LOGARITHMIC FORM FOR RAPID COMPUTATION OF THE OBLIQUE GRID

Sherman A. Wengerd, Lieutenant, U.S.N.R.

THE METHOD of mapping from oblique aerial photographs which has been developed by the Canadian Topographical Survey and the later trimetrogon method as developed in the Alaskan Branch of the U. S. Geological Survey have proved that the mapping of large areas quickly, inexpensively, and relatively accurately with oblique aerial photographs is entirely feasible. Both methods process large numbers of photographs quickly and efficiently with a minimum of attention to any single oblique used. Admiration which many workers hold for these mass operations resulting in small scale maps of so much of the earth's surface in such a short time with so little control is adequately registered in any photogrammetric discussion. Many useful attributes of single oblique aerial photographs have virtually been forgotten in the mêlee of total war. Single aerial obliques have not been more fully exploited for accurate location of control points on one datum plane for several reasons, namely: A single photograph seldom covers enough area—or enough of the area to be immediately controlled—to warrant spending much time or effort; a number of photographs covering an area to be controlled require computation and construction of several grids to gain the accuracy desired; the computations as described in standard reference texts look more involved than they really are; few people consider the single aerial oblique worthy of much attention for precise mapping because relief makes simple graphic plotting of detail impossible. For certain special control problems and for the individual worker who must do his own photogrammetry prior to field work, data from a single oblique aerial photograph can be an invaluable aid and shortcut.

The theory, computation, and construction of the oblique grid have been presented in as many variations as there are authors who have mentioned oblique mapping:

(Canadian Topographic Survey 1928 and 1932; McCurdy 1940; Breed and Hosmer 1940; Bagley 1941; Sharp 1942; Eardley 1942; Smith 1943; Church 1944; Forbes 1944)

Little original research has been published on the Canadian grid since the 1928 and 1932 papers of the Canadian Topographic Survey. Perusal of the above mentioned publications will give any student an excellent background in formula derivation and theory of the oblique grid, hence few repetitions will be made in this short paper. A lucid outline for computation and construction of grids is found in McCurdy's "*Manual of Aerial Photogrammetry*" 1940. The symbolism used in that work has been followed rather closely in the computational form presented here.

Design of an accurate easily-worked computational form for the oblique grid was initiated by the writer in the Pacific area when it was anticipated that it would be necessary to lay many battle mosaics of poorly charted islands and atolls for which no accurate ground control was available. It was found that most of the islands and atolls have fairly accurate observation spots and are correctly oriented on the charts available. Detail, except the more prominent capes and hills, was found to be so poorly located that radial triangulation of vertical photographs was virtually impossible unless an inadequate number of control points was chosen so that much freedom could be exercised in making the templates fit. It was found, however, that the observation spot plus another

GRAPHIC RECTIFICATION OF OBLIQUE PHOTOGRAPH

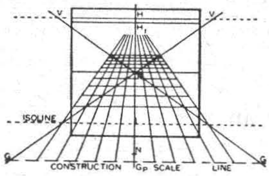
Date *18, August, 1943* Area *Frobisher Bay-Baffin Island* Photograph *0-24*

COMPUTATIONS

Formulae

1. $D = K\sqrt{A}$
2. $\tan\theta_1 = \frac{PH_1}{f}$
3. $\theta = D + \theta_1$
4. $PH = f \tan\theta$
5. $HG_P = \frac{A \sec\theta}{S}$
6. $PG_P = HG_P - PH$
7. $HV = f \sec\theta$
8. $G_PG = \frac{HV}{PH} PG$
9. $\lambda = \frac{1}{2}(90° - \theta)$
10. $PI = f \tan\lambda$
11. $PN = \frac{f}{\tan\theta}$

Diagram



P = principal point
H_1 = apparent horizon
H = true horizon
I = isocenter
N = nadir (plumb point)

Scale along isoline equals $\frac{f}{H}$

Symbols		Values
f	focal length	*6.098* inches
A	altitude	*10,140* feet
K	constant	58.82
PH_1	distance P to H_1	*3.215* inches
D	dip of horizon	*1°39'52"*
θ_1	apparent depression	*27°47'58"*
θ	true depression angle	*29°27'50"*
PH	distance P to H	*3.445* inches
S	scale in feet to 1 inch	*1,000* feet
HG_P	distance H to G_P	*11.917* inches
PG_P	distance P to G_P	*8.472* inches
HV	distance H to V	*7.004* inches
G_PG	distance G_P to G	*17.224* inches
λ	one-half Tilt	*30°16'05"*
PI	distance P to I	*3.559* inches
PN	distance P to N	*10.794* inches

log K	1.769 523	(1)
log $\sqrt{A}$	2.008 020	
log D	3.777 543	
D	5992 seconds	
D	1°39'52"	

log PH_1	0.507 181	(2)
colog f	9.214 810	
log tan θ_1	9.721 991	
θ_1	27°47'58"	

D	1°39'52"	(3)
θ_1	27°47'58"	
θ	29°27'50"	

log f	0.785 190	(4)
log tan θ	9.752 002	
log PH	0.537 192	
PH	3.445 inches	

log A	4.016 040	(5)
log sec θ	0.060 147	
colog S	7.000 000	
log HG_P	1.076 187	
HG_P	11.917 inches	

HG_P	11.917	(6)
PH (−)	3.445	
PG_P	8.472 inches	

log f	0.785 190	(7)
log sec θ	0.060 147	
log HV	0.845 337	
HV	7.004 inches	

log HV	0.845 337	(8)
log PG_P	0.927 990	
colog PH	9.462 808	
log G_PG	1.236 135	
G_PG	17.224 inches	

90°	89°59'60"	(9)
θ (−)	29°27'50"	
(divide by 2)	60°32'10"	
λ	30°16'05"	

log f	0.785 190	(10)
log tan λ	9.766 115	
log PI	0.551 305	
PI	3.559 inches	

log f	0.785 190	(11)
log cot θ	0.247 998	
log PN	1.033 188	
PN	10.794 inches	

Slide rule checks

Notes: *Computations for a photograph with hypothetical focal length, altitude, and PH_1 distance.*

Computer *Wengerd* Date *5-25-45*

FIG. 1. Logarithmic computation form for the graphic rectification of a single oblique aerial photograph. Computations made for a photograph taken at 10,140 feet; 6,098-inch metrogon lens; PH distance 3.215 inches; to be plotted on a rectilinear grid to a scale of 1-inch equals 1,000 feet.

carefully chosen point, with the resulting scale and line orientation, taken from a chart inaccurate in detail, coupled with numerous points gained from an accurately computed and carefully constructed oblique grid laid over a good horizon oblique, gave control which could be gotten in no other way over enemy territory. The distribution of enough control points so gained and placed upon an accurate polyconic projection made feasible the radial triangulation of vertical photographs which otherwise might not have had so much value in an uncontrolled stapled mosaic. If no time was available for radial triangulation, enough points could be picked so that print-to-print ratios could be closely controlled and angular discrepancies inherent to the matched mosaic avoided. If still more speed was necessary in mosaic control prior to assembly, the shorelines could be sketched quickly onto a rectilinear projection with the aid of an oblique grid and the vertical photographs laid to that shore line at the flight scale.

Preliminary tests proved that an oblique photograph, showing a clear or easily reconstructible horizon, plus an altitude corrected in flight through the application of high altitude pressure-temperature tables, and an accurately calibrated focal length, yields a wealth of good control data. Specific tests of ground control points triangulated in the Arctic and picked on 6 inch oblique photographs with distinct horizon, of known altitude, and known focal length, proved that distances and angular relations between stations located from within one-half inch of the bottom to one inch beyond the principal point of nine by nine inch aerial obliques taken at 10,000 foot altitude could be relied upon to be accurate enough so that only a plane table triangulation net or geodetic triangulation would detect errors. It was found to be expedient to plot control points to a scale at least 50% larger than the isoline scale after corrections for curvature and refraction were applied to each point as outlined by Forbes, PHOTOGRAMMETRIC ENGINEERING, Vol. X, No. 3, pp. 202–205. Forbes has found that distances from nadir to selected sea level points could be gained quickly to within an accuracy of 1% far beyond the principal point on aerial obliques with sharp horizons, PHOTOGRAMMETRIC ENGINEERING, Vol. X, No. 3, pp. 204–205. Construction of the final mosaic at a scale equal to or smaller than the isoline scale was found advisable.

The specific tests made by the writer involved measurement, computation, construction of the acetate grid, and plotting of points on linen-backed boat sheet paper in half a day under stable temperature and relative humidity. The points were later plotted on a polyconic projection laid out on vehisote board which has a negligible coefficient of expansion. Utilization of an accurate form which could be worked quickly with a minimum of theorizing was considered essential in completing the work in the shortest possible time. Choice of logarithms for computation was made because slide-rule and long hand computation using simple functions are not conducive to accuracy without careful thought during every step involved in rapid computation of the eleven basic formulae. Derivative theory during computation slows down greatly the completion of the task for all but the few who can carry complex numerical sequences mentally while operating the twenty-inch slide-rule necessary to attain accuracy commensurate with that gained in using logarithms. The form, if worked using six-place logarithms, gives results within .001 of an inch without interpolation and can easily be completed in 15 to 20 minutes by one trained in rapid handling of log tables. It is believed that enough data is given in descriptions and diagram on the form to enable a worker to deduce derivation of the formulae should none of the standard photogrammetric references be available. Further explanation of the terms and an order of grid construction will be found best given by McCurdy, Manual of Aerial Photogrammetry, pp. 69–70.

The computation sheet is divided into seven main parts as follows:

1. The heading.
2. Formulae in order of solution are placed in the upper left; each formula is numbered to coincide with a number found in the upper right corner of each computation "box."
3. A diagram is placed in the upper center with basic definitions.
4. Symbols with short description are found on the upper right with a blank column into which are entered four basic values and the computed values.
5. A large computational section consisting of eleven "boxes" occupies the central area. Formulae are arranged in flow sheet order to insure speed and accuracy.
6. A small section labelled "*Notes*" is placed at the bottom left for miscellaneous figuring.
7. A slide-rule check section is included on the lower right.

Subtraction of numbers is an operation more subject to human error than addition of numbers in any geodetic or photogrammetric computation involving logs. Cologs and reciprocal functions have been used wherever possible in the formulae to speed up computation. It will be noted that choice of the isoline as construction scale line results in plotting directly to isoline scale of the photograph and further that values PGp and PI should be exactly the same. If the isoline is always chosen as construction scale line, formula #10 may be left unsolved, or can be solved as a check. Though choice of the isoline as a construction line for direct graphic plotting of oblique lines on the acetate grid is mathematically accurate, actual mechanical difficulties in drawing or scratching fine lines from short construction line intercepts to the point H almost necessitate use of a construction scale line at right angles to the principal line some distance below the isoline.

Selected Bibliography to Which Reference Is Made

1. Canadian Topographical Survey 1928. A Graphical Method of Plotting Oblique Aerial Photographs. *Bull. of the Topographical Survey.* Dept. of the Interior, Dominion of Canada.
2. Canadian Topographical Survey 1932. The Use of Oblique Aerial Photographs for Mapping. *Topographical Survey Bulletin* No. 62, Dept. of the Interior, Dominion of Canada. (Reprinted in: "Manual of Photogrammetry" American Society of Photogrammetry, pp. 578–596, 1944.)
3. McCurdy, P. G., 1940. *Manual of Aerial Photogrammetry,* Chapter VII, Oblique Mapping, pp. 65–76. Hydrographic Office, U. S. Navy Department, Washington, D. C.
4. Breed, Charles B., and Hosmer, George L., 1940. *The Principles and Practice of Surveying.* Vol. II. Higher Surveying. Part III Photographic Surveying Sec. 281 Oblique Aerial Photographs, pp. 407–430—5th ed. John Wiley and Sons, New York.
5. Bagley, James W., 1941. *Aerophotography and Aerosurveying,* pp. 239–251. McGraw Hill Book Co., Inc., New York.
6. Sharp, H. Oakley, 1942. *Photogrammetry,* Chapter 9, Oblique Aerial Surveying, pp. 75–78, John Wiley and Sons, Inc., New York.
7. Eardley, A. J., 1942. *Aerial Photographs: Their Use and Interpretation,* Chapter 8, Oblique Aerial Photographs for Reconnaissance Mapping, pp. 184–193, Harper and Brothers, New York.
8. Smith, H. T. U., 1943. *Aerial Photographs and Their Applications,* Chapter 7. Planimetric Maps from Oblique Photos, pp. 178–192. D. Appleton-Century Co., Inc., New York.
9. Church, Earl, 1944. *Elements of Aerial Photogrammetry,* Chapter VII. Perspective Grid Method for Constructing Planimetric Maps from High Oblique Photographs, pp. 76–85. Syracuse University Press, Syracuse, New York.
10. Forbes, Alexander, 1944. Short Cuts in Long-distance Photogrammetry, Photogrammetric Engineering, Vol. 10, No. 3, pp. 192–205.