A GRAPHIC METHOD OF MEASURING VERTICAL ANGLES FROM OBLIQUE PHOTOS

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THE measurement of vertical angles is an essential step in height determination from oblique aerial photos. For making this measurement precisely, use of the photo-alidade is standard procedure. Where this instrument is not available, however, graphic methods may be used to obtain somewhat less refined results. The graphic solution outlined below is believed to possess advantages of directness and simplicity over others that have been proposed.

The nature of the problem of measuring vertical angles may be visualized with the aid of the stereo-sketch, Figure 1. The desired vertical angle, α, is obviously measured in a vertical plane containing the plumb line and the picture point in question, and the problem is to rotate this plane into the picture plane for convenience of measurement.

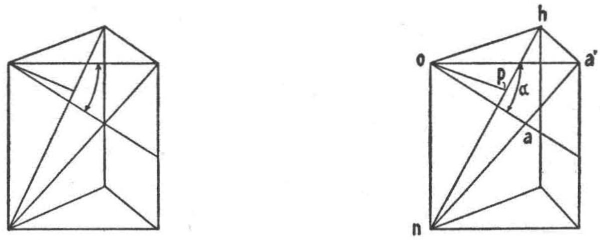

FIG. 1. Stereo-sketch illustrating the solid geometry of an oblique photo in skeleton form; *o* is the perspective center, *n* the nadir point, *P* the principal point, *nha'* the picture plane, *noh* the principal plane, *oha'* the horizon plane, and *noa'* a vertical plane through the plumb line *on* and a picture point *a*, to which the vertical angle α is to be measured.

In Figure 2, the principal plane, the horizon plane, and the vertical plane through the picture point are shown as rotated into the picture plane. The principal plane is rotated about the principal line, *nh*, as an axis, through an angle of 90°. The horizon plane *ho'a'* is rotated about the true horizon line *ha'* as an axis, through an angle of more than 90°. The vertical plane *no''a'* is rotated about *na'* as an axis, through an angle of more than 90°. In constructing this diagram, the photo plane is plotted first, and then the principal plane. Next is the horizon plane, which is controlled by points *h* and *a'*, and by the line *ho'* which must equal *ho*, as may be seen in Figure 1, and thus is determined by swinging an arc of radius *ho* from *h* as a center. Returning again to Figure 1, it is seen that lines *no''* and *o''a'* in the vertical plane through the picture point must equal *no* and *o'a'*, respectively, and their intersection point, *o''*, is therefore determined by the intersection of arcs having these radii; it is evident also that these lines must form a right angle.

By drawing Figure 2 on cardboard at an enlarged scale, trimming along the solid lines, and folding along the axes of rotation so that *o*, *o'*, and *o''* come together, a three-dimensional model useful for visualizing the problem and its solution may be had.

The method used in constructing the diagram and folding model of Figure 2 may of course be used as a graphic solution for the vertical angle, but is lacking

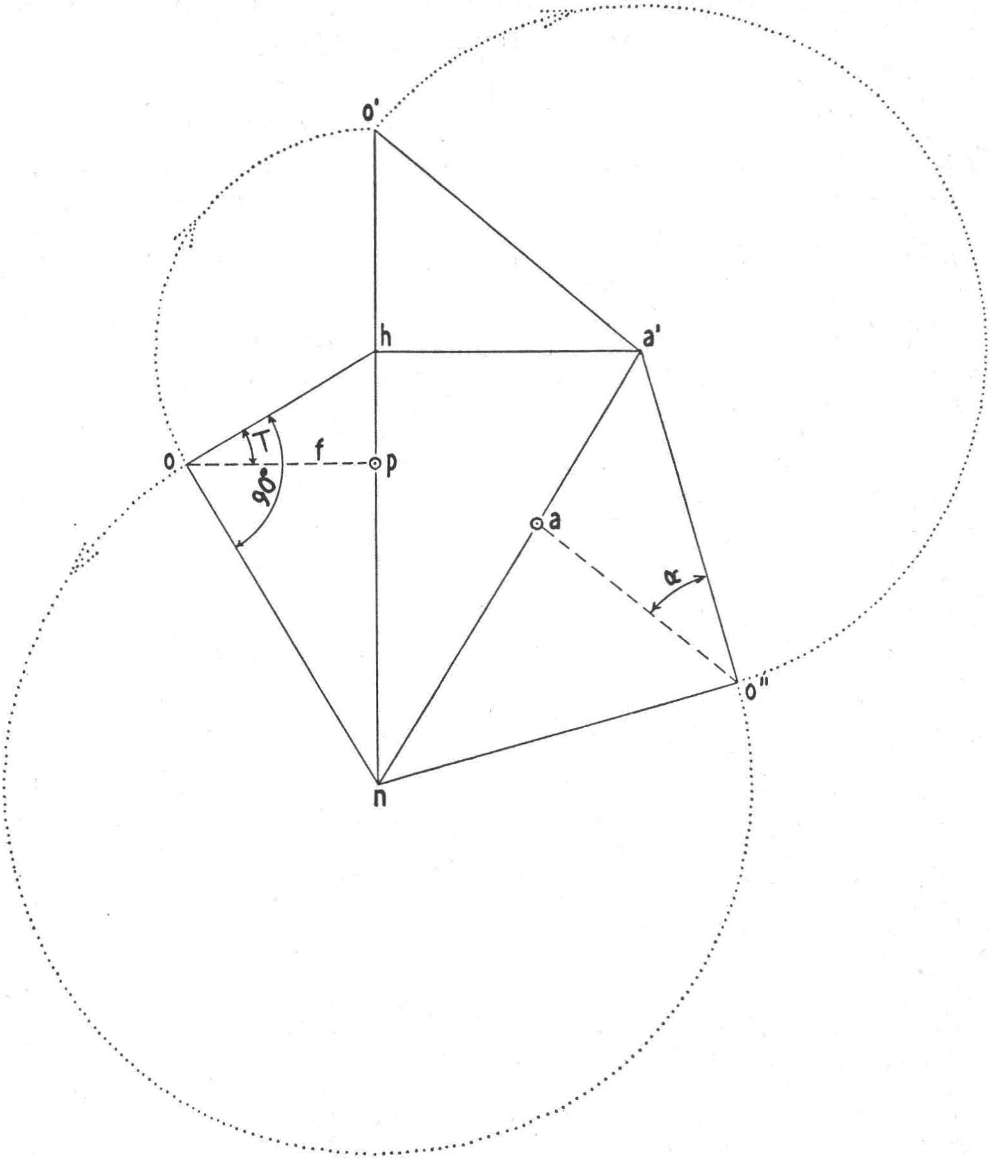

FIG. 2. Diagram showing the principal plane, *noh*, the horizon plane, *ho'a'*, and the vertical plane *no''a'* rotated into the picture plane *nha'*. Focal length is represented by *f*, depression angle of the camera axis by *T;* other lettering same as in Figure 1. Dotted lines are arcs used for construction purposes.

in ease and simplicity. A more direct approach is indicated by the fact that the triangle *no''a'* is a right triangle which must have points *n* and *a'* at its corners, and its side *no''* equal to *no*. Obviously only one right triangle can be drawn through *n* and *a'* having its side *no''* meeting this specification, and the right-angled corner of this triangle must line on an arc of radius *on* from *n* as a center. This triangle may be drawn simply by adjusting any right-angled drafting triangle so that its sides touch *n* and *a'*, and its corner lies on the arc indicated.

Steps in applying the above method are shown in Figure 3A, and are outlined below. It is assumed that the depression angle of the camera axis has first been determined, and that the true horizon and the principal line have been drawn on the photo. The photo is then fastened in position either on a sheet of drawing paper, or under a transparent overlay. Construction lines are then drawn as follows:

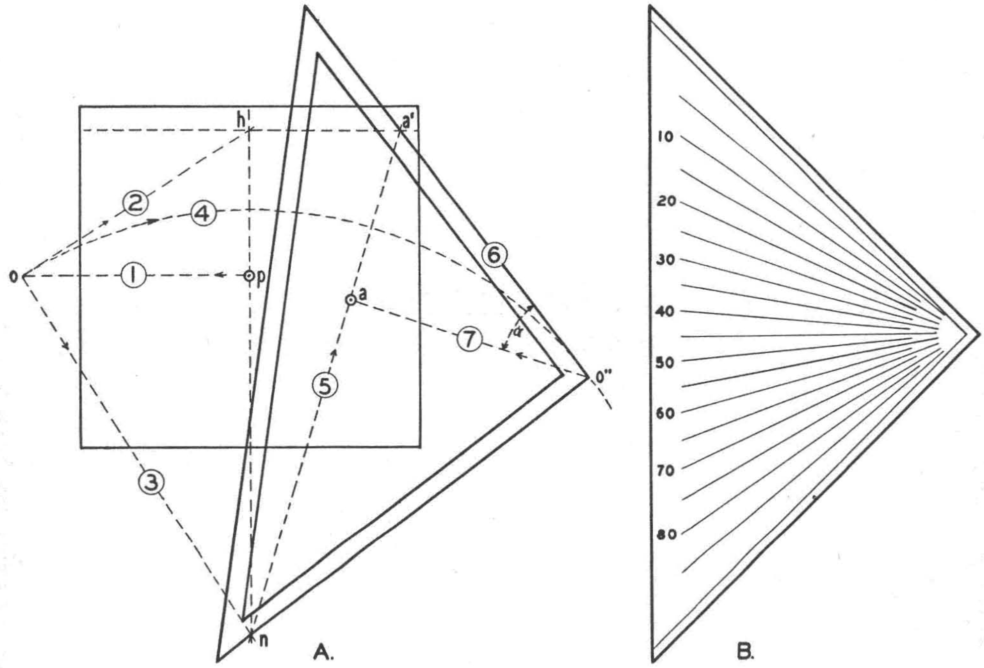

FIG. 3A. Diagram showing procedure for measuring vertical angle α to photo point a. The single solid line represents the photo and the double solid line a right triangle. Dashed lines are construction lines, and numbers indicate order in which they are drawn.

B. Skeleton diagram of template combining right triangle and protractor, for use in place of triangle in A.

(1) Line op equal to focal length is drawn from the principal point P perpendicular to the principal line.

(2) From o, the perspective center, a line is drawn to h, the intersection point of the principal line and the true horizon.

(3) A line on is drawn from o perpendicular to oh; this line is the plumb line, and its intersection n with the principal line locates the nadir point.

(4) From n as a center, an arc of radius on is drawn from o.

(5) A line is drawn from the nadir point n through the picture point a to intersect the horizon line at a'.

(6) A right triangle is moved with its sides in contact with points n and a' until its corner cuts the arc drawn from o, thus locating the point o''. A line is then drawn from o'' to a'.

(7) A line is drawn from o'' through a, and the angle $ao''a'$, or α, is measured with a protractor.

Angles to additional points on the same side of the principal line may be measured in similar fashion. For points on the opposite side of the principal line, the triangle is, or course, reversed. Where many angles are to be measured, it is more convenient to combine the triangle and the protractor into a template as shown in Figure 3B.

The above method is particularly well suited to Tri-metrogon obliques, and for other high obliques where the focal length is not so great or the depression angle of the camera so small as to extend the necessary construction lines over so large a space as to become awkward.