PARALLAX WEDGE MEASURING DEVICES*

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Introduction

THERE is a definite need for an inexpensive device that permits rapid and precise measuring of parallax. This need is greatest in the field of military photo-interpretation for the measurement of heights of buildings, aerials, other objects of interest, and spot elevations; and in the field of forestry for the measurement of tree heights and approximate slope gradients.

The parallax wedges described below provide a very inexpensive, rapid, and accurate means of measuring differences in parallax. These wedges consist of two rows of dots, which when fused stereoscopically and superimposed over a stereoscopic aerial view, appear as a single row of dots slanting downwards through the stereoscopic model. The dots provide a sequence of reference marks, of fixed vertical intervals, which may be shifted about in the model, and against which objects may be compared in elevation. For instance, the height of a tree would be determined by noting the difference in reading between the image dot that lay on the ground at the base of the tree and the image dot that was level with the top of the tree.

The parallax wedges are neither more nor less accurate than the conventional parallax bar. They are not suitable for contouring maps, but rather for determining relative spot elevations and heights. They are very simple and inexpensive. Because they are far easier to use than the parallax bar, a relatively inexperienced observer can generally measure parallax more accurately with them than with the bar.

Vectograph Height Scaler

The original form of the parallax wedge measuring device was developed by R. T. Kriebel of Polaroid Corporation for use with Vectographs in tree height measurement trials conducted by the Harvard Forest. Vectographs are three dimensional photographs prepared in the form of a single print by the Polaroid process. The height scaler consists of two rows of dots (Figure 1), one row polarized in one plane, and the other row polarized in a plane at right angles to the first. When superimposed upon a Vectograph and viewed through three-dimensional viewers (glasses with the two lenses polarized at right angles to one another), the rows of dots merge in a single row which slopes down into the stereoscopic image. The chief disadvantages of the Vectograph technique are the cost of preparing the prints, and the slight loss in resolution characteristic to the process.

Simple Parallax Wedges

The same principle of two diverging lines of dots printed on a transparent material has been adopted by the author to the use of simple stereoscopic pairs. If the two lines are about an inch and a half to two and a half inches apart (Figure 2), they can be merged under a simple lens stereoscope. If six or seven inches apart, they can be merged under a mirror stereoscope.

The design of the parallax wedge is simple. Three dimensions must arbitrarily be determined: (1) The average spacing between the two diverging lines, (2)

* The cooperation of R. T. Kriebel of the Polaroid Corporation and R. D. Garver of the U. S. Forest Service was instrumental in developing the devices described in this paper.

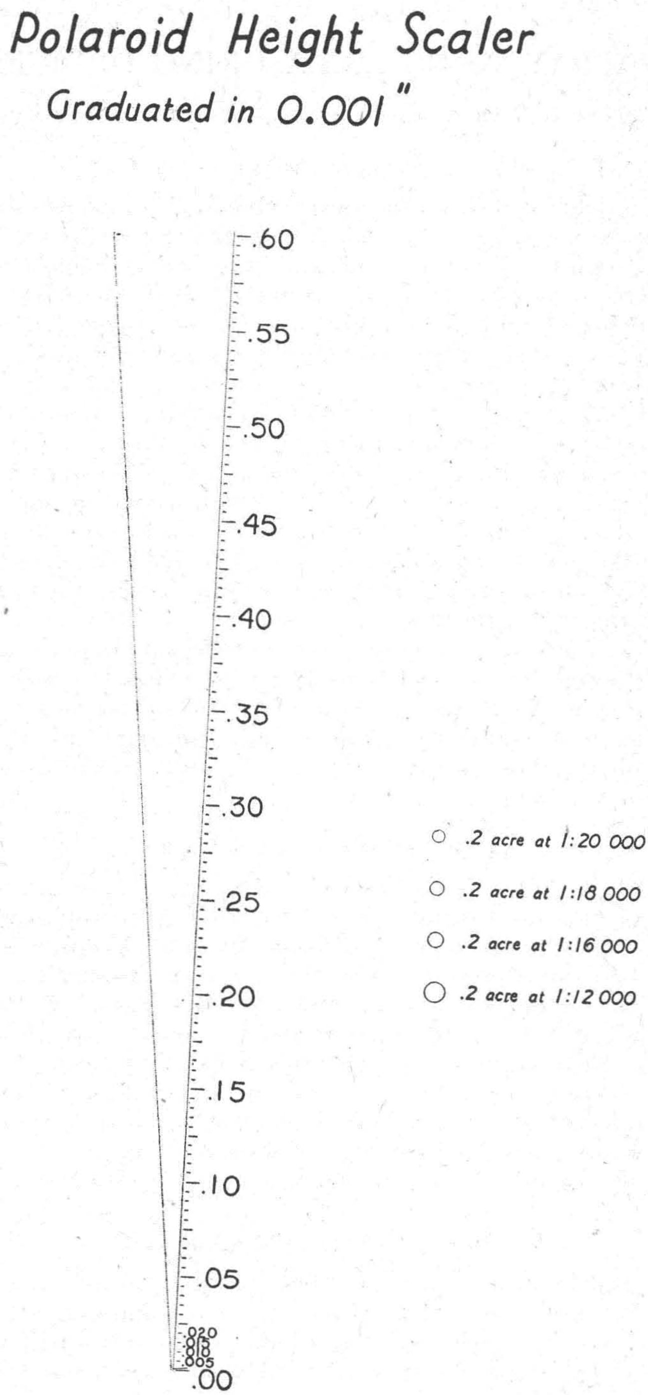

FIGURE 1

the amounts of parallax differential to be represented by each successive pair of dots, and (3) the desired slope of the merged rows of dots.

The spacing between the two diverging lines is dependent upon the stereoscope used and the width between the eyes of the observer. The average observer with a pocket stereoscope can best use a wedge in which the center points of the two diverging lines are $2\frac{1}{4}$ to $2\frac{1}{2}$ inches apart. To merge the diverging lines under a mirror stereoscope, they must be at last six or seven inches apart. The exact distance depends upon the stereoscope used.

In general, it is best to have each successive pair of dots one one-thousandth of an inch closer together than the preceding pair. This is easly accomplished. For instance, if the wedge is to consist of one thousand dots in each line and each pair of dots are to be one one-thousandth of an inch closer together than the preceding pair, then the two lines should be one inch closer together at one end than they are at the other. For a wedge designed for use with a pocket stereoscope, the two lines might well be 1.8 inches apart at one end and 2.8 inches apart at the other. The wedge can then be scaled to read parallax differentials directly. For instance, if the fused dot at the same elevation as the base of a tree reads 2.016 inches and the dot at the same elevation as the tip of the tree reads 2.003 inches, the difference in parallax between the top and bottom of the tree is 13 thousandths of an inch.

The apparent slope of the fused line of dots is controlled by the spacing of the dots in the diverging lines. If the dots are closely spaced, the line will slope steeply; if widely spaced, the slope will be gradual. An apparent slope of about 10 degrees appears to be quite satisfactory.

A number of different models have been constructed with different types of reticules spaced at various intervals. Satisfactory types of reticules, besides dots, include dots joined with a fine line, small circles, short dashed horizontal lines, and long solid calibrated lines.

USE

Training required: As with all phases of stereoscopy, young men can master the use of parallax wedge more easily than middle-aged men. Some people cannot measure parallax by any method. The average observer can use the device accurately within two days, and many young men can make reasonably accurate readings with less than ten minutes practice. The ease in use is due to the fact that it is far easier for the average person to see where a sloping row of dots intersects the stereo-image than it is for him to place a single dot in the same plane as the image.

Orientation of photographs: Satisfactory readings may be made when the photographs are oriented by eye. For best results, however, the photographs should be oriented by the stereo-base method. That is, both principal points should be marked on both photographs, and the four points lined up with a ruler.

Orientation of wedge: This is quickly and easily accomplished by rotating the wedge slightly until the corresponding reference marks on the two rows of slots are opposite.

Reading elevations on ground: Readings may be made simply by observing what reference dot most nearly coincides in elevation with the point of ground in question. Another method is to run the eye up the photograph to a point where the line of dots is well above the ground. Then, if the eye is moved rapidly back again, the line of dots will appear to break where it contacts the ground. This break provides an easy way to determine which reference mark is closest in elevation to the ground level. In measuring ground levels under dense woods, the use of the pseudoscopic image is invaluable, as small clearings in the forest appear as spires. The tops of these spires represent the ground level and can be easily and accurately measured.

Reading tree heights and other elevated points: Here also, use can be made of the sloping row of dots to obtain an accurate reading. If the row of dots is placed to the side of the object being measured, it is a simple matter to slide the wedge sideways and see whether the line goes into the object or over the top. By moving the wedge up and down, the dot which just touches the top of the object can easily be found.

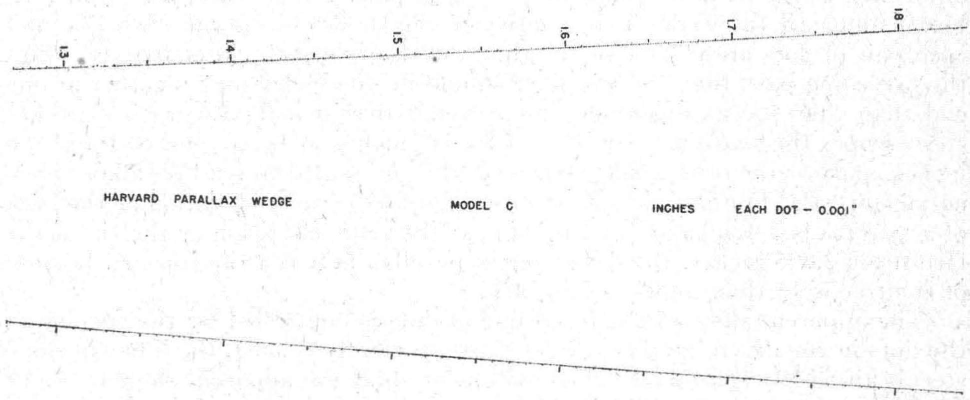

FIGURE 2

Accuracy: The parallax wedge is just as accurate as the distance between any two consecutive reference points. In the wedges in use, this is one-thousandth of an inch. Empirical tests at the Harvard Forest show that an untrained observer can measure tree heights with the parallax wedge on photographs of a scale of 1:12000 with an average error of less than three feet.

Table 1 gives the expectable accuracy of parallax height measurements with the wedge devices. This is for photographs taken with an 8.25 inch focal length lens, and a stereo base of 3.60 inches. It is based upon the assumption that the average untrained observer can read parallax with an average error of two thousandths of an inch. Extensive empirical tests carried on at the Harvard Forest demonstrate that the values given below are rather conservative, and that skilled observers can do somewhat better than indicated. These tests, dealing with the measurements of trees of known height, will be published elsewhere.

Conversion of readings: Since the readings obtained with the parallax wedge are readings of parallax differentials comparable in every way with the readings obtained by other floating dot devices, they may be converted to actual elevation or height by the parallax formula:

$$ho = \frac{f \times P}{RF(W+P)} \quad \text{or} \quad ho = \frac{H \times P}{W+P}$$

Where ho is the height of the object, f is the focal length of the camera, P is the differential parallax, RF is the representative fraction or natural scale of the photograph, W is the photo distance between the centers of the stereo pair in line of flight, and H is the height of the camera above the ground.

For instance, the actual difference in feet represented by successive pairs of dots one one-thousandth of an inch (0.0000833 ft.) closer than the adjoining pair would be calculated as follows for photographs taken with an $8\frac{1}{4}$ inch lens (0.6875 ft.) to a scale of 1:20,000, assuming that the photo distance between centers of the stereo pair in question is 3.60 inches (0.300 ft.):

$$\frac{0.6875\times 0.0000833}{\frac{1}{20,000}(0.3000833)}=3.82\text{ ft.}$$

Under the above conditions, then, each successive set of dots would merge into a single image 3.82 ft. higher than the preceding merged dot.

TABLE 1. EXPECTABLE ACCURACY OF PARALLAX HEIGHT MEASUREMENTS FOR DIFFERENT SCALES

Scale (R.F.)	Average error (Feet)
1:4000	1.5
6000	2.3
8000	3.1
10000	3.8
12000	4.5
14000	5.4
16000	6.1
18000	6.9
20000	7.6
22000	8.4
24000	9.2
26000	9.9
28000	10.7
30000	11.5