NOTES ON PARALLAX AND STEREO-ELEVATIONS

By Louis Desjardins

THE present short paper will attempt to clarify the derivations and import of the writer's new parallax formula, appearing in his article on "Contouring and Elevation Measurement on Vertical Aerial Photographs," in PHOTOGRAMMETRIC ENGINEERING (v. IX, no. 4), p. 218.

The formula given is:

$$de(ft) = \frac{f(ft)\ S_r b_r(mm)\ dp(mm)}{b_s(mm)\ b_n(mm)}$$

in which de is the difference in elevation between two points on the ground, whose photo images are recognized on the stereo model, f is the focal length, $1{:}S_r$ is the scale of a radial or templet assembly, b_r is the distance between photo centers on this assembly, dp is the difference in parallax between the two points in question, and b_s and b_n are the values of the photo bases as corrected for the two elevations in question.

The advantage of this formula is that all its factors are exactly determinable, hence it permits precise parallax-elevation calibration, a feat not achieved by any use of standard parallax tables, or any standard formula for dp.

In its simplest form, the new formula becomes:

$$de = \frac{fBdp}{b_1 b_2}$$

in which b_1 and b_2 are photo bases referred to lower and upper limits respectively of the interval represented by de, and B is the camera base. The derivation is quite simple: $dp = b_1\ de/A_2$ (standard parallax formula) in which A_2 is the camera altitude above the upper limit. But $A_n/f = B/b_n$, in which A_n is the camera altitude above any given level and b_n the photo base for that level. Hence $A_2/f = B/b_2$ and $A_2 = f\ B/b_2$. Hence $dp = b_1\ de\ b_2/f\ B$, or $de = f\ B\ dp/b_1\ b_2$.

To understand the applicability of this formula, one must clarify the relationship between "parallax" and "photo base." These terms in their strictest meaning are for practical purposes identical and synonymous, and it would have been quite accurate to have used the letter p each time in the place of b. Parallax, as used in photogrammetry, may be defined as the photo intercept of the parallax angle. Fig. 1 illustrates the two camera stations M and N, camera base B, photo plane, and ground with two levels 1 and 2. In the lower of these levels is situated a point R, which is seen from the two stations in directions which make an angle of r (NS is parallel to MR). The true parallax angle is r', substituting the point R' lying in the same level with R but making the angle $R'MN$ a right angle. For practical purposes r may be considered the parallax angle, for, though not equal to r', its photo intercept p_r equals p_1 the intercept of r'. It has been proven in well-known publications that on truly vertical photos all ground points lying in the same level, no matter in what part of the stereo model, will have the same parallax intercept or parallax as we use the term. But p_1 and p_2 are actually b_1 and b_2 (photo bases). Hence the parallax or effective photo base for any point in the model is found by measuring the photo base on either of the photos, provided that this is corrected to the same elevation by adding or subtracting the parallax differential between the point in question and the transferred (not central) principal point end of the photo base.

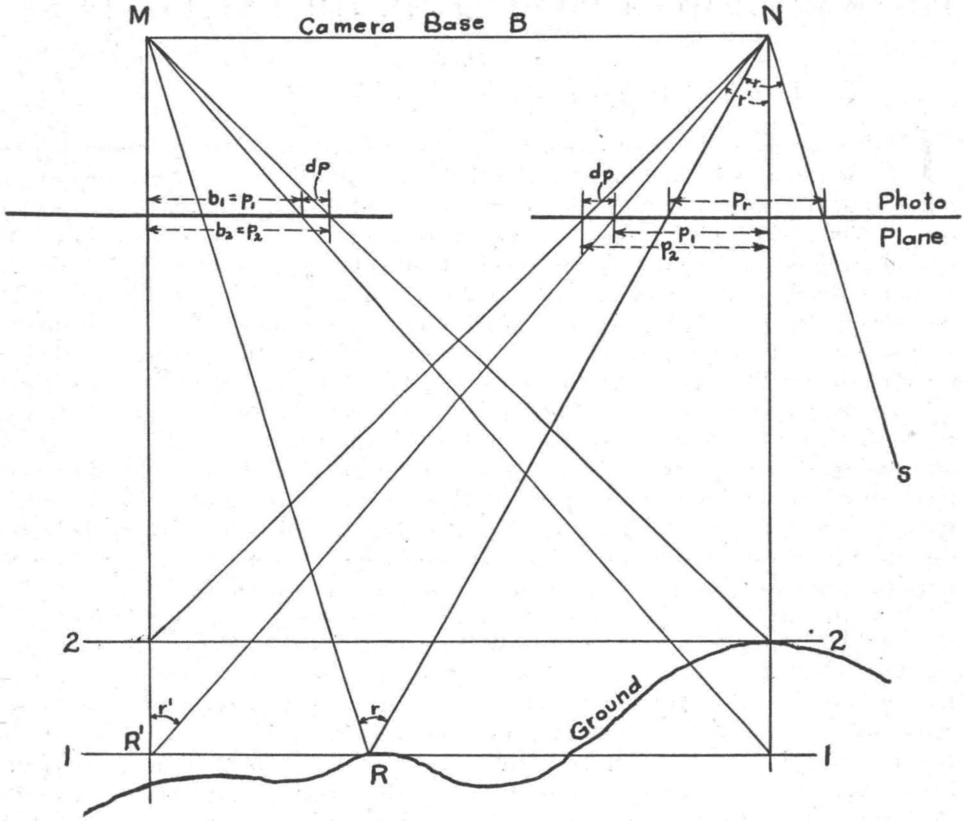

FIG. 1.

The key to the applicability of the formula, then, is the fact that $b_2=b_1+dp$ and dp is always measurable. Any single control point of known elevation, wherever located in the stereo model, will permit one to get correct values of b for all other points in the model. The precise manner that this works, as made into a routine in the contour methods described in the other paper, is seen by recasting the formula:

$$de_{cn} = \frac{fBdp_{cn}}{b_c b_n}$$

the subscripts referring: c to any control point of known elevation, and n to any point whose elevation is to be determined. dp_{cn} is measurable (by parallax scale or bar). b_c is gotten from: $b_c=b_p\pm dp_{pc}$, the subscript p referring to one of the principal points where the value of b_p is measurable, (dp_{pc} is also measurable). Lastly and in similar manner b_n is gotten from: $b_n=b_c\pm dp_{cn}$.

The above relationships incidentally disclose why a workable exact formula for dp, when de is known or given, is impossible, because when dp is unknown the b factors are incapable of determination. This stumbling block in one form or another is present whatever "dp" formula one attempts to use, and even though such a formula may be mathematically precise, one is still helpless when one cannot evaluate the factors. The same difficulty accounts for errors resulting from the use of standard parallax tables. The fact that $b_2-b_1=dp$ is the most clarifying idea one's mind can take hold of in this work.