THE SIGNIFICANCE OF THE CALIBRATED FOCAL LENGTH

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ABSTRACT

The equivalent focal length as defined by first order equations of geometrical optics represents a limiting value as the lens aperture and field of view approach zero. An equivalent focal length as visually measured at the National Bureau of Standards on an optical bench is measured to the plane of best axial definition. This value takes into account the finite aperture of the lens but applies only to the field in the immediate neighborhood of the axis. When a lens is tested photographically on the Bureau's precision lens testing camera the equivalent focal length is customarily measured to the plane of best average definition. For a lens mounted in a camera the calibrated focal length is measured. This focal length terminates at the picture plane of the camera and is computed to distribute the errors of distortion in an approximately uniform manner over the entire field. This is the value of most interest to the photogrammetrist. The corresponding reports issued by the National Bureau of Standards indicate the particular focal length that is measured.

IN CONNECTION with the testing of precision airplane cameras designed for the production of negatives required for precision topographic mapping by photogrammetric methods, the National Bureau of Standards has introduced the term "Calibrated Focal Length"[1] to apply to an adjusted value of the equivalent focal length which serves to distribute the inaccuracies resulting from distortion more or less uniformly over the entire negative, instead of concentrating them almost entirely upon the marginal portion. That the derivation and significance of this particular focal length is not fully understood is implied by definitions and explanations regarding it that have recently appeared in publications of the Society.[2] It therefore appears desirable to restate the considerations which lead to the establishment of the calibrated focal length.

For the photogrammetrist the definition of the focal length that is most directly applicable to his work is embodied in the equation

$$= \frac{y'}{\tan \alpha} \qquad (1)$$

where f is the equivalent focal length, y' is the distance in the image plane measure from the axial point to the image of an infinitely distant object, and α is the angle between the infinitely distant axial point and the respective object point. It is evident that this equation gives the metrical relation that is used by the photogrammetrist in transferring data from a photograph to a map.

When an attempt is made to determine by test the equivalent focal length of a lens, making use of the relation of equation 1, it immediately becomes apparent that the equation must be restricted by further limitations imposed upon the definitions of the different symbols if the equation is to define the equivalent focal length precisely. Photographic lenses are seldom free from distortion and, in the presence of distortion, the ratio $y'/\tan \alpha$ has different values for different values of α, that is for different parts of the field. Furthermore the selection of the picture plane in which y' is measured requires judgment. In the

[1] PHOTOGRAMMETRIC ENGINEERING 4, p. 173 (1938), J. Research N.B.S. 22, p. 209 (1939), RP1177.

[2] PHOTOGRAMMETRIC ENGINEERING 8, p. 259 (1942).

PHOTOGRAMMETRIC ENGINEERING 9, p. 36 (1943).

EDITORS NOTE: These two articles will be reprinted in the forthcoming *Manual of Photogrammetry.* Corrections have been made in them so as to agree with the present discussion.

neighborhood of the axis of the lens the imagery is of substantially equal sharpness over a considerable range of distance measured along the axis (particularly when tested by a photographic method) and furthermore a question may arise as to whether or not one should select the picture plane for maximum sharpness at the center or should use best average sharpness over the entire field as a criterion.

The lens designer is chiefly concerned with lenses as specified by radii, thicknesses, and indices and almost always selects the image plane containing the paraxial focal point as the picture plane to define the focal length. He further escapes the uncertainty arising from the presence of distortion by taking the limit in this plane of the ratio $y'/\tan\alpha$ as α approaches zero. This selection has the merit of leading to a unique value and also to the value that is most readily computed. It is the value that is defined by the first order equations of geometrical optics. The value defined in this manner almost always differs from a measured value determined for an actual lens because of the different aberrations and of the differing effects which, as has been mentioned, lead to uncertainty in the choice of picture plane.

Before the construction of the precision testing camera at the National Bureau of Standards, equivalent focal lengths were customarily measured visually on the optical bench. With an optical bench, when proceeding in the usual manner, one ordinarily selects the image plane which gives the best image on the axis and uses relatively small values of α in determining the setting of the nodal slide. This gives a value approximating that of geometrical optics although differences will in general arise because of the effect of diffraction and spherical aberration, these effects being excluded by the limiting equations selected in lens design. It has been the custom at the National Bureau of Standards to apply the term "Equivalent Focal Length," without qualifications, to a measured value obtained by this method.

The user of a photographic lens is customarily most interested in the plane of best average definition. This plane cannot be readily determined on the optical bench but is easily located photographically by means of a precision testing camera. Lenses to be used in an airplane camera for photogrammetric purposes, when not of excessive focal length, are tested on this camera and the equivalent focal length, if the lens is not mounted in a camera, is measured to the plane of best average definition, this fact being stated in the report. This method of measurement is designed to give an approximation to the limit of $y'/\tan\alpha$ as α approaches zero because this seems to be the most natural equivalent focal length to serve as a basis for the measurement of distortion.

The following tabulated values of distortion measured in millimeters illustrate the differences in the resulting values of distortion when referred to the equivalent and the calibrated focal lengths, both values of the focal length applying to the same picture plane.

Distortion referred to equivalent focal length.

0	5°	10°	15°	20°	25°	30°	35°
.00	.00	.00	.00	+.01	+.02	+.07	+.05

Value of equivalent focal length 209.31 mm.

Distortion referred to calibrated focal length.

0	5°	10°	15°	20°	25°	30°	35°
.00	−.01	−.01	−.02	−.02	−.02	+.02	−.02

Value of calibrated focal length 209.39 mm.

When a complete camera in focal adjustment is submitted to the Bureau for test the picture plane in which the measurements are made is that determined by the particular focal setting of the camera and the Bureau exercises no choice. As before the equivalent focal length will be computed to give a good approximation to the limit of $y'/\tan \alpha$ as α approaches zero. Consequently this gives the metrical relation of equation 1 applying to the center of the negative. The values of distortion referred to this focal length are given in the test report. However, photogrammetric apparatus is not commonly equipped with compensating means for eliminating the effect of distortion. Consequently, in the presence of distortion it is advantageous to have a different value of equivalent focal length which will correspond to an average value of $y'/\tan \alpha$ over the entire plane. This does not involve a change in the focusing of the airplane camera but only the use of a different value of the equivalent focal length which applies when setting the negative in the multiplex or other mapping apparatus or when evaluating distances by computational methods applied to linear measurements made on the negative. To the photogrammetrist this is the most important of the several equivalent focal lengths that have been discussed. Consequently it has been given a special name and is referred to as the "calibrated focal length." It is not customary to give a value of the calibrated focal length except when a complete camera is submitted for test. The maximum value of the distortion corresponding to the calibrated focal length will, in general, be considerably less than the corresponding distortion referred to the equivalent focal length applying to the axial portion of the picture.