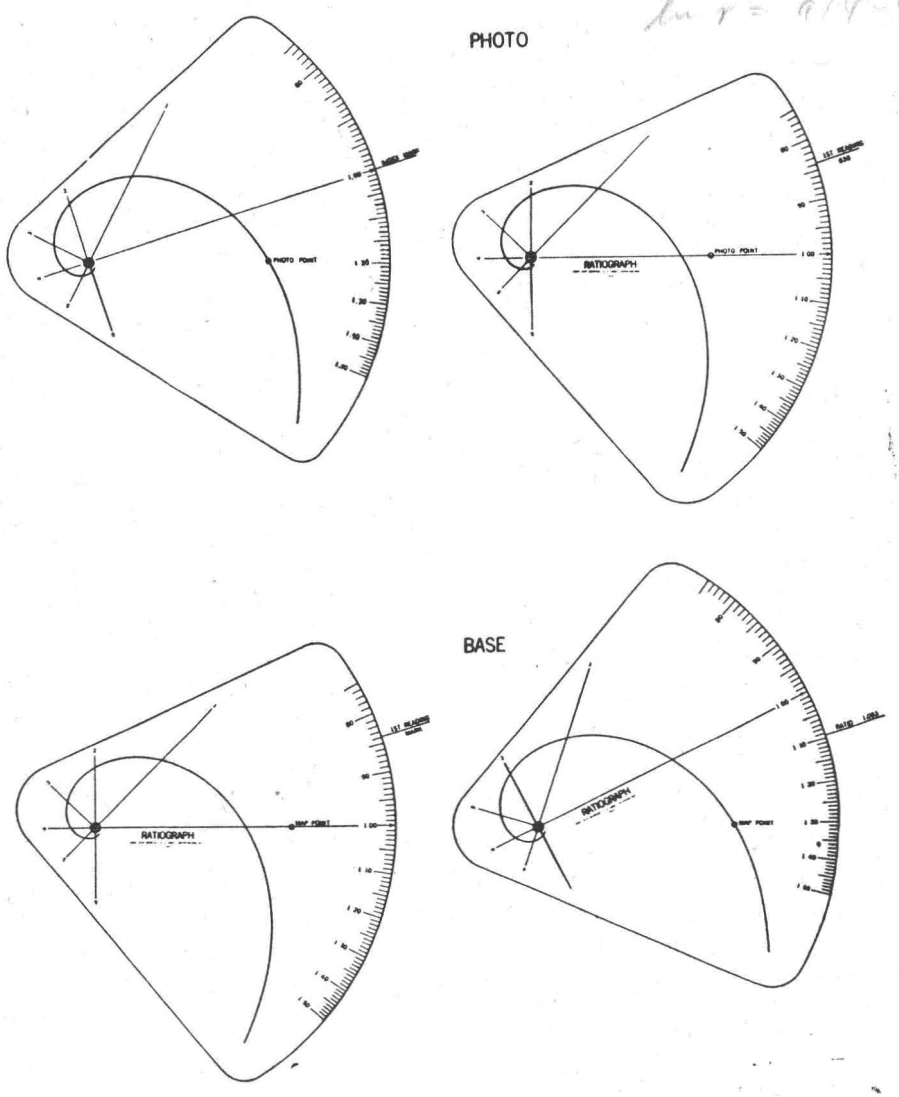

FIG. 2

position the mark at the ratio scale will indicate the ratio of the photograph to the map base (Figure 2).

The reference lines mentioned above are simple arbitrary lines for use in place of the 1.00 line as a reference point when the photo distances or map distances are so short as to otherwise require a considerable rotation of the device.

THE TEMPLET RATIO

The Ratiograph has been modified to permit determination of photo ratios to map base by using only the slotted templet.

By this method, each templet stud must be tacked in position at the completion of the radial triangulation. A small circular rubber stamp, on a shaft, which fits snugly over the templet stud, has been devised to permit stamping the posi-

tion of the templet stud with relation to the templet as established by the radial control net.

The device, instead of having one principal curve, for this method, contains four such principal curves. These curves are constructed from the same mathematical formula, using the various radii which depend on the length of the slot and the diameter of the stamp for marking the final position of the control point. Thus, the curves are equidistant apart when measured along a line radial to the principal point of the device.

It will be evident that when the device is superimposed on a templet, properly centered in such a position that the extremities of any given slot fall exactly between the two outer principal curves the photo control point is in the same position as though the actual photograph were used and would fall on the principal curve of the original device. Rotation of the device until the stamped circle falls precisely between the two inner principal curves establishes the position of the map control point at the same point where it would occur on the line of the principal curve if the original device were used. Accordingly, the ratio may be read directly by marking the 1.00 or arrow on the ratio scale at the first setting and reading the ratio direct at this mark after the device has been rotated to its second position (Figure 3).

Obviously this modified Ratiograph depends, in construction, upon the dimensions of the templet slotter and of the stamp to be used for marking the map positions of the control points. It is not appropriate for use with the metal templet spider but has very definite advantages when the slotted templet method is employed.

Analysis for Approximate Tilt

The average ratio, determined by dividing the sum of the ratios by the number of points involved, provides a fair approximation of the enlargement or reduction to be applied to the respective photographs to convert to the desired scale.

This average ratio may be adversely affected by excessive relief. Accordingly, it is necessary to observe the relative elevations of the various points in order that the ratios may be adjusted for relief to permit analysis of the approximate air tilt.

Relief displacement for the rather common domestic camera of $8\frac{1}{4}$ inch focal length amounts to .007 in ratio for each 100 feet above or below the datum plane of the photograph, which may be considered as the elevation at the principal point. The ratio adjustment for elevation is plus for increased elevation and minus for decreased elevation. The axis of tilt may be secured by a graphical determination of the line of common scale as indicated by the final point ratios corrected for relief affects. The application of the relief adjustment and the resulting axis of tilt is illustrated by the following typical example.

×Elev. 250	×Elev. 300	×Elev. 500
1.012 Initial Ratio	.998 Initial Ratio	.995 Initial Ratio
+0.010 Elev. Adj.	+ .007 Elev. Adj.	− .007 Elev. Adj.
1.022 Adjusted Ratio	1.005 Adjusted Ratio	.988 Adjusted Ratio

−2°00′ TILT
−0°30′ TIP

Principal Point
Elev. 400

×Elev. 350
1.023 Initial Ratio
+0.003 Elev. Adj.
1.026 Adjusted Ratio

AVERAGE RATIO
1.009

×Elev. 400
.991 Initial Ratio
.000 Elev. Adj.
.991 Adjusted Ratio

×Elev. 100
1.009 Initial Ratio
+0.021 Elev. Adj.
1.030 Adjusted Ratio

×Elev. 700
1.034 Initial Ratio
0.021 Elev. Adj.
1.013 Adjusted Ratio

×Elev. 800
1.024 Initial Ratio
−0.028 Elev. Adj.
.996 Adjusted Ratio

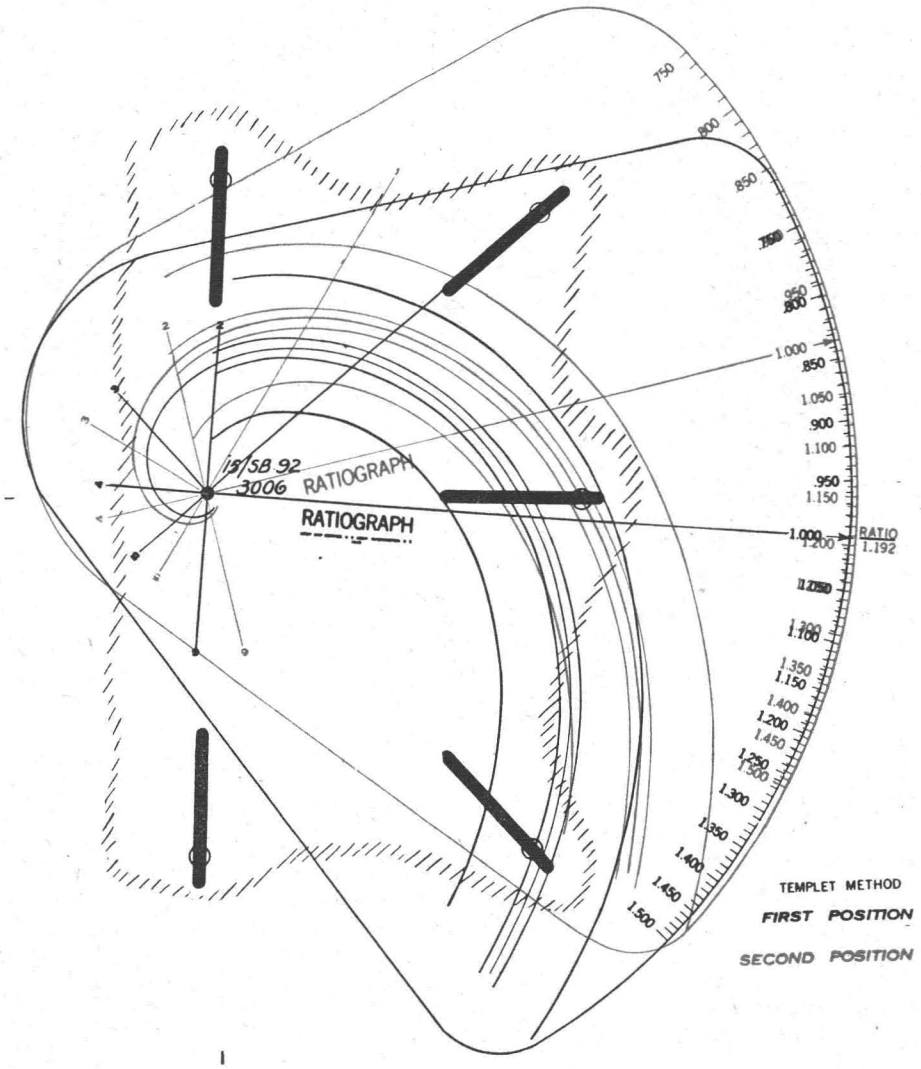

FIG. 3

The Circular Tilt Calculator

It will be evident that if the control points on a photograph are not symmetrically located with relation to the principal point, the ratios, adjusted for relief affects, cannot be expected to show directly the amount of air tilt without considering the distance between points.

To facilitate determination of the tip and tilt from the ratios at various locations a circular slide rule has been devised. This slide rule is based on the fact that one degree of tilt introduces a change of .017 in ratio across eight inches of an exposure made with a camera having a focal length of 8.25 inches. With this

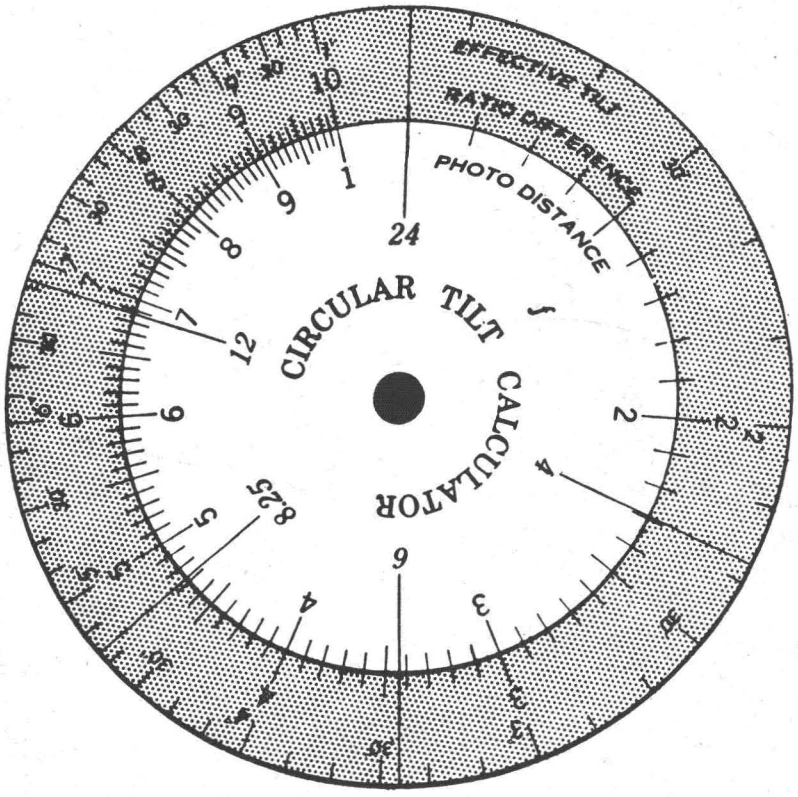

Fig. 4

instrument it is necessary only to scale the approximate distances between control points on the photograph, set the distances obtained on the inner gradation of the slide rule at the ratio difference, determined by subtracting the two ratios involved, and reading the tilt in degrees on the outer circle of the rule at the line for the focal length of the camera used in making the exposure (Figure 4).

The tip and tilt, in degrees, thus determined for each photograph, are customarily recorded in the upper right hand corner of the print to facilitate preparation of the requisition for laboratory work.

Rectification

Rectification data may be furnished to the laboratory in any one of several ways, dependent on the rectifying equipment available.

For example, some cameras are provided with mechanism for rotating the negative to the axis of tilt in which case the azimuth of the tilt axis must be furnished as well as the total tilt normal to the tilt axis instead of the component tilts in the two directions.

Not infrequently, it is customary to furnish the laboratory with the amount of tilt as measured laterally and the amount of tip as measured fore and aft. In this case, the tilt data are usually furnished for the upper and right hand edges of the print using plus or minus signs to indicate whether the sides are to be raised or depressed.

For rectifying equipment, which does not permit moving the negative in the plate holder to place the isocenter on the tilt axis adjustment of the average ratio may be made to overcome the discrepancies otherwise introduced.

Each projection camera used for rectification processes should be calibrated by trial method unless the exact focal length of the lens and the other principal distances are known.