SHORT-CUTS IN LONG-DISTANCE PHOTOGRAMMETRY*

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AERIAL surveys are based as far as possible on vertical photography with ample overlap. Sometimes an area to be mapped is so large and so far from a suitable base of supplies that complete vertical coverage is impossible. In such a situation the method of mapping from high oblique photographs described by O. M. Miller[1] affords fairly high precision with great economy of flying time and photography. More recently the trimetrogon system has been developed to cover large areas with a combination of vertical and oblique photography and it serves its purpose well.

Sometimes it happens that a map is urgently required of an unsurveyed area appearing only in the distance of one or more high oblique photographs, taken for the purpose of mapping nearer areas in the foreground. For example, the wing pictures of a tri-metrogon survey designed to cover a continental area including the principal islands off the coast, may reveal in the remote distance outlying islets and reefs too far from the camera to be accurately mapped by ordinary methods. An unexpected occasion for navigating these waters may render it important to know the approximate positions of these reefs and islets before it is possible to organize a new aerial survey. The question arises as to how far into the distance of a photograph we can place sea-level objects with a useful degree of approximation, and what the probable margin of error will be for a given distance viewed from a given altitude.

Recently in the Hydrographic Office, U.S.N., revision of an old chart of northern waters called for the use of a high oblique of a distant area. An unsurveyed group of islands appeared in the distance of a photograph taken from 10,000 feet with a 12-inch camera; the islands extended from 24 to 30 nautical miles from the position of the airplane, yet even from this distance it was evident that the old chart was grossly in error and could be materially improved by means of measurements on this photograph, relating the islands to a known area in the foreground of the picture. The best method of making such measurements would be with a photo-alidade or a photo-goniometer, but in absence of such instruments, simple methods were developed which were probably as accurate as the uncertainty of the precise tilt of the camera permitted. Since the area was, and still is, unsurveyed, there was no direct means of checking the accuracy of the results of the measurements. Therefore an experiment was made with other photographs showing a large group of islands including several accurately surveyed landmarks, well distributed at various distances from the camera. Two methods of estimating distance without instruments other than a pocket lens, ruler and slide-rule were compared with each other, and the estimated distances were compared with the known distances based on the survey. The results of this test are encouraging enough to warrant a brief report and discussion.

It is a familiar fact that when an object is so distant as to appear but a few degrees below the horizon, its distance is much less subject to precise measurement that its azimuth. Miller[2] has emphasized that at 5° below the horizonal an error of 1′ in the vertical angle introduces about 12 times the horizontal displacement (toward or away from the observer) that the same error in horizontal

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(1) Alexander Forbes: Northernmost Labrador Mapped from the Air, American Geographical Society Special Publication No. 22, New York, 1938.

(2) O. M. Miller: Photogrammetric Engineering, Vol. VIII, No. 1, p. 55, 1942.

angle will introduce in the transverse direction. The present communication will deal primarily with estimation of distance, since azimuth measurements under like conditions are inherently so much more precise that, relatively speaking, they present no problem.

The first method of measuring distance was based on the construction of a Canadian perspective grid to fit the focal length of the camera and the altitude and tilt of the test photograph. The second was based on measurement of the distance of the desired sea-level point below the apparent horizon as it is imaged

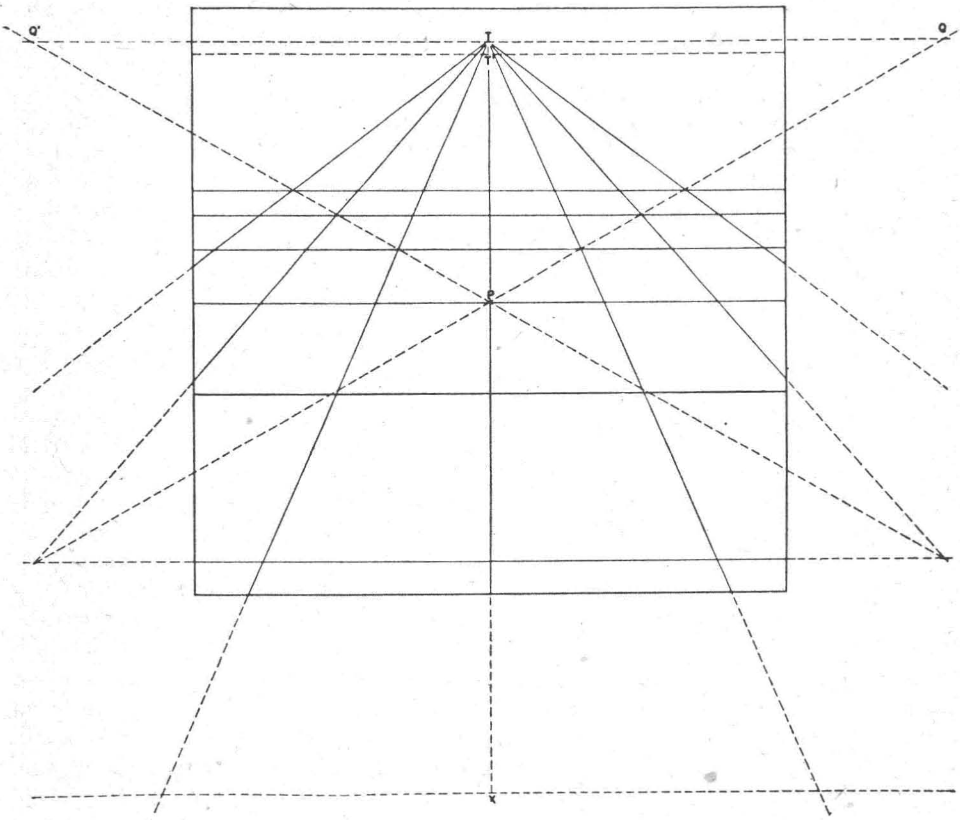

FIG. 1. Construction of grid for tilt of 60° from vertical.

in the photograph. In both methods the first step is to determine the distance from the nadir (plumb point) to the apparent position of the distant object on the plane tangent to the earth's surface at the nadir, which for brevity will be called the datum plane. The second step is to find the correction that must be added to allow for curvature of the earth and atmospheric refraction. Only points at sea level will be considered, and for the present the utility of the procedures will thus be limited to shore lines and other objects on the surface of the sea.

THE GRID EXTENSION METHOD

In the first method a perspective grid is prepared by the standard method for rectifying high obliques (Fig. 1), a process involving a few simple formulae (See Hotine,[3] Bagley[4]). The first step is to determine the tilt, for which a prereq-

(3) M. Hotine: Surveying from Air Photographs, Pp. 66–68.
(4) J. W. Bagley: Aero-photography and Aero-surveying, Pp. 247–248.

uisite is knowledge of the horizon dip. Let QTQ' represent the true horizon on the photograph and on the grid that is to be superimposed on it, and let T' be the intersection of the visible horizon trace with the principal line; let P be the principal point, f the principal distance (approximately the focal length) and t and t' the respective vertical angles from P to T and T'.

The dip is the vertical angle between true and apparent horizons, i.e. the difference between t and t'. It depends on the height of the air station (camera) and is found by the formula: dip (in seconds) $= 59\sqrt{H}$, where H is altitude in feet. Since $\tan t' = PT'/f$ and $PT/f = \tan t$, the procedure is to measure on the photograph the distance PT', divide by f, find the angle in a table of tangents, add the dip angle, find the tangent of the corrected angle, multiply it by f and place the true horizon at the resulting distance from P on the grid.

The true and apparent horizons are drawn as parallel lines at the top of the grid; the principal line is drawn perpendicular to these and the principal point, P, is marked thereon at the proper distance from T, as ascertained above. The principal line is continued down to a point, X, at a distance from T which serves to establish the size of the squares on the ground that are to be represented by the perspective grid. Let H = altitude in feet, and L = the side of a square on the ground in feet; let θ = tilt of the camera axis measured from the vertical (i.e. the complement of t).[5] Select a convenient unit, d, for spacing the grid lines on the construction base line. Measure $TX = (dH \operatorname{cosec} \theta)/L$. Through X draw the base line parallel to the horizon and lay off on it the points of intersection of the meridians of the grid spaced at the unit distance, d, right and left from the principal line. Draw the meridians from their vanishing point, T, on the true horizon to these measured intersections. They represent parallel lines on the ground.

The next step is to find the vanishing points, Q and Q', of the diagonals through the grid squares. For this purpose the distance $TQ = f \operatorname{cosec} \theta$ is laid off on the true horizon to right and left of T. The first two diagonals are drawn from Q and Q' through P. Intersections of these two with the meridians will establish the corners of squares, and through these corners lines parallel to the horizon are drawn to complete the quadrilaterals of the grid. The first few transverse lines can be drawn by means of the intersections provided by the first two diagonals. Additional diagonals from Q and Q' can then be drawn through other corners thus established, and new transverse lines can be added by means of intersection of these diagonals with the meridians. The grid is thus built up and its construction is carried into the upper (or more distant) part of the picture as far as is feasible, i.e., until the lines are so close together that small errors in draftsmanship cause large errors in grid transformation. The procedure is illustrated in Fig. 1.

The grid can be drawn on frosted acetate which is transparent enough to reveal nearly all details in the photograph when placed over it. Details are more clearly seen through a grid photographed on a film positive. For greater accuracy, the grid may be drawn two to four times the desired scale, then reduced by photography and put on a film positive for use over the photograph. The grid thus prepared is secured over a print of the photograph on non-warping paper. It is of cardinal importance that the line representing the apparent horizon, T', on the grid be matched as accurately as possible to the horizon imaged on the photograph. Point P on the photograph should lie exactly on the principal line of the grid.

(5) The word "tilt" is used in the sense established by authorities on photogrammetry, Hotine, Bagley and Sharp, to mean the angle between the camera axis and the vertical. This must be clearly distinguished from the meaning given in the War Department Technical Manual on Aerial Photography, 1941, "Motion around a horizontal axis parallel to the line of flight," which is a particular directional component of the total or true tilt.

If a number of photographs are used, all taken within 1° of the same tilt, a single grid drawn with the mean tilt of the series may be used without serious error, *provided* the horizon is accurately matched and not the principal point; specifically, a grid drawn for a tilt of 60° from the vertical applied to a photograph taken with a tilt of 59° or 61° from 10,000 feet altitude will introduce a distance error of about 300 meters at 20,000 meters from the plumb point, and 370 meters at 30,000 meters away.

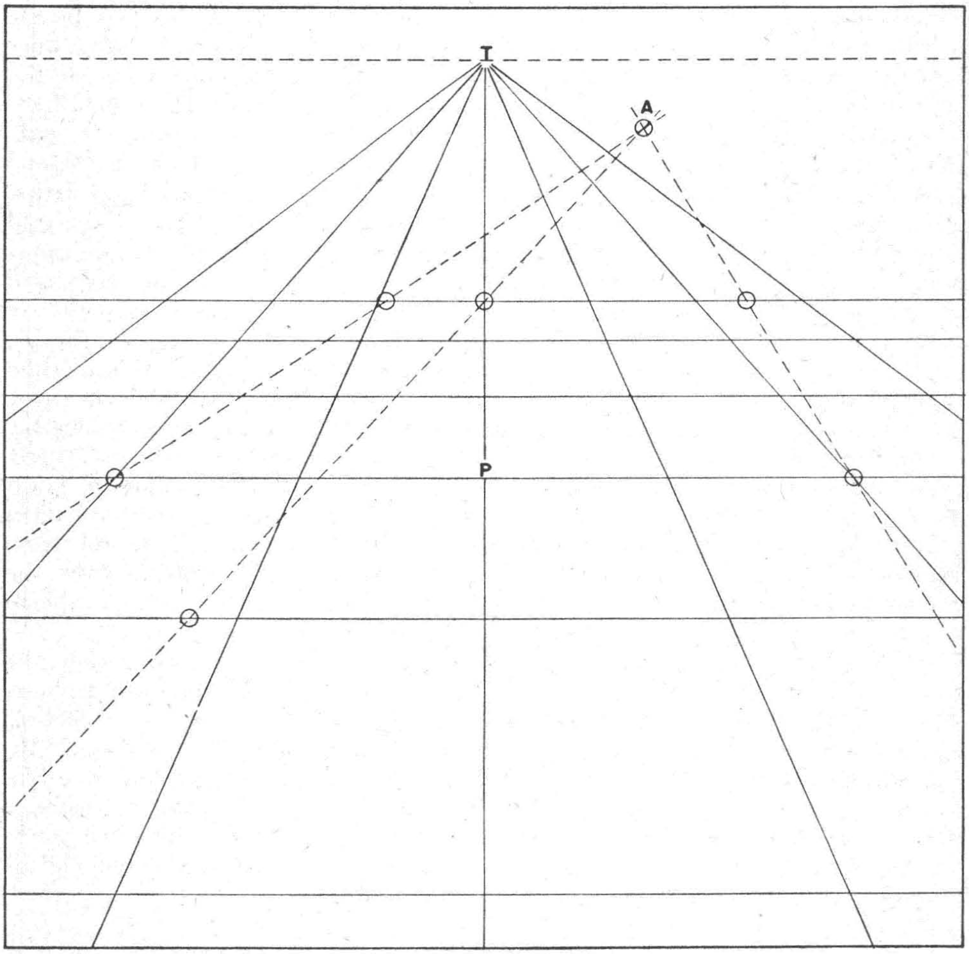

FIG. 2. Projection of a distant point (A) from coordinates on grid.

To make a rectified map from an oblique photograph, prepare a rectangular grid whose squares represent imaginary squares on the ground. The quardrilaterals on the grid, when superimposed on the photograph, outline the positions of the imaginary squares on the ground. A sea-level shore line is transferred from the photograph to the map by identifying corresponding points with the aid of the grid. The map scale is determined by the size of the squares on the rectangular grid used for drafting the map in relation to the size of the squares on the ground which the squares represent. For example, if $L=1000$ feet and the map is drawn on a rectangular grid with corresponding squares of 1 inch, the map scale will be 1:12,000.

We may now proceed to the special use of the grid for triangulating distant points. Let us call the remote point whose distance is required the "object point." When the grid has been properly secured over the photograph, the object point can be triangulated as follows: If the point lies near enough the principal line to permit direct measurement of its abscissa this is done by laying a straight edge (or better a line ruled on acetate) over the point and over the central vanishing point. The position of this line between the radiating grid lines at once gives the abscissa. If the connecting line lies outside of the grid, this direct measurement cannot be made. The position of the point is fixed with added control by placing three lines converging on the point; these may be the meridian measuring the abscissa and two others based on ranges, or three lines based on ranges. The range marks are points on lines which converge at the object. They should be selected as far apart as possible along the range lines. The converging lines should be so placed as to intersect at the "strongest" angle that is compatible with placing the range points far enough apart. Since a strong convergence entails using the lateral margins of the grid the desired separation of the range points can only be had by extending the construction of the grid laterally well beyond the limits of the photograph in the foreground. The range points are identified on a rectangular grid corresponding to the perspective grid and the apparent position of the object point is found by the intersection of three straight lines drawn through the three pairs of range points (see Fig. 2).

If the measurement of points is carried far into the distance, this method becomes increasingly inaccurate, for minute errors in draftsmanship will cause the three lines to cross in a triangle, and because the angle of convergence becomes increasingly weak with longer shots, the triangles become increasingly elongated, and the distance measurement correspondingly indeterminate. Actually, it has been found possible by careful drawing to make three lines meet practically at a point in the plot from a 6-inch focal length photograph from 10,000 feet, in the case of points as much as 25,000 meters distant from the camera. When greater distances than this are to be measured, it is preferable to use the second method, to be described below.

It should be emphasized that the procedure thus far described is only the first step, giving the apparent position of the object point on the datum plane. The distance of this apparent position from the plumb point will be called m; its complete determination involves relating N, the plumb point, to the grid by extending the intersection of the principal plane with the datum plane to a distance $=H \tan \theta$ from P. When both points are placed on the grid the distance between them can be scaled, thus completing the first stage of distance determination. Before describing the final step in the finding of true distance, let us consider the second method of performing the first stage.

The Depression Method

The second method of estimating the distance of remote points may be called the depression method. By means of precomputed curves it can be applied far more rapidly than the grid extension method, and for distances at which the range lines cross in elongated triangles, it is probably more accurate.

The depression method is based on plane trigonometry and consists in measuring the distance of the image of the object point below the apparent horizon on the picture, deriving from this the angle of depression, adding that to the dip of the horizon to get the true angle of depression, and multiplying the cotangent of that angle by the altitude of the air station.

The procedure is illustrated in Fig. 3, in which O is the air station (camera), N the nadir or plumb point, and A is the object point from which light rays follow a curved path to O, due to atmospheric refraction. The apparent position of the object point (a) is where the tangent to the curved path of light meets the datum plane. It is evident that in the triangle ONa, the side ON is the altitude, H, of the air station. If the line Oa is inclined at an angle ϕ below the horizontal, the distance Na is $H \cot \phi$. The line Oa, tangent at O to the curved path of the light rays, represents the apparent direction of the object point as imaged in the photograph.

When the object point is imaged close to the principal line on the photograph, the angle ϕ between the ray from O to a and the horizontal can be simply obtained by reference to f, the focal length of the camera. When the image of the object point is more than about 10% of the focal length to one side of the principal line, it is necessary to correct for the obliquity of the vertical plane containing the image and the perspective center to the principal plane of the

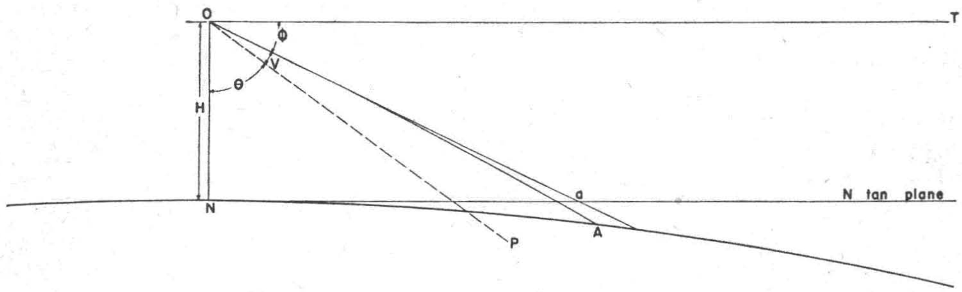

FIG. 3. Diagram to illustrate the principle of estimating distance from measured depression. O, air station; OT, true horizontal; N, nadir or plumb point; N tan plane, nadir tangent plane or datum plane; OP, optical axis; A, object point on earth's surface; a, apparent position on datum plane. θ, camera tilt from vertical; V, angle of apparent position (a) above principal point; ϕ, depression below true horizontal.

camera. To do this the tangent of the angle of lateral displacement from the principal line is found by dividing the measured lateral distance by f. The measured vertical distance of the image below the horizon trace is then multiplied by the cosine of that angle. The depression, thus reduced to its equivalent value on the principal plane, is then used in the rest of the operation. Tables I & II show how distance m is related to the variables involved, in the case of the 6-inch focal length.

The actual procedure is to measure the vertical distance of the object point image below the horizon trace on the photograph as accurately as possible. The lateral displacement (horizontal distance of the object from the principal line) is then measured, and since precision in this is not important, the measurement can be made very quickly. If the lateral displacement is less than 10% of the focal length, it can be ignored; if more than that, the angle of displacement should be found and the corrected depression derived as described in the preceding paragraph. The depression below the apparent horizon (thus corrected if necessary) is subtracted from the distance of the horizon above P, the principal point on the photograph; this distance divided by the focal length gives the tangent of the angle V, between the camera axis and the ray to point a. Angle V subtracted from $90° - \theta$ (camera tilt measured from horizontal) gives ϕ, the angle of depression of ray Oa below the true horizontal. The distance (m) of point a from the plumb point is then found by the formula $m = H \cot \phi$.

In this operation, we have arrived by a short-cut at the distance from the plumb point to the apparent position of the object point in the datum plane. The other dimension required to fix the position of the point must be found by

TABLE I. UNCORRECTED DISTANCE (m) IN METERS FOR DIFFERENT VALUES OF ALTITUDE, TILT AND DIP

H in feet	Tilt (θ) 55°					Tilt (θ) 60°				
	Depression in inches below horizon trace on photograph									
	0.3	0.5	1.0	1.5	2.0	0.3	0.5	1.0	1.5	2.0
5,000	27,400	19,040	10,500	7,053	5,189	25,655	17,595	9,628	6,463	4,764
10,000	47,430	34,160	19,680	13,420	9,970	44,770	31,950	18,160	12,395	9,200
15,000	64,310	47,550	28,125	19,440	14,540	61,080	44,620	26,060	17,980	13,440
20,000	79,280	60,220	36,040	25,165	18,905	75,770	56,205	33,545	23,340	17,540

TABLE II. UNCORRECTED DISTANCE FOR DIFFERENT VALUES OF TILT AND DEPRESSION AT 10,000 FEET ALTITUDE

Depression in inches	Tilt (θ) from vertical			
	55°	60°	65°	70°
0.30	47,430	44,770	42,610	41,015
0.40	39,840	37,340	35,310	33,870
0.50	34,160	31,950	30,145	28,840
0.60	29,910	27,830	26,210	24,910
0.80	23,790	22,060	20,720	19,780
1.00	19,680	18,160	17,040	16,290
1.20	16,640	15,370	14,420	13,770
1.50	13,420	12,395	11,640	11,130

azimuth measurement on the photograph or by alignment with known points in the foreground. This can be done easily with far greater precision than is possible in the distance measurement. The procedure is as follows: Let Z be the desired bearing, i.e. the horizontal angle between a and the principal plane; let x be the abscissa of the point a, i.e. its lateral displacement from the principle line on the photograph; let Y be the vertical angle from P to the point where a horizontal line through a crosses the principal line. Then $\tan Z = (x \cos Y)/(f \cos \phi)$. The grid extension method has the advantage of fixing both dimensions in one manoeuvre, but it requires careful draftsmanship and is a slower process.

CORRECTION FOR CURVATURE AND REFRACTION

Having determined the distance of the apparent position on the datum plane by either method, it remains to complete the operation of obtaining the true distance. This is done by adding the correction for curvature and refraction. For this correction Mr. O. M. Miller has furnished a "simple approximate formula":

$$\Delta M = \frac{(1 - 2j)m^3}{2RH},$$

where ΔM is the increment due to curvature and refraction, j is the coefficient of refraction, generally assumed to be 0.070, R is the radius of the earth (6,367,-000 meters assumed in this paper), m is the apparent distance of the object point from the plumb point, assuming it to lie in the datum plane (Na in Fig. 3), and H is the altitude of the air station (camera). The approximate true distance, M, is $m + \Delta M$.

The significance of this correction is illustrated in Fig. 3. In this diagram the height of the air station in relation to the earth's radius is greatly exaggerated, in order to separate the lines enough for recognition. The correction, ΔM, represents the difference between the distances Na and NA.

Precomputed Curves for the Depression Method

The use of the depression method can be greatly expedited by means of precomputed tables and curves. The first set of curves show the camera tilt plotted against the distance of the apparent horizon below the top of the picture in the principal line. The curves must be drawn for the type of camera used (focal-length and angular field of view), and for the altitude of the air station. Curves drawn for the 6-inch focal-length camera (9×9 inch photograph) are shown in Fig. 4. These curves may be used for the initial step in either the grid extension or the depression method, for in either case the first thing to find out is the tilt.

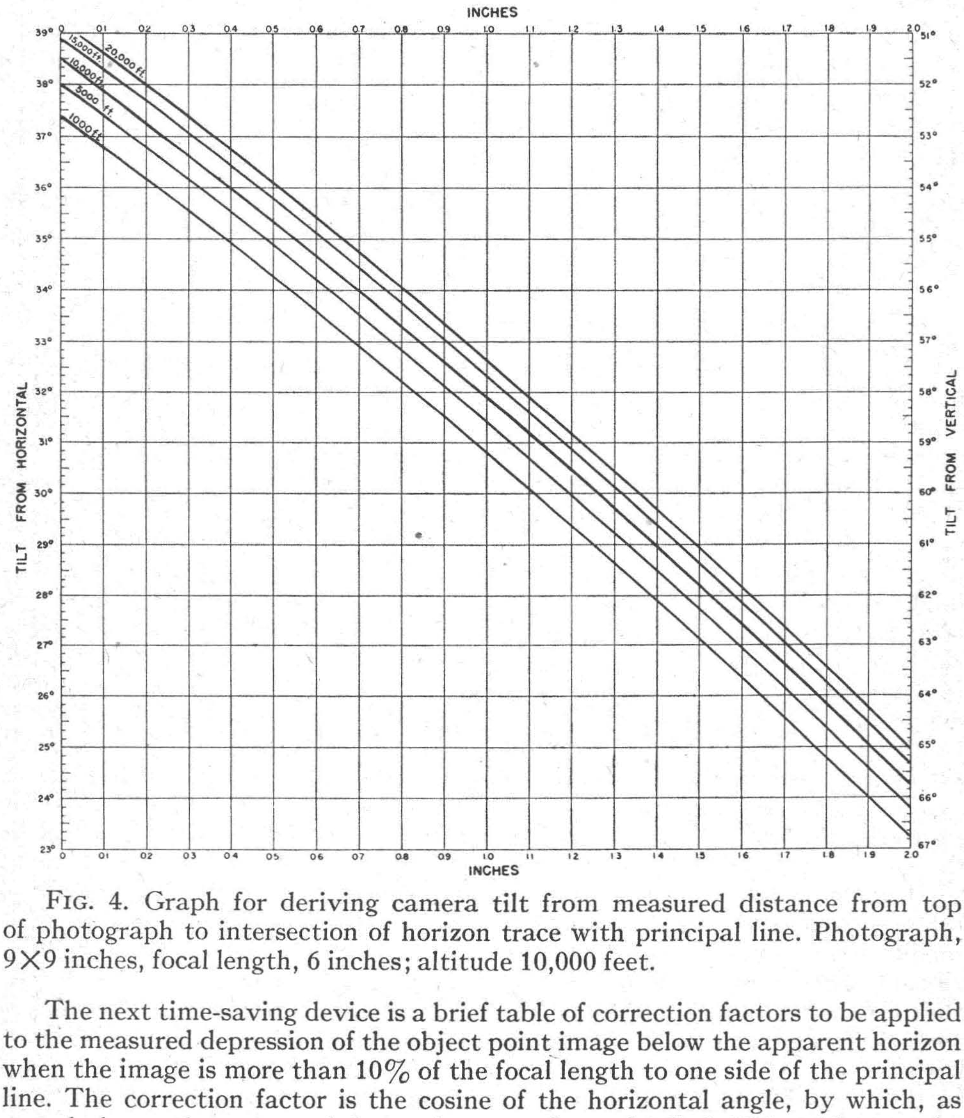

Fig. 4. Graph for deriving camera tilt from measured distance from top of photograph to intersection of horizon trace with principal line. Photograph, 9×9 inches, focal length, 6 inches; altitude 10,000 feet.

The next time-saving device is a brief table of correction factors to be applied to the measured depression of the object point image below the apparent horizon when the image is more than 10% of the focal length to one side of the principal line. The correction factor is the cosine of the horizontal angle, by which, as stated above, the measured depression must be multiplied. This table must be prepared for the focal length employed. Table III has been prepared for $f=6.0$ inches. It is entered with the lateral displacement in inches, and the corresponding cosine of the angle of lateral displacement is directly read off and applied with a slide rule to the measured depression of the object point below the horizon trace.

TABLE III. LATERAL DISPLACEMENT

Inches	Factor for depression
0.6	.995
0.8	.99
1.0	.99
1.1	.98
1.2	.98
1.3	.98
1.4	.97
1.5	.97
1.6	.97
1.7	.96
1.8	.96
1.9	.95
2.0	.95
2.2	.94
2.4	.93
2.6	.92
2.8	.91
3.0	.89
3.2	.88
3.5	.86
4.0	.83

Next comes a family of curves, each giving the distance m (Na in Fig. 3) in terms of measured depression for one value of camera tilt. It is a simple matter to plot a curve for every 5° of tilt over the range of tilts employed, and not very arduous to plot one for every degree in the range most commonly used. Fig. 5A shows four curves for the 6-inch focal-length camera (9×9 inch picture) at 10,000 feet for the following tilts (measured from vertical) 55° 60° 65° 70°. These values embrace the tilts employed in high oblique photography when properly done; the best photographs for mapping are those between 55° and 60°. If a large majority of photographs are taken with tilts in this range and a curve is plotted for every degree from 55° to 60°, then when the tilt is determined (for example at 57° 40′), it is easy by interpolation between the adjacent curves for 57° and 58° to read the value of the desired distance (m) corresponding to the actual tilt and the measured depression. If greater precision is required, a curve can be drawn for the exact value of tilt. To facilitate the drawing of such a curve, the necessary points can be found on another group of curves in which distance (m) is plotted against tilt (θ), and each curve represents a different value of measured depression (object point below horizon). Such a family of curves, drawn to supplement those in Fig. 5A, is shown in Fig. 5B.

The curves shown in Fig. 5 are only valid for the 10,000 foot altitude; for other altitudes other similar curves are required. Differences in altitude require modification for two reasons: one is the change in scale, which is directly proportional to altitude; the other is the change in horizon dip, which introduces errors that are much smaller but whose correction involves trigonometric computation. With a 6-inch focal length at a tilt of 60°, the horizon dip, as measured on the photograph, changes approximately 0.01 inch for every 1000 feet of altitude in the range between 10,000 and 20,000 feet. Taking 0.005 inch as the least measurable distance on the picture, we conclude that for variations of altitude up to 500 feet, it is neither necessary nor feasible to make corrections for change in horizon dip. But a change of 500 feet at 10,000 will cause a 5% change of scale, which is far from negligible. In practice, for the best results attainable with

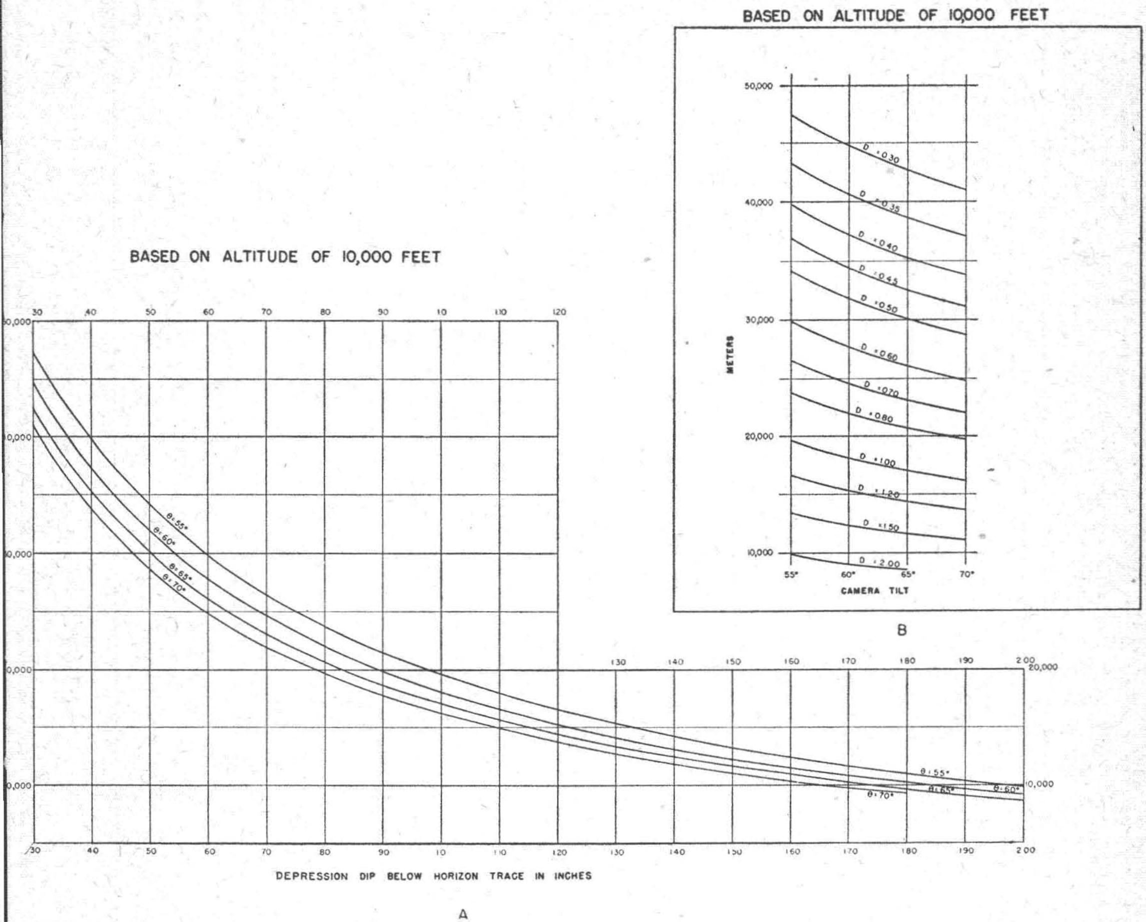

FIG. 5. Graphs for deriving uncorrected distance in meters (apparent distance on nadir tangent plane) from dip (measured distance below horizon trace.) Based on 6-inch focal length and 10,000 foot altitude.

this method, there should be a set of curves for every thousand feet over the range of altitudes employed, unless it is possible to arrange flight plans adhering closely to specified altitudes such as 10,000 or 15,000 feet. A curve for the nearest thousand feet may be used and corrections for small deviations can be very simply made by changing the value of m, derived from the curve, in the ratio of the actual altitude to the altitude for which the curve is drawn.

The curve for rapid finding of the correction for curvature and refraction is shown in Fig. 6. Since the correction is independent of focal length or camera tilt, and depends solely on altitude and apparent distance, m (assuming that we may neglect the small and unknown effect of variation in refraction), and since for a given value of m the correction is a linear function of H (altitude), a single curve will suffice for actual use. The curve is most readily used for different altitudes by having different altitude scales at the margin. For intermediate values, a simple factor may be applied.

An example will help to clarify the procedure. In a 6-inch metrogon (9×9 inch) photograph, from 10,000 feet, a sharp sea horizon is found to be 0.82

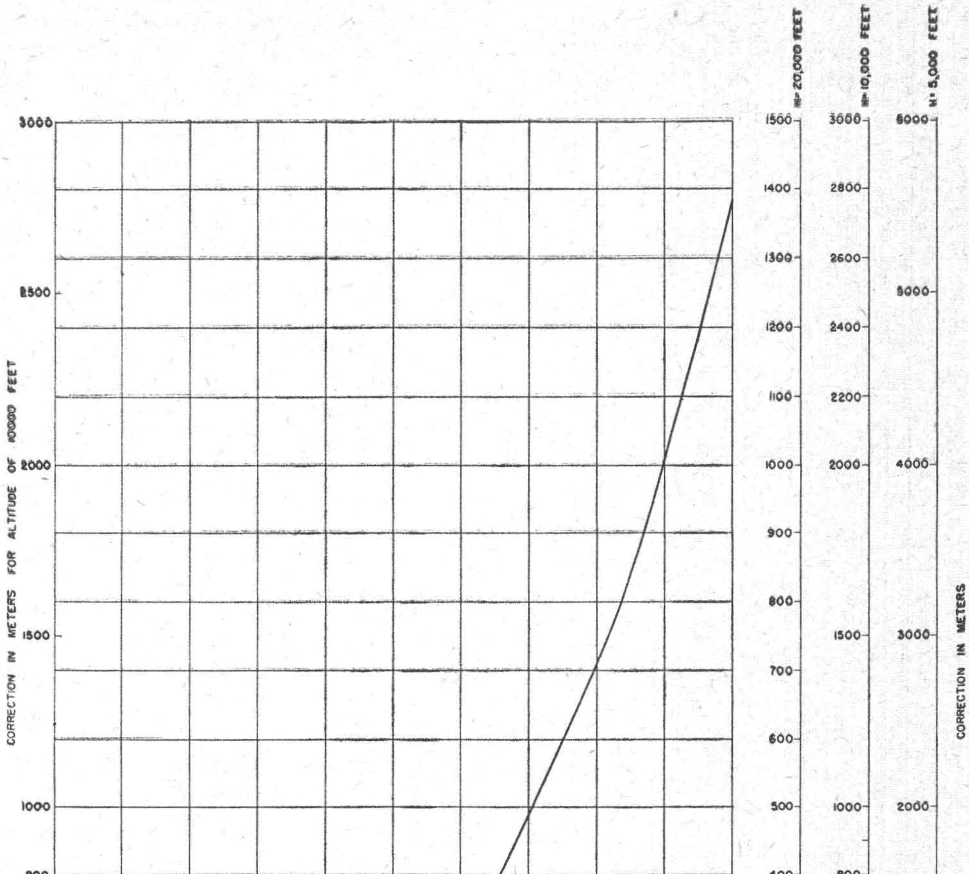

FIG. 6. Graph of correction for curvature and refraction to be added to apparent distance to find true distance from plumb point. (Use marginal scale for altitude of air station. For intermediate altitudes multiply correction by a factor based on the law that correction varies inversely with altitude.

inch below the top of the picture in the principal plane. From the curve in Fig. 4, it is found that the tilt is 56° 51′ from vertical. If many points on this photograph are to be measured, a curve should then be drawn parallel to those in Fig. 5A, connecting the points where the curves in Fig. 5B cross the meridian representing 56° 51′. If only a few points are desired and a hasty approximation will suffice, the result may be had directly from the nomograph in Fig. 5A by interpolation. Suppose the point to be measured is 1.8 inches to the right of the principal line and 0.75 inch below the horizon, from Table III it is found that the depression 0.75 must be multiplied by the factor 0.958, giving a corrected depression of 0.718. A vertical straight edge at the abscissa 0.718 in Fig. 5A reaches a point 37% of the way from the curve for 55° to that for 60° (representing the intermediate value for 50° 51′) at a level whose ordinate is shown in the margin to represent 25,100 meters. This is the value of m (uncorrected distance). The curve in Fig. 6 is then entered from the bottom with the value of m 25,100, as abscissa, and by means of the scale at the side, representing the altitude of 10,000 feet, the value of the correction ΔM for curvature and refraction is found to be 355 meters. This, added to m gives the true distance, M, as 25,455 meters.

Results of Test

The precision attainable by the methods described has been tested by careful measurement of 30 widely distributed points on four high oblique photographs

Table IV

Angle from N	Distance	Error	At 10,000 Feet Distance		Error		Percentage Error
			Feet	Meters	Feet	Meters	
50°	1.19H	.0009H	11,900	3,630	9	3	0.08
60°	1.732H	.0019H	17,320	5,280	19	6	0.11
65°	2.144H	.0030H	21,440	6,540	30	9	0.14
70°	2.747H	.0048H	27,470	8,380	48	16	0.17
75°	3.732H	.0089H	37,320	12,230	89	29	0.24
80°	5.671H	.0205H	56,710	18,600	205	62	0.36
85°	11.430H	0.835H	114,300	34,800	835	274	0.73
86°	14.30H	.131H	143,000	43,600	1,310	399	0.92
87°	19,08H	.233H	190,800	58,200	2,330	709	1.22
88°	28.64H	.525H	286,360	87,400	5,250	1,600	1.83

taken from approximately 10,000 feet with a 6-inch metrogon camera. Two of the pictures showed well defined horizons of level and presumably low land; the other two offered sharply defined sea horizons. On one photograph, 13 points were measured by both methods, grid extension and depression. On the other three photographs, 17 points were measured by the depression method alone. The area shown in these photographs is partly surveyed, most of it being covered by a precise triangulation supplemented by overlapping vertical photographs. By making a preliminary mosaic from these vertical photographs, approximately controlled by reference to the triangulation, the points used in this test have been placed in relation to one another with a probable error of well under 150 meters in almost all cases; where extension by uncontrolled mosaic has extended several miles beyond the triangulated control, the uncertainty is greater.

It must be understood that a limit to the possible precision of long-distance photogrammetry is imposed by the degree of precision with which tilt can be determined. When tilt determination depends on a visible horizon the limit is set by the precision with which the horizon can be defined. Even the most clearly defined horizon can hardly be placed in relation to other features of the photograph more accurately than within 0.005 inch. It is proper to assume that at best there is a probable error of this amount; with an ill-defined horizon it may be greater.

TABLE V

Photograph	Tilt	Depression Inches	Abscissa	Corrected Depression	Actual Distance meters	Distance by Depression	Error %	Distance by Grid	Error %
I	57°04′	1.25	0	1.25	15,550	15,580	+0.2	15,470	−0.6
		1.03	3.0	0.92	19,730	20,530	+4.0	20,000	+1.4
		0.94	1.8	0.90	20,420	20,945	+2.5	20,400	−0.1
		0.82	2.3	0.77	23,550	24,045	+2.1	23,700	+0.6
		0.70	0.7	0.70	26,050	26,075	+0.1	25,700	−1.3
		0.665	0	0.665	26,380	27,120	+2.8	26,140	−0.9
		0.615	3.1	0.545	31,080	31,660	+1.9	31,750	+2.2
		0.54	0.6	0.54	31,550	31,870	+1.0	31,100	−1.4
		0.42	3.05	0.374	41,350	41,980	+1.5	41,345	−0.01
		0.36	1.6	0.35	42,400	43,765	+3.2	41,590	−1.9
		0.35	0.3	0.35	43,100	43,815	+1.65	43,765	+1.5
		0.30	2.7	0.274	50,900	51,040	+0.2	50,560	−0.67
		0.17	2.7	0.155	71,300	69,870	−2.0	69,430	−2.6
II	56°55′	1.005	0	1.005	19,200	19,150	−0.25		
		0.68	0.1	0.68	26,140	26,810	+2.6		
		0.62	1.0	0.61	28,960	29,225	+2.5		
		0.31	1.1	0.305	48,450	48,270	−0.4		
III	56°45′	1.95	0	1.95	10,100	9,820	−2.8		
		1.13	2.8	1.02	18,900	19,000	+0.5		
		0.97	0	0.97	20,120	20,135	+0.1		
		0.99	2.3	0.924	20,800	20,590	−1.0		
		0.75	0.8	0.743	24,990	24,935	−0.2		
		0.75	1.8	0.72	25,540	25,455	−0.3		
		0.76	2.9	0.684	27,000	27,020	+0.1		
		0.62	0	0.62	29,400	29,440	+0.14		
		0.51	0.1	0.51	33,820	33,690	−0.4		
		0.44	0.3	0.44	37,800	37,570	−0.6		
		0.35	0.5	0.35	44,900	44,100	−1.6		
IV	56°24′	1.01	2.4	0.933	20,120	20,480	+1.8		
		0.885	2.1	0.832	22,100	22,750	+3.1		

The effect on distance measurements of this assumed minimum probable error (±0.005 inch) in placing the horizon, at various distances from the plumb point is shown in Table IV. It is based on the assumption of a photograph with 6-inch focal length at a tilt of 60° from the vertical. The first column gives angle of measured object from the vertical; the second, the distance from the plumb point in terms of H (altitude of air station); the third, the error resulting from a 0.005 inch error in placing the horizon at each distance, also in terms of H. The remaining columns show the actual corresponding figures in both feet and meters for the particular case of a photograph from 10,000 feet altitude. The table deals only with the uncorrected distance (m) at which a straight ray meets the datum plane. It will be seen that for points nearer than the principal point the probable error is negligible, but that for points within 1° of the apparent horizon, the error will be of the order of a mile.

The results obtained in the measure of actual points in the four test photographs are shown in Table V. In the case of each photograph, the test points are listed in the order of their actual distances from the plumb point. Photographs I and IV were those with land horizon; II and II each had a sharp sea horizon extending across approximately the middle third of the picture. The tilt from vertical is recorded for each photograph. In the column designated "depression" are recorded the vertical distances of the object points below the horizon trace. The abscissa is the horizontal distance from the principal line. The corrected depression is derived by means of the factor in Table III (cosine of horizontal angle). The actual distance in meters is measured from the preliminary chart of the area. The distances calculated by depression and by grid extension are compared with the true values, and in each case the percentage error is listed in an adjacent column.

The greatest error is 4% in the case of the second point listed, as measured by depression. This point was 8 miles from the nearest triangulated control station and related to the latter by uncontrolled mosaic drawn from several vertical photographs. Measurements to this point on a single clear oblique are probably more reliable than the mosaic, and since both depression and grid measurements agree in giving a greater distance than that derived from the mosaic, and since the difference between them is only 2.55%, we may reasonably discard the value 19,730 and assume the distance measured on the grid extension to be more nearly correct. In only two other cases does the discrepancy exceed 3%. Omitting the discarded case from the series, as being inadequately controlled, and considering the remaining 29 measured points, we find the apparent errors in measurements by the depression method are grouped as follows:

0 —1.0%	13
1.0—2.0%	7
2.0—3.0%	7
3.0—4.0%	2

In short, a clear majority show discrepancies of less than 2%. The average discrepancy is 0.96%. The average discrepancy in the case of the grid extension method in the 12 points so plotted, amounts to 1.16%.

One would expect the percentage error to increase with increasing distance. Such a trend, if it exists at all, is scarcely perceptible in this series, until a distance of over 70,000 meters is reached, when the error by both methods is 2% or more.

It is noteworthy that the least probable error, due to uncertainty of tilt (Table IV) even assuming no other source of error, reaches 1% between 86° and 87° from the vertical, at which angle the distance m is about 50,000 meters. The error in actual measurements amounts to about 1% within the range of distances in which the inherent error is less than that amount.

The conclusion to be drawn from these figures is that in situations in which an estimate of distance, reliable to within 2%, is useful and in which no better source of information is available, the methods described here can be of practical value.