USEFUL GRAPHICAL CONSTRUCTIONS ON AERIAL PHOTOGRAPHS

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CONTENTS

A. Introduction

THE AIM of this paper is to present a number of graphical constructions, mostly for use with oblique photos, which may be of practical use to those unequipped with elaborate mapping devices, who have occasion to perform surveying operations or prepare maps of limited extent from aerial photos. Included are certain operations which may prove of use even to professional photogrammetrists, namely, the determination of the plumb point and the plotting of fully corrected radial lines on vertical photos. The writer acknowledges a considerable debt to Mr. P. H. Blanchet, whose original work has been the basis of some of the constructions developed here, especially in connection with the isoscale line (a term original with Mr. Blanchet).

Some of the material in this paper was first prepared in the form of a manuscript intended for geologists, as an aid to the field use of oblique photos, and planned for publication in the Bulletin of the Geological Society of America, but the publication of this has been delayed to permit the preparation and prior publication of the present paper as a more suitable medium for the first technical presentation of a large amount of new material essentially nongeological in character.

I. Constructions for Single Obliques

B. The Principal Triangle

Many useful graphical operations with oblique photos require the location of the following lines and points—principal line, true horizon line, principal point or photo center, isocenter, and photo plumb point. The graphical procedure for locating all these is illustrated in Fig. 1. The photo is first mounted over a larger sheet of blank paper to allow for extension of the construction lines, especially of the principal line and true horizon line (Or, perhaps better, a large sheet of transparent paper may be laid over the photo and used for all construction lines). The starting point is the principal point, *P*, located at the center of the photograph. The principal line is drawn through this perpendicular to the apparent horizon at H'. The true horizon is a line perpendicular to the principal line and parallel to the apparent horizon, or if this possesses appreciable curvature, parallel to a straight line tangent to it at H'. To locate the true horizon, one must find the distance between H' and H, usually the same on all photos taken on a single flight. It is found graphically by the following steps: Erect a perpendicular to the principal line at P, on which lay off PC equal to the focal length, f, of the taking camera. Draw CH' (not drawn on Fig. 1), and lay off an angle $H'CH$ equal in minutes of arc to $59/60\sqrt{A}$, in which A is the camera altitude in feet. CH is laid off on the upper side of CH', and H is the point of inter-

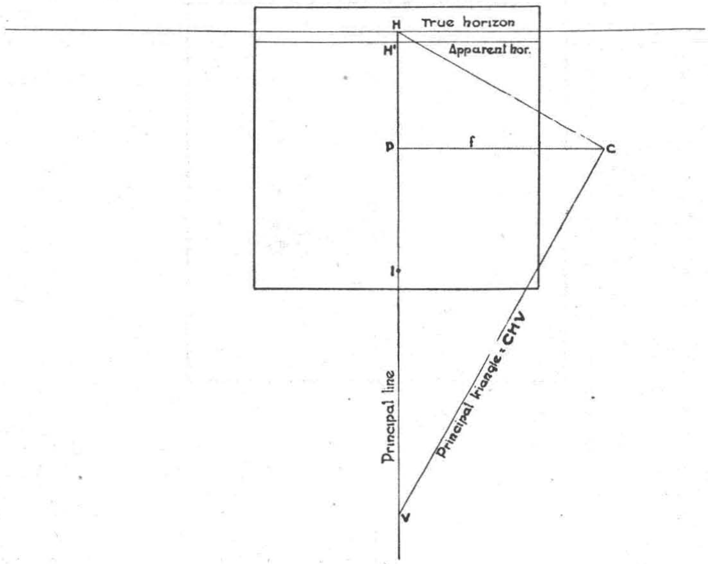

FIG. 1

section with the principal line. The true horizon line is then drawn through H, and should be extended a considerable distance to either side beyond the photo. In routine work $H'H$ is treated as constant, so that there is no need for drawing CH'. It is important, however, to draw CH, because this facilitates the graphical determination of the isocenter and other points. The isocenter I, is found by laying off on the principal line a distance HI equal to HC. The plumb point V, is found by drawing CV perpendicular to CH, V being the intersection with the principal line.

The construction triangle, CHV, will be referred to as the principal triangle. Though this triangle is drawn lying in the photo plane, it actually consists of lines lying in the principal plane, which can be visualized by imagining it swung up 90°, using the principal line as a hinge, and bringing the line PC perpendicular to the photo. C then lies at the perspective center (position of the camera lens), the line CH lies in a horizontal plane, and the line CV is a plumb line. (It will be recalled that the principal plane by definition is the vertical plane containing the photo axis.)

For a proof that the distance from H to the isocenter equals HC, see PHOTOGRAMMETRIC ENGINEERING, Vol. X, No. 4, p. 247. No further proof should be necessary that the point V, as determined, is the photo plumb point.

C. Datum Elevation Correction

Before introducing the graphical constructions involving ground points recognized on the oblique photos, we must consider the effect of topographic relief on the positions of these points. Since most of the constructions will require the use of corrected points in the place of the original photo points, we must acquaint the worker with the best means of determining the corrected positions. (All photo points of purely geometrical character, such as principal point, isocenter, etc., are in a different category, and will not receive this topographic correction. In the particular operation of constructing radial lines or

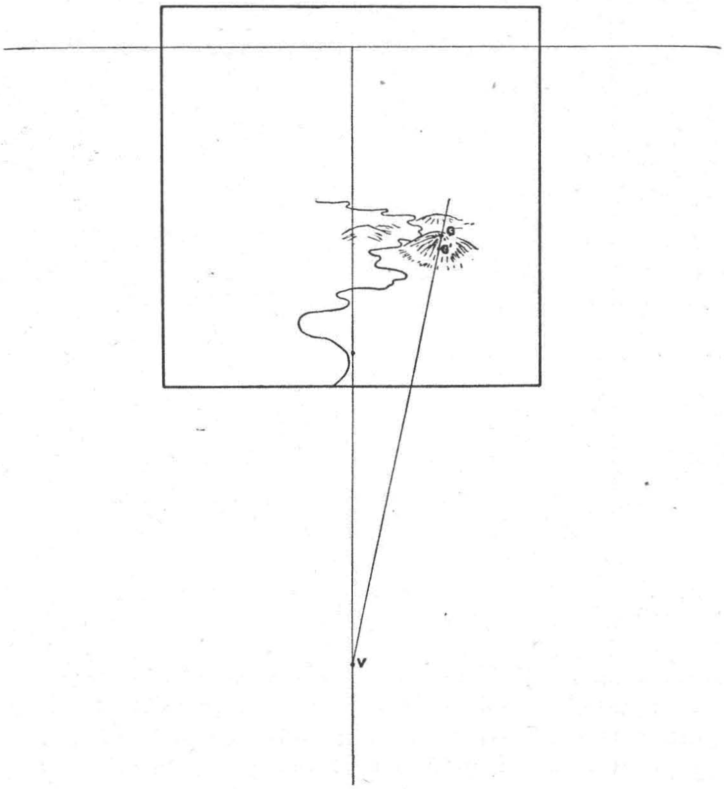

Fig. 2

direction lines whose origin is the photo plumb point, the use of the original and corrected photo points would give identical results, and hence the original points can be used.)

The simple geometry of an oblique photo is that which regards the photo as an oblique view of a horizontal plane seen from some point above. If the earth's surface within this view is that of an area of level ground, then all ground features will be located on the photo at the positions in accordance with this simple geometric plan. But if the earth's surface possesses relief, various photo positions are displaced, and must be corrected by referring each one to its plumb-line projection into an imaginary horizontal plane.

In the extended views of landscapes with relief, as one sees them on oblique photos, one can usually discern some dominating natural level, or can easily judge the average level, to use as this imaginary projection plane. If several shore lines or streams are visible, these can be selected to represent it, and this should be given first choice. Second choice would be a dominating plateau surface. Third choice would be an average topographic level in the absence of a single dominating natural level surface. The level selected will be referred to as the horizontal datum level of the photo.

Since a corrected photo point is that which in nature lies on a plumb line through the original point, its position on the photo must therefore lie on a line passing through the original photo point and the photo plumb point *V* (Fig. 2). *V* is actually the photo vanishing point of all vertical lines in nature. The exact position, on this line through *V*, on which to place a corrected photo point, as *G'* for *G*, is a matter of judgement as one makes a careful study of the topography while viewing the photo in a manner that gives a "three-dimensional" effect, as described below.

"Three-dimensional" viewing

The most realistic "three-dimensional" viewing of a single photo is obtained in the following manner. Prepare some three or four spectacle-stock spherical meniscus lenses, of focal lengths ranging from 6″ to 18″, each fastened to a one-

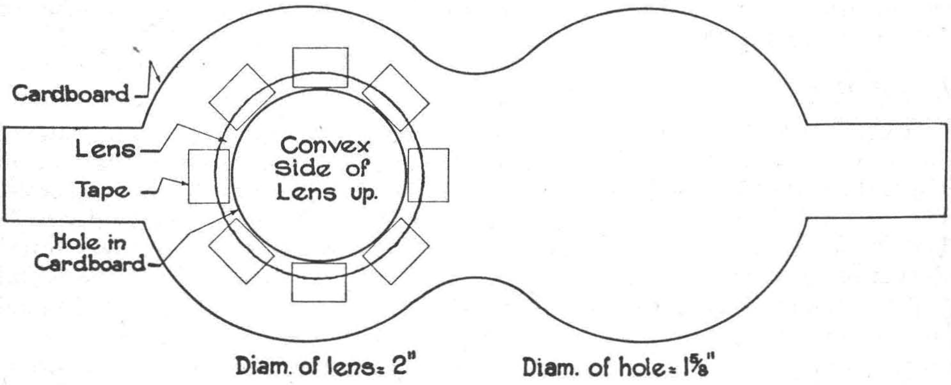

FIG. 3

holed frame of thick cardboard of the shape illustrated in Fig. 3. The photo must be held steadily in a bright light when viewing, and the viewing device held directly in front of the eyes. Notice that this permits one eye only to view the photo, but allows the other eye complete freedom to remain open and function in harmony and naturally with the seeing eye. When a lens is selected whose focal length is such that placing the photo at this distance from the eye (i.e., permitting the eyes to focus at "far" vision) causes the photo to subtend approximately the same angle to the eye as the original scene in nature subtended to the camera, the ocular conditions of viewing the original scene in the dimensions of nature are reproduced and a vivid three-dimensional effect is experienced. This is proven by noting that while actually only one eye sees the photo, the angle of convergence between the two eyes is involuntarily forced to change as one's attention passes from background to foreground objects in the picture. This method of viewing increases remarkably the ability to interpret the content of the photo (especially true of photos of ordinary life), and it permits a very accurate judgement of the spatial relations of all topographic features. The correction of photo points to a datum-level plane should be made while viewing in this manner.

D. Isoscale Line and Map Plotting

Camera height to map scale

A good idea of the basis for the following graphical constructions involving measurements to scale of horizontal distances, and map plotting to scale, from oblique photos, may be gotten from considering the relations between a photo and a map, to any definite scale, of the same terrain as seen in the photo. Suppose that the same camera which took the photo of the ground were also to take a photograph of the map. To make these two photos essentially identical, it is merely necessary that the camera over the map restore the relative position that it originally had with reference to the ground. For instance, let us assume that the original position was at an altitude of 20,000 feet vertically above a ground point A, with the camera axis pointing north, with a tilt angle of 60°. The photograph of the map should be made with the lens above the map position of A, at a height given by 20,000 feet to the map scale, also pointing north, and 60°.

Our problem, however, is that of making the map, and the method will be understood by keeping in mind this relation between such a map and imaginary photograph of the same, though the method uses the original photo in place of the imaginary photo.

Isoscale line

Fig. 4 shows the relations between the photo and map. Seen in the plane of the principal triangle (principal plane), the camera-lens plumb line is CV, on which the distance CV' is laid off to equal the camera altitude to the map scale. The map will then intersect this plumb line at V' in a plane perpendicular to it. The line M_1M_2, through V' and perpendicular to it will be the trace of the intersection of the map and principal plane. But the intersection of the photo and principal plane is HV. The intersection of M_1M_2 and HV is S, the common point of principal plane, map plane, and photo plane. A line S_1S_2 can now be drawn in the photo plane, through S, perpendicular to HV, to mark the intersection with the map plane. This is called the isoscale line, because being in both planes, it is the photo line having the map scale.

To illustrate this scale relation, measure the distance SR equal to one mile to scale. Draw RH, and various lines t_1, t_2 and t_3, parallel to SR, each intercepted by RH and VH. It is recognized at once that H is a horizon vanishing point, and that RH and VH represent parallel lines on the ground (or map), hence t_1, t_2 and t_3 all represent one mile. It is seen that only $t_s(RS)$ is one mile to the map scale.

Map construction plane

We may now swing the map plane into the photo plane using the isoscale line S_1S_2 as a hinge. This will permit constructions belonging in the map plane to be made on our construction sheet. For instance, we may draw a map line R_1R_2 parallel to the map trace of the principal line which coincides with VH. R_1R_2 is the map equivalent of the photo line RH. The several lines t_1, t_2, etc., should be equal in length on the map. The line $t_2(UT)$ of the photo, for example, will actually become $UmTm$ on the map, the point Um being found on a line through I and U. This simple relation (to be proven in the following paragraph) affords a simple constructional means of plotting the map location of any desired photo point.

Role of isocenter

The simple construction to be used for map plotting involves a double role of the isocenter, made possible because of the fact that the isocenter on the map

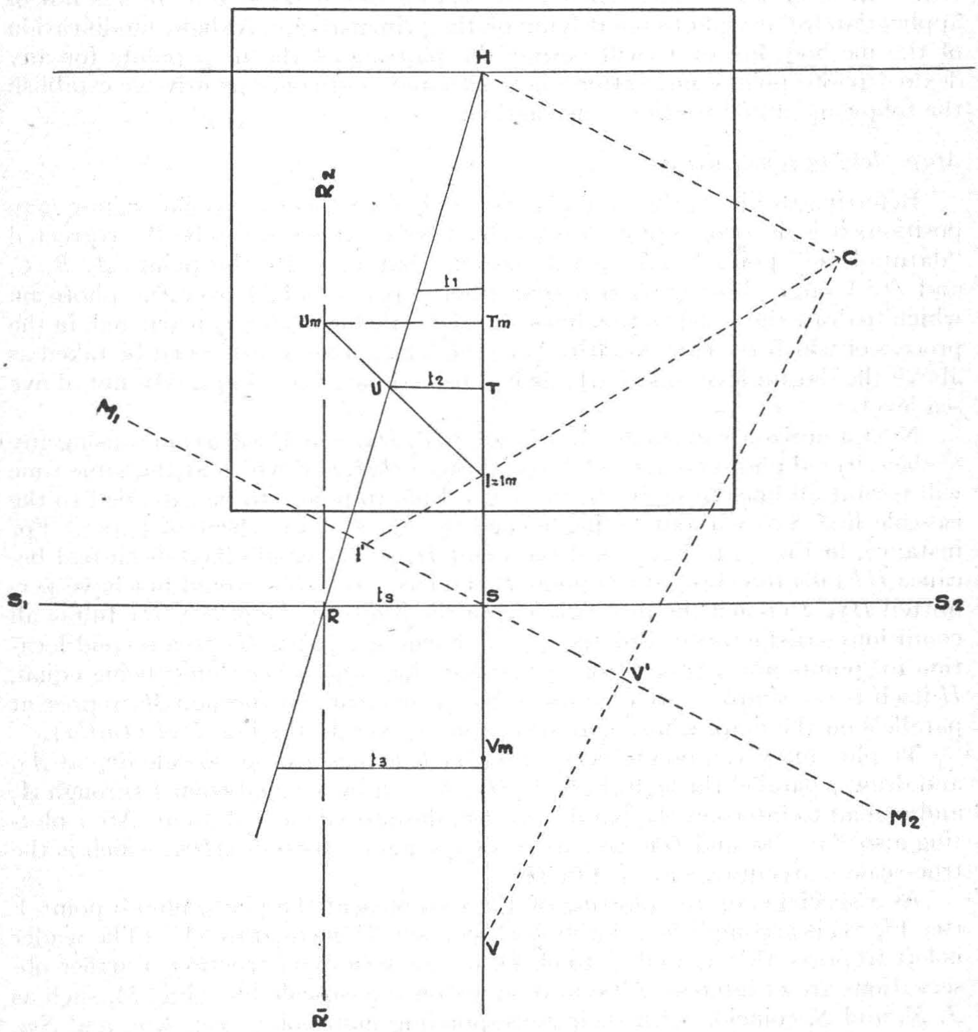

FIG. 4

(seen originally at I' on Fig. 4) becomes coincident with the isocenter on the photo (I), when the map plane is swung into the photo plane. This is because SI' equals SI. The dotted lines in Fig. 4 lie in the plane containing the principal triangle (principal plane), which plane intersects the map along the line M_1M_2. The point I' is found by prolonging the perspective ray CI to the line M_1M_2. To show that $I'S=IS$, is merely to note the similar triangles $I'IS$ and CIH in which $HC=HI$.

It is an established principle in photogrammetry that radial lines through the isocenter of a photo make the same angles with each other as the corresponding lines on the ground or on a map. Since the principal line on the map was swung into coincidence with the principal line on the photo (*SH*) and since *Im*(=*I'*) was swung into coincidence with *I*, the map line *ImUm* must take the same direction as the photo line *IU*. Hence we draw *IU* as a photo line, and consider it also as a map line, prolonging it to intersect the other map line (R_1R_2) at *Um*. The reader will see by testing this method that it will fail of application for any photo point lying on the principal line. A slight modification of the method, however, will permit the plotting of the map points for any desired photo points, no matter where situated, and consequently we establish the following actual routine construction.

Map plotting construction

Referring to Fig. 5, the photo is first studied to select the points whose map positions it is desired to plot. As explained before these are to be the corrected "datum-level" points, and we will assume that they are the points *A*, *B*, *C*, and *D*. A large sheet of transparent tracing paper is laid over the photo on which to draw the construction lines. The isoscale line S_1S_2 is constructed, in the process of which the camera altitude to the selected map scale must be taken as above the datum level (as nearly as can be estimated or computed), not above sea level.

Next a horizon vanishing point is selected, *Hx*, which will avoid causing any of the selected photo points to fall too close to *HxI*, and which at the same time will permit all lines from *Hx* to the several photo points to be extended to the isoscale line S_1S_2 without falling beyond the edges of the sheet of paper. (For instance, in Fig. 5, to have used the point *H* for *Hx* would have been bad because *HI* falls too close to the point *B*. To have used *Hy*, would not have permitted *HyC* to reach the line S_1S_2 within the limits of the paper. *Hx* fulfils all conditions satisfactorily. But there is no harm in moving *Hx* to a second location for points not mapped from a first location; and other things being equal, *H* itself is the simplest point to use.) All the lines drawn through *Hx* represent parallels on the map, whose map direction is given in the line *HxI* (*ImHx*).

To plot any given point, as *A*, draw *HxA*, extend to the isoscale line at *As*, and draw a parallel through *As* to *ImHx*. Also draw a radial from *I* through *A*, and extend to intersect the parallel at *Am*, the desired plotted point. After plotting also *Bm*, *Cm*, and *Dm* we may draw the figure *AmBmCmDm*, which is the true-scale map equivalent of *ABCD*.

As a special case, the plotting of *Vm* to represent the photo plumb point *V* (see Fig. 4) is accomplished simply by laying off *SVm* equal to *SV'*. (The reader is left to prove this as well as to check by the other construction. Further observations are of interest. All photo points on the isoscale line (Fig. 5), such as *J*, *K*, and *S*, coincide with their corresponding map points *Jm*, *Km*, and *Sm*. Since the photo point *I* also coincides with the corresponding map point *Im*, the average photo scale between *I* and *S* is the same as the map scale. But whereas the map scale is uniform in all directions, the photo scale is not uniform in any other direction than along lines perpendicular to the principal line. The reader may find it interesting to show that a photo point on the principal line between *I* and *S*, such as *E*, falls on the map at *Em*, as shown, likewise that a point on the principal line below *S*, as *F*, will all at *Fm*. Compare with the relation between *V* and *Vm* given above.

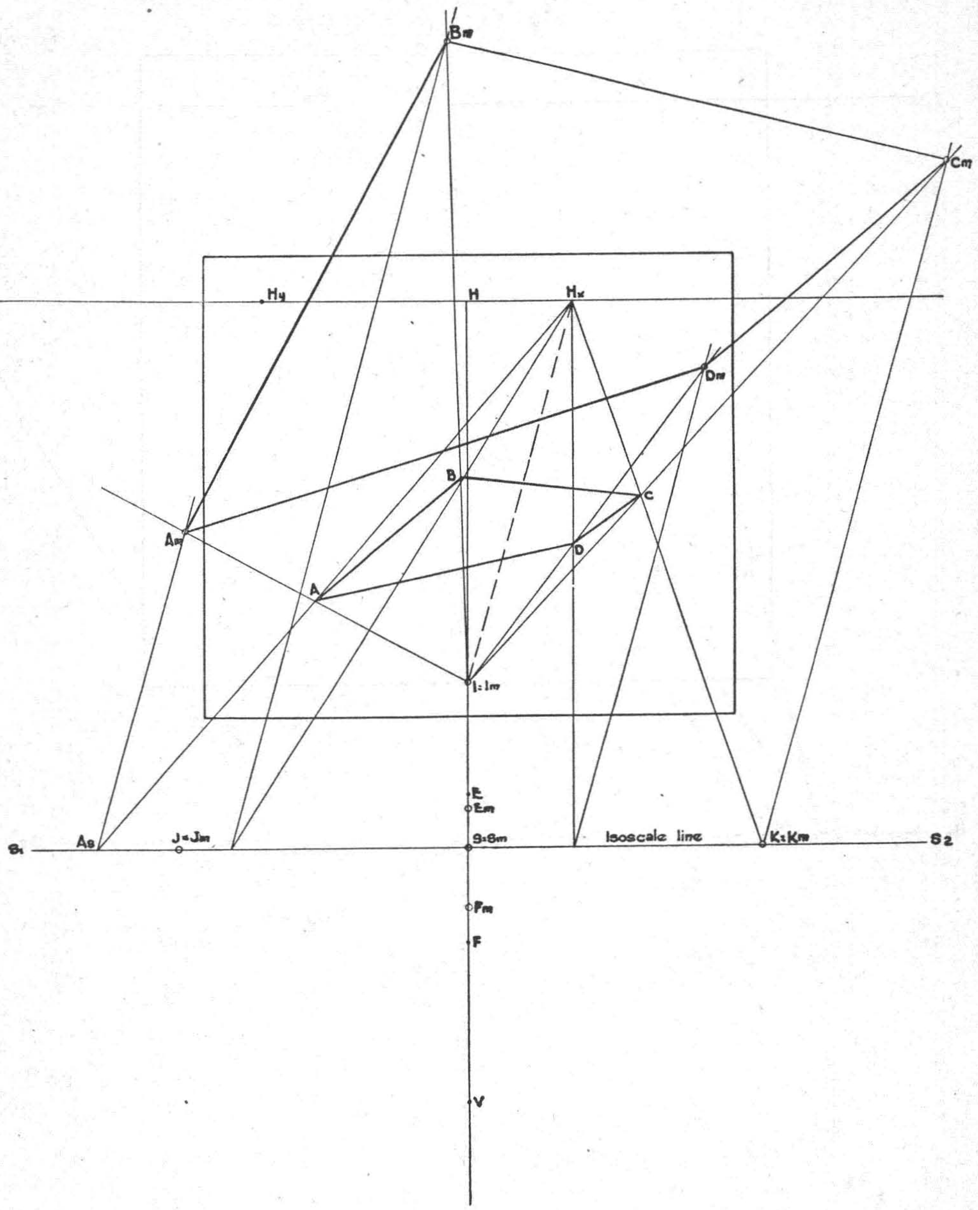

FIG. 5

To measure distances between photo points

To measure the distance between any two photo points it is believed that the simplest construction is to plot as a map the points in question to a selected scale, and then scale off the distances between the resulting map points.

To find camera altitude

To find or check the camera altitude when any horizontal distance contained in the photo is known, proceed as in Fig. 6. Suppose the distance AB is

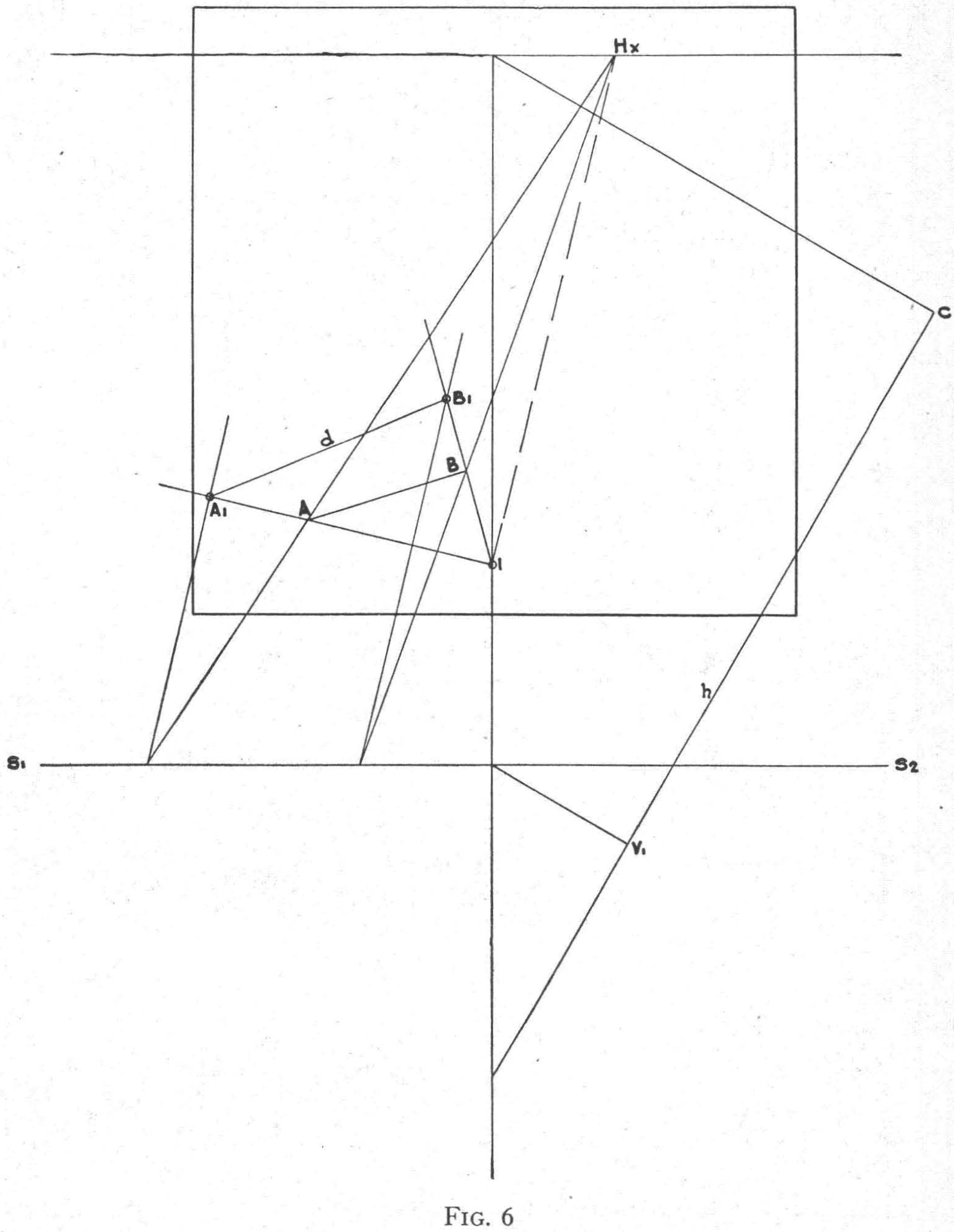

FIG. 6

known to be d. Assume any isoscale line S_1S_2 which will cause the camera altitude, h, to be represented by the line CV_1. Plot A_1B_1, the map representation of AB. The known distance d is then shown to the scale assumed as A_1B_1. h is then found by computation, as: $h=(CV_1/A_1B_1)d$, in which CV_1 and A_1B_1 are the actual lengths of the respective construction lines.

E. Map Plotting Grid

The isoscale-line principle makes possible the construction of a plotting grid whose lines are engraved on two transparent sheets, which, used together, can be made to fit exactly any oblique photo, for any desired map scale. The engraved lines on two sheets are identical, hence only one need be prepared, and two copies provided (in which case one will be reversed when used). Fig. 7 shows how the original drawing for the sheet may be prepared. A horizon line is drawn, near one end of which a vanishing point Hx is marked. A temporary line AB is drawn lightly in pencil near the bottom of the sheet, parallel to the horizon line, and along AB equal distances are laid off to provide points through which lines are to be ruled from Hx.

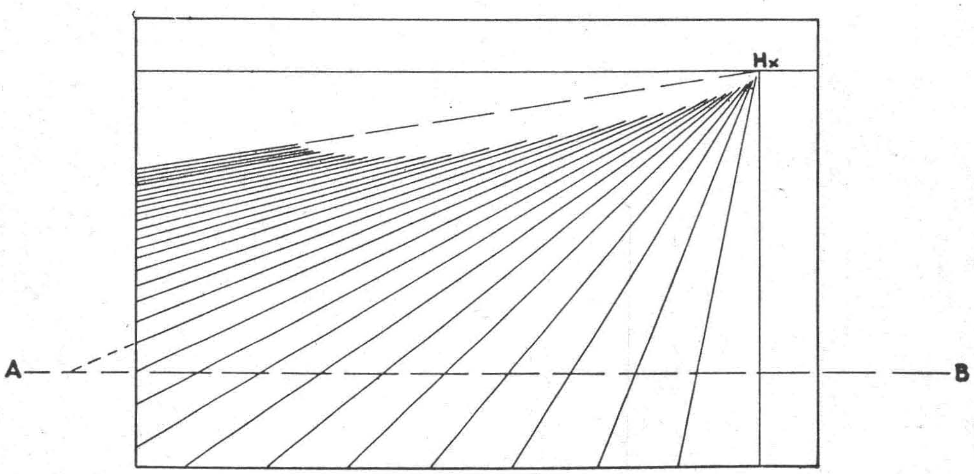

Fig. 7

To use the two grid sheets on an oblique photo (Fig. 8A), first draw the true horizon line, and the isoscale line to the desired map scale, using the camera altitude above the photo datum level. On the horizon line the points Hx and xH are located by making $xHH = HHx = HI$ (I is isocenter) $= HC$ (of principal triangle). Then place one of the grid sheets with its vanishing point coinciding with Hx, and horizon lines coinciding, and the other sheet reversed with the vanishing point at xH.

The corresponding map grid is ruled in squares as shown in Fig. 8B. These squares are to be made of such size that their diagonal, shown as "*d*," must equal the intercept "*d*," on Fig. 8A of any two adjacent grid lines upon the isoscale line.

F. Elevation Measurement

It will be recalled from Fig. 2 that routine practice in dealing with photo points requires the determination of a corrected point lying within the plane of datum elevation. This furnishes us with two photo points (one original and one corrected) for each ground point. The difference in position between these two can give us a measure of their difference in elevation, to be determined by a simple graphical construction to a scale measure.

In Fig. 9 we will let A be an actual photo point, and Ad the corrected datum position of the same point. S_1S_2 is the isoscale line constructed for the datum level containing *Ad*, and accompanied by *CVd*, the distance to scale, equal to the camera altitude above this datum. Our object now is to find another isoscale line pertaining to the level of the higher point *A*, and accompanied by a point *Va*, and giving an elevation difference measured to scale as *Vd Va*. (It must be pictured in the mind that this involves two maps to the same scale, one lying in the horizontal plane of *Ad*, and the other in the horizontal plane of *A*, and

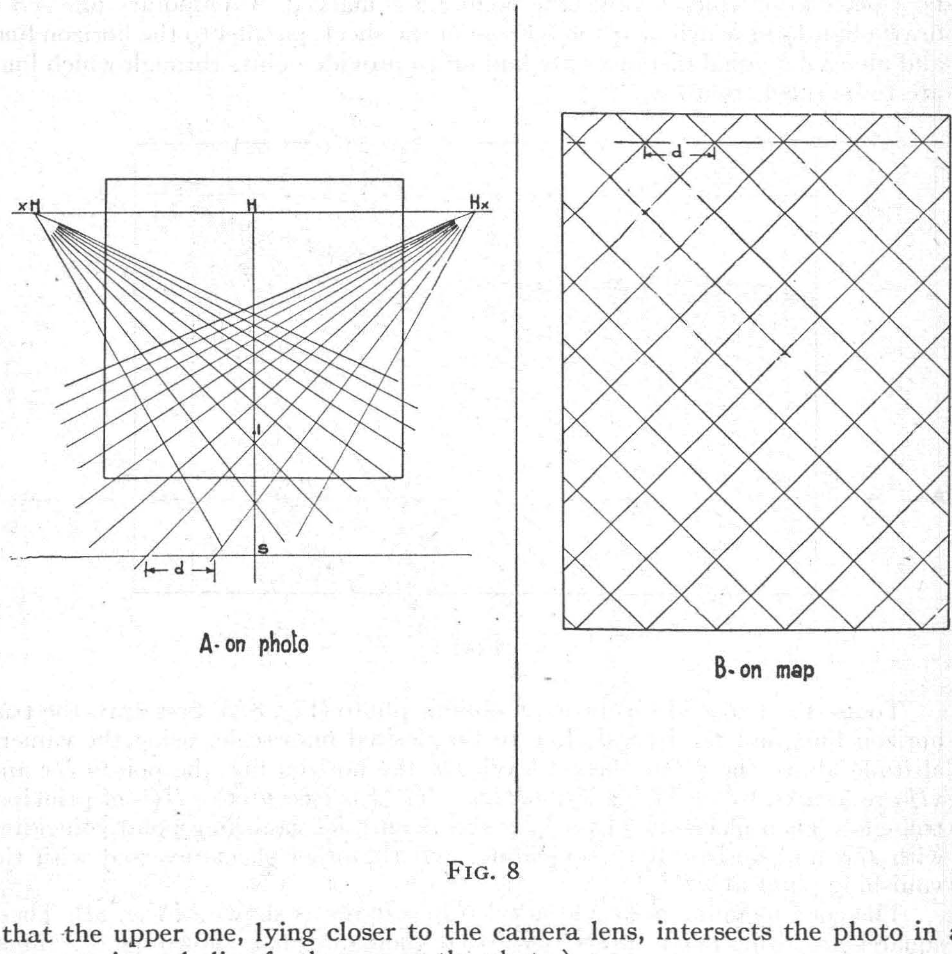

FIG. 8

that the upper one, lying closer to the camera lens, intersects the photo in a separate isoscale line farther up on the photo.)

A suitable horizon vanishing point *Hx* is used that will provide an appreciable angle between the lines *HxA* and *HxAd*. The line *HxV* is drawn to represent the photo trace of the vertical plane passed through *Hx* and the camera perspective center, and causing *HxV* to contain the photo image of all possible vertical or horizontal lines lying in that plane. Let *HxAd* be drawn and extended to the isoscale line at *E*, and let *HxV* intersect the isoscale line at *F*. If a photo line *AdBd* is drawn as shown, lying in the datum plane and perpendicular to the photo principal line, the true-scale measure of the distance *AdBd* is seen on the isoscale line as the distance *EF*. But if in the higher plane the corresponding line *AB* is drawn, it will represent the same actual length (equal to *EF*), because *AAd* and *BBd* both represent plumb lines. The lines *HxE* and *HxV* represent parallel lines on the lower map, and HxE' and HxV represent the identical lines on the upper map. To locate the upper isoscale line all we need is the line $E'F'$, intercepted by these two last, at the place where $E'F'$ becomes equal to EF. Hence we may draw EG parallel to HxV, and find E' at the intersection of EG and HxA. (It need merely be noted that VHx and EG are *not* map lines showing

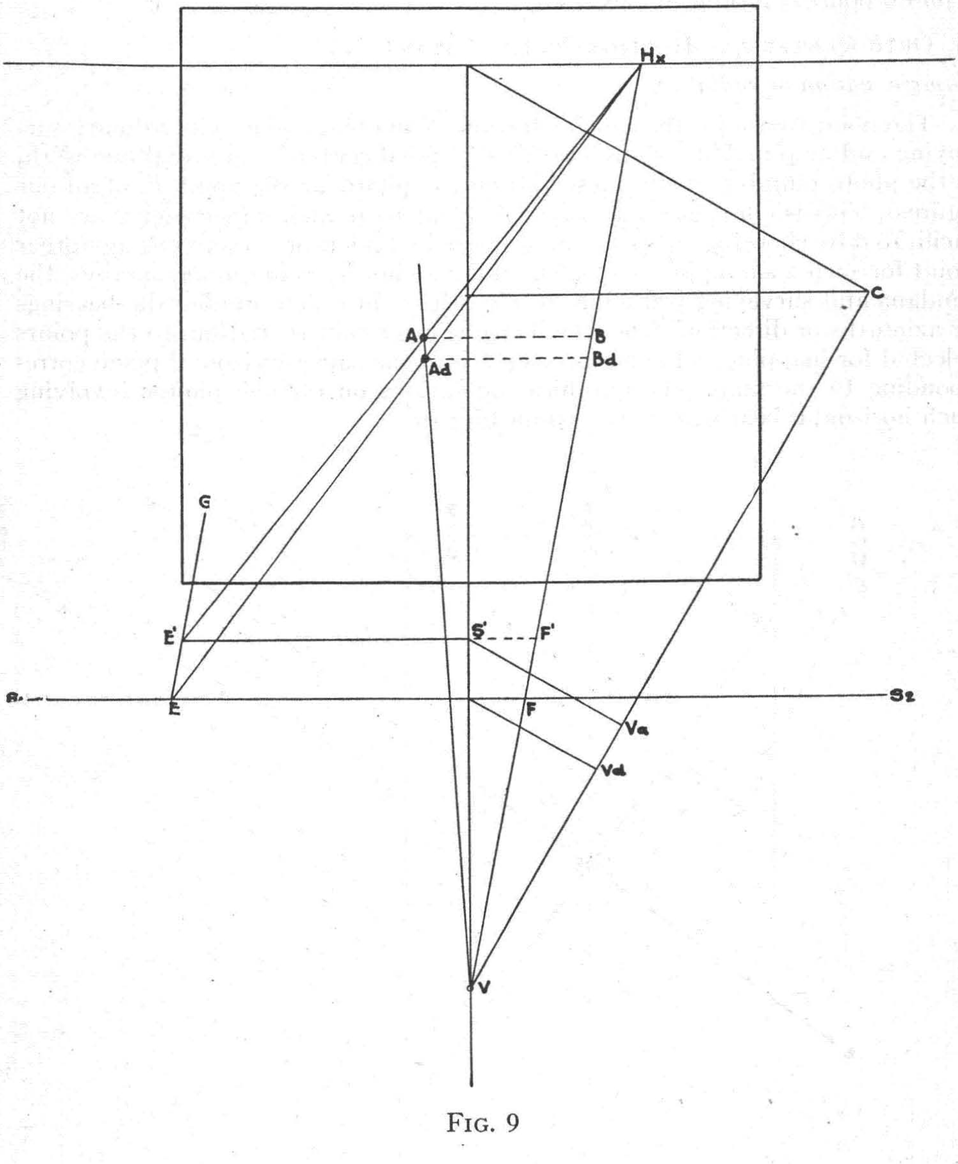

FIG. 9

the above mentioned parallel lines in the true direction—they are photo lines having no particular direction so far as the map is concerned. The only purpose of drawing GE parallel to HxV is to provide a convenient means of making $E'F'$ equal to EF while E' fulfils the further condition of passing through HxA.)

In Fig. 9 the lines shown by dashes are not necessary in the routine construction, whose steps are now summarized as follows: we start with the principal triangle, the isoscale line S_1S_2, and the two points A and Ad. First select a convenient point Hx and draw HxV. Draw $HxAd$ extending to E. Draw EG parallel to HxV, and draw HxA extending to intersect EG, at E'. Draw $E'S'$ and $S'Va$, and measure the desired elevation (difference between A and Ad) to scale as $VdVa$. To read actual elevations above sea level, a graduated scale may be placed along the line VC after a point Vn is determined on it corresponding to a photo point N of known elevation.

G. Orthocenter and Horizontal Directions

Camera station or radial center

The point to use for the camera station of an oblique photo in ordinary surveying and map making, as well as for the radial center in all radial line work, is the photo plumb point (represented on the photo by the point V, of all our figures). This is emphasized to point out that mapping and surveying are not facilitated by choosing either the photo principal point or isocenter or any other point for such a surveying center. On oblique photos, as in ground surveys, the fundamental surveying procedure resolves itself into determining the bearings or azimuths or directions from the instrument or camera station, to the points selected for mapping, all being projected into the same horizontal plane corresponding to the map. The graphical operations on oblique photos involving such horizontal bearings are shown on Fig. 10.

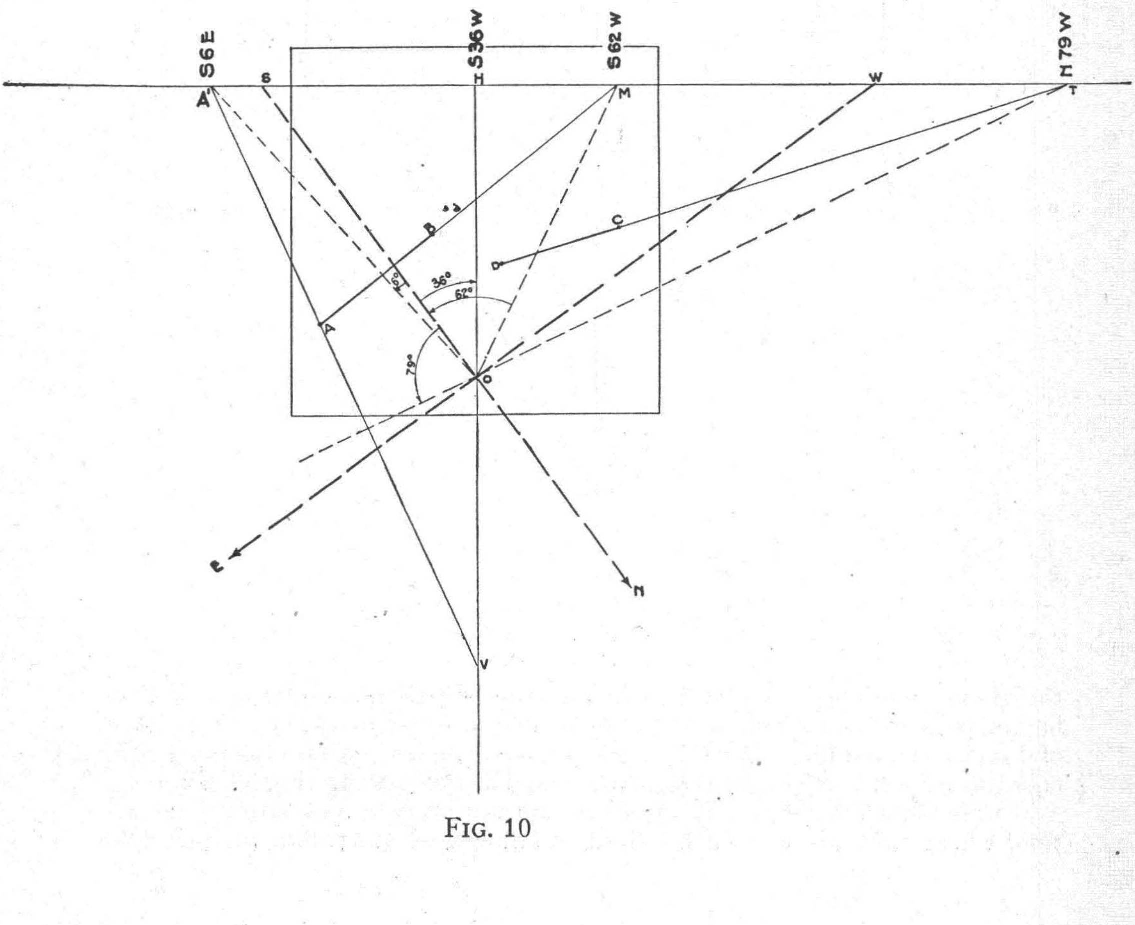

Fig. 10

Orthocenter

Before discussing the constructions of Fig. 10, however, we must discuss the point O, known as the orthocenter, and which represents the camera station as described above (that is, it represents the photo point V), and serves as a center for the simple construction of the bearing lines in question. On Fig. 10, the orthocenter O falls at the position of the isocenter (I of Fig. 1), hence is determined by laying off HO equal to HC of the principal triangle. (In their properties the orthocenter and isocenter are not to be confused—see PHOTOGRAMMETRIC ENGINEERING, Vol. X, No. 4, pp. 247–251.) The orthocenter and all lines drawn to it are only drawn upon the photo for convenience—the relationship will be made clear by imagining these lines to lie in a plane which should be swung up out of the photo plane using the true horizon line as a hinge, and brought to a horizontal position that will place the point O into coincidence with the perspective center (represented by C of Fig. 1). With this in mind, one can see that any point on the camera plumb line, as V, must fall at O when projected orthographically on to this horizontal plane and the projection swung into the photo plane. In Fig. 10 all lines actually belonging in this horizontal plane, have been made dashed, while lines belonging in the photo plane are continuous.

Vanishing points

The term vanishing point has long been familiar as applied to any point on the true horizon line to which an indefinite number of photo lines can be drawn, which will represent parallel lines on the ground (or map). On any given photo, there is only one such point to mark any particular direction. An azimuth line from the camera station in that direction will be the line on the photo from V to that vanishing point, and the corresponding line in our horizontal plane will be the line from O to the same point. All lines in the horizontal plane passing through O make the same angles with each other as the corresponding lines do passing through the plumb point of the camera station on the ground or on the map.

Orienting an oblique photo

An oblique photo is sufficiently oriented when the vanishing point of any single direction whatsoever is located. This will permit simple constructions (Fig. 10) of any desired bearing or azimuth line through any photo point, or the direction determination of any photo line, or the direction from the camera station to any photo point.

In Fig. 10 it is assumed that the vanishing point, M, of a given direction, $S62° W$, is known, as shown. (This might have been found, for instance, by prolonging some photo line AB known to have that direction.) Draw MO, lay off the 62° angle in the proper direction going from west to south, and draw the line OS, locating S, the vanishing point of due south. The west vanishing point, W, is found from a line from O at right angles to the last. Determine next the bearing of the photo axis or principal line, OH. This is done by measuring the angle SOH, found to be 36°, hence at the point H we may write $S36° W$. To determine the bearing of any point on the photo as seen from the camera station, illustrated by the point A, draw VA, prolonging to the horizon line at A', thence draw $A'O$. If this makes an angle of 6° going east from OS, then the bearing is $S6° E$. To determine the bearing between any two points on the photo, for instance, the bearing of D from C, prolong CD (backward) to the horizon line at T, thence draw TO and prolong. If the measurement shows an angle of 79° with

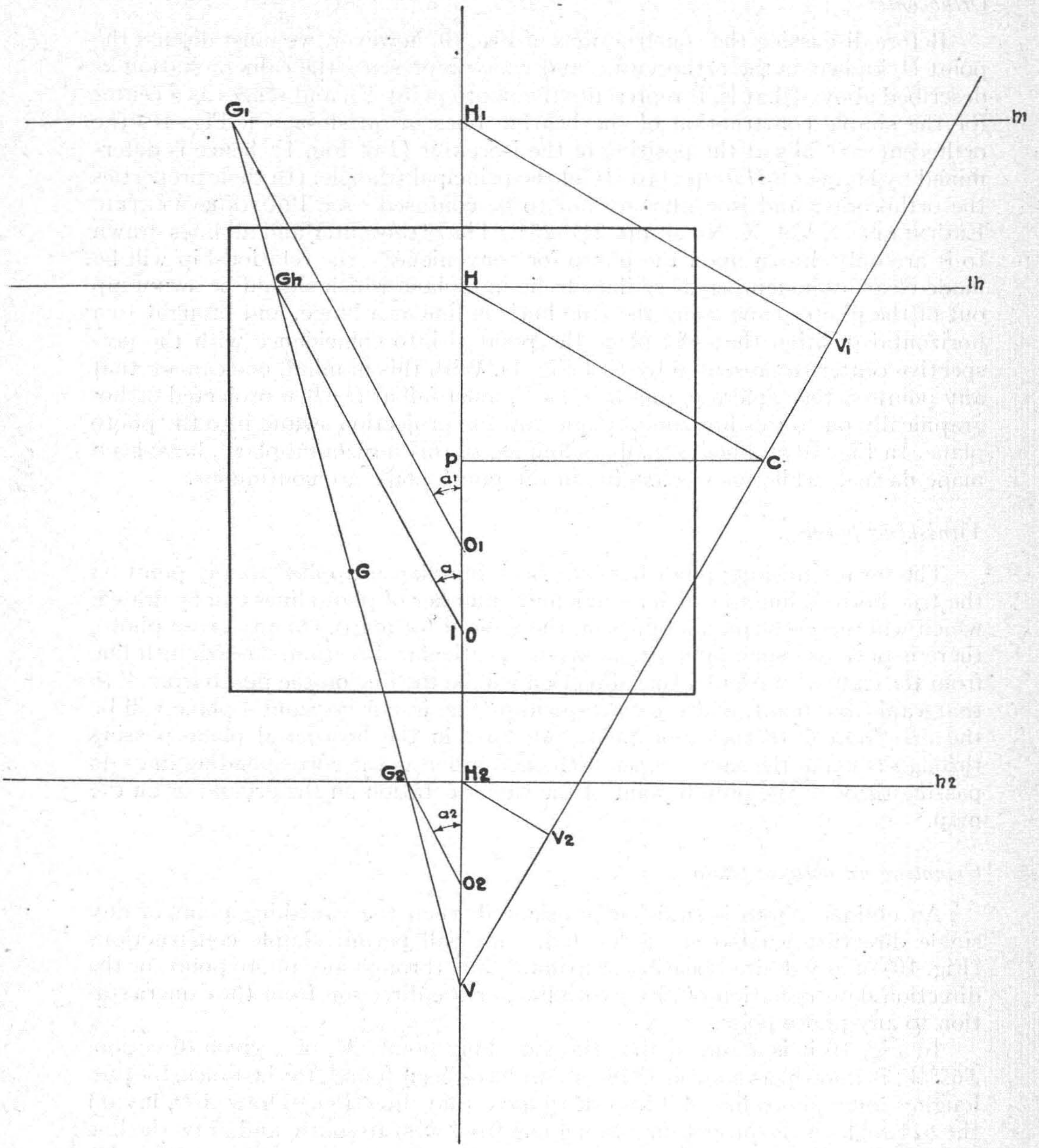

FIG. 11

OS, in the direction from S toward E, the required bearing is $S79°E$, and the point T will mark the vanishing point of $N79°W$.

Artificial horizons

The term "artificial horizon" is already in use to designate any line on a photo (h_1 and h_2 on Fig. 11) parallel to the true horizon line and used as a substitute for it in certain constructions. It is the intersection with the photo of any horizontal plane other than that passing through the perspective center. As we will make use of such artificial horizons in some of the constructions to follow (especially for plumb-point determination and corrected radial-line plotting) it will help to set forth their pertinent geometric properties. In Fig. 11 the traces of the artificial horizontal planes mentioned are seen in the principal plane as the lines H_1V_1 and H_2V_2. If the first horizontal plane is swung into the photo plane with its artificial horizon (h_1) as a hinge, the point V_1 (orthographic projection of C into that plane) will fall at O_1 as an orthocenter, ($H_1O_1 = H_1V_1$). Similarly the orthocenter of the second plane will fall at $O_2(H_2O_2 = H_2V_2)$. The reader can easily see that the horizontal bearing from the camera station to any photo point as G will have to be the same in any horizontal plane into which it is projected. Hence the same construction applies in all cases with artificial horizons, for finding this bearing line, as was applied when using the true horizon, namely, to draw the line VG, intersecting with the horizon line in question (for example at G_1) from whence a line to the corresponding orthocenter (G_1O_1). (The reader should easily see independent proofs from simple plane geometry that the angles a, a_1 and a_2 of Fig. 11, are equal—also that $\frac{H_1H}{HV} = \frac{O_1O}{OV}$ and $\frac{H_2H}{HV} = \frac{O_2O}{OV}$.)

II. Constructions for Overlapping Obliques

H. Radial Line Plotting

The plotting of a continuous map from two or more overlapping oblique photos is best initiated by the preparation of an assembly of radial-line templets, exactly as in mapping with vertical photos. The photo plumb points must be used as centers for the radial lines, as the attempted use of other centers would provide no way of eliminating displacements due to differences in relief. The radials through the camera plumb point are unaffected by relief.

Each photo templet will consist of a sheet of transparent paper, usually considerably larger than the photo, especially if two or more obliques have a common camera station, with or without a central vertical photo, in which cases a single templet serves all photos. For convenience, all necessary construction lines pertaining to the photo (horizon line, principal triangle, etc.) may be drawn in the first instance on the templets, which should show also the photo principal lines.

The method of drawing radial lines to represent the selected photo points is the orthocenter method, illustrated in Fig. 10 in the case of the point A. This radial is drawn in two steps—first the construction line VA' passing through A, thence the radial itself, OA', which should be extended a considerable distance and labeled plainly as pertaining to point A.

In principle, radial-line plotting with obliques is the same as with verticals—sufficient numbers of photo points common to two or more photos are selected to provide ample tying together of the photos to fix the location of each point, and hence control the map. The only difference is that the radial centers of verticals fall within the photo and are transferred stereoscopically from one photo to another as tie points along with other selected common points. But with obliques the radial centers are usually outside the photos and cannot be so transferred and fixed, and the determination of their location depends on fixing a sufficient number of other points.

A rule may be laid down that to make one templet serve two or more photos taken from the same camera station only one tie point is necessary between each photo and the next one adjacent. This is illustrated in Fig. 12, which shows three such related obliques. The point A is marked as a common point to the first two photos, and B common to the last two. The templet sheet may first be laid over

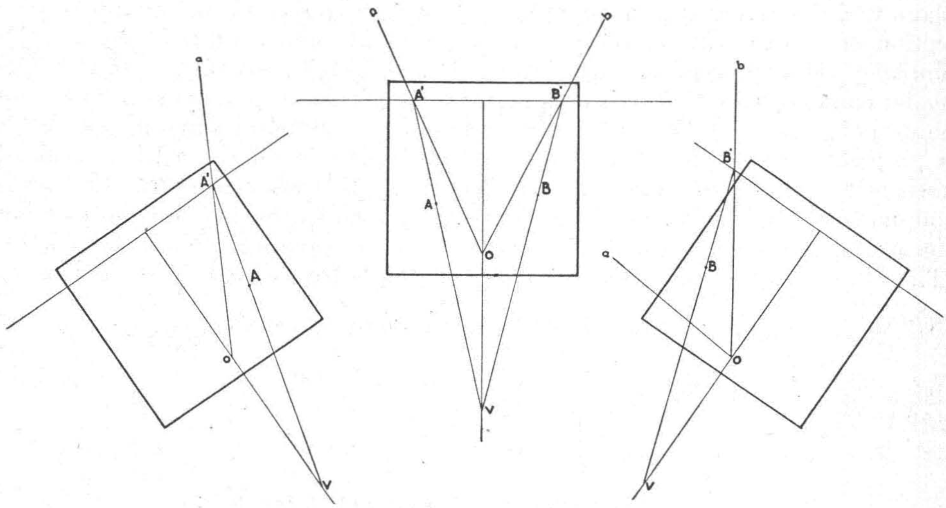

FIG. 12

the left hand photo, and the radial center marked at O. The radial Oa is drawn (and before lifting the sheet, radials may be drawn for as many other selected points as this photo contains). The sheet is then lifted and placed over the next

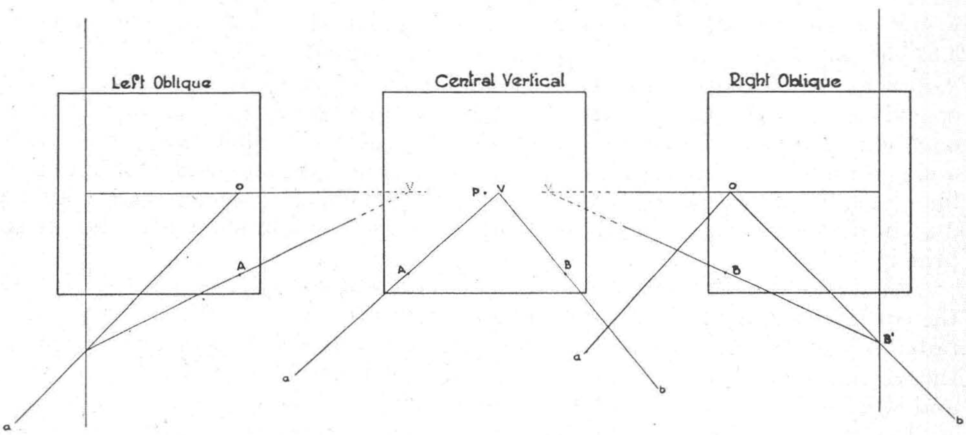

FIG. 13

photo merely by superimposing the point O, and the line Oa over the point A'. In like manner Ob is drawn (and other radials). Lastly, the third photo is worked after superimposing the line Ob.

This same rule may be applied to sets of trimetrogon photos after the plumb point is marked on the central vertical photo. (See method for getting this plumb point later in this paper.) Fig. 13 shows three mutual trimetrogon photos, on the central of which the plumb point has been determined at V. By steps strictly analogous to those of Fig. 12 the templet is moved from one photo to the other, the only difference being that the radials may be directly drawn though the original photo points on the central vertical photo. The only caution is not to use the central principal point of this photo for the radial center.

A second rule may be laid down that to permit surveying by radial-line templets, each templet should contain at least two rays for points represented by rays occurring on two other templets (or in other words, at least two three-way intersections per templet). This rule is identical to that for the requirements in working with strictly vertical photos, and further discussions of radial line techniques will not be given here.

If the reader has forgotten a statement made earlier, it is worth repeating that all photo points selected for radial lines are the original points, not those corrected for a datum elevation (because it should be very easily demonstrated that any such "corrected" points would give identical radials).

I. Map Plotting

A templet assembly, whose templets are prepared as described in the last section, will provide a plot to some accurate scale, or to a predetermined scale, of all photo plumb points and selected photo tie points, and can be made to show direction lines of all photo principal lines.

Further filling in of map details can proceed by quick approximate methods utilizing one photo at a time by methods illustrated in Figs. 5 and 6 (isoscale plotting) or Figs. 7 and 8 (grid plotting). These will require the use of datum-corrected photo points. If there are considerable portions of each photo not duplicated on any other photo (as on standard Canadian obliques) there is no alternative to these methods. But when the obliques provide considerable overlapping (100% with trimetrogon obliques) the filling in of map details can proceed by a better method of two-photo construction, using original photo points, and being a virtual continuation of templet-radial construction.

The templets themselves may be used, or fresh transparent sheeets, if desired. A recommended procedure is as follows; use transparent paper not only for the templet of photo constructions but also for the map. First mark the points to be plotted on each of the two photos, accompanying each point by a small identification number, as shown for one of the photos by Fig. 14. Next lay a separate templet sheet over each photo, and on each one, as in Fig. 14, mark the position of the orthocenter O, and rule the principal line an indefinite distance, and also the horizon line. Then with the edge of a ruler held against the point V, line up against each of the points in turn, and for each make a tick mark on the horizon line, as shown, giving each the corresponding number. When this is completed for both photos, fasten the two templet sheets to the under side of the map-plotting sheet, each one with the orthocenter coinciding with the map plumb point and with the principal line coinciding with that on the map. Then proceed as in Fig. 15, which shows the map plot of the two photo plumb points as A and B, and the principal lines as arrows. The two horizon lines aa' and bb' are visible through the map paper, each containing its row of numbered tick marks. Then by laying the edges of rulers against A and B, and against the matched tick marks, the intersections will give in each case the desired map point.

J. Elevation Measurement

The graphical method of elevation measurement pictured in Fig. 9 used a single photo and required an estimate of a datum elevation point for use with each original photo point. We are now ready for an elaboration of this method using two photos and finding the true datum point graphically. This time, however, we can work independently of any topographic datum level in the picture,

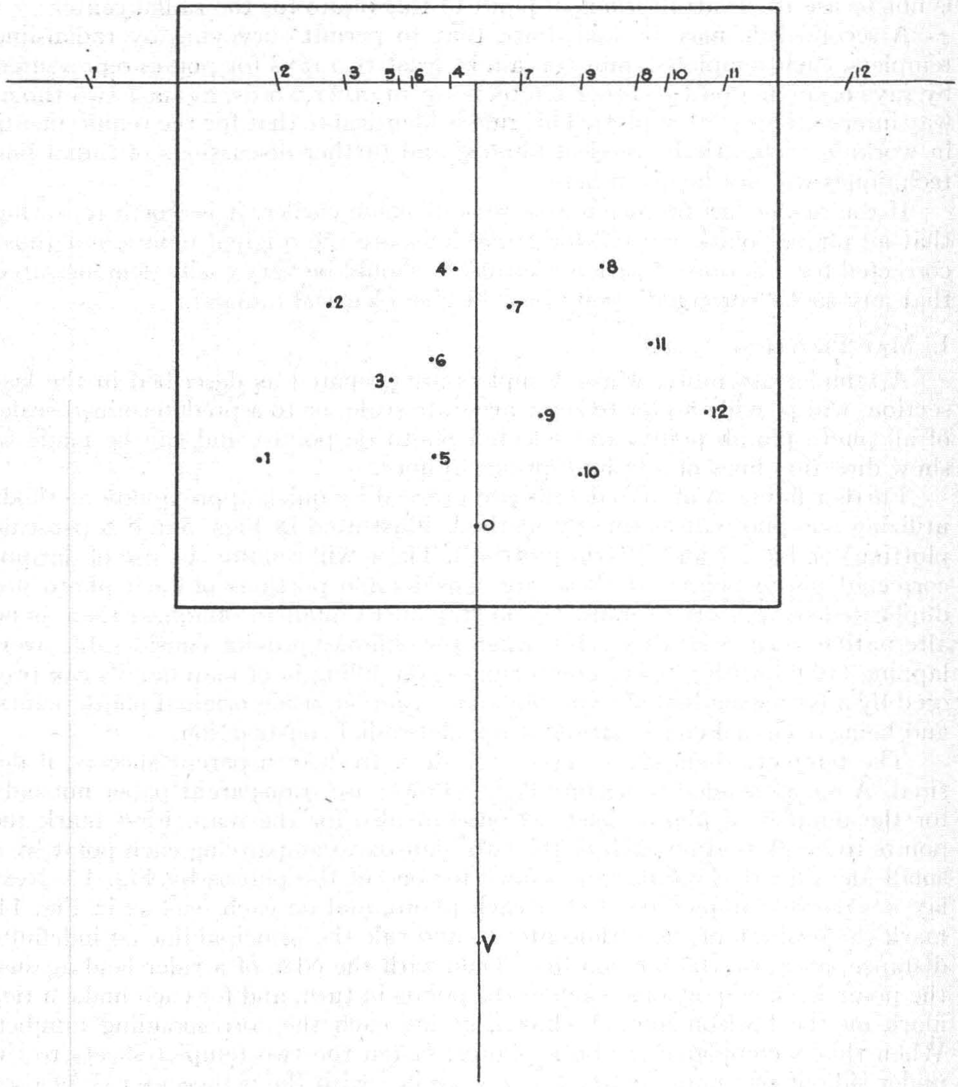

FIG. 14

and measure elevations directly in terms of sea level, provided that we know the elevation of the camera above sea level.

The construction is performed over one, only, of the two photos (whichever is convenient) and accuracy depends on careful, fine drafting in everything involved. It requires first the map plotting of the point whose elevation is to be measured.

On a sheet of transparent construction paper placed over one of the two photos, let us say "Photo A" (Fig. 16), we construct an isoscale line for sea level S_1S_2 (by scaling CV' equal to the camera altitude above sea level, to the map scale). On the map (lower part of Fig. 16) the plotted camera stations (plumb points) are at A and B, and the point to be measured is surveyed in at G, which point is also seen on the photo as G. A suitable horizon vanishing point, Hx, is used. HxV is drawn intersecting S_1S_2 at A'. HxO is drawn, making an angle r with the principal line.

On the map, the direction line Ax is drawn making the same angle r with the principal line Aa. Then the base line m_1m_2 is drawn through A perpendicular to Aa. A line is now drawn on the map from G parallel to Ax, intersecting m_1m_2 at G'. Returning to the photo, a distance equal to AG' is laid off on the isoscale

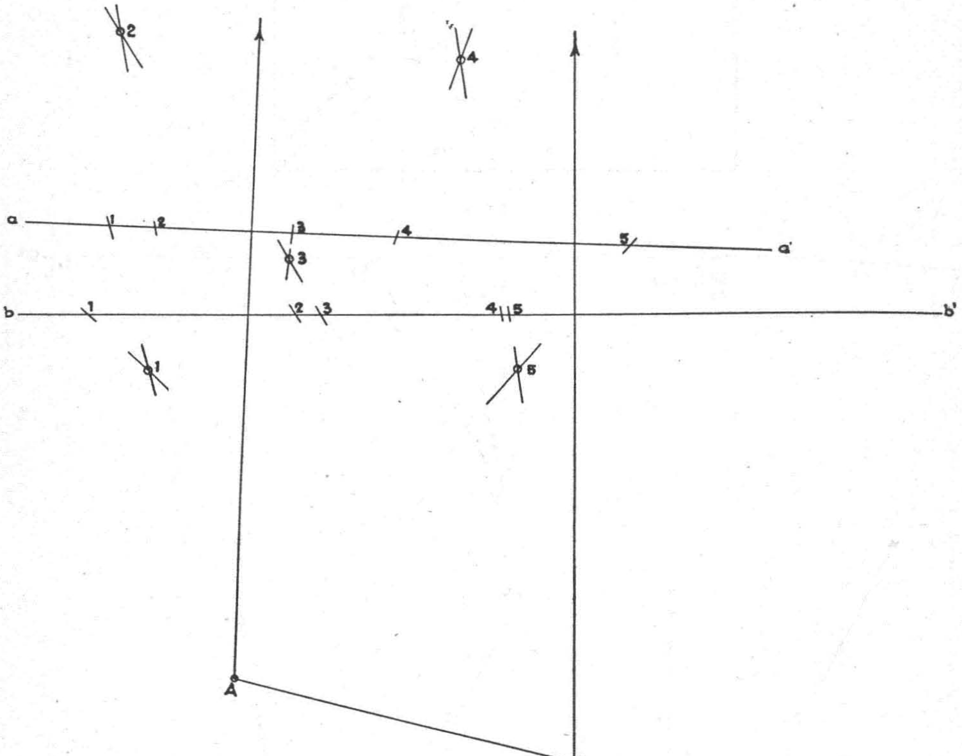

FIG. 15

line by placing the end representing A at A', and establishing $A'G'_s$. The sea level plumb-point photo position of G will lie on a line through G and V, so VG can be drawn. If a line is now drawn from G'_s to Hx, its intersection with VG will give the sea level position, G_s, as a photo point. The measure of the elevation of G_sG, is found in the same manner as in Fig. 9, by drawing G'_sQ parallel to VHx, Q being at the intersection with HxG, thence drawing QR, and RVg. The map-scale measure of the elevation of G above sea level is $V'Vg$. Though this ideally should give an accurate measure, in practice there may be sources of error. Therefore, if the photo contains a point of known elevation, for instance G in Fig. 16, a graduated scale along the line VC should be shifted if necessary to bring the figure for this elevation to the point Vg, even though the zero figure (sea level) is no longer at V'.

It is seen that in performing these constructions, the lines VG, and HxG'_s, and determination of G_s, are unnecessary. Fig. 16 shows construction lines

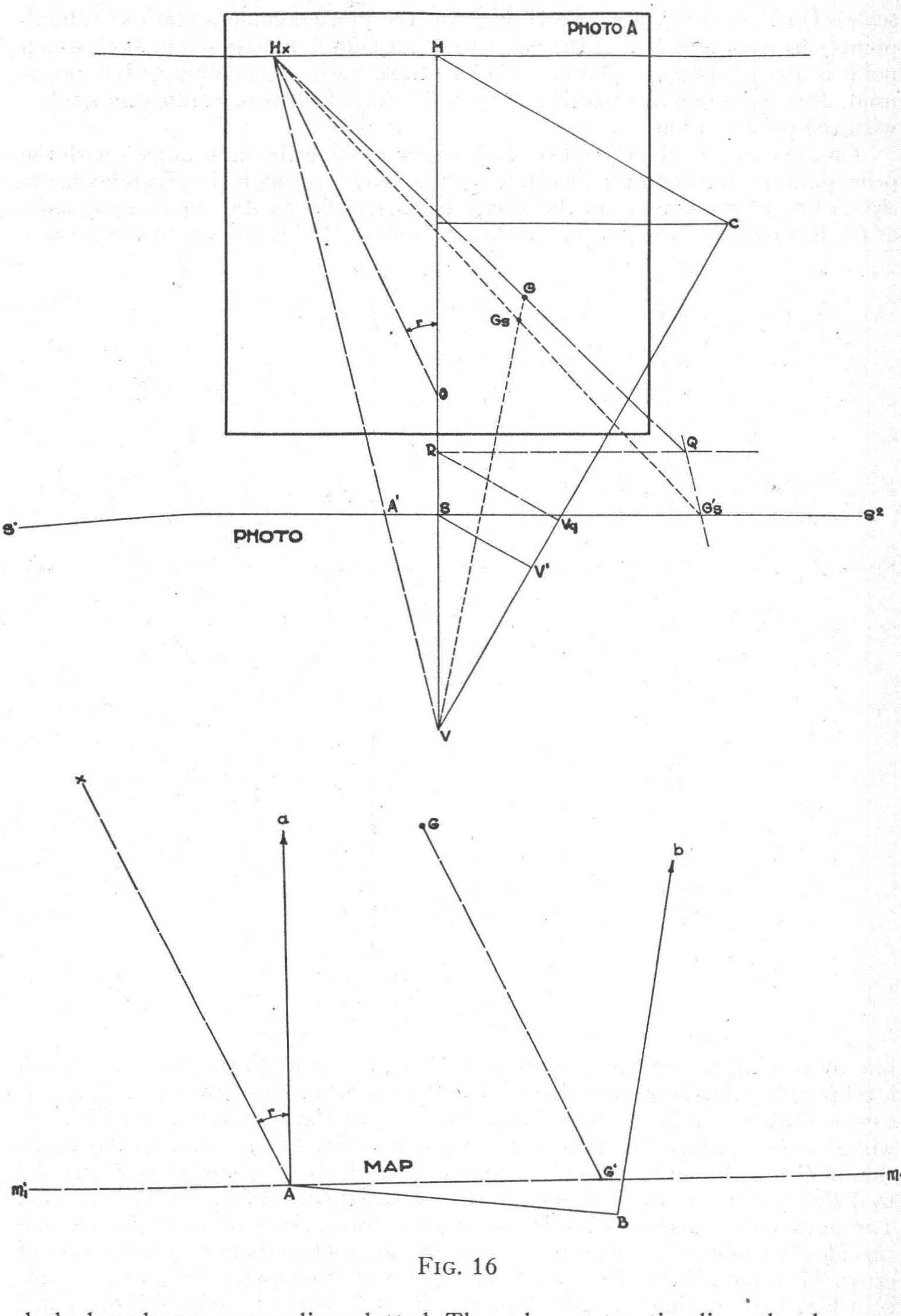

FIG. 16

dashed, and unnecessary lines dotted. The only construction lines that have to be repeated for each point measured are those corresponding to GG', measurement of $A'G'_s$, HxG, G'_sQ, QR and RVg.

The points to observe in understanding the proof of the above are as follows: On the map the direction of the line Ax from the camera plumb point is established by the orthocenter line OHx, on the photo, to the vanishing point—hence the lines HxG and HxG_s will both be seen on the map as the line GG' parallel to Ax. On the photo the line VHx is recognized as corresponding to VHx of Fig. 9, where it will be remembered EF and $E'F'$ were shown as being the scale measure of the distance between the two parallel lines in the direction of the isoscale line. But in the map, Fig. 16, the distance apart in such a direction is constructed as AG'. Hence on the photo the same distance (corresponding to EF of Fig. 9) is laid off as $A'G'_s$. The rest of the construction is strictly identical with Fig. 9.

The reader may possibly be left a little puzzled from all this as to what role is played by the second photo (Photo B) in this construction. The answer is that the role of Photo B is completely discharged by its share of the placement of the point G at the position on the map where it is found.

III. Constructions for Vertical Photos

The following operations on vertical photos become possible by virtue of our treating them strictly as obliques, and involve only certain appreciable effects

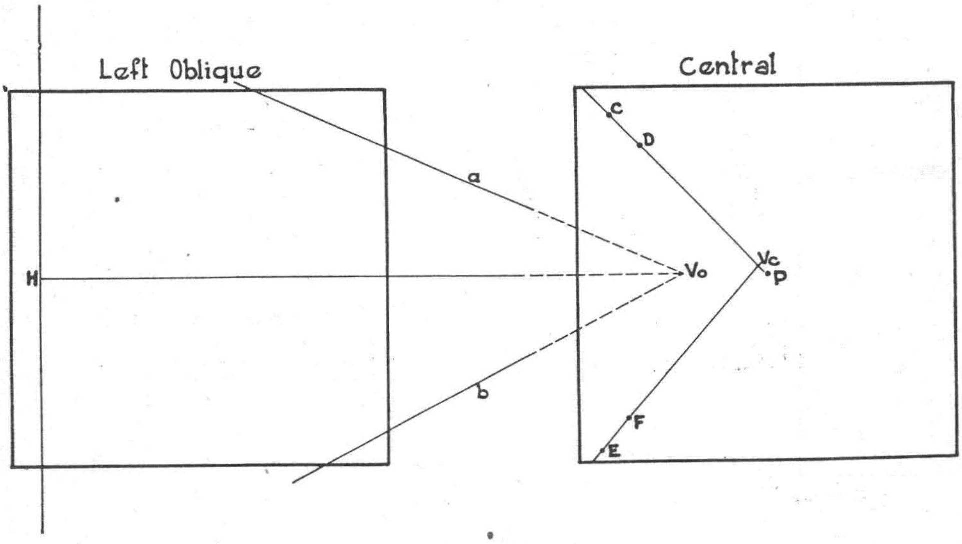

Fig. 17

of even very slight tilt. One of the operations given here furnishes a rather quick determination of the plumb point on ordinary vertical photos in the absence of control data, in other words, solving the tilt to a degree of precision that errors resulting from its use should be inappreciable. It is an approximation method, necessarily, in the total absence of control. The chief usefulness of these plumb point determinations is set forth in the last section of this paper in the operation for drawing radial lines (for templets, etc.) corrected for both relief and tilt.

K. Plumb Point on Trimetrogon Verticals

The simplest method of finding the plumb point on the central photo of a trimetrogon group of three, is illustrated in Fig. 17, showing the left oblique and central photos. On the oblique the plumb point Vo is found (perhaps by

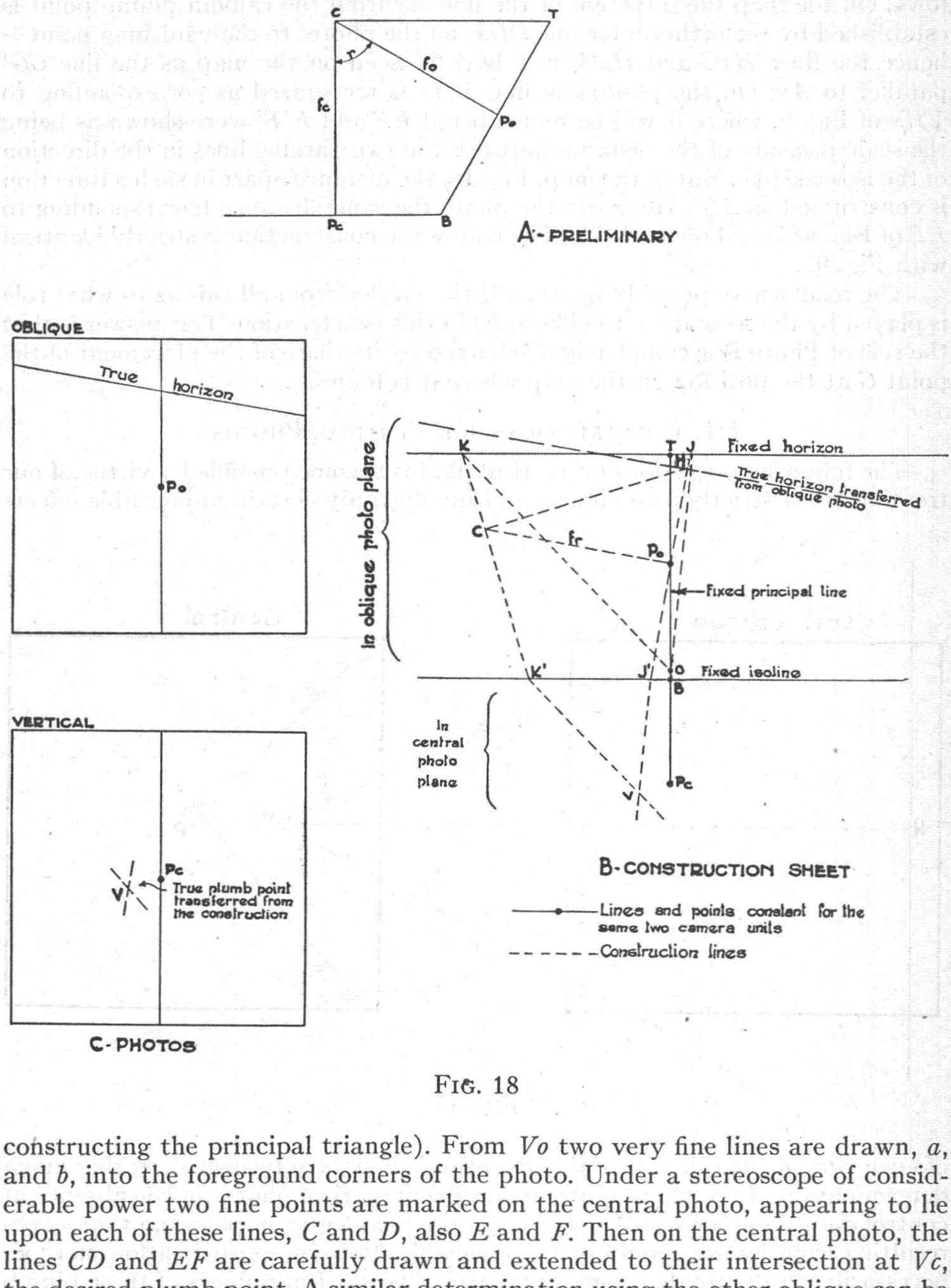

FIG. 18

constructing the principal triangle). From *Vo* two very fine lines are drawn, *a*, and *b*, into the foreground corners of the photo. Under a stereoscope of considerable power two fine points are marked on the central photo, appearing to lie upon each of these lines, *C* and *D*, also *E* and *F*. Then on the central photo, the lines *CD* and *EF* are carefully drawn and extended to their intersection at *Vc*, the desired plumb point. A similar determination using the other oblique can be made and an average point midway between the two intersections (*Vc*) so determined, may be adopted. However, if either horizon determination is considered more trustworthy than the other, the resulting plumb point determination on that side should be given preference.

Alternative method

An alternative method, somewhat longer but requiring no stereoscopic transfer of points, is as follows, and shown in Fig. 18. Preliminary steps are shown in part *A* of the figure. From a perspective center *C*, lay off the exact focal length of the central photo, $CPc(=f_c)$, and that of the oblique $CP_0(=f_0)$, making an angle, *r*, equal to the angle determined as fixed between the two camera axes. (This is 60° in correctly mounted trimetrogon cameras.) Draw a perpendicular to *CPc* through *Pc*, and another to *CPo* through *Po*, these meeting at *B*. Extend *BPo* to meet a parallel to *PcB* through *C* at *T*.

On a transparent construction sheet, part *B* of Fig. 18, lay off distances equal to *TPo*, *PoB* and *BPc* in sequence on a straight line, to be known as the "fixed principal line." Through this line draw indefinite perpendiculars at *T* and *B*. The one through *T* is the "fixed horizon line," and that through *B* the "fixed isoline." Lay off on the fixed principal line *TO* equal to *TC* of Fig. 18A.

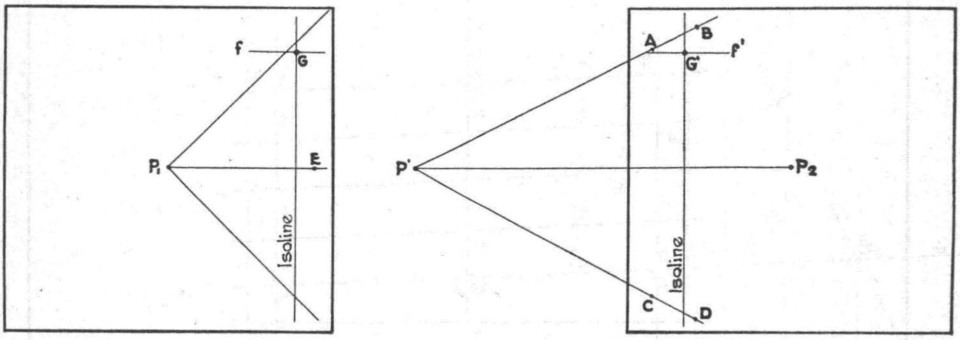

FIG. 19

(*O* is a "fixed orthocenter.") It is noted that all these preliminary constructions on Fig. 18B are constants for all photos taken by the same two cameras mounted as a unit.

Next on the photos, Fig. 18C, locate the principal points *Po* and *Pc*, through which draw "fixed principal lines" as shown, which are in line with the fiducial marks if the cameras are properly mounted, but at least are perpendicular to the isoline as determined on each photo.

(The isoline is best understood on Fig. 18A, where it is cut by the plane of the photo axes at the point *B*. The isoline is the common isoscale line of the two photos—being the intersection of the two photo planes when their mutual orientation is preserved and their perspective centers are brought into coincidence. Hence it is perpendicular to the plane of the two photo axes, and on either photo is perpendicular to the trace of this plane. If it is desired to determine or check the isolines, the following method may be used, illustrated in Fig. 19. On either photo draw two fine lines from the principal point P_1 into the two corners toward the other photo. Under a stereoscope of considerable power, mark two points on this other photo appearing on each of these lines, thus *A* and *B*, and *C* and *D*. Draw *AB* and *CD*, and find their intersection at P', thence draw $P'P_2$, and under the stereoscope mark a point *E* on the first photo seen on this last line, and draw P_1E. On each photo draw a short line *f* or f', these parallel to and the same distance from the respective central lines. Under the stereoscope mark the apparent intersection of *f* and f' as *G* and G'. The isolines pass through *G* and G' perpendicular to the central lines which are the fixed principal lines.)

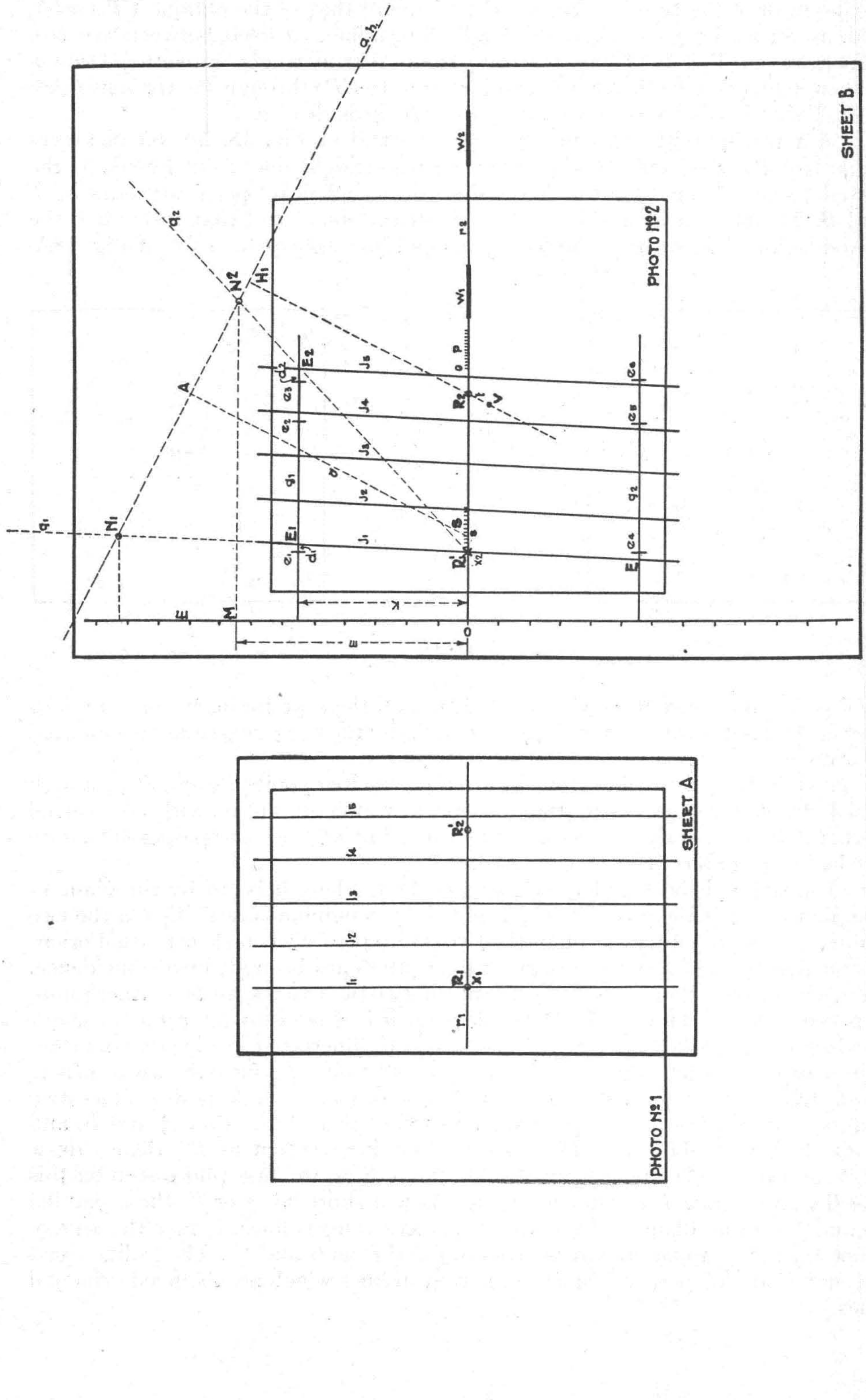
SHEET B
PHOTO No 2
SHEET A
PHOTO No 1
N1
N2
A
H1
E1
E2
M
S
V
w1
w2
q1
q2

FIG. 20

The photo isolines are not necessary in our construction—only the fixed principal lines. On Fig. 18B, the isolines, however, combine as a single line through *B*. The oblique photo must show the true horizon line (Fig. 18C). The remaining construction steps are the only ones that must be repeated for each photo. The construction sheet is laid over the oblique photo with *Po* and the fixed principal lines in coincidence, and the true horizon is traced. Then on the construction sheet (Fig. 18B) draw JJ', passing through *Po*, and perpendicular to this horizon line, intersecting it at *H*. Draw *PoC* equal to *CPo* of Fig. 18A, and perpendicular to JJ'. Draw *CH*, and then KK' perpendicular to *CH*, and passing through *C*. The points *J*, J', *K*, and K' are to lie on the permanent lines (fixed horizon and fixed isoline) as shown. Draw *JO* and *KO*, and to these last two respectively draw parallels through J' and K', whose intersection is *V*, the desired plumb point. This is transferred to the central photo (Fig. 18C) after placing the *Pc* points and fixed principal lines in register.

The proof of this construction involves recognition of the broken principal triangle on Fig. 18B, seen as $HCK'VJ'H$. The object is merely to find the lower apex *V* of this principal triangle, since we already know that this apex marks the photo plumb point. In going from above to below the fixed isoline we go from the oblique photo plane to the central photo plane. The points *J* and *K* are vanishing points on a fixed horizon which marks the trace of a fixed horizontal plane through the perspective center constantly parallel to the central photo. The orthocenter of this horizon plane falls at *O*, hence *OJ* and *OK* are direction lines that will apply in the central photo plane. Hence the two sides of the principal triangle change to these directions below the isoline and the triangle apex falls at *V*. While we actually found the plumb point of the oblique photo on the central photo, we know that the two photos share the same ground plumb point. It is noted that the point *O* would fall at *B* (Fig. 18B) if the two focal lengths (*fo* and *fc*, Fig. 18A) were exactly the same.

The reader is invited to try these instructive exercises: Repeat the construction of Fig. 18B with the principal triangle on the right side, then, with the left one also drawn, prove strictly by plane geometry that the three lines at *V* must necessarily intersect in one point.

L. Plumb Point Determination on Ordinary Vertical Photos

The writer has already published (Photogrammetric Engineering, Vol. IX, No. 4, pp. 214–224) a method of adjusting and manipulating a series of "floating lines" to appear in the stereoscopic view of vertical photos as a visible horizontal plane, capable of being apparently raised or lowered to intersect the topography at any desired level, and adjusted against the effects of tilt to appear horizontal with reference to all physiographic criteria of natural horizontality seen at any level in the topography. The equipment and operations now to be described enable one to find the point on the photo marking the foot of the perpendicular from the camera to this "floating-line" plane. This will come so close to the true photo plumb point, that, unless the floating-line adjustment were very badly made, it would be practically impossible to distinguish the two points. Since the aim has been to devise the simplest and most practical routine steps, those here established do not consist of graphical constructions alone, but contain measurements derived by slide-rule at points where the equivalent constructions would be more difficult.

Equipment

The equipment consists of two sheets (*A* and *B*, Fig. 20) whose approximate sizes and shapes with respect to the photos are as shown. These are both made of transparent film on which the following elements are photographically engraved: First there are the lines r_1, l_{1-5}, r_2, g_{1-2}, e_{1-6}, and m. An "x" is engraved at X_2. The line m is accompanied by an engraved metric scale graduated to millimeters, whose zero is at the line r_2 from which it reads in both directions. There is a short scale graduated to millimeters at s, whose zero is at X_2. These two scales should be clearly legible to the naked eye. A third scale, graduated to tenth millimeters and legible only under a 5 or 7 power magnifier, is placed along the line r_2 farther to the right, and is referred to as p (parallax scale). Its zero end is at the left side. The preparation of all these lines and scales on the sheets must be accomplished with extreme precision, that is, to the hundredth millimeter. The lines r_1 and r_2 should be very fine, and l_{1-5} should appear fine, but plain, in a low power stereoscope. These last must be exactly at right angles to r_1, and the marks e_{1-6} must be so placed that when Sheet A is superimposed over Sheet B they will fall exactly in register with the corresponding l lines when X_1 is placed over X_2. The lines g_1 and g_2 are equidistant from and parallel to r_2, their suggested distance being exactly 8 cm. from r_2, this distance being designated k, and used as a constant in one of the slide-rule equations.

Sheet B also contains two slots, w_1 and w_2, exactly centered along the line r_2, and of exactly the width of the thickness of ordinary pins. With a pin stuck into the table or board through each slot, the sheet may slide back and forth over the photo in the direction of r_2.

The several lines j_{1-5} shown on Sheet B are not part of Sheet B, but each consists of a straight engraved line of exactly the same strength as the l-lines, on an individual narrow strip of film. These are to be mounted to Sheet B by means of small pieces of cellulose tape, and while viewing under the stereoscope to be adjusted to appear to fuse with the l-lines to produce the "floating-lines." (This operation is described later.)

Selection of points R_1 and R_2

The constructions to follow become theoretically exact when the photo points R_1 and R_2 (Fig. 20) are the isocenters, but a close approximation of results is achieved by using other points near the isocenters when these are not known. The differences in results are inappreciable in photos having only the average amount of tilt—but when the first plumb-point determination reveals more than average tilt, the operations can be repeated using for R_1 or R_2 the isocenter location resulting from the first determination.

An idea of the order of magnitude of error of the plumb point resulting from using a point R other than the isocenter is seen in the proportion (not a true equation) $(VV'/VP)=(IP/PR')$, in which V and V' are true and determined plumb points, I true isocenter, P principal point used as R, and R' transferred R-point from the other photo.

As a routine procedure, a row of consecutive photos should be worked as follows: Begin by a preliminary plumb-point determination on photo 2 from photo 1 by using the principal point on each one as R. This will give a preliminary isocenter on photo 2—taken midway between the preliminary plumb point and the principal point. Then make a determination on photo 1 from photo 2 using the preliminary isocenter of the latter and the principal point of the former. Unless the tilt of photo 1 is very great, this determination can be accepted as final. Then make a second determination on photo 2 from photo 1, using isocenters. Then, in order, make determinations on 3 from 2, on 4 from 3, etc., each time using the isocenter of one photo and principal point of the other, repeating with isocenters of both only in cases of unusual tilt.

Preparation of photos

The preparation of the photos requires marking on each one the point to be used for R (which will usually be the principal point in the first instance), marking it by a very fine needle prick, and transferring to the opposite photo very exactly under a magnifying stereoscope. Small circles around the points will show their positions.

The two photos are mounted for stereoscopic viewing, with the lines R_1R_2' and $R_1'R_2$ approximately in line. Sheet A is then mounted over photo 1 with the point X_1 exactly over R_1, and the line r_1 exactly over R_2'. Sheet B is not mounted rigidly over photo 2, but pinned through the slots w_1 and w_2 to permit sliding, with the line r_2 always exactly over R_1' and R_2. But previous to installing sheet B, an index marker, consisting of a fine black line against a white background, must be mounted on the photo under the average position to be taken by scale p. All readings made on this scale, are of this index.

Adjustment of floating lines

Figure 20 shows a total of five lines to appear as floating lines to mark the horizontal plane to which the plumb point is determined. The adjustment is made by individually shifting each of the lines j_{1-5} until they constitute a visible horizontal plane (seen fused with lines l_{1-5}), which agrees with topographic criteria at any level or in any part of the stereoscopic model. A more detailed discussion of such criteria is given in PHOTOGRAMMETRIC ENGINEERING, Vol. IX, No. 4, pp. 214–220. In the course of the adjustment, the floating-line plane may be freely raised or lowered as a whole by sliding the sheet, thus bringing it to the actual level of the topographic features against which it is checked or adjusted. The aim is not to adjust it merely to look level, because the tilt of the photos may make this untrustworthy, but to make it look parallel to the natural level as interpreted in the topography. There is but one restriction in the adjustment—the first line, j_1, must pass over the point X_2.

Satisfactory adjustments should be possible with only four or three floating lines more widely spaced, instead of the five shown. The writer has illustrated with five because this is the best equipment when the floating plane is used also for contouring according to the method described in the article mentioned just above.

Parallax readings

After the movable lines, j_{1-5}, are adjusted, a naked-eye inspection is made at the marks e_1, e_3, and e_6 to select the two of these at which the greatest parallax displacements of the lines appear. On Fig. 20, this displacement is obviously greater at e_1 and e_3 than at e_6, so this last is rejected, and measurement is to be made at only the first two. In each case the measurement (d_1 or d_2) is the difference between two readings made to the hundredth millimeter on scale p under a 5 or 7 power lens—one reading made when the floating line (j_1 or j_5) at the intersection with g_1 is brought (by sliding the sheet) to appear at the level of the ground, and the other when the mark e_1 or e_3 is fused with l_1 or l_5 and brought to appear at the level of the ground. We will designate by E_1 and E_2 the points at the intersections of g and j lines.

Artificial horizon

After the readings for d_1 and d_2 are made, the sheet B is slid to bring the floating lines to the approximate level of R_2 and fastened. The pins are removed from w_1 and w_2, and a sheet of transparent paper is laid over the whole (this sheet somewhat larger than sheet B, especially extending farther to the left). This sheet may be the same one used later as a templet for the corrected radial lines (last section of this article); in fact, the determination of the plumb point and drawing the radial lines become one continuous process.

Starting from X_2, two indefinite lines are to be drawn, q_1 and q_2, with the ruler held against the points E_1 and E_2 respectively, but observing this rule: If the E-point lies to the right of the corresponding e line, q extends in the same direction as that E point from X_2 (the case of both q lines of Fig. 20). But if E lies to the left of e (true for instance at e_{4-6}) the corresponding q line would start at X_2 and go in the other direction. It is possible for one q line to take one direction while the other takes the other direction. Suppose, in the particular set-up of Fig. 20, a parallax determination were made at e_4. In the first place this should give the same value of d as at e_1. But at e_4 the position of E would be to the left, and the q line would have to extend in the opposite direction, that is, in the direction shown for q_1—so the use of a parallax measure made at either e_1 or e_4 amounts to precisely the same thing.

The next construction step is to find the points N_1 and N_2 on q_1 and q_2 respectively through which to draw the artificial horizon, $a.h.$ Any such point, N, is to be located opposite a reading on the m scale, found quickly by the slide rule by this formula:

$$m = \frac{sk}{d} \cdot \tag{1}$$

k is a constant, being the distance between the g and r lines, d is the parallax difference in question, and s is any convenient small number, usually 1, but may be 2 or 3, or .5, .4, etc., selected to enable the formula to yield locations for N_1 and N_2 conveniently placed upon the construction sheet. The same value for s must be used in the computation of each N point. Having found the point on the m scale on the same side of r_2 as the q line, find N by running a normal line out from the scale to the q line.

Finding the plumb point

The plumb point V will be found on a line drawn through P_2 (the principal point of photo 2) perpendicular to the artificial horizon $a.h.$ The distance from P_2 to V, t, measured on the other side of P_2 from $a.h.$, is computed on the slide rule by this formula:

$$t = \frac{f^2 s}{ab} \cdot \tag{2}$$

f is the focal length of the taking camera. s has the same value as used above. a is the length of a line, SA (Fig. 20), drawn perpendicular to $a.h.$ from S, a point on the s scale expressing in millimeters the value of s. b, to be theoretically exact, is the distance X_2P_2 when the floating lines appear at the level of the ground at the plumb point to be determined, but the distance X_2R_2 can be used for b with no appreciable difference in results in most cases.

In both the formulas given, all measurements are to be in millimeters.

Explanations and proofs

A considerable part of the underlying theory of the above method of plumb-point determination is seen in Fig. 21, which repeats the main construction lines of Fig. 20, greatly reduced to permit extending these to the true horizon, $t.h.$

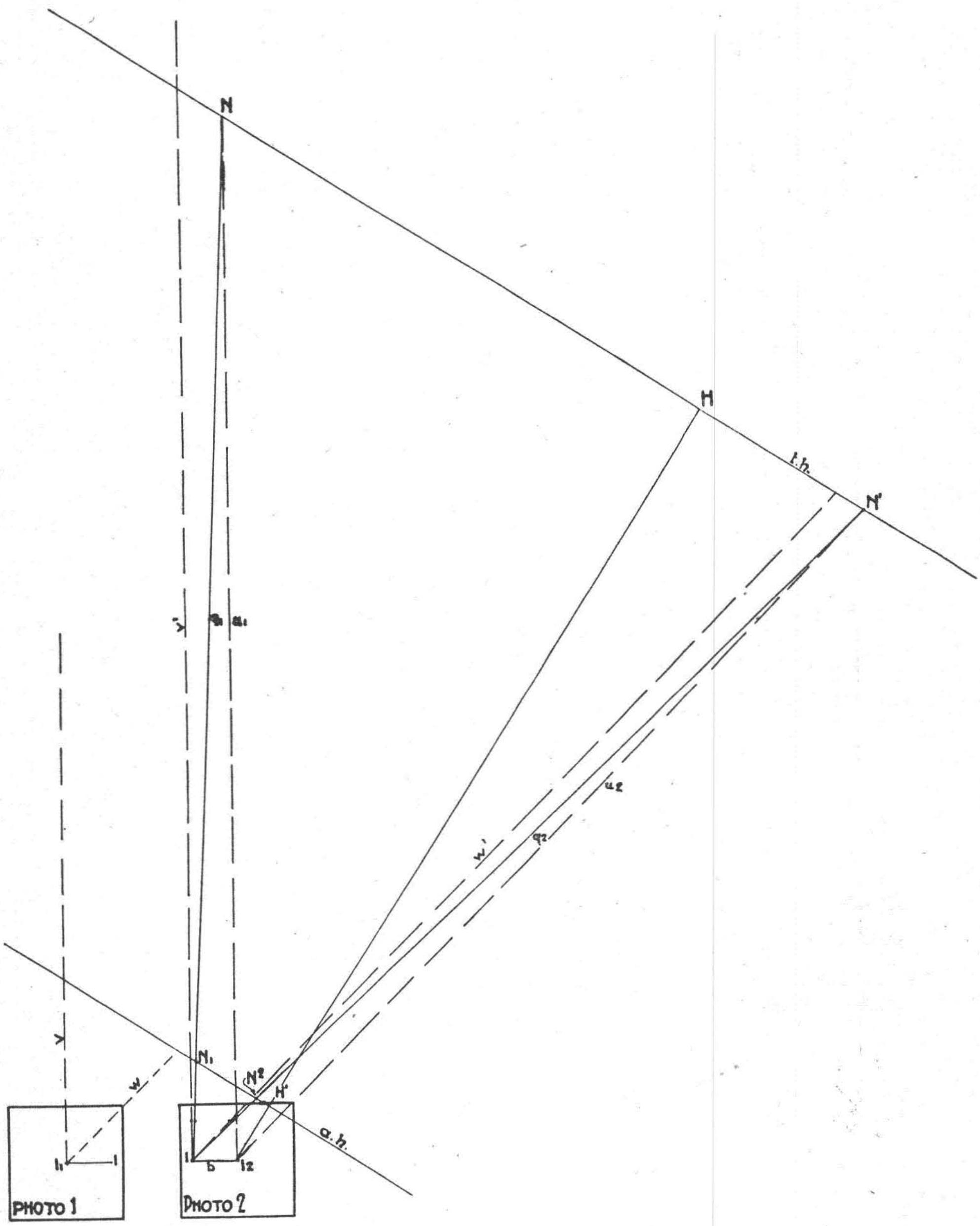

FIG. 21

On photos 1 and 2, both, we have a common base line II, the points I being isocenters or approximately so. On photo 1 the line v (l_1 of Fig. 20) makes a right angle with this base. This is interpreted as representing a true right angle on the ground because the angle is made at the isocenter. On photo 2 we may consider a line u_1 drawn from I_2 at right angles to the base, and interpret it as representing a ground line parallel to v. But v is transferred stereoscopically to photo 2, not as the parallel v', but actually as the slightly oblique line q_1, which is

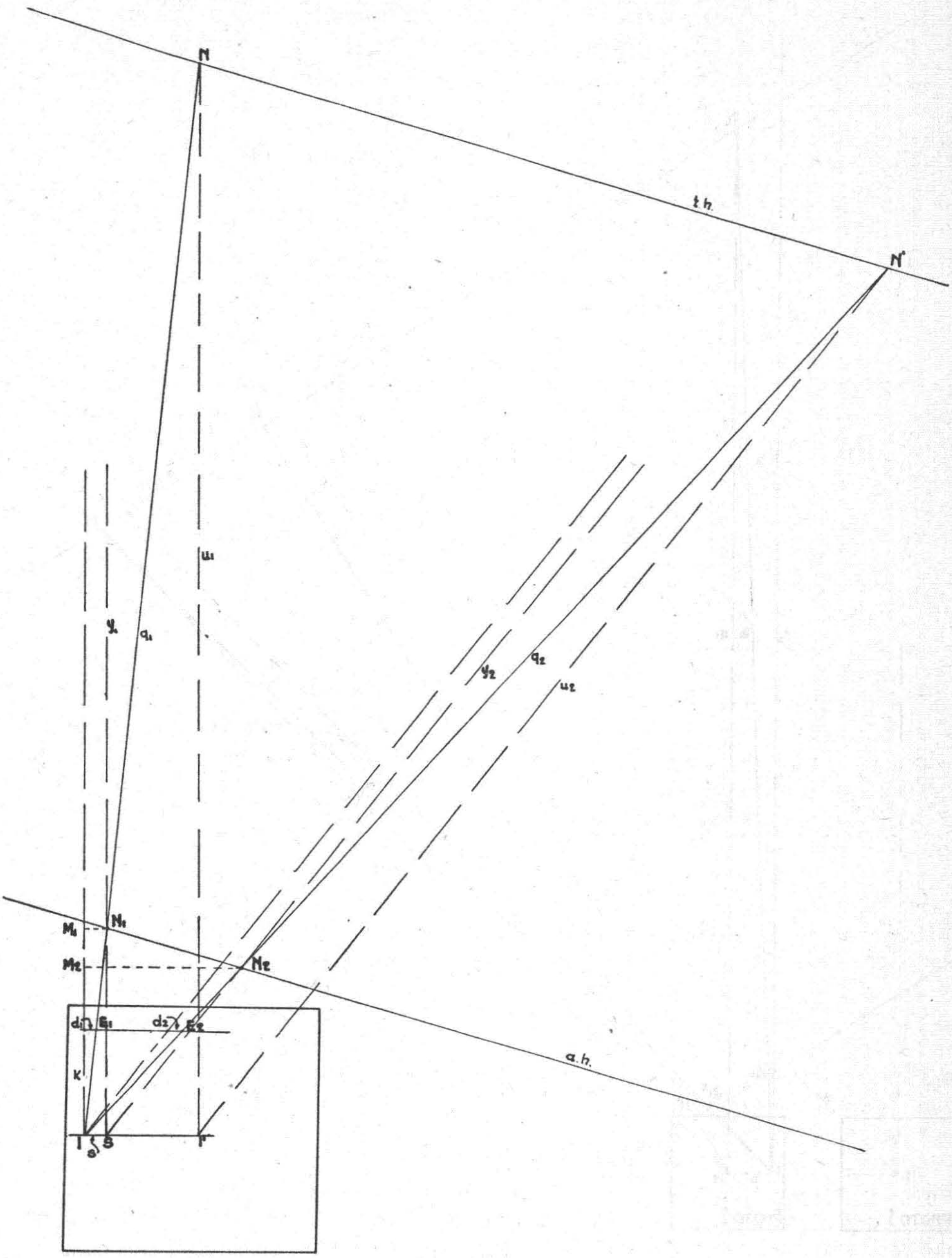

FIG. 22

interpreted as representing a parallel to u_1; and the fact that they converge, meeting at N, indicates that N is their vanishing point, and must lie on the true horizon of photo 2. In similar manner, a second point on the horizon, N', is found by the intersection of q_2 and u_2, the latter drawn from I_2 parallel to w' or to w.

As the true horizon is situated at too great a distance to be handled on the construction sheet, we must obviously work with an artificial horizon, *a.h.*, parallel to it, and capable of showing all its geometric properties proportionately reduced. The manner of making this reduction by means of formula (1) for m, is best shown by a separate diagram, Fig. 22. This contains the principal elements of Fig. 21 with the addition of the selected point S on the base line, at a distance s from the left isocenter, and of the two lines y_1 and y_2, drawn through S parallel respectively to u_1 and u_2. The intersection of q_1 and y_1 gives N_1 and the intersection of q_2 and y_2 gives N_2. Similar triangles are easily seen giving these proportions:

$$\frac{IN_1}{IN} = \frac{IS}{II'} = \frac{IN_2}{IN'},$$

proving that N_1N_2 is parallel to NN' and is hence an artificial horizon. But it is not desirable to use the method of intersecting q_1 and y_1, or q_2 and y_2, to locate N_1 and N_2, because there is usually too small an angle between these lines. It is better to find measures for the distances IM_1 and $IM_2(m_1$ and $m_2)$, and use these to locate N_1 and N_2 on q_1 and q_2. Similar triangles will be seen giving

$$\frac{m_1}{k} = \frac{s}{d_1}, \quad \text{also} \quad \frac{m_2}{k} = \frac{s}{d_2}, \quad \text{hence we get} \quad m = \frac{sk}{d}. \tag{1}$$

Figure 23 illustrates what might be called the strategy used to insure the capture of an artificial horizon no matter what direction it might take, or side of the photo on which it might lie. With only three places, A, B, and C, at which a parallax difference need be measured, we are provided with six different directions, represented by the indefinite lines a, b, c, a', b', and c', along which an artificial horizon may be intercepted. But it takes only two such lines to intercept any given horizon, and hence only two such parallax measures are required. The two best to use will be those two on which the intercept falls closest to the central point I. But this very proximity of the horizon produces a relative increase in the parallax displacement on these two lines compared with the third line. Hence a preliminary inspection of the displacements will always permit a successful selection of two for measuring.

Fig. 23 also illustrates the relation of a parallax measure at B pertaining to an artificial horizon h which intercepts b' in the opposite direction from B. In the first place in regard to the amount of the parallax displacement at B, it can be shown that this is the same amount as it would have been on the same side as h, or at B', were it possible to measure here. Let us assume that an artificial horizon point at N would require a given parallax measure at B', this being the distance between the displaced line and a true direction line. But both these lines are straight lines and pass through I. Hence, extended an equal distance beyond to B, their separation would again be equal to that at B'. Hence the parallax associated with N in one direction can be measured at B lying in the other direction.

Lastly from Fig. 23 we can show the reason for the rule which tells whether N lies in the same or opposite direction from A, B, or C. If we draw a line through the other isocenter point I', as g, parallel to any of the true direction lines associated with a, b, or c, it is seen that such a line g in every case lies to the right of the true direction line. But the displaced line, a, b, or c, converges toward g, and hence lies on the side toward it, as you go toward the horizon, where it meets g at a vanishing point. Therefore, the horizon lies in the direction of a convergence (to the right), or opposite from a divergence (to the left).

The derivation of the formula for t is the last thing to explain (Fig. 24). The use of the formula theoretically requires that the factor b be measured along a line whose right end is the principal point, not the isocenter. In routine work the line $R_1'R_2$ (Fig. 20) is drawn with R_2 at the principal point (P_2). But if R_2 is at the isocenter, a separate line may be drawn from X_2 to P_2 and this used for b in the formula, and the point S shifted to this line. This is mentioned for theoretical accuracy, while in practice the difference in final result yielded by this change is inappreciable, and this step may be ignored.

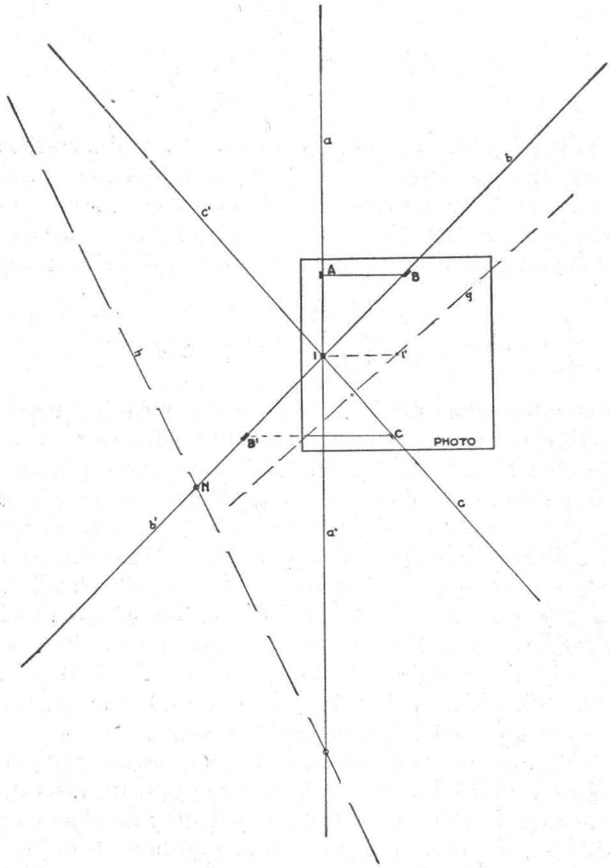

FIG. 23

Figure 24 shows the derivation of the formula. In addition to lines used on Fig. 20, we have added the true horizon, the line HX, and the principal triangle HCV, the last accompanied as usual by the graphical measure of the focal length f. By similar triangles it is seen that $t/f = f/a'$, where a' is the distance PH. We also see that $a'/a = b/s$. The first equation becomes $t = f^2/a'$, and the second $a' = ab/s$. Substituting for a' we get $t = (f^2s/ab)$ (2). (Exercise for the reader: Prove the assumption that HX passes through A.)

M. Plotting of Fully Corrected Radial Lines

The determination of the plumb point on a vertical photo is logically followed by radial-line templet making, in which this plumb point is used for the radial center, and all radials are completely freed from effects of both tilt and relief. The steps described below may economically follow on the same trans-

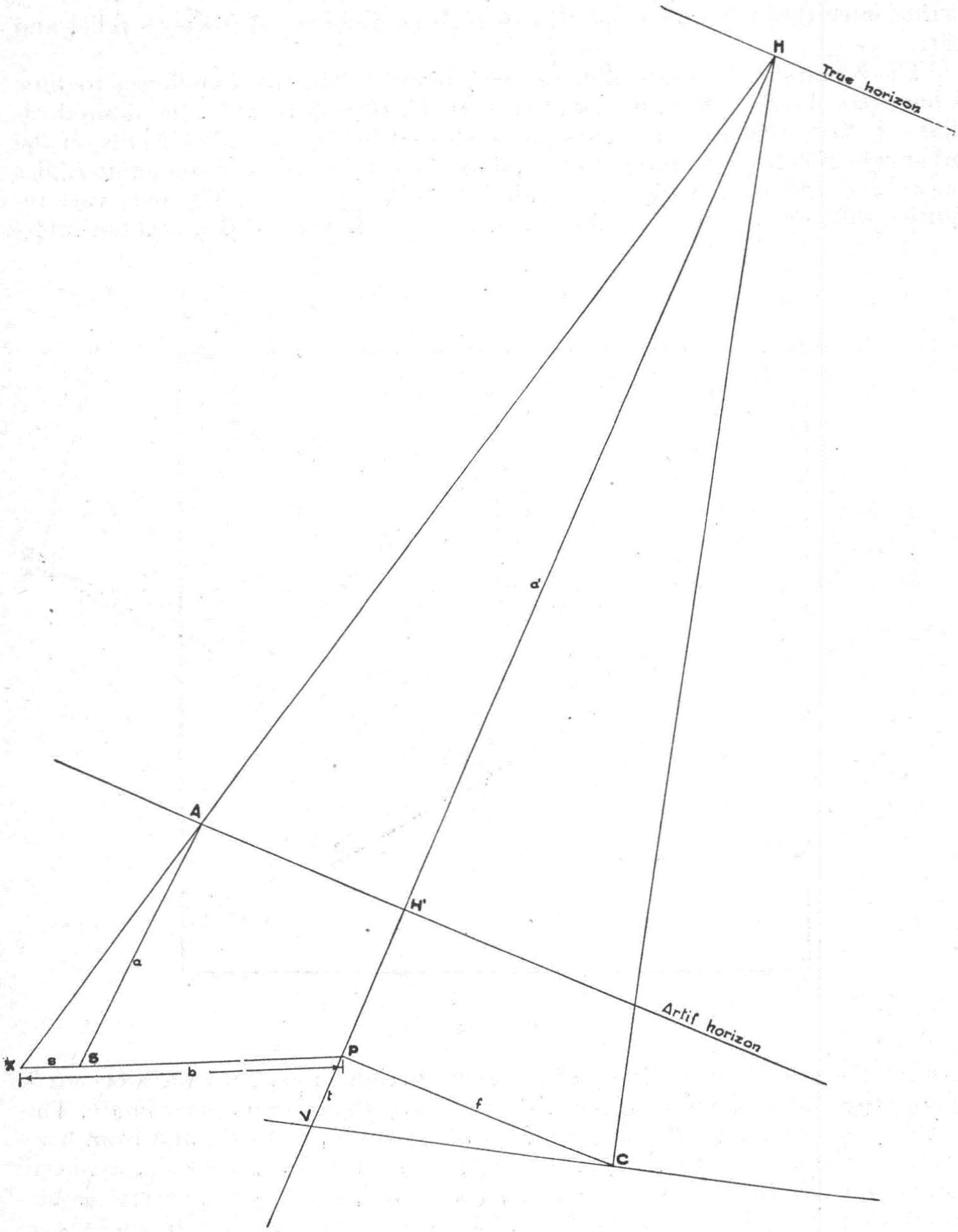

FIG. 24

parent overlay sheet used as described above for drawing an artificial horizon and plotting the plumb point.

In Fig. 25 it is assumed that the plumb point has been located at V, with the principal point at P, the principal line *p.l.*, and an artificial horizon h, intersecting the *p.l.* at H. Let A, B, D, and E be four points to which radials are to be drawn. The construction is as follows:

Draw PC equal to the focal length f, and perpendicular to the *p.l.* Draw CV. Drop a perpendicular HQ to CV, and on the *p.l.* lay off $HO=HQ$. O is the orthocenter through which radials can be drawn corrected for both relief and tilt.

The common case will be illustrated by point A. The rule to follow is to draw a line from V through A, intersecting h at A', then draw OA', the desired radial (a). Note that in some cases, as illustrated by D, the radial d falls on the other side of O from h. In other cases, illustrated by E, the horizon point E' lies between E and V. But these cases all follow the rule given. The only case requiring different treatment is that illustrated by B, where VB would not inter-

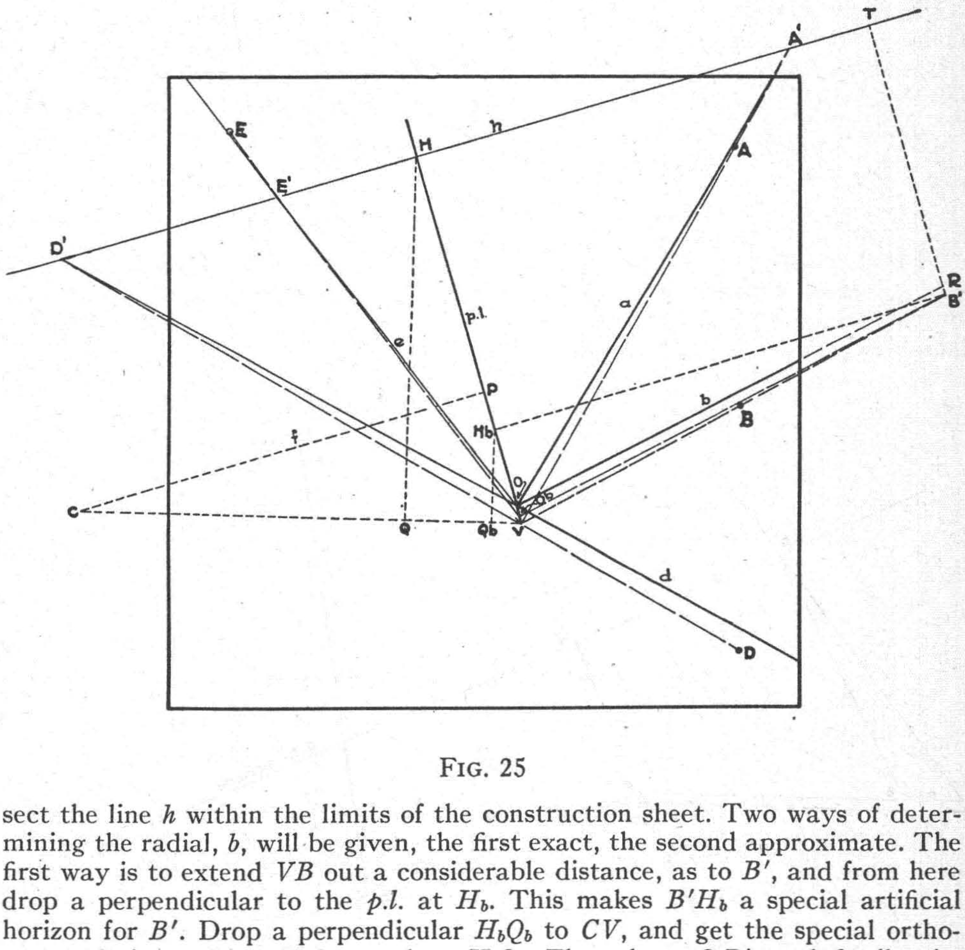

FIG. 25

sect the line h within the limits of the construction sheet. Two ways of determining the radial, b, will be given, the first exact, the second approximate. The first way is to extend VB out a considerable distance, as to B', and from here drop a perpendicular to the *p.l.* at H_b. This makes $B'H_b$ a special artificial horizon for B'. Drop a perpendicular H_bQ_b to CV, and get the special orthocenter O_b by making H_bO_b equal to H_bQ_b. Then draw O_bB', and finally the desired radial b through O parallel to O_bB'. The second, approximate, method is to draw or estimate by eye the position of a line $B'T$ perpendicular to h, on which to estimate a point R cutting $B'T$ in the same proportion by which O cuts VH. Then draw b as OR.

It is emphasized, to avoid any possible misunderstanding, that the radial center on the templet for the corrected lines a, b, d, and e, is the orthocenter O, and that this orthocenter is actually an orthographic projection of the plumb point V, and hence this radial center corresponds to the camera plumb point on any templet assembly, control plot, or map.

The proof of this construction merely requires familiarity with the method of horizontal direction determination, illustrated on Fig. 10, in the case of point A, where the operation consisted of drawing the line VA' through A, thence $A'O$, the last line giving the direction. The only difference is in the use of an artificial horizon for the true horizon. One should visualize, on Fig. 25, a horizontal plane passed through h, then swung down into the photo plane using h as a hinge. The plumb line intercept of this horizontal plane, first seen at Q, is brought into the position O. A given photo line through V, as VA, is seen projected into the horizontal plane as OA', because V projects to O, and A' is common to both lines. Hence OA' gives the true direction of VA.

It is pointed out that in this construction using an artificial horizon, the point O does not fall at the isocenter. The reader might find it instructive to find the isocenter I, in Fig. 25 (by a bisector of the angle at C), then to prove that QO is parallel to CI, and also that the true horizon will cross the *p.l.* at a point H' making $(H'H/HV) = (IO/OV)$.

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