A SIMPLE PHOTO-RECTIFIER FOR HIGH OBLIQUES

Alexander Forbes, Captain, U.S.N.R.

INTRODUCTION

THE rectification of high oblique aerial photographs has been achieved by various methods. Photographic rectification, commonly used with near verticals and low obliques, has the great advantage of automatic reproduction, which renders the result independent of draftsmanship. If the optical system is good, the substitution of photo-rectification for manual methods gives greater precision; if an intricate shore-line is to be mapped, it gives greater speed and saving of man-power. High oblique photographs, which include the horizon, offer the advantages of great increase in area covered and of ready determination of tilt and swing when a sea horizon is clearly photographed. The chief obstacle to photo-rectification lies in the problem of lens distortion, which with high obliquity becomes serious. The standard procedure for rectifying low obliques is to tilt the lens so that its plane passes through the line of intersection of the picture plane and the map plane. This serves well with photographs whose tilt is not over 45°. When a wide-angle camera is used with tilts of from 55° to 65° and the large and well-defined area extending to 75° or more from the nadir is to be mapped, the obliquity of rays passing through the lens of the rectifier will cause serious distortion in the more distant parts of the map, both because of the increased angular distortion and because at high obliquity a given angular distortion causes a large spatial distortion on the map plane. Substitution of a

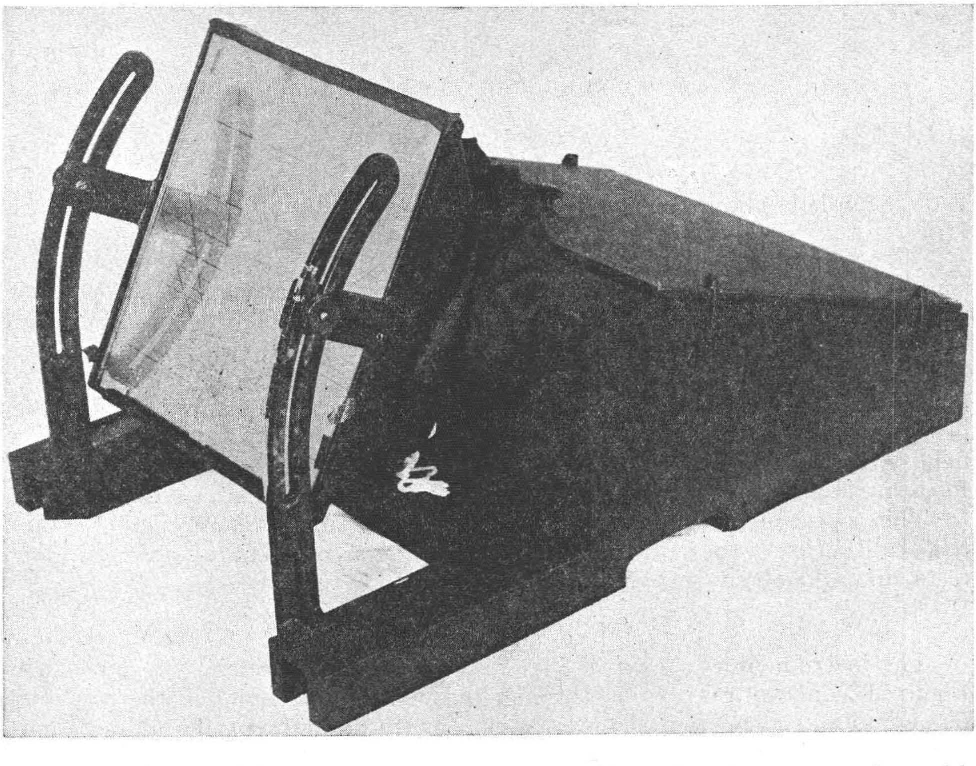

FIG. 1. Hydrographic Office pin-hole photo-rectifier, showing perspective grid between glass plates mounted in negative box.

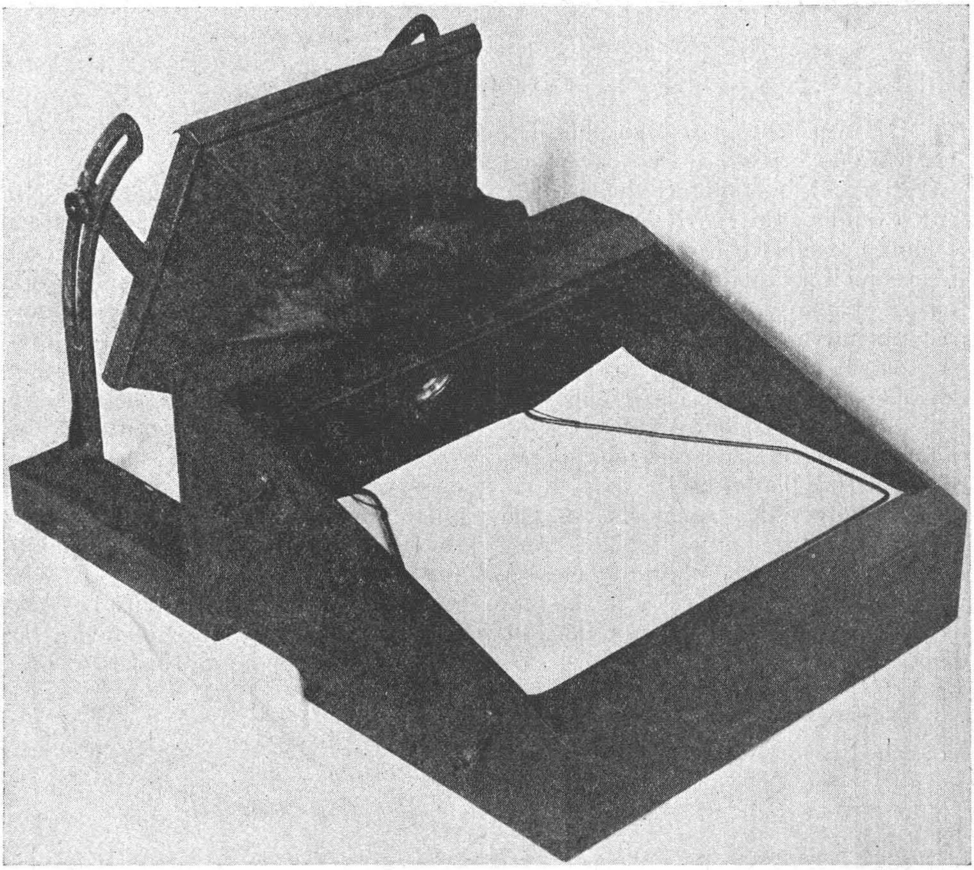

FIG. 2. Pin-hole photo-rectifier with lid removed from map box, showing paper secured on map plane by spring clip.

simple pin-hole for a lens eliminates this distortion and reduces the geometry of rectification to its simplest terms.

The present communication deals with a pin-hole rectifier developed at the Hydrographic Office of the Navy to expedite reconnaisance mapping of shorelines from high obliques. The instrument was designed for portability and speed of operation, rather than precision, being intended for an advance base or a field party with limited laboratory facilities. It was constructed in 1944 in the Pattern Shop of the Naval Gun Factory at the Washington Navy Yard.

This report includes a study of illumination, optimum aperture for the pinhole, calibration of the instrument, analysis of the precision obtainable, mapping tests, operational aids and the speed attainable in actual use.

DESCRIPTION OF INSTRUMENT

The instrument is shown in Figures 1 and 2. The principle is clarified in Figure 3, combining a section through the perspective center in the principal plane and an elevation of the mechanism for tilt adjustment. The camera lens is replaced by a pin-hole at the perspective center. The geometrical relations of photograph and datum plane are simply reproduced. The box with the pin-hole and negative is tilted at the same angle to the map plane as that at which the camera was tilted in relation to the ground when the picture was taken. Bromide paper or film is laid flat on the map plane; the negative is illuminated from behind, and its rectified image is thus projected and photographed on a plane parallel to the datum plane and at a scale determined by the distance of the map plane from the pin-hole divided by the altitude of the air station. For simplicity of construction the negative box was made with a fixed focal length of 6.0 inches and designed for 9×9 inch photographs taken with metrogon lens cameras (K-17), commonly used in the trimetrogon system of aero-surveying. The map plane is at a fixed distance from the perspective center, and thus for any given altitude of air station the scale is fixed. The only adjustments provided are for tilt and swing.

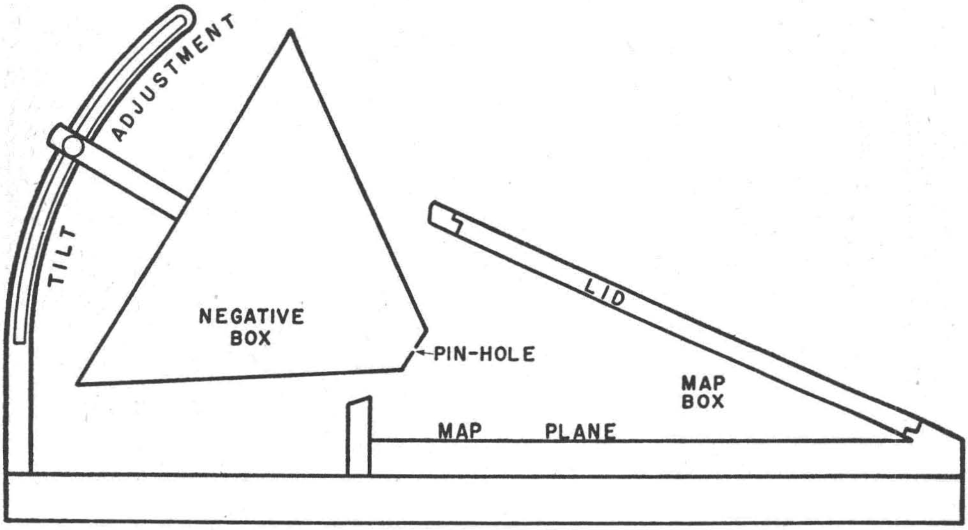

FIG. 3. Diagram of photo-rectifier showing vertical section in principal plane and elevation of mechanism for tilt adjustment.

The essential features of the rectifier are as follows: The negative (or tracing of shore-line on acetate) is held flat between glass plates; the pin-hole is accurately placed at the perspective center, i.e. 6.0 inches from the negative on a line perpendicular to the negative at its principal point, which coincides with the center of the glass plates; the negative box (carrying plates and pin-hole) is pivoted on a horizontal axis passing through the pin-hole, and this axis is parallel to the map plane; the tilt is adjustable from 90° to 42° from the vertical; the map box, containing the horizontal map plane, is light-tight during use.

To provide a simple means of varying the size of the pin-hole, there is a socket in the negative box into which is fitted a bronze plug with the pin-hole drilled in its center. Three such plugs are provided having pin-holes of 0.018, 0.022 and 0.030 inch in diameter. Definition is greatly improved, at the cost of introducing a slight distortion, by substituting for a simple pin-hole a very thin lens designed by Dr. J. G. Baker of the Harvard Observatory Laboratory. This lens has a centrally contained aperture of pin-hole size. Two such lenses have been made and fitted in plugs similar to those with simple pin-holes; thus the change from one pin-hole to another or from pin-hole to lens can be made in a few seconds. One of the Baker lenses has an aperture of 0.016 inch, the other 0.036 inch. The smaller one affords much the better definition; the purpose of the larger one is to reduce the time of exposure with a dense negative. This it does at the cost of considerable definition in the foreground but with little loss in the distance. A comparison of the definition obtained with the 0.018 inch pin-hole and the 0.016 inch aperture lens is shown in Figures 4 and 5 in which the same perspective grid was rectified, first with the pin-hole, then with the lens. In this and many other tests the grid was drawn for a tilt of 60°, since this is the standard tilt of wing pictures in trimetrogon photography and very near the optimum tilt for high oblique photogrammetry with the type of camera for which the rectifier was designed.

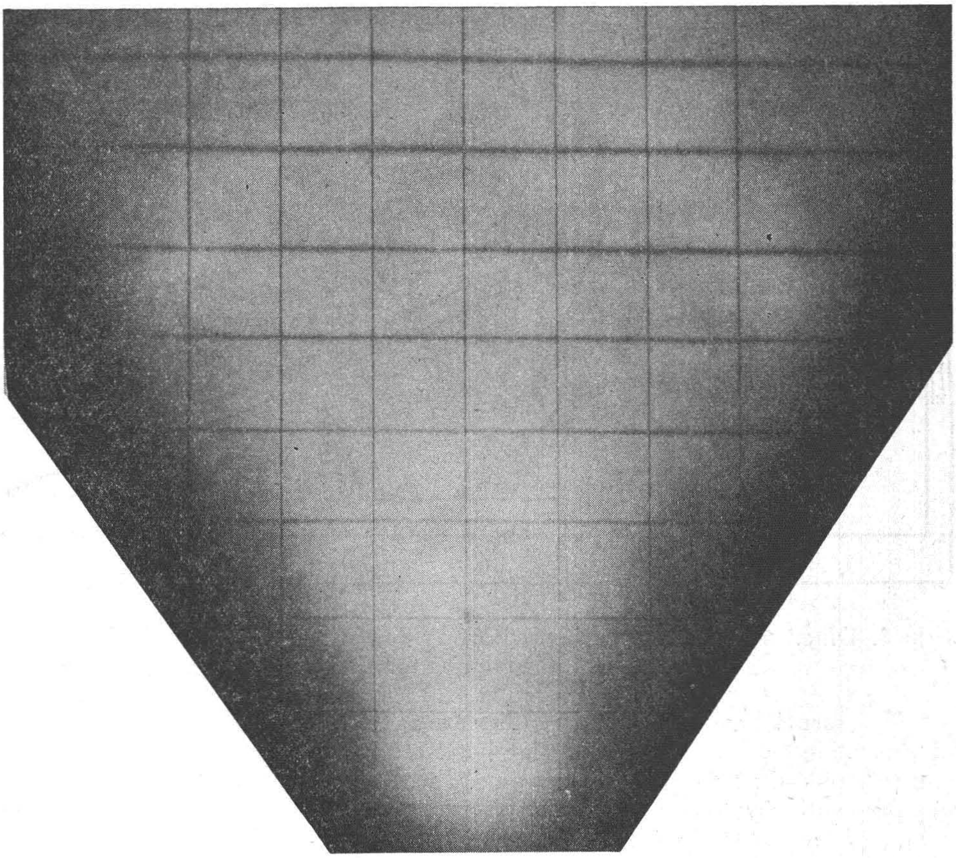

FIG. 4. Rectification of perspective grid with 0.018 inch pin-hole. Grid drawn for 60° tilt. Adjustment in rectifier for 60° tilt. Squares on map plane represent distances of 0.5 H. Exposure of 10 minutes on Agfa commercial orthochromatic film, with graded optical filter to equalize exposure. Contact print made from film.

Accuracy in tilt adjustment is enhanced by having rigid metal arms extend beyond the negative plane to a distance of 9 inches from the axis of rotation. A mark on the end of each arm is set in the proper position in relation to marks on the curved guide plates and the arms are secured by screw clamps.

Accuracy of rectification depends on the precision with which the principal point of the photograph is centered at the principal point of the instrument. To this end the central point of the glass plate is marked with a cross; the principal point of the photograph must be marked on the negative or tracing and this point must be accurately superposed on the cross marking the principal point on the glass plate.

Adjustment for swing is made by placing the horizon trace parallel to the top of the glass plate on which the negative or tracing is secured. This is most easily done by laying a sheet of cross-section paper on a light box and laying the glass plate on the paper with its edge parallel to one line. The negative or tracing is then secured to the glass plate, carefully centered and with the horizon trace parallel to the nearest line on the cross-section paper. The glass plates are $9\frac{1}{2} \times 9\frac{1}{2}$ inches square and this size allows room for 9×9 inch negatives to swing as much as 6°. If the swing of the photograph is more than this, the corners of the negative or of a tracing of it may be trimmed to permit further rotation.

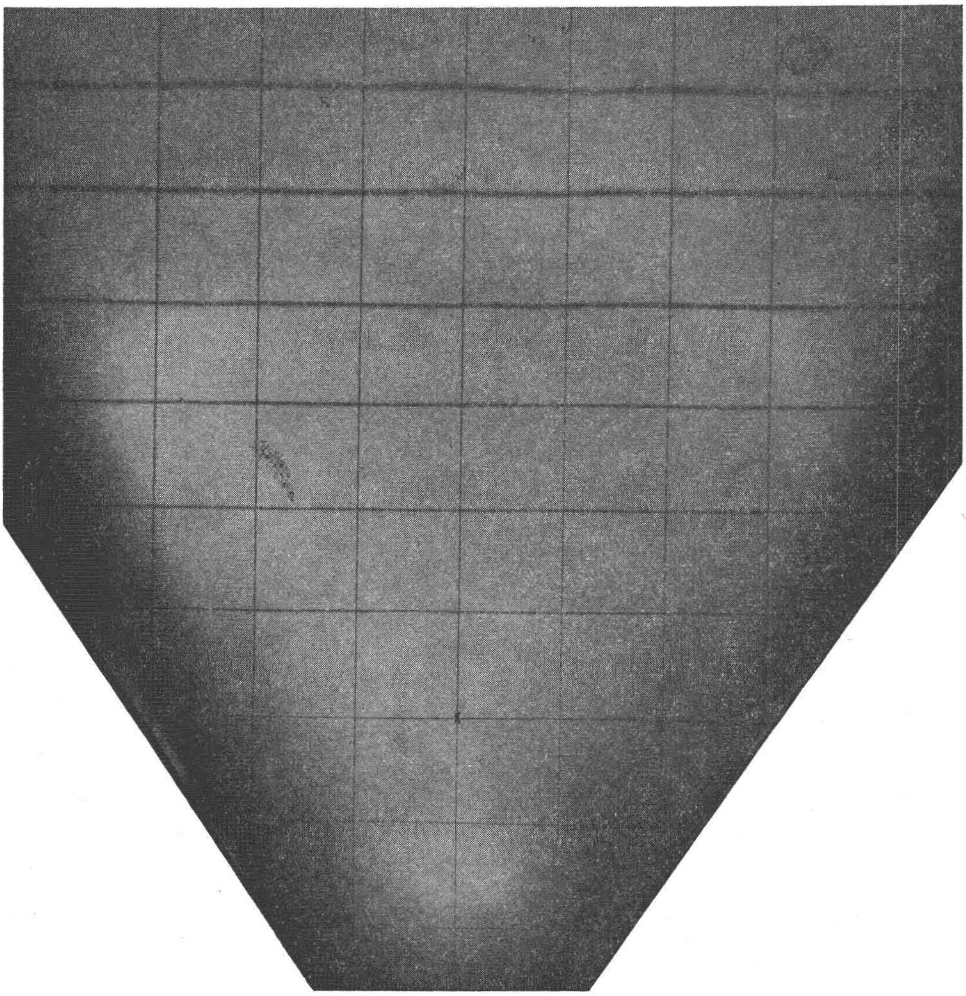

FIG. 5. Rectification of perspective grid with 0.016 inch aperture Baker thin lens. Other details as in Figure 4.

Scotch masking tape serves as a hinge along one edge of the pair of glass plates between which the negative is held flat. When the negative is correctly secured to the plate that has the principal point marked on it, the other plate is swung into position and secured with additional strips of tape. This arrangement expedites the mounting of negatives for rectification.

The map box may be constructed with the map plane at any desired distance from the pin-hole. The closer it is placed the brighter will be the illumination, hence the shorter will be the required time of exposure. The scale of the resulting map is directly proportional to the vertical distance from pin-hole to map plane. The problem is to select a distance that will give a convenient scale, clear enough definition and brief enough time of exposure for rapid mapping operations. The distance selected is 2.0 inches. This gives a scale one-third as large as that of a vertical photograph taken with the 6-inch camera lens for which the instrument was designed; e.g. an oblique photograph from 10,000 feet will be rectified into a map on a scale of 1:60,000. Illumination at this distance is sufficient for practical purposes; an acetate tracing illuminated by a bank of fluorescent lamps gives a clear image on film in a 5-minute exposure. In the case of Figures 4 and 5 the exposure was 10 minutes for each. The horizontal floor of the map box is large enough to accommodate a film or sheet of bromide paper 11×14 inches, the larger dimension being transverse to the principal line.

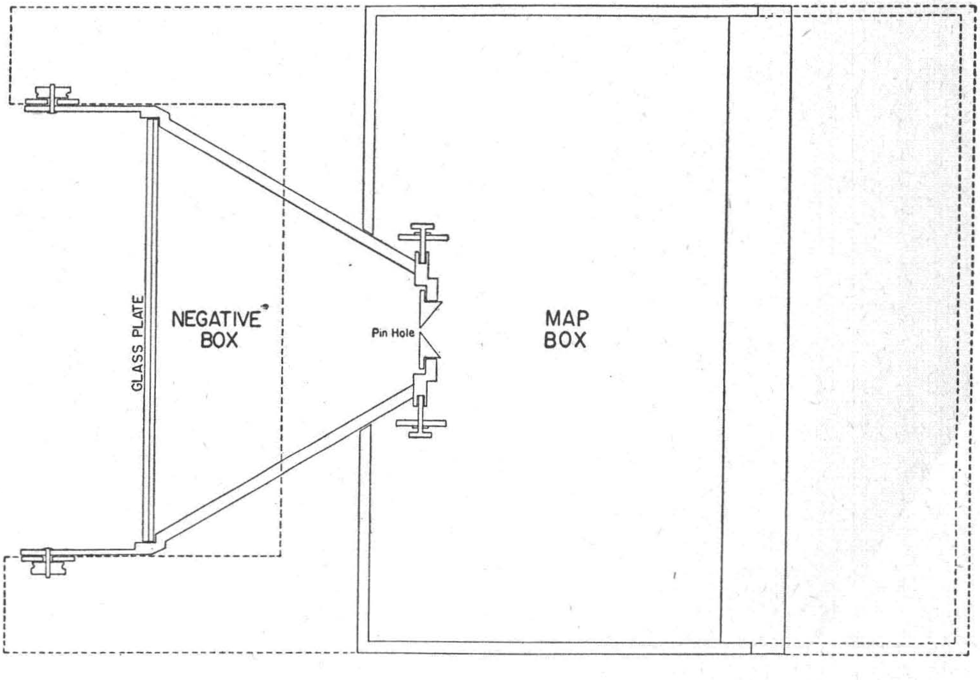

FIG. 6. Horizontal section of photo-rectifier through optical center (pin-hole), with negative box in horizontal position, i.e., with tilt adjustment at 90°.

It is most important that the film or bromide paper should lie flat on the floor of the map box. A slight local elevation of the surface will cause great distortion, especially in the distance where the angle from the vertical is large. A spring clip is provided around the margin to hold the film or paper flat; but this should be supplemented by a thin layer of adhesive material on the floor of the map box. Rubber cement may be used with paper, and "stay-flat" (Agfa) with film. The latter substance lasts for several days without renewal, remaining sticky enough to hold film that is pressed firmly upon it.

The lid of the map box is light-tight. The rotation of the negative box around the horizontal axis through the perspective center necessitates narrow openings at the junction between negative box and map box, through which stray light would leak if not prevented. The simplest way to stop this leakage is to wrap black cloth around the junction and tie it securely with string. As an added precaution it is well to reinforce this with a second black cloth thrown over the map box during exposure and wrapped around the inner layer of cloth at the junction.

Most of the construction is of masonite panels on a wooden base, with metal fittings where necessary. A few details of the design are shown in Figures 6 and 7, horizontal and transverse vertical sections through the perspective center; i.e. in the two planes perpendicular to that of Figure 3. Many of the dimensions in this design are not critical. It is only essential that the axis of tilt rotation should pass through the perspective center and be parallel to the map plane, and that the glass plates with the negative should fit snugly in a frame that holds them in the correct geometrical relation to the perspective center, and the lower edge of the plates should be parallel to the map plane to serve as a reliable guide in placing the horizon trace.

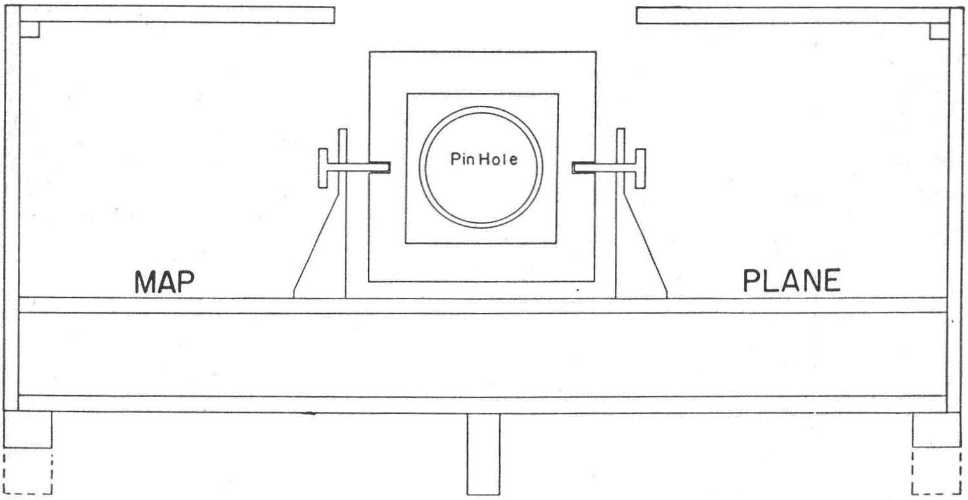

FIG. 7. Vertical transverse section of rectifier through optical center.

The instrument is not difficult to construct, it could be built in almost any well-equipped machine shop in two or three weeks, if a simple pin-hole is used. A lens on Dr. Baker's design could easily be made in any good optical shop. The weight of the instrument at the Hydrographic Office is about 20 pounds.

ILLUMINATION

A bright source of illumination is needed because of the small size of the pinhole required for good definition. Otherwise the duration of exposure is so great as to rob the instrument of its chief merit—speed. Direct sunlight affords powerful illumination. Objections to this are its absence at night and in cloudy weather, and the difficulty in diffusing it evenly over the map field. Even with a diffusing plate of opal glass between the sun and the instrument, the illumination is too much localized, giving over-exposure of the area nearly in line with the sun's rays. A blue sky away from the sun or a cloudy sky affords diffuse illumination, but the lack of uniformity, which prevents standardizing of exposure, and the inconvenience of moving the rectifier from the dark-room to a window are drawbacks.

By far the most satisfactory illumination has been obtained with a bank of three desk-type "Dazor" fluorescent lamps, held parallel and as close together as possible in a simple wooden box with a 11×14 inch opal glass plate in front of the lamps. Each lamp has two parallel tubes, and the six tubes, nearly evenly spaced and backed by their three reflectors, deliver through the opal glass a strong, well-diffused light. The tubes stand vertically on the table close to the plane of the negative.

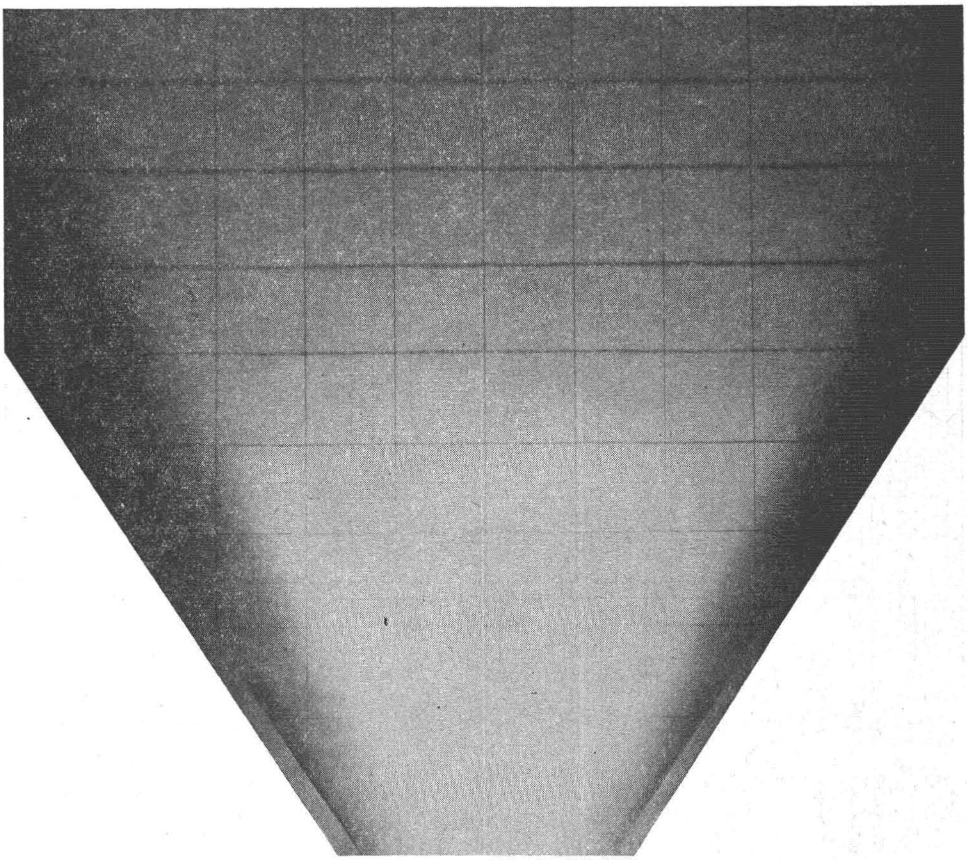

FIG. 8. Rectification of 60° perspective grid with 0.016 inch aperture lens, without optical filter, showing over-exposure in foreground.

A major problem of illumination arises because a uniform source of light does not give equal exposure all over the map plane. The exposure in the foreground is very much greater than in the distance, both because of closer proximity to the pin-hole light source and because of the more nearly perpendicular angle of incidence. Also the exposure is greater near the principal line than at the sides, for similar reasons. Figure 8 shows the unequal exposure in different parts of a rectifier perspective grid. To compensate for this inequality it is possible to screen by hand with a card having a convex lower edge, held between the light and the negative, moved quickly over the foreground and increasingly slowly over the distance. With practice it is possible in this way to give a fairly even distribution of exposure. It is far better to use a graded filter, made by exposing a film to light decreased at a proper rate from top to bottom. The filter must be secured in the correct position over the negative during exposure.

Several attempts to produce filters by exposing film through the pin-hole in reversed geometric relation were not successful. Effective filters were finally made in the following way. As a preliminary test a small fixed source of light was set up in a dark room; a strip of commercial ortho film at a measured

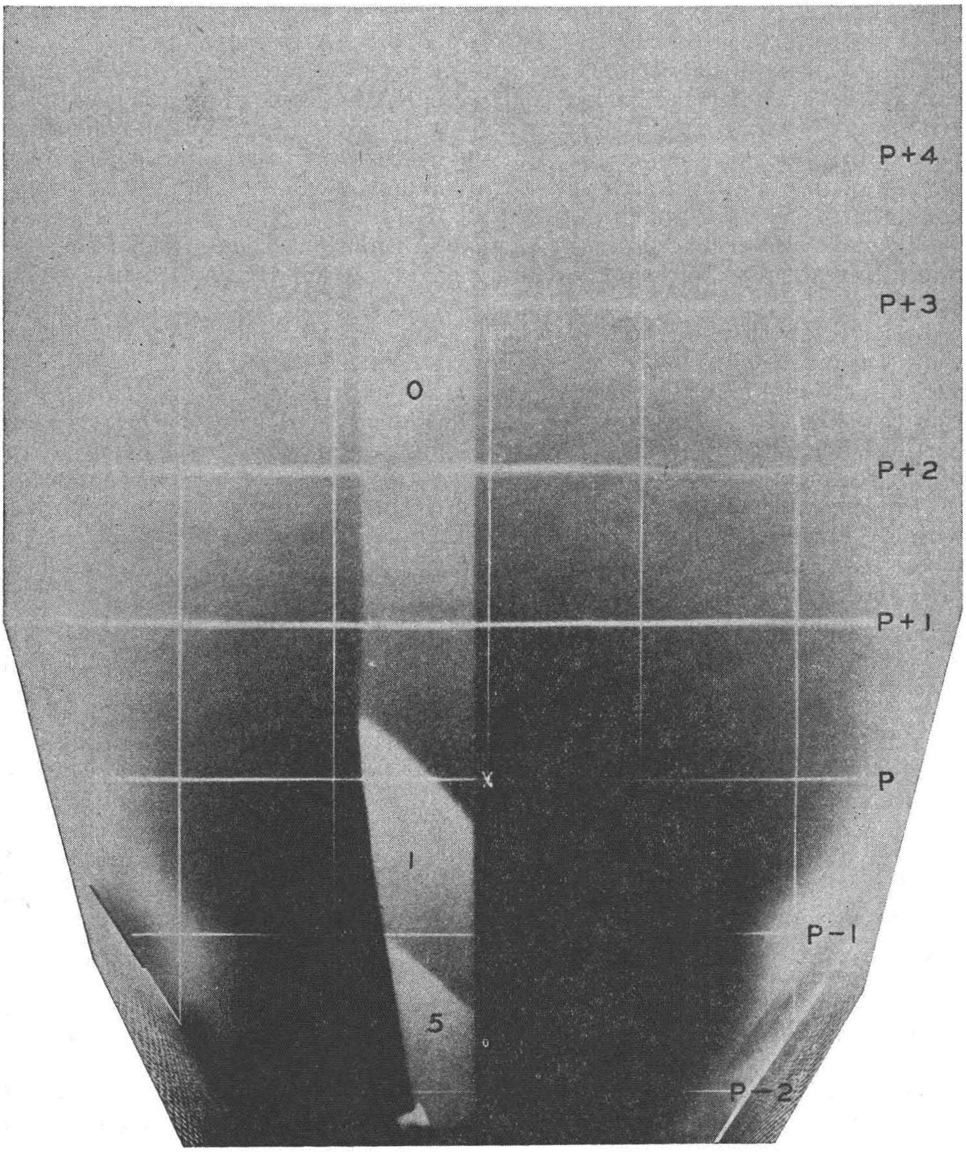

FIG. 9. Trial exposure with strip of experimental optical filter over part of grid. Tilt 60°. Principal point marked with cross; transverse parallels marked according to distance in inches from principal point. Numbers on the oblique bands indicate exposure in seconds, used to make the filter.

distance from the light was exposed in successive oblique bands for durations ranging from one to 60 seconds and then developed. A perspective grid (tilt 60°) drawn on acetate was then mounted in the rectifier with the banded strip superposed on it as a test filter and rectified on bromide paper with exposure to daylight. The result of such a test with three bands is shown in Figure 9. The grid is drawn to produce one-inch squares on the map plane, when correctly rectified. The transverse parallel marked P in Figure 9 passes through the principal point. For convenience the parallels above and below the principal point are designated $P+1$, $P+2$, $P-1$, $P-2$, etc., as shown in Figure 9. To the left of the center line the effect of the filter is evident. Whereas over much of the field there is heavy over-exposure, the area from $P-1.7$ to $P-2.3$, covered by that part of the filter exposed for 5 seconds (marked 5), shows the correct exposure for good contrast between line and background; similarly the 1-second filter is good for a zone from $P-0.6$ to $P-1.5$. The area marked 0 was covered by the unexposed part of the filter; on the remainder of the field there was no filter at all. Comparison of this unfiltered area with the 0 band shows that even unexposed film cuts off a considerable amount of light.

To make a filter a according to these indications, it was necessary to expose a film to gradually increasing amounts of light, so that the part covering each parallel would have the exposure necessary to give the density of film found to be correct in the above test. A specially designed cam mounted on a revolving shaft was arranged to lower a screen over the surface of a film at the properly varied speed required to give the right exposure at each level. During this operation the sides of the film were screened to reduce the density of the marginal portions. The developed film served as the filter to be secured over the negative during rectification.

Figures 4 and 5 were made with one of the most successful filters. Figure 5, with the Baker lens, shows nearly equal exposure all over the map plane except in the marginal sectors amounting to about 10 per cent of the width on each side, and contrast between line and background is visible all the way to the margin in the film on which the rectification was made, though not in the print from it reproduced in the figure.

Size of Aperture

Whether a simple pin-hole or a pin-hole lens is used, the size of aperture to be selected is a compromise between definition and speed. Experiments have been made with pin-holes of various sizes from 0.012 inch to 0.030 inch. Comparison of rectified grids obtained with pin-holes of .012 and .017 inch shows a slight advantage in sharpness of definition in the case of the smaller aperture. When the pin-hole was increased to .022 inch there was a considerable loss of definition, and at .030 inch the blurring was bad enough to offset the gain in speed except for crude and hasty approximations. The image of a point source of light formed by a pin-hole depends on the interference of waves starting at the pin-hole and is not a straight proportionate projection of the hole. This probably explains the greater loss of definition between .017 and .022 than between 0.12 and 0.17 inch.

Substitution of the Baker pin-hole lens (.016 inch) improved the definition far more than reducing the aperture of a simple pin-hole from .017 to .012 inch. The distortion introduced by the lens was so slight as to be negligible; indeed it was doubtful if a measurable distortion could be detected with the means that were available. Therefore this lens afforded the most practical optical system for mapping purposes. The Baker lens with the larger aperture (.036 inch) was designed to focus on the distance rather than on the foreground and gives nearly as good definition in the distance as the .016 inch aperture lens; thus it is especially well adapted to speed, for the distance is what requires prolonged exposure with a small aperture. The quickest way to get all the geographical information from a dense negative, in which shore-line in both foreground and distance are desired, is to rectify once with the small aperture lens and once with the large aperture, using no filter with either one. Both exposures will be of graded density, stronger in the foreground than in the distance, but the small aperture will give sharp definition in the foreground, and if the exposures are correctly timed, even though the distance is under-exposed with the small aperture and the foreground over-exposed with the large aperture, the readable portions will overlap in the middle distance.

Calibration of Tilt Adjustment

Calibration of the tilt adjustment was effected by rectifying a series of perspective grids drawn on frosted acetate for various tilts from 52° to 75°. In this way also the accuracy of rectification was tested. In constructing these grids the parallels were placed by computation, instead of by the usual method of connecting the intersections of diagonals with meridians, for with the latter method great skill in draftsmanship is required to maintain proper spacing beyond the principal point.[1]

With each grid the position of the tilt adjustment for correct rectification was estimated and the grid was photographically projected through the pin-hole or lens upon film or bromide paper on the map plane. The resulting approximately rectified grid was then carefully measured under a lens. In evaluating the result, both ordinates and abscissae were measured; by ordinates are meant distances in the direction parallel to the principal plane; by abscissae, distances perpendicular to the principal plane; the principal point is taken as the origin. The ordinates were the more sensitive indicators, but the abscissae were more reliable, for slight errors in draftsmanship and minute local elevations of the recording surface above the map plane caused magnified and misleading errors in ordinates. The main reliance was placed on the abscissae, i. e., on careful measurement to determine whether the meridians diverged or converged on receding into the distance. Repeated trials were made until the position of the tilt adjustment was found at which the rectified meridians were parallel. The correct position was then marked on the curved guide plates on both sides. A calibration curve was drawn relating degrees of tilt to inches on the guide plates above and below the mark established for 60°, as a reference point, and this curve was used for settings intermediate between the marked values placed at the 5° intervals; the settings were made with a ruler graduated to 0.02 inch. Since approximately 0.16 inch on the scale represents 1° of tilt and the ruler can be read nearly to 0.01 inch, it is not difficult to adjust the rectifier tilt to the nearest 0.1°.

The accuracy with which it is possible to rectify shore-lines and other sea-level features is shown by the approximation to perfect squares when a properly drawn grid is projected through the pin-hole or lens. Table I shows the measurements obtained from rectification of a 60° grid both with the .016 inch aperture lens and with the .018 inch pin-hole, at the best setting of tilt adjustment. The average dimensions of the rectangles were carefully measured in different parts of the field. Measurements of both abscissae and ordinates are tabulated. The

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[1] Even with careful drawing an error of about .01 inch in the ordinate of parallel $P+4$ in the 60° grid used to make Figures 4 and 5 resulted in a displacement of nearly .05 inch in the ordinate of this parallel on the map plane.

increase or decrease in abscissae per square is given as dX/Y, and the corresponding change in ordinates as dY/Y; these values are obtained by dividing the measured change by the number of intervening squares on the ordinate; a positive value means larger dimensions in the distance than in the foreground. The change per square is thus a measure of departure from perfect rectification.

TABLE I. MEAN DIMENSIONS OF SQUARES IN INCHES

0.016 inch Lens				0.018 inch Pin-hole			
At	Abscissae	Ordinates From		At	Abscissae	Ordinates From	
P−2	.975	P−2 to P−1	1.00	P−2	.985	P−2 to P−1	1.005
P−1	.975	P to P+3 (mean)	.996	P−1	.984	P to P+3 (mean)	.994
P	.9765	P+3 to P+6 (mean)	.987	P	.986	P+3 to P+6 (mean)	.981
P+5	.975			P+5	.979		
P+6	.975			P+6	.978		
$\frac{dX}{Y}=0$		$\frac{dY}{Y}=-.002$		$\frac{dX}{Y}=-.001$		$\frac{dY}{Y}=-.004$	
Mean abscissa in whole field			.975	Mean abscissa			.982
Mean ordinate in whole field			.995	Mean ordinate			.990

It will be seen that with the lens the tilt was correct, as judged by the practically perfect parallelism of the meridians, yet there was a slight decrease in ordinates amounting to .002 inch per square. With the pin-hole at the same tilt adjustment, there was a decrease in abscissae of .001 inch per square, and in ordinates of .004 inch per square. It is also noteworthy that the mean value of ordinates in the whole map plane was 2 per cent greater than that of abscissae in the case of the lens, and 0.8 per cent greater with the pin-hole. Part or all of this difference between abscissae and ordinates in the case of the pin-hole may be due to inaccurate draftsmanship in constructing the perspective grid. The fact that there is a greater discrepancy in the case of the lens, when the same grid was rectified at the same tilt adjustment, suggests a slight distortion introduced by the lens; it amounts to about 1 per cent and does not introduce serious errors.

The practical effect of such errors as those revealed in these tests upon mapping may be judged by inspection of the above figures. An error of 1 per cent in reconnaissance delineation of a shore-line is not very serious. Its local consequences within a limited area, such as a single square, will probably be unimportant, but the cumulative effect in the larger area of the entire map plane is more significant. For example, an excess of 1 per cent in ordinates over abscissae would, in the 8 inches from $P-2$ to $P+6$, amount to 0.08 inch. In aerial mapping from 10,000 feet with the 6-inch camera, the rectified photographs are on a scale of 1:60,000; thus an error of 0.01 inch within a square amounts to only 50 feet on the ground, but an error of .08 inch amounts to 400 feet. Such cumulative errors can be largely eliminated by transferring the rectified image to a true grid. A rectangular grid can be prepared on tracing paper with squares carefully drawn at a uniform scale, having the squares equal to the mean of the squares in the rectified grid. This true grid is then superposed on the rectified grid and photograph, one square at a time, and the outlines are thus traced; in this way the errors will be reduced to the local dimensions within each square. This operation introduces a considerable delay in production of the final map; whether it is worth doing in any given case depends on the size of error to be eliminated and on the relative importance of accuracy and speed in the project.

SOURCES OF ERROR

A difficulty inherent in mapping from high oblique photographs is the obscuring of shore-lines by intervening high land. When the farther shore of an island disappears behind a hill, obviously it cannot be placed except by extending the flight line until it can be photographed from a direction whence it is not obscured. Sometimes it is easy to see that the shore-line is hidden; at other times even the most careful examination leaves the matter in doubt. Then the only certain solution is photography from a different angle.

Other sources of error arise from uncertainty of the precise altitude of the plane (affecting only scale), error in tilt determination, error in swing, and difference in focal length of the camera from the 6-inch distance between pinhole and principal point on the negative in the rectifier. A major change in altitude will change the horizon dip and consequently will modify tilt determination, but the change due to any ordinary error in altimetry is so small as to be negligible. Specifically, with a 6-inch focal length and a tilt of 60°, the change in horizon dip at an altitude of 10,000 feet is .01 inch, measured on the photograph, for a 1,000-foot change of altitude. The probable error in placing the horizon on the photograph, under the most favorable conditions, is nearly .005 inch, and this is equivalent to a 500-foot error in altitude in its effect on tilt determination; therefore, the probable error in altimetry, rarely exceeding 100 feet, is insignificant compared with the inherent uncertainty as to tilt. It should be noted that the assumed error of .005 inch in placing the horizon represents an error of 2.3′ in tilt, and this is within the limits of accuracy provided in the tilt adjustment of the instrument.

Tilt Error

In a majority of aerial photographs the ideal condition for finding the tilt—a sharply defined sea horizon—is not obtained; either the horizon appears on a land mass of uncertain altitude, or is hazy and ill defined, or it is actually obscured by clouds. Often the position of the horizon can be placed with certainty to within 0.02 or 0.03 inch of its true position, but no nearer. The question arises as to the effect of the resulting error in tilt on the rectified shore-lines. An error of 0.03 inch in placing the horizon, with a 6-inch focal length at approximately 60° tilt, will introduce a tilt error of 14′.

To assess the planimetric errors in rectification we may plot a curve of errors in ordinates on the map plane arising from an assumed error of 0.1° (or 6′) in tilt determination. This curve is shown in Figure 10. It is based on the standard condition, viz. 6-inch focal length with tilt approximately 60°, and shows the excess of distance along the principal line, measured from the projection of the principal point, resulting from adjusting the rectifier tilt for 0.1° more than the actual tilt. Foreground points, i.e. those between the principal point and the plumb point, also project to excessive distances from the principal point, and since this excess is toward the plumb point and therefore expressed as a negative ordinate, these points are so plotted in the curve. From inspection of the curve it is seen that when a 6-inch camera is used at 10,000 feet with a tilt of about 60°, an error of 0.1° causes an ordinate error of about 50 feet on the ground in the extreme foreground. This error decreases to zero at the projection of the principal point; in the distance it increases more rapidly, amounting to 75 feet at 70° from the nadir, 190 feet at 75° and 520 feet at 80°. Beyond 75° the rectifier

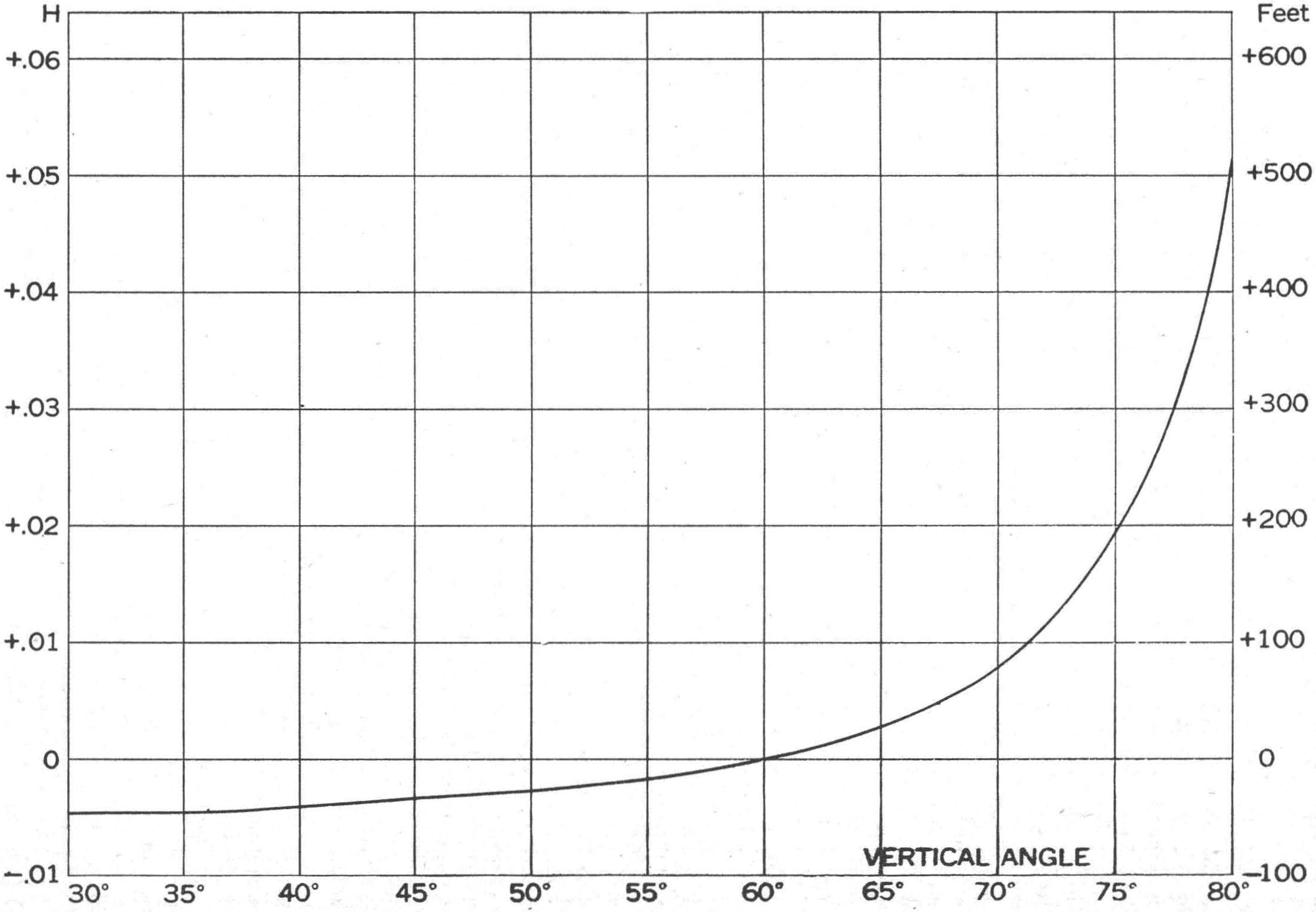

FIG. 10. Graph of errors in ordinates (distances from the projection of the principal point measured parallel to the principal plane) resulting from an error of 0.1° in tilt adjustment in rectifying a photograph taken at 60° tilt. Abscissae, vertical angles from nadir; ordinates, errors in planimetric ordinates, measured in fractions of air station altitude at the left, and in feet on the datum plane with a photograph from 10,000-foot altitude at the right.

does not project good enough detail for plotting distances; we must recognize this as the point beyond which uncertainty of tilt may introduce errors of more than 0.02 H, and errors of like magnitude may arise from imperfect definition.

Swing Error

Error in swing is apt to arise from the same causes as error in tilt—horizon obscured by haze, clouds or high land; consequently the effects of the two sources of error are usually compounded. But for analysis we may consider the case in which there is no tilt error and see what distortions will result from swing error alone.

The effect of swing error has not been computed by trigonometry, but the order of magnitude was determined by a simple experiment. A 60° perspective grid was placed in the rectifying camera at the proper tilt and rectified on film. It was then rotated 1° around the principal point, on which it was still carefully centered, and a similar but distorted print was made on another film. The two films were superposed with their principal points and principal lines carefully matched, and the divergence of the transverse grid lines was observed. It was found that the transverse line crossing the principal line 75° from the plumb point was displaced about 0.17 inch at the extreme lateral margin of the map plane. In a photograph from 10,000 feet this represents a displacement of 850 feet.

Unless the horizon is badly obscured the swing error is not likely to exceed 20′; in this case at 75°, which we consider the limit of the good plotting area, the worst displacement would be less than 300 feet, and this amount would only be found at the extreme margin of the picture area. Near the principal line the displacement would be negligible. Even if the horizon is completely obscured and the only clue to swing is a bank of stratus or cumulus clouds, it is usually possible to estimate swing to within 30′; therefore, errors in swing will in general not cause serious distortion.

Error Related to Focal Length

Metrogon lens cameras, generally referred to as of 6-inch focal length, actually have focal lengths varying by as much as 0.1 inch or more. In the case of five cameras used in the survey of Frobisher Bay which furnished material for this study, the average focal length was 6.058 inches. The distorting effect of an excess of 0.10 inch in camera focal length on projection with the rectifier set for 6.00 inches is calculated by means of the following formulae, furnished by Mr. Robert Singleton. Let x = the abscissa of any point on the photograph measured to right or left of the principal line, X = the corresponding abscissa on the map plane ($H = f = 6.00$ inches); y = the ordinate on the photograph measured from the principal point, Y = the corresponding ordinate on the map; D = the distance on the photograph from the true horizon to the principal point; df = the excess of camera focal length, specifically 0.10 inch.

$$\frac{dX}{X} = \frac{fdf}{D^2 + f^2}, \quad \text{and} \quad \frac{dY}{Y} = \frac{2fdf}{D^2 + f^2}.$$

It is evident that the percentage errors of both abscissae and ordinates are constant all over the map plane, and for the ordinates they are twice as large as for the abscissae. Table II shows the actual displacements of ordinates reduced to the map plane in the rectifier, where $H = 2.0$ inches, for various values of vertical angles from the nadir. These displacements are also translated into feet on the datum plane in terrain photographed from 10,000 feet altitude.

Table II. Focal Length 6.10 Inches Instead of 6.00. Tilt 60°

Vertical Angle	Ordinate	Displacement of Ordinate		Excess over Displacement of Abscissa	
	Inches on Map from P	Inches on Map	Feet ($H=10{,}000$)	Inches on Map	Feet ($H=10{,}000$)
30°	−2.31	−.057	−285	−.0285	−142
40°	−1.79	−.044	−220	−.022	−110
50°	−1.08	−.027	−135	−.0135	− 67
60°	0	0	0	0	0
65°	+ .826	+.022	+110	+.011	+ 55
70°	+2.03	+.051	+255	+.0255	+127
75°	+4.00	+.10	+500	+.050	+250
78°	+5.94	+.149	+745	+.0745	+373
80°	+7.90	+.198	+990	+.099	+495

Since the abscissae are also enlarged by the excess of focal length, in a constant percentage which is half as great as the proportional enlargement of ordinates, the actual distortion is minimized to that extent. Balancing half of the enlargement of ordinates against the enlargement of abscissae, as a mere change of scale, we find a residual increment causing actual distortion, which is tabulated in the last two columns of Table II. Since even at 80° from the nadir the distorting ordinate displacement, due to a 0.10-inch excess of focal length, amounts to less than 500 feet at a scale of 1:60,000, that amount of uncertainty as to focal length will not introduce serious errors in reconnaissance surveying; the mean error found in the cameras used in Frobisher Bay will introduce only 58 per cent of the tabulated errors, i.e., a maximum of less than 300 feet.

For the greatest precision the focal length of the lens employed should be ascertained, and if it is greater than 6.00 inches the glass plates should be shimmed to place the negative at the same distance from the pin-hole. This will eliminate the focal length distortion. In the instrument as constructed, such shimming can be carried to 6.10 inches or more, and this will suffice for nearly all metrogon lenses.

If the rectifier is to be used with photographs taken with cameras of still greater focal lengths, e.g., $6\frac{3}{8}$ inches or $8\frac{1}{4}$ inches, this can be done by a method for which I am indebted to Professor Philip Kissam and Mr. Robert Singleton.[2] A change of tilt and a shift of the principal point of the photograph by amounts computed by simple formulae will result in true rectification. The formulae for these changes are as follows: Let f = focal length of camera with which the picture was taken; f' = the focal length in the rectifier (6 inches); t = the tilt of the camera taking the picture; t' = the desired tilt for the rectifier; d = the shift of the principal point, p, of the negative along the principal line from the cross at the center of the glass plate.

Then:
$$\sin t' = \frac{f' \sin t}{f} \quad \text{and} \quad d = \frac{f}{\sin t}(\cos t' - \cos t).$$

If frequent use is made of photographs taken with focal length of $8\frac{1}{4}$ inches in the 6-inch rectifier, Table III may be used.

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[2] The basic formulae on which this method is based, were mentioned in Brock patents and are reported to have been derived from an earlier source. Their application to this transformation is believed to have been first proposed by Mr. Singleton.

TABLE III. FOR RECTIFYING OBLIQUES TAKEN WITH $8\frac{1}{4}$-INCH FOCAL-LENGTH CAMERA IN 6-INCH RECTIFIER

Tilt of camera *t*	Tilt of rectifier *t'*	Shift of principal point toward horizon *d in inches*
66°	41°41′	3.06
67°	42°09′	3.14
68°	42°32′	3.22
69°	42°55′	3.305
70°	43°14′	3.395
71°	43°36′	3.485
72°	43°57′	3.57
73°	44°16′	3.655
74°	44°30′	3.755
75°	44°45′	3.855
76°	45° 0′	3.96
77°	45°14′	4.06
78°	45°28′	4.165
79°	45°42′	4.27
80°	45°54′	4.375

Earth's Curvature and Refraction

Since the photo-rectifier projects upon a level map plane and without appreciable refraction, the earth's curvature and atmospheric refraction will introduce errors in the projection of distant points at sea level. The errors are negligible in the area nearer than the principal point in the case of a 60° tilt. If we limit our expectation of good rectification to the area within the 75° vertical angle, the errors due to curvature and refraction will still be small compared to other probable errors. Specifically, at 75° from the nadir in a photograph from an altitude of 10,000 feet, there is a radial displacement of about 120 feet; from 20,000 feet the displacement is about 420 feet, to be added to a horizontal distance of 74,500 feet from the plumb point; i.e., the error is about 0.5 per cent of the actual distance. In reconnaissance work from 10,000 feet or even 15,000 feet, this source of error may be neglected. In photography from higher altitudes the error may become significant at distances near the limit of good definition in the rectified print. The necessary correction can then be easily applied.[3]

MAPPING TESTS

The tests in rectifying perspective grids, described above, establish the degree of instrumental precision of which the device is capable, and the calculations show the theoretical magnitudes of probable errors due to various causes. It is desirable to supplement this with an actual mapping test with photographs taken in a routine survey. For this purpose three high oblique photographs were selected from a series taken in a survey of Frobisher Bay. They show a group of islands—Bishop, Hill and Faris—from three different directions, all taken from approximately 10,000 feet, with tilts between 56° and 58.5°. One of these photographs is reproduced in Figure 11; the other two cover most of the same group of islands, one with its principal plane 67° to the right, the other 127° to the left of the principal plane of the photograph shown in Figure 11. In none of the photographs was there a sea horizon, only a line of nearly level land of unknown altitude. If, as is probable, the altitude of this land is not over 1,000 feet, it would

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[3] Cf. Forbes. PHOTOGRAMMETRIC ENGINEERING, Vol. X, No. 3, p. 202, 1944.

not introduce a tilt error as great as 0.1°, and this, we have seen, will not greatly distort the planimetry at vertical angles less than 75°.

Oblique photographs of these islands offer a severe test of a mapping procedure for two reasons:—steep cliffs close to the shore obscure the shore-line in some photographs, and often it is hard to tell by inspection how much of the shore-line is obscured; the tidal range of 30 feet or more and the correspondingly large difference between spring and neap tides, renders the position of high-water mark on a gradually shelving beach quite indeterminate, and the water is so clear that shoals near the surface are hard to distinguish from dry land. Therefore discrepancies in placing the shore-line, in photographs from different angles, are to be anticipated, due to difficulties in photo-interpretation.

Tracings on frosted acetate were made from these three photographs, the shore-line at high tide being delineated as nearly as it could be placed by study with a hand lens. The three tracings were rectified on film with the Baker pinhole lens. The focal length of the camera used was not recorded, and no allowance was made for its possible excess over 6.0 inches. From each of these photo-rectifications a pencil tracing on acetate was made over a light box. These three

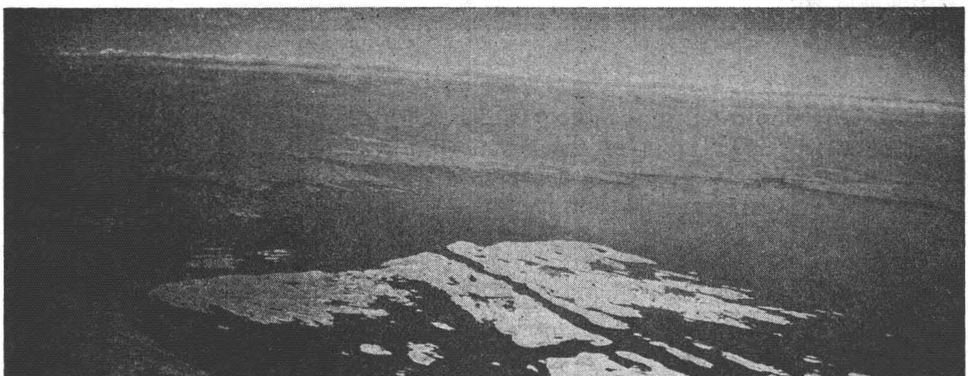

FIG. 11. Oblique photograph of islands in Frobisher Bay with K-17 camera; tilt 56.8°.

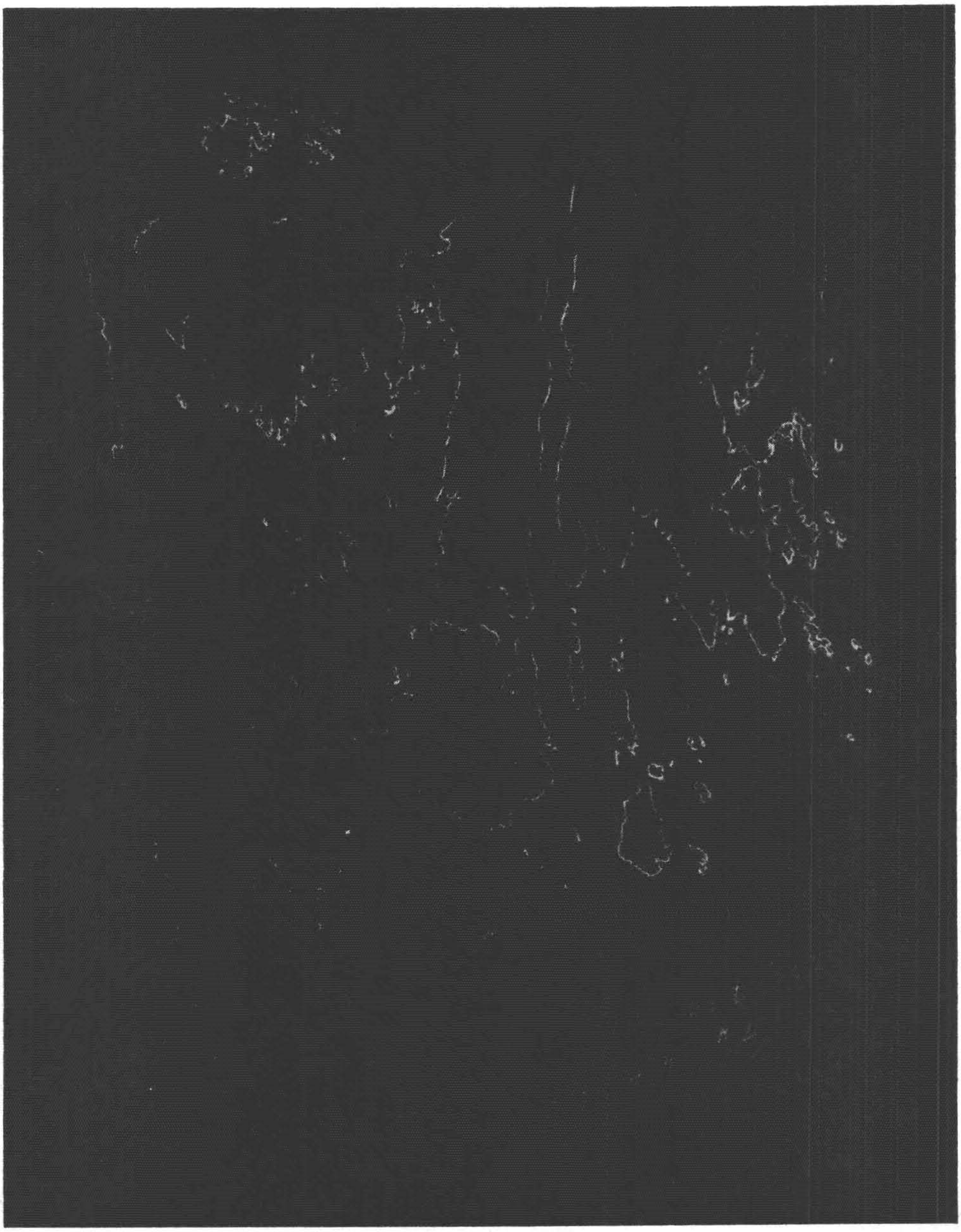

FIG. 12. Composite print of rectifications of three overlapping photographs of area shown in Figure 11, see text. The projections of the principal lines appear faintly.

tracings were then superposed with the shore-lines matched as closely as possible, taped together and from them a contact print was made on bromide paper. This composite print is reproduced in Figure 12, to show how closely the three tracings agreed.

The agreement of most of the salient points in the three rectifications to within .04 inch shows a very close approximation in scale and thus indicates that

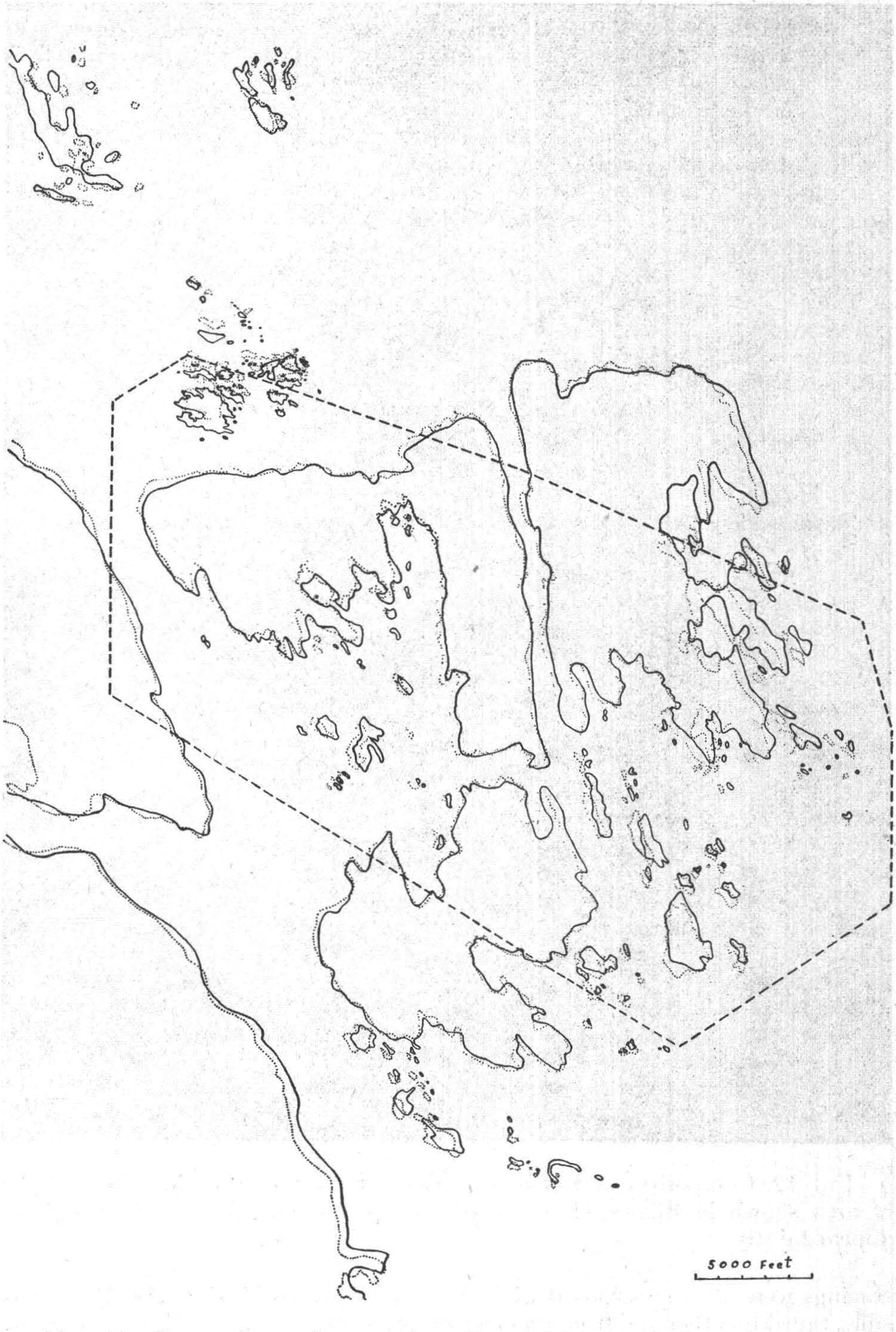

FIG. 13. Comparison of mean shore-line (high water) from the composite in Figure 12 with H. O. Chart No. 5830. The photo-rectification is the solid line; tracing from chart, dotted.

all three tilt adjustments were very nearly correct. Wherever discrepancies of 0.10 inch or more appear they result from uncertainty in tracing the shore-line from the photograph, either because of concealment by high ground or because of confusion due to tidal flats. Wherever the shore-line is bold and well-defined in the photograph, the three rectifications agree in locating it within .08 inch, and almost everywhere much nearer than that.

From the composite print an acetate tracing was made, the average of the three shore-lines being traced except where it was evident that the true shore-line was concealed in one of the pictures and visible in another; then, of course, the more reliable photograph was followed.

The area covered by these photographs was also surveyed by ground control triangulation and "vertical" photography, and from this combination of data a chart had recently been compiled by standard photogrammetric methods in the Hydrographic Office. This chart (scale 1:75,000) was placed in a Saltzman projector and enlarged till the shore-line matched the composite tracing of the rectified obliques as nearly as possible. From the enlargement ratio required to achieve this matching, it was deduced that the scale of the rectified photographs was 1:60,250. Figure 13 shows the agreement between the composite rectification and the enlargement from the chart. Here again the major discrepancies are due to photo-interpretation of high-water mark at the margins of shallow tidal basins; the well-defined shores agree nearly everywhere within .06 inch or less except in the more remote parts of the photographs. At this scale, .06 inch represents 300 feet on the ground. Even at 9.5 nautical miles from the camera, an island in one of the pictures is rectified to a position precisely in its proper bearing (error not over 150 feet) and only 1,300 feet in error as to distance. This is 79° from the nadir, and thus is far beyond the area deemed suitable for accurate rectification.

In evaluating the results of this test, it should be noted that in a place where the tidal range varies from 38 feet on extreme springs to 11 feet on occasional neap tides, on a beach so flat that the low water and high water marks are more than 3,000 feet apart, the most accurate survey can scarcely place the mean high-water shore-line within less than 200 feet. It may be fairly said that the rectifier, carefully used, can place the shore-lines of large areas quickly and with reasonably close approximations for a reconnaissance survey, and can indeed approach the accuracy of vertical photography over an area considerably larger per photograph.

Figure 14 shows how large are the areas on the datum plane covered by different zones in an oblique at 60° tilt with the 6-inch focal length camera of 9×9 inch field, compared with the area included in a vertical photograph taken with the same camera at the same height, and adopted for the purpose of the curve as the unit of area. Abscissae represent limiting vertical angles of the area utilized; ordinates are included areas. It will be seen that if only the foreground half of the oblique photograph is used, i.e., only to the principal point at 60°, the area covered is 1.25 times the area covered by the vertical photograph; if the oblique is used up to 75° from the nadir, the ratio is increased to 5.1:1; at 79° it is 9:1, and even at this distant range approximations may be close enough for usefulness in reconnaissance mapping. A similar curve for a tilt of 55° is shown as a dotted line; it is noteworthy that if the limit of cartography is set at 75° the areas covered at these two tilts are equal to within 0.5 per cent.

An important element in the usefulness of high obliques is their ability to bridge wide water gaps and their related capacity for strengthening control by means of extended azimuth lines traversing large areas that require many overlapping photographs if verticals alone are used. Here it should be noted that

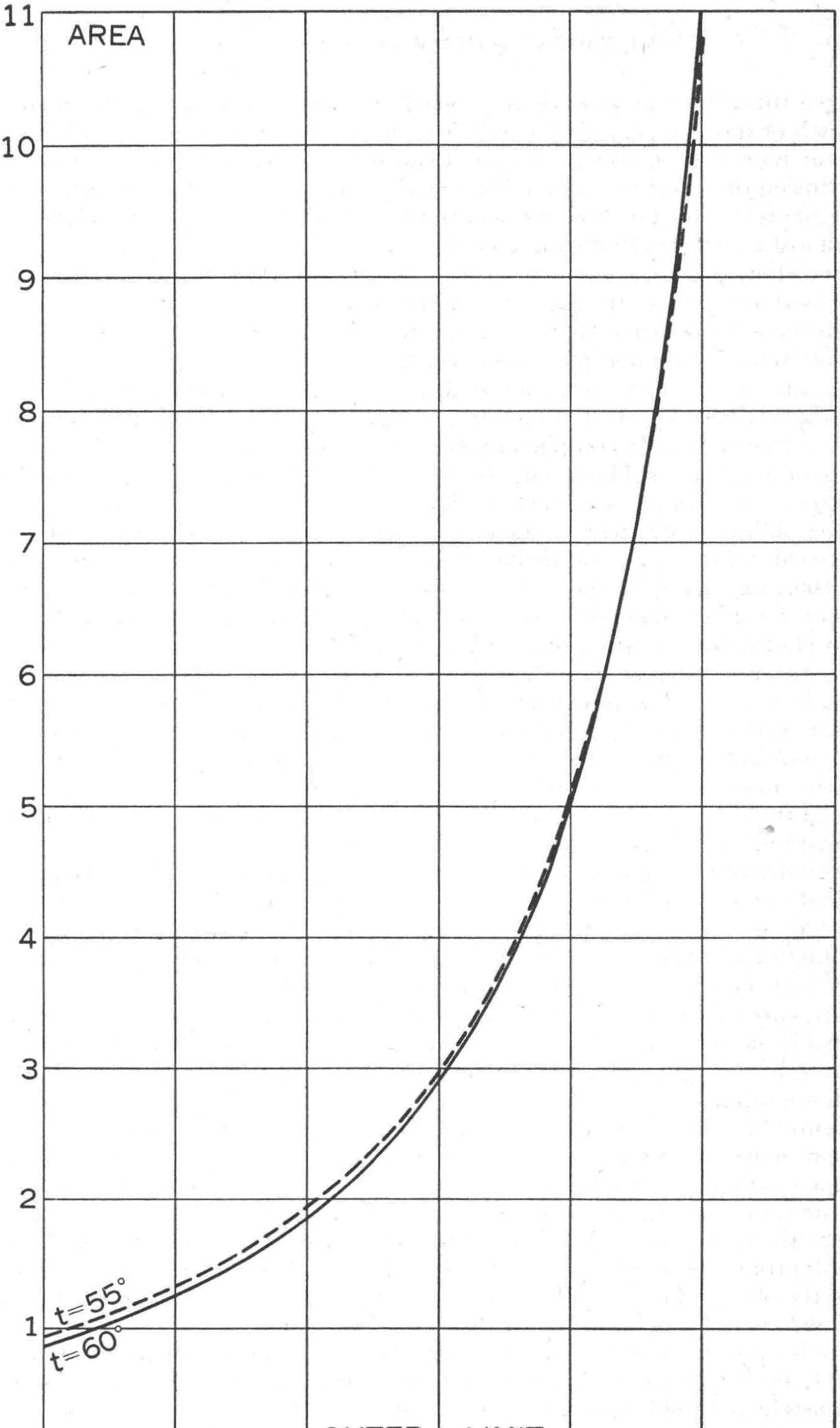

FIG. 14. Graph showing area on datum plane covered by rectifying high oblique from foreground to various transverse lines, compared with area covered by same camera in vertical photograph from the same altitude. Solid line, tilt 60°; dotted line, tilt 55°. Abscissae are vertical angles from nadir of limiting transverse lines.

while errors in distance from the air station increase rapidly as the vertical angle increases beyond 75°, there is no corresponding increase in azimuth errors. At a vertical angle of 80° a given angular error will cause nearly six times as great a horizontal displacement in ordinates (distance from nadir) as in abscissae (based on azimuth). For this reason it is clear that in mapping an area only appearing in the distance of a high oblique, accuracy can be greatly increased by taking another photograph of the area with the camera axis nearly perpendicular to that of the first photograph.[4]

The advantages of the high oblique in bridging water gaps and strengthening control were demonstrated in the photographs of another part of Frobisher Bay where rows of islands lie with their long axes in a NW–SE line. Between the islands of each row the gaps are small, but between adjacent rows they are larger. An area some 30 by 15 miles had been covered with about 140 "vertical" photographs from 15,000 feet and a much smaller number of high obliques from 10,000 feet. No ground control of this area was available, and no radial line plot had been made from the verticals; the area is still unsurveyed. Two methods of hasty reconnaissance use of the photographic material were compared,—an uncontrolled mosaic from the "vertical" photographs and a compilation from photo-rectifications of selected obliques. For the mosaic 69 verticals were used; they were taken in four flight lines in which the overlap of successive pictures was about 57 per cent, and the side-lap of adjacent flight lines varied from 15 to 50 per cent. For photo-rectification, 15 of the obliques were selected, their principal planes crossing the area in various directions. Of these, eleven were used to make the compilation and two more for corroboration.

The uncontrolled mosaic resulted in inconsistencies amounting to about 4,000 feet. By this is meant that when a U-shaped plot was made from two parallel flight lines linked together by intervening islands at one end, then intervening islands at the other differed by about 4,000 feet according to whether they were plotted from the right or the left arm of the U. Such discrepancies signify either distortion by tilt or change in altitude between photographs, or both, and the difficulty in adjustment is increased by the wide gaps between the rows of islands.

Comparison of photo-rectifications of adjacent pictures resulted in a few discrepancies, nearly as large as in the mosaic, in some of the more distant regions, i.e. beyond 75°; but by a process of adjustment to be described in the next section, and by proper use of azimuth lines, the uncertainties could easily be eliminated to an extent that was not possible with the verticals without far more elaborate control methods than simple overlapped tracing. When a final plot was made from the rectified obliques adjusted by these methods, the residual discrepancies amounted to about 500 feet at the most. A more thorough analysis would probably reduce the errors still further without the use of any ground control whatever.

OPERATIONAL AIDS

Since a major advantage claimed for the simple photo-rectifier is speed in obtaining approximate results, such devices and procedures as will expedite its use should be considered. A set of precomputed curves for finding the tilt has proved a time-saving convenience. Curves of this sort have been published in a previous communication;[5] they are reproduced in a slightly modified graph in Figure 15. Camera tilt is plotted against measured distance of the horizon trace on the photograph from either the principal point or the top of the picture for a

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[4] Cf. O. M. Miller. PHOTOGRAMMETRIC ENGINEERING, Vol. VIII, No. 1, p. 55, 1942.

[5] Forbes: Loc. cit., Fig. 4, p. 199.

6-inch focal length and for representative altitudes of the air station. The ordinate scale at the right expresses tilt in the standard sense, i.e., from the vertical; on the left are the complementary angles. The abscissae as shown at the bottom of the graph are perpendicular distances from the principal point to the horizon trace. In the K-17 camera, for which the rectifier is designed, the top of the picture is 4.50 inches from the principal point. It is much quicker to rule a line across the top of the picture where the margin is interrupted by the fiducial mark and to measure from this to the horizon trace than to find and mark the principal point and then measure from that. If the swing is not more than 3° or 4°, and if the measurement is made vertically from where the horizon crosses the principal line, the error due to swing will be unimportant. Therefore, the measured distance from the top of the picture is indicated at the top of the graph. For values of altitude other than those for which the curves are drawn, visual interpolation between curves will give the tilt nearly enough for such precision as is feasible.

Mention has been made of a process of readjustment when two or more oblique photographs of the same area are rectified. A single photograph of accurately known tilt, correctly rectified, will suffice for a true delineation of shorelines, within the limits of precision estimated above. Probable errors can be greatly reduced by combining the results of two or more photographs, especially if the horizon is so indistinct as to leave tilt somewhat indeterminate; the value of such combination is greatest if the photographs are taken from widely different directions.

When the rectifications of two overlapping photographs of a shore-line fail to agree, it may be due to a difference in altitude between the two air stations or to an error in tilt adjustment in one or both photographs or to a combination of both causes. Difference in altitude merely alters scale, and if the tilt adjustment is correct for both pictures the outlines can be made to match by simple enlargement or reduction. A few trial measurements, transverse and parallel to the principal plane, will reveal whether the difference in scale is equal in all directions. If it is not, then there is a discrepancy of tilt. The differentiation between scale inequality due to change of altitude, and tilt distortion depends on comparison of foreground and background areas. Tilt distortion is least in the foreground but rapidly increases in the distance; scale difference is the same all over the map plane. Even if the two pictures were taken in nearly the same direction, comparison on this basis may serve to evaluate a tilt error. If one picture has a bit of clear sea horizon or other valid clue to tilt, the rectification of the other picture may be adjusted to it by enlarging or reducing it, if necessary, till the foreground scales agree, and then judging the tilt error from the rate at which the remaining discrepancies increase with distance from the nadir.

Since the analysis is best done with a pair of photographs taken with camera axes at right angles, the procedure will be described for this case. Select the photograph with the least uncertainty of tilt and take the foreground of its rectified image as standard for scale. (Call this photograph No. 1.) Since the principal planes of the two photographs are nearly perpendicular, abscissae in No. 1 correspond to ordinates in No. 2, and vice versa. Select two points appearing in both pictures and as near the foreground of both as possible, so placed that the line between them is nearly transverse to the principal plane in No. 1, and parallel to the principal plane in No. 2. Comparison of the measured distances between these points in the two rectifications will give the best available clue to scale relation between them.

The tilt distortion is then found by selecting a series of points in No. 2 be-

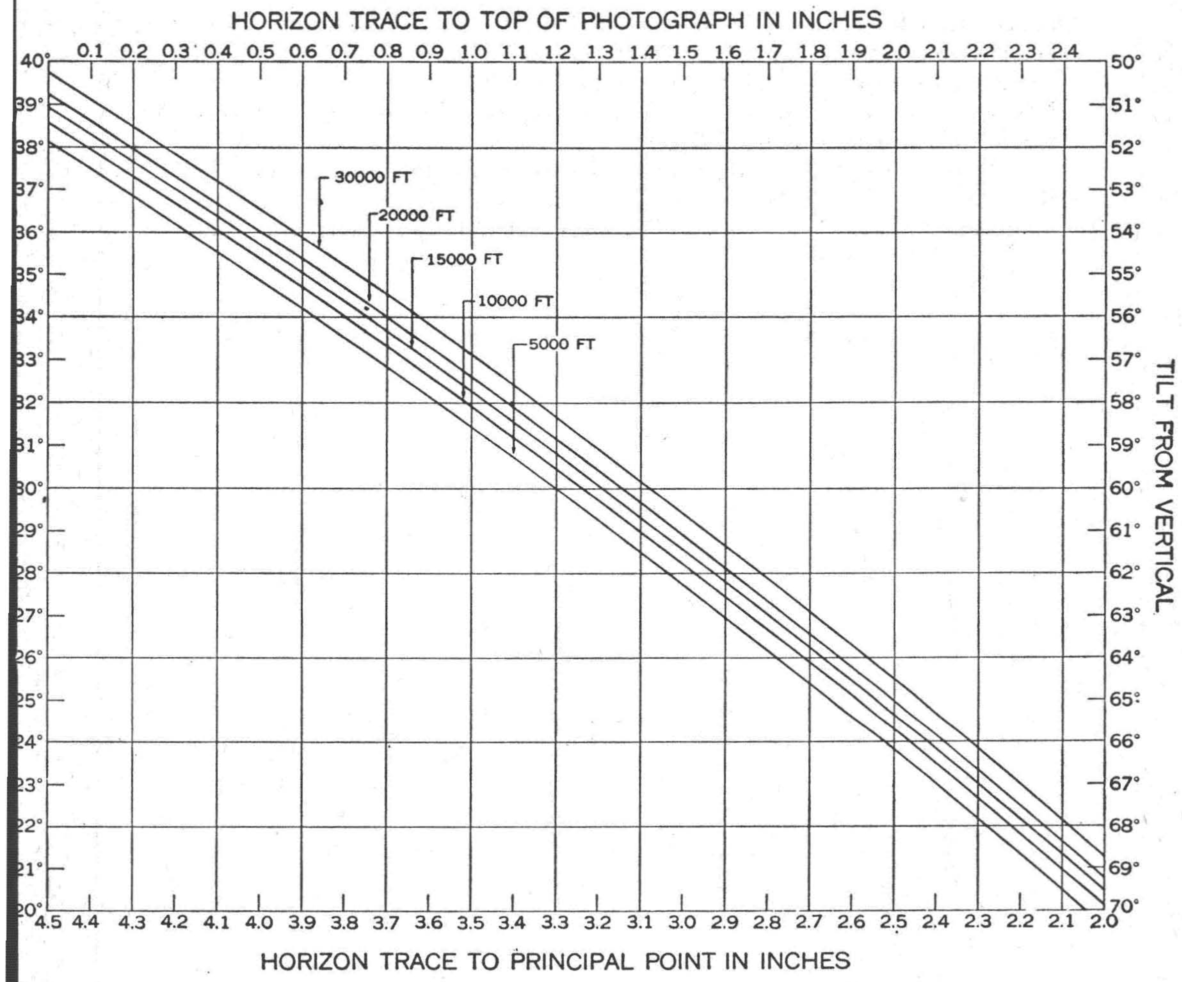
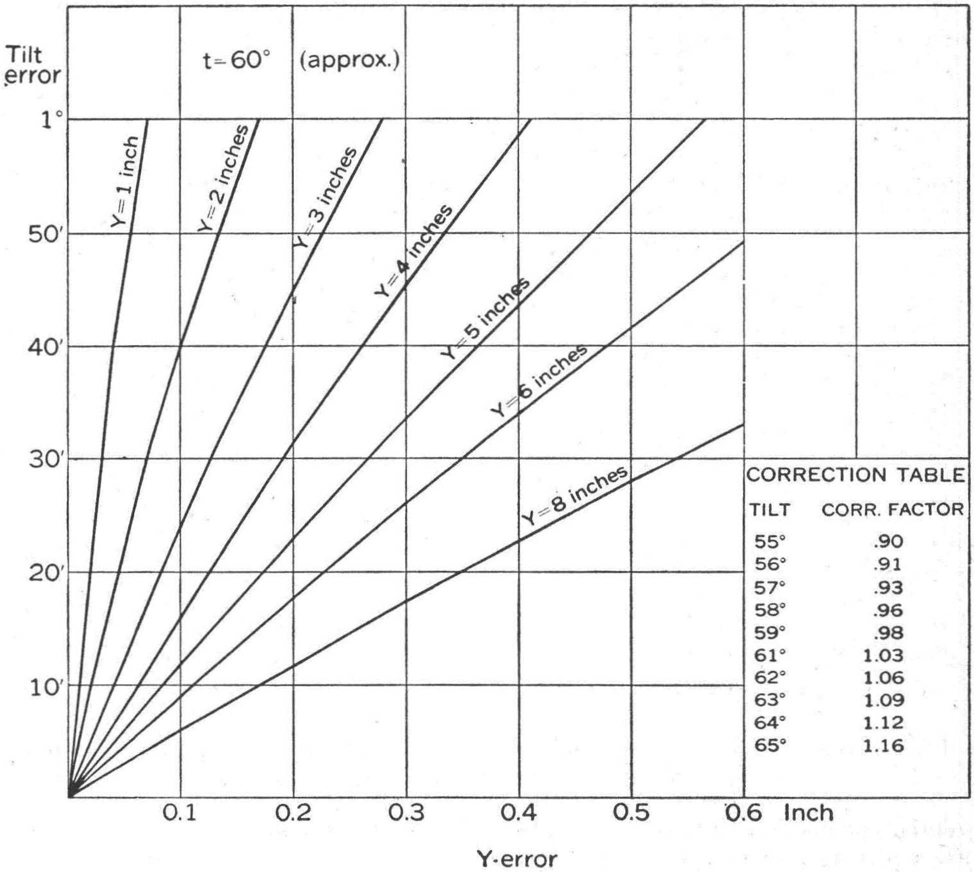


FIG. 15. Precomputed curves for finding tilt with 9×9 inch photograph, 6-inch focal length, for 5 different altitudes of air station.

yond its principal point, appearing also in No. 1, including one or two as far from the nadir as possible. The distance of each point beyond the transverse line at the principal point, P, is carefully measured and recorded. By matching the two rectifications, the principal line, principal point, and the transverse line at that point in No. 2 are located and marked on No. 1, and the corresponding distances from that transverse line to the selected points are measured in No. 1, for comparison with the corresponding distances in No. 2. If the tilt adjustment of No. 1 was correct, this comparison will show whether or not the tilt adjustment of No. 2 was correct, and if not, by how much it was in error.

If the foreground scales agree closely enough to show no appreciable difference in altitude between the two air stations, the tilt error of No. 2 can be quickly found by reference to the curves in Figure 16, in which tilt errors are plotted against ordinate errors for various values of the rectified ordinate (distances from the transverse line at *P* on the map plane). The difference between distance beyond *P* in No. 2 and the corresponding distance in No. 1 is found by subtraction for each of the selected points. If the points selected are close to integral numbers of inches beyond *P*, the curves may be used without interpolation. For example if a point 4 inches beyond *P* in No. 2 shows an excess of .26 inch over the corresponding distance in No. 1 it indicates a tilt error of 40′. A single well measured pair of distances will suffice, but it is well to use a series of 3 or 4 points to get a valid average. If points cannot be found close to integral inches from *P*, interpolation can be used between adjacent curves. These curves are drawn for a tilt of 60°; if the tilt differs from 60° by more than 1°, a correction factor should be applied; this will be found in the table appearing in Figure 16. The measured differences are multiplied by this factor and the curves are then used for reference as described above.



CORRECTION TABLE

TILT	CORR. FACTOR
55°	.90
56°	.91
57°	.93
58°	.96
59°	.98
61°	1.03
62°	1.06
63°	1.09
64°	1.12
65°	1.16

FIG. 16. Curves for finding tilt error from comparison of ordinates in photo-rectification with corresponding distances in a "cross-fire" rectification or other map known to be correct. Abscissae are errors in measured ordinates; ordinates are tilt errors deduced from the measurements. Each curve represents an ordinate value (distance from projection of principal point) on map plane, at which the error is found. Correction factors are for tilts other than 60°; measured errors should be multiplied by these factors.

If the foreground measurements show a scale difference, then the ordinate measurement must be correspondingly corrected before the tilt error is found.

Suppose, for example, the scale in the foreground of No. 2 is found to be 2 per cent larger than that of No. 1, indicating 2 per cent lower altitude of the air station. Then the distances beyond *P* of the selected points in No. 2 must be reduced by 2 per cent before comparison with the corresponding distances in No. 1. The curves are then used as described above.

When the tilt error in No. 2 has been found, it is a simple matter to readjust the tilt in the instrument and repeat the rectification. Excessive ordinates signify excessive tilt and call for a reduction in tilt adjustment. When scale and tilt in No. 2 are adjusted to harmonize with No. 1, the rectified image can be used to corroborate shore-line delineation in the area of overlap and to extend the map to such additional area as it covers.

SPEED OF OPERATION

The simplest way to rectify an oblique in this instrument is to place the negative between the glass plates of the negative box and project the image upon bromide paper; a rectified positive print is thus obtained in a single operation. Unfortunately this takes much too long for rapid mapping. It has already been seen that the distance requires many times as much exposure as the foreground and that to get an even print, a graded filter is needed. Using such a filter with a moderately dense negative, exposures as long as 16 hours have been needed to bring out proper detail on bromide paper. The time of exposure is reduced about 15-fold by using Eastman commercial ortho film instead of paper; for the same negative one hour sufficed. The Baker lens with the larger aperture reduces the required exposure about 5-fold but at the cost of some loss of foreground detail.

Film must be viewed and studied over a light box, and this may not be convenient. If a positive print be desired, this can be quickly made from the rectified film, itself a diapositive, by printing on direct positive paper. This is very sensitive, and about 10 seconds' exposure suffices for best results; the development requires less than 10 minutes. The positive print on paper can thus be had far more quickly than by direct printing from a negative whose density requires several hours' exposure.

Unless the original negative is unusually thin, time can be saved by tracing the shore-line on frosted acetate laid directly over the negative on a light box. A medium soft pencil will make a fine line dark enough to throw a clear shadow, and the exposure required will be about a tenth as long as with an average negative. Exposure of 3 minutes gives good contrast on commercial ortho film over most of the field, and 6 minutes will bring out the weaker marginal parts. Tracing the shore-line from a 9×9 inch negative, carrying the work to 76° or 78° from the nadir, requires anywhere from 2 to 30 minutes, depending on the extent and intricacy of the shore-line. Unless there are many small islands in the picture, it can usually be done quite accurately in less than 20 minutes.

The speed attainable in such a mapping project involves two kinds of time measurement, the time required to carry a single photograph through the entire process of rectification, and the amount of overlap that will permit work on a number of photographs to proceed almost simultaneously. Suppose a photogrammetric unit is prepared to use the negative of an oblique as soon as it is developed and dried. The following time schedule is based on actual experience; at some points it could be further speeded with practice.

Before the acetate tracing of the shore-line is set up in the rectifier, the horizon trace and the principal point must be located and marked on the tracing. The precision of what follows depends on the care with which this is done, and the process should not be hurried; 3 to 5 minutes are needed for this step alone.

This includes measuring the tilt with the aid of the curves in Figure 15, and writing it at the top of the tracing for reference when the tilt adjustment is made in the instrument. Since in any project a number of photographs will probably be used on an overlapping schedule, it is fair to take as the time for tracing the shore-line a liberal average of 20 minutes, rather than the extreme case of 30 minutes. The time required for removing one tracing from the glass plates and securing the next one with its principal point carefully centered and the horizon trace in correct alignment, is 8 minutes.

At this point the tracing is ready for transfer from the light box to the dark room for setting up in the rectifier. With lights on, the tracing between the glass plates is placed in the negative box, the filter is secured over the plates and the tilt adjustment is made. These steps require 4 minutes. Then with an overhead ruby light, the film is loaded into the map box, pressed down, and the lid is secured and covered with black cloth, all done in two minutes. Exposure is then begun at once, and we may assume an exposure of 6 minutes, to bring out marginal details.

If a double dark-room is provided, with arrangement for rapid transfer of the exposed film from a compartment where the camera is set up, to an adjoining compartment where development can proceed while the lights are on in the camera compartment, development can begin within less than half a minute from the termination of exposure, and the reloading of the rectifying camera can begin at once. Development requires 6 minutes, hypo and washing about 30 minutes more, and drying time can be reduced to about 30 minutes with the aid of an electric fan.

Adding up all these times it is seen that from the arrival of the original negative at the photogrammetric unit till the rectified film is dried and ready for the cartographer is less than two hours.

Handling a number of photographs on an efficient overlapping schedule requires a properly organized team. Assuming that only one rectifier is available, the only necessary bottleneck is the time required to set up the tracing, adjust the tilt, load film and expose it. This need not exceed 12 minutes; therefore photographs can be passed through the mill at approximately 5 per hour. The arrangements necessary for such a schedule include enough draftsmen to make tracings as fast as they can be fed into the rectifier, and at least 3 pairs of glass plates. The ideal arrangement would be to have one man specialize on placing the principal point and the horizon trace, operations that require precision and judgment, and on measuring tilt and mounting the tracing in the glass plates. Two draftsmen, doing two photographs simultaneously, could then trace the shore-lines quickly enough to maintain schedule. One man should operate the rectifier, adjusting tilt, loading film and timing the exposure. He should hand the exposed film to an assistant in the developing room and proceed at once to rectify the next tracing.

Additional personnel with knowledge of photogrammetry should check on the matching of overlapping rectified tracings, adjust scale for changing altitude and correct faulty measurements of tilt, by the method described above. If the horizon is indistinct in many of the pictures, intelligence will be required in making these adjustments, and several of the tracings may have to be rectified again after the true tilt has been evaluated. It will not be necessary to redraw the horizon trace, but merely to reset the tilt adjustment to its corrected value.

If we limit our estimate of area adequately covered to that within 75° of the nadir, and assume a standard tilt of 60°, and an altitude of 10,000 feet,

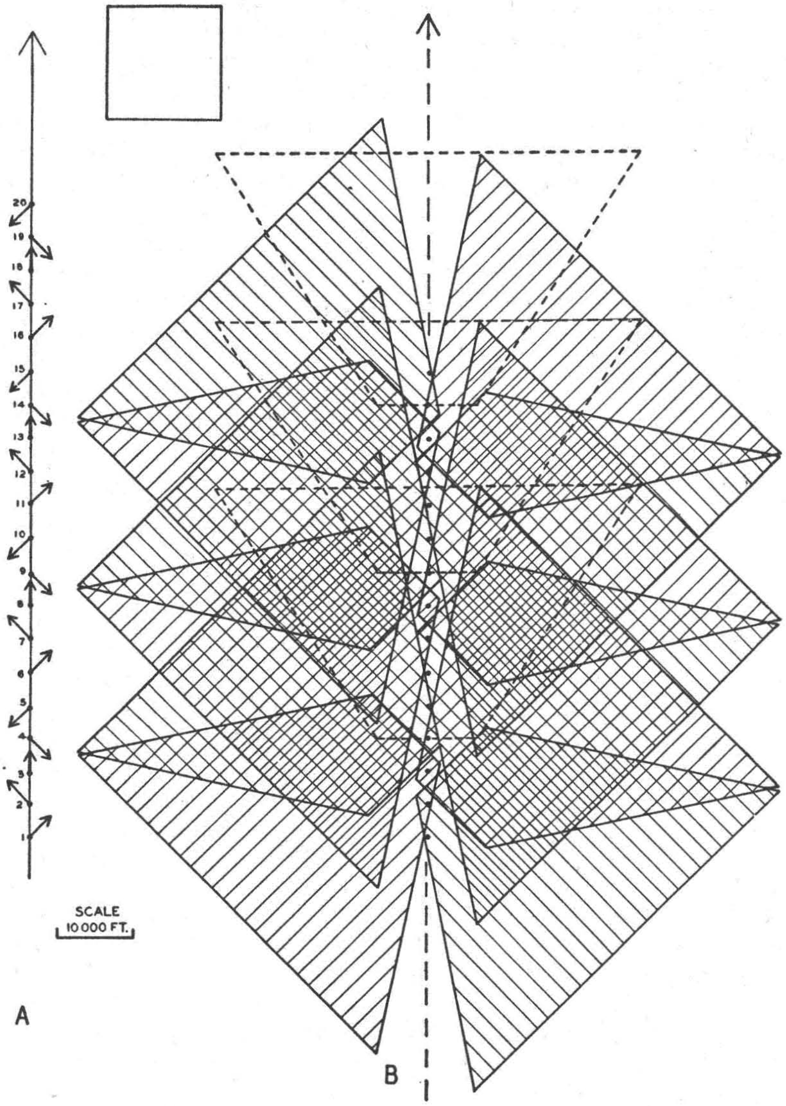


FIG. 17. Proposed arrangement of aerial photography to utilize rectified obliques with K-17 camera at 60° tilt.

A shows flight line with photographs taken at equal intervals, aimed as shown by arrows.

B shows areas covered by obliques rectified from foreground to 75° from nadir. Each flaring quadrilateral shows the area covered by a single picture (cf. Figures 4 and 5). Hatch lines are drawn parallel to the principal plane of each photograph; the extent of cross-fire is shown by their density. Solid outlines show areas covered by photographs diagonal to flight line; supplementary views along flight line are shown in dotted lines.

The square shows the area covered by a vertical from the same altitude. The scale line shows 10,000 feet on the datum plane, assuming an altitude of 10,000 feet. At higher altitudes the value of the line would increase proportionately, i.e., the length of the line equals the altitude, whatever its value.

each photograph will cover 4.1 square miles. A photographic plan in which the camera is aimed 45° from the flight line, half of the pictures forward and half back,[6] offers the advantage cf cross-fire. A possible plan that would make good use of rectified obliques is shown in Figure 17. It is assumed for convenience that the plane flies in a straight line at an altitude of 10,000 feet with a speed of 120 mph, and that high obliques can be taken in any desired direction at intervals of 25 seconds. Figure 17A shows the flight line with the air stations and direction of aim of the camera at each one. In each cycle of five pictures the sequence is first diagonally forward right, then left, then straight ahead, then diagonally backward right, then left. The distance between successive air stations is 4,400 feet. Figure 17B shows the pattern of coverage if each picture is taken with a tilt of 60° and rectified out to 75° from the nadir. Considering first only the diagonal views on one side of the flight line (e.g., the right side), such a pair as Nos. 1 and 4 provide a good foreground overlap suitable for mutual checking of tilt by the method described above. Nos. 1 and 9 have a mutual overlap of 74 percent, and therefore the abscissae of one can be used to check the more doubtful ordinates of the other over most of their combined area. Considerable areas are further strengthened by being covered by four different photographs.

The overlap between the right and left series of pictures is a small intermittent pattern straddling the flight line. Once in each cycle there is a line of contact but no overlap. To strengthen the overlap between right and left halves of the pattern, once in each cycle a picture is taken straight ahead along the flight line (or straight back if the plane design renders it easier). An even more important use of these pictures is to provide rigidity of control by long azimuth lines that can be extended along the flight line far beyond the 75° limit set for good rectification. Such lines do not depend on a good horizon or other means of precise tilt determination; they are valid wherever a straight line can be drawn through distinct landmarks distributed from the foreground into the distance. The complete pattern covers a band with a mean width of over 15 miles, with generous overlap everywhere except in small triangles near the outer fringes. Flying at 15,000 feet, a similar pattern would be covered with the interval between pictures increased to 37.5 seconds, and the band would be increased to a mean width of about 23 miles.

Vertical photography with the same camera and the usual 60 percent overlap covers a strip 2.84 miles wide from 10,000 feet, and 4.27 miles wide from 15,000 feet. Thus in an hour's flight at 10,000 feet the oblique pattern covers 1,800 square miles as against 340 square miles with verticals alone, and the same ratio of 5.3:1 obtains for any altitude.

In the laboroatory we have seen that though a single photograph requires nearly two hours for complete rectification, a long series can be put through the rectifier at 5 per hour. This means that if conditions are favorable for tilt measurement, and the rectified pictures can be matched well enough for the task in hand without repeated printing or corrected tilt adjustment, the material can be delivered to the cartographer at a rate of 63 square miles per hour, from a 10,000-foot altitude, or at 142 square miles per hour from 15,000 feet. The added time required for readjustment when poor horizons introduce significant errors in tilt, cannot be predicted accurately, but it is probable that rarely would the output per hour be reduced more than 50 percent.

If the reconnaissance is in an area wholly devoid of ground control, judicious use of "cross-fire" and of long azimuth lines will reduce errors approximately

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[6] Cf. O. M. Miller. Geographical Review, Vol. XXI, p. 201, 1931.

to the scale error due to whatever uncertainty there may be as to altitude. This scale error can be minimized by making a flight at an altitude within the range of a radio altimeter over a part of the area to be mapped, taking vertical photographs and measuring altitude simultaneously. Landmarks in these pictures will then provide a scale whose percentage accuracy is almost equal to that of the radio altimeter. For comparison of the laboratory schedules with verticals and obliques, if we assume that a draftsman can make overlapped tracings from verticals at 10 photographs per hour and the standard 60 percent overlap is maintained, the output of one draftsman will be 32.5 square miles per hour from a 10,000-foot altitude, or 73 square miles per hour from 15,000 feet. If the flight strip is divided between two draftsmen, their combined output will equal that of our hypothetical team with the rectified obliques.

Since "verticals" may actually have tilts of as much as 3° or 4°, an uncontrolled mosaic may well have cumulative errors that in a long series will greatly exceed those that escape detection when horizon obliques are properly used. Thus the advantages of the oblique pattern lie in better control where ground control is absent and in great economy of flying time and photograpny.

Probably the particular pattern shown in Figure 17 and evaluated in this discussion is not the best that could be devised, but it is offered as one that is fairly simple and has certain advantages in both precision and economy.

THE RÔLE OF OBLIQUES

Cartographers generally prefer verticals (in practice near-verticals) to obliques, for their use is simpler; yet sometimes obliques are the only mapping material available. The comparisons just made show that where ground control is meagre or absent, high obliques can make good much of the deficiency and are therefore preferable to near-verticals for precision, as well as economy.

The merit of obliques in bridging water gaps between islands too far apart to be properly related in vertical photographs, has been emphasized in the mapping test, described above.

A cloud layer frequently covers an island, rendering vertical photography impossible, but there may be clearance enough for good oblique coverage. In such a case, if the cloud is not too close to the ground, a single well-placed oblique may give the entire shore-line of an island. If the clouds are too low or the island too large, overlapping obliques taken in a single circuit of the island, or less, flown at a suitable distance off-shore, will give a complete map of the perimeter.

In time of war enemy-held territory can be photographed by long-range obliques with much less risk to planes and personnel from hostile interference.

For all of these reasons any instrument or procedure that increases either the speed or the precision with which geographical information can be derived from high oblique photographs, is of potential value to cartography. It is believed that in reconnaissance surveying, when greater precision than this instrument provides is not required, the photo-rectifier can serve a useful purpose.

SUMMARY

1. To meet the need for quick rectification of high oblique aerial photographs in reconnaissance surveying, a rectifying camera has been constructed, tested and used for shore-line delineation in sample mapping projects. It operates by transillumination of the photographic negative or a tracing from it on acetate, projecting the image through a simple pin-hole or a thin pin-hole lens, at the same tilt as that of the camera when the picture was taken, and thus photographing the rectified image on a level map plane close enough to the pin-hole to give adequate contrast in an exposure of a few minutes.

2. The instrument has been tested for precision of rectification, and with its tilt adjustable to the nearest 0.1°, is shown to be capable of rectifying sea-level features out to 75° from the nadir with horizontal errors less than 2 percent of the altitude of the air station.

3. Possible sources of error are analyzed, and their probable cumulative effect is shown to be not much greater than that which is inherent in the instrument. Such cumulative errors can be greatly reduced by "cross-fire," i.e., photographing an area from two viewpoints with camera axes at right angles to each other, thus checking distances in one photograph by the more accurate azimuth measurements in the other.

4. Two mapping tests have been made from routine oblique photography, one in an area accurately surveyed with ground control and vertical photography, the other in an area covered with both oblique and vertical photography, but otherwise unsurveyed, and with large water gaps to be bridged. The first test showed agreement with the actual survey to the degree of accuracy predicted on theoretical grounds. In the second test rectification of 13 obliques gave much more concordant results than an uncontrolled mosaic from 69 overlapping "verticals." In an area about 30×15 miles, the discrepancies in superposed rectifications from obliques nowhere exceeded about 500 feet.

5. A flight plan to take advantage of high oblique photography is proposed and the organization of a team for using the material by photo-rectification is described. It is estimated that the flight plan at a 10,000-foot altitude will cover 1,800 square miles per hour if the airplane speed is 120 mph, and a team of draftsmen and photographers with one or two photogrammetrists of only moderate experience, comprising a total of 7 or 8 men with one rectifier, could convert the developed photographs into a shore-line map of good reconnaissance accuracy at a rate of 63 square miles per hour.

6. Certain advantages of high oblique photographs are discussed, including economy of flying time and photography, extending control where ground control is weak or absent, bridging water gaps and penetrating areas where cloud cover precludes vertical photography. When these conditions are cogent, and when for any reason oblique photographs are the only material available, the photo-rectifier is a useful adjunct to reconnaissance mapping.

RESOLVING POWER

The Publications Committee is planning on publishing an errata sheet for the "Manual of Photogrammetry." It is not intended to include minor mistakes such as misspelled words. Any one knowing of errors in formulae or computations, incorrect mathematical signs, etc., is invited to submit them to the committee.

Maurice Perrier has been appointed manager of the eastern Division of Fairchild Aerial Surveys, Inc. He returns from three years' service as Chief of the Photogrammetric Division of the U. S. Army Map Service. His service with Fairchild dated from 1924.

The American Standards Assoc. announces that specification "Z38.2.12.—1943—American Standard Method of Determining Photographic Speed and Speed Number" has been prepared by R. T. Pierce of the Weston Electrical Instrument Corporation.