THE MAXIMUM VALUES OF TILT AND RELIEF WHEN COMPILING RADIAL LINE PLOTS USING THE PRINCIPAL POINT

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IN AERIAL photogrammetry, horizontal positions are usually determined by means of a radial line plot, prepared with slotted or "lazy daisy" templets. As is well known, radial lines drawn from the principal point on tilted photographs of terrain having relief do not represent true directions. The effect of tilt is radial from the isocenter, while the effect of relief is radial from the nadir point. Radial line plotting is based on the "radial line assumption" to the effect that if the tilt is kept within certain values, and if the relief is not excessive, then errors introduced by drawing radials from the principal point will be negligible.

The rigorous calculation[1] of the errors due to the radial line assumption gives rise to a rather long equation in which it is difficult to determine the tolerances that may be permitted in either tilt or relief if the resulting error in position of the radial line is not to exceed a predetermined amount. Obviously, it would be desirable to have some formula or set of charts so that with a given tolerance in the position of the radial line the maximum permissible relief or tilt could readily be determined.

Hart[2] has investigated the validity of the radial line assumption and shows that the effect of tilt distortion on lines drawn from the principal point is negligible and that the relief distortion is the governing factor in producing errors in radial lines. He shows that the maximum lateral error caused by relief of any line drawn from the principal point may be calculated from the formula

$$e = f \tan i \frac{h}{H} \tag{1}$$

where e is the lateral error, f is the focal length of the lens, i the angle of tilt, h the relief, and H the height of the air station above the datum. Kowalczyk, Fish, and Dill have also developed the same formula.[3] This formula as derived by Hart gives the lateral error for the worst position on the photograph, and can be written in this simple form by making certain mathematical assumptions. These assumptions do not invalidate the formula for rough approximations if the photographs are small and the tilts are not excessive.

Hart draws a series of curves that show the maximum lateral error plotted against tilt for the various values of the expression $f\,h/H$, and draws his conclusions as to the limitations in using the center point for various focal lengths and ratios of h/H (relief to flying height). This is not convenient for determining the maximum allowable relief, since the expression involves three variables.

[1] Bagley, J. W., "Aerophotography and Aerosurveying," McGraw-Hill, 1941.
Bagley, J. W., "Pivot Points for Radial Line Plotting," PHOTOGRAMMETRIC ENGINEERING, Vol. XI, No. 1.

[2] Hart, C. A., "Air Photography Applied to Surveying," Longmans, Green and Co., 1940.

[3] Kowalczyk, C. E., Fish, L. F., and Dill, A. P., "Manual of Photogrammetry," Pitman Publishing Corp., New York, 1944.

However, if the formula is rewritten as

$$e = \left(\frac{f}{H}\right) \tan ih \tag{2}$$

it is in more convenient form, and e is seen to be a function of the scale at which the plotting is done, the angle of tilt, and the relief. Notice that contrary to the widely accepted notion that the error in radial line plotting may be re-

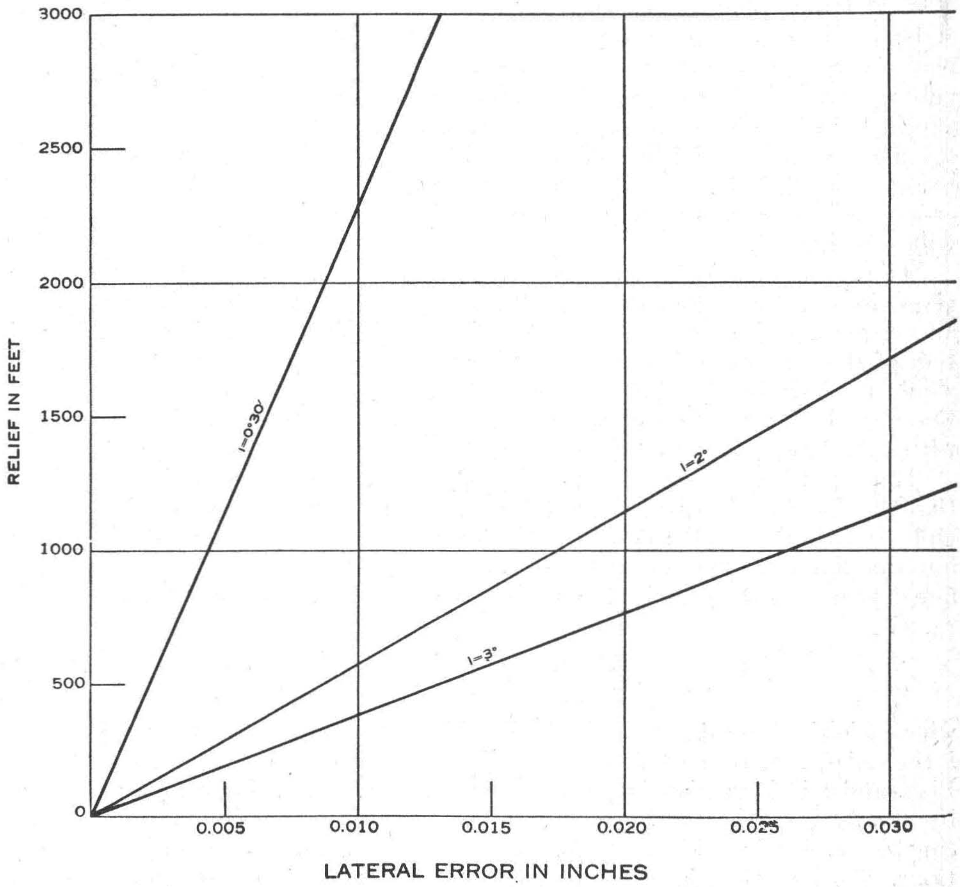

FIG. 1. Curves showing relief plotted against maximum lateral error for a mapping scale of 1:24,000.

duced by using a wide-angle lens, actually the focal length of the lens is immaterial, since if a long lens is used the flying height must be increased in order to obtain the specificed scale, thereby reducing the relief effect.

Using equation (2), curves have been prepared in which relief is plotted against the lateral error for various tilts and map scales. Having determined the maximum lateral error in radial lines that will satisfy the accuracy requirements of the work at hand, it is a simple matter to determine from the curves the maximum relief that may be tolerated at a given plotting scale and maximum tilt.

Figure 1 is drawn for a plotting scale of 1:24,000 and shows curves for tilt angles of 3°, 2°, and 0°30′. To use the chart, decide the permissible lateral error, and enter the chart along this line, following it vertically upward until it crosses the curve for the maximum value of tilt. Proceed horizontally to the left, where the maximum relief may be read directly from the ordinate scale. For example, when mapping at a scale of 1:24,000 it is desired to determine the

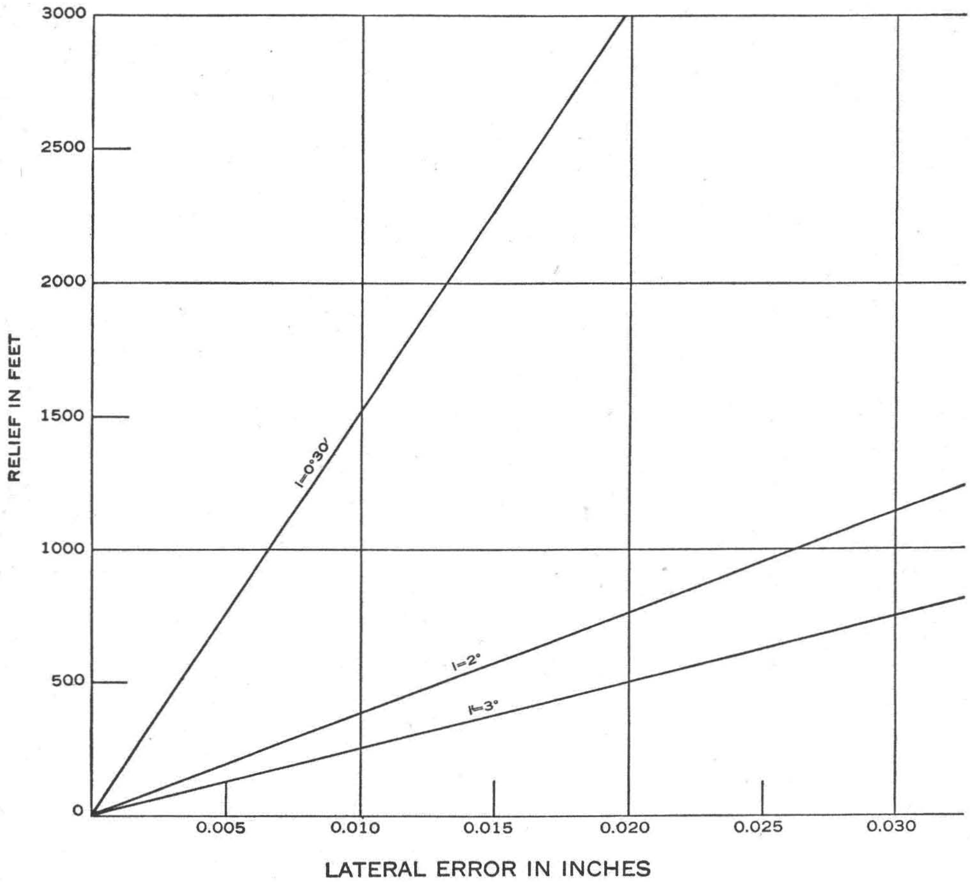

FIG. 2. Curves showing relief plotted against maximum lateral error for a mapping scale of 1:15,840.

maximum relief that may occur without introducing any error exceeding 0.01 inch in radial lines with photographs whose tilt will be 2° or less. Follow the 0.01 inch line upward until it intersects the curve for 2° of tilt, and then proceed horizontally to the left, where 573 feet is found to be the maximum relief that may occur without having errors that exceed the tolerance.

Figure 2 shows the curves for a plotting scale of 1:15,840.

The value of 3° was selected as the maximum value of tilt likely to occur in photographs taken for standard mapping. Since it is not unreasonable to expect 2° to be the maximum tilt permitted under specifications drawn in the near future, a curve was drawn for that value to show how this reduction in tilt will increase the relief that may be handled without difficulty under these conditions.

As automatic pilots become available to commercial concerns, there should be a further reduction of the maximum tilt encountered. Tests run in Great Britain just prior to the war indicate that with the automatic pilot maximum tilt values may be kept to 0°30′ or less. Accordingly, curves are drawn for a

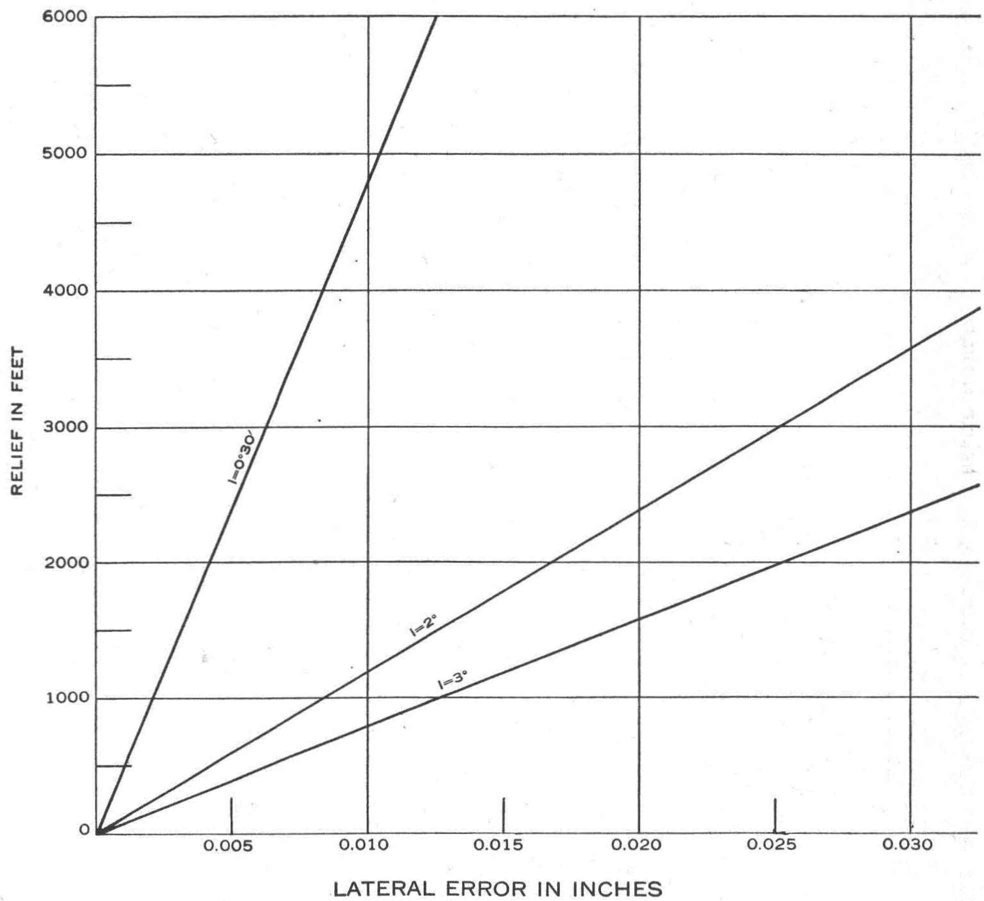

FIG. 3. Curves showing relief plotted against maximum lateral error for a mapping scale of 1:50,000.

tilt of this amount, and it is of interest to note the tremendous increase in permissible relief. With a plotting scale of 1:24,000 and a tolerance of 0.01 inch in lateral position of a radial line, the maximum relief becomes 2,295 feet instead of 573 feet. This vastly increases the areas where plotting may be done accurately without recourse to tilt determination.

Aerial surveying finds a wide field of usefulness in exploration and reconnaissance work where compilation scales are much smaller than in standard mapping. Figure 3 has been prepared to show the permissible relief for varying values of maximum tilt, when the plot is compiled at a scale of 1:50,000. It is of interest to note that even with tilts as large as 3°, relief as great as 750 feet will cause no difficulty. If the automatic pilot should prove capable of keeping tilts to 0°30′ or less, relief as great as 4761 feet may occur without producing errors of more than 0.01 inch in the position of any radial line drawn from the principal point.

It should be noted that the error caused by drawing radials from the principal point is directly proportional to the plotting scale and to the relief. Accordingly the maximum permissible relief for compilations made at 1:31,680 will be exactly twice that for a scale of 1:15,840, and it is not necessary to prepare curves for every possible scale.

The discussion up to this point has been based on a formula derived by making certain assumptions. These assumptions are permissible for most purposes if the tilt is less than 2° and when the length of the radial lines does not exceed that normally encountered on a 9×9-inch picture, but if the tilt is large or the plot is made at a larger scale than the original 9×9 photographs the formula used becomes inexact. The greater the tilt, or the greater the enlargement of the plot, the greater the discrepancy between the actual errors and those calculated by the formula.

This inexactness arises from the assumption that the effect of tilt distortion is negligible for lines drawn radially from the principal point. Hart and Kowalczyk, Fish, and Dill show that the lateral error, e_T, in radial lines so drawn may be calculated from the formula

$$e_T = \frac{L}{4} \cdot i^2 \cdot \sin 2\theta \qquad (3)$$

where L is the distance from the isocenter to the point, i is the angle of tilt, and θ is the angle measured at the isocenter between the principal line and the radial line to the point. It will be noticed that this error increases as the square of the angle of tilt, so it is sixteen times as great for a tilt of 4° as it is for a tilt of 1°, showing that it must be considered when tilts get large. This error increases directly as L increases, and therefore becomes appreciable if the points are far out towards the edges of large pictures such as are obtained with multi-lens cameras, or if the plot is at a larger scale than the original photographs. This error is a maximum when θ is 45° (sin 2×45° equals 1) and the computations to be described were made assuming θ to be 45° so that the resulting errors would be maximum values.

Using equation (3), the errors due to tilt distortion in radial lines drawn from the principal point were computed for angles of tilt of 0°30′, 2° and 3°. Values of L—the distance to the point from the isocenter—varied from 0 to 15 inches. On a 9×9-inch picture the maximum value of L is around 6.5 inches, while on an 18×18-inch print (2-diameter enlargement) the maximum will be about 13 inches, so that the results should be applicable for all cases except perhaps the extremities of a composite print.

Figure 4 shows the results of these computations with the lateral error in inches plotted as abscissa and the distance L as ordinate. To use the graphs, determine the maximum value of L for the pictures at hand, and enter the graph opposite this amount. Proceed horizontally to the curve for the maximum value of tilt permitted under the specifications, where the lateral error in inches may be read from the horizontal scale.

The value obtained from Figure 4 is an error in the radial line assumption that was not included in the earlier graphs. If accurate knowledge of the permissible relief in a given area is desired and it is felt that tilt distortion may may not be negligible, the procedure is as follows:

Find the maximum value of L for the photographs to be used and from Figure 4 obtain the error resulting from tilt distortion for the maximum value of tilt permitted under the specifications. This value is subtracted from the total acceptable error in radial lines. The difference so obtained is then used in Figure 1, 2, or 3 (depending on the scale of the plotting) as the maximum

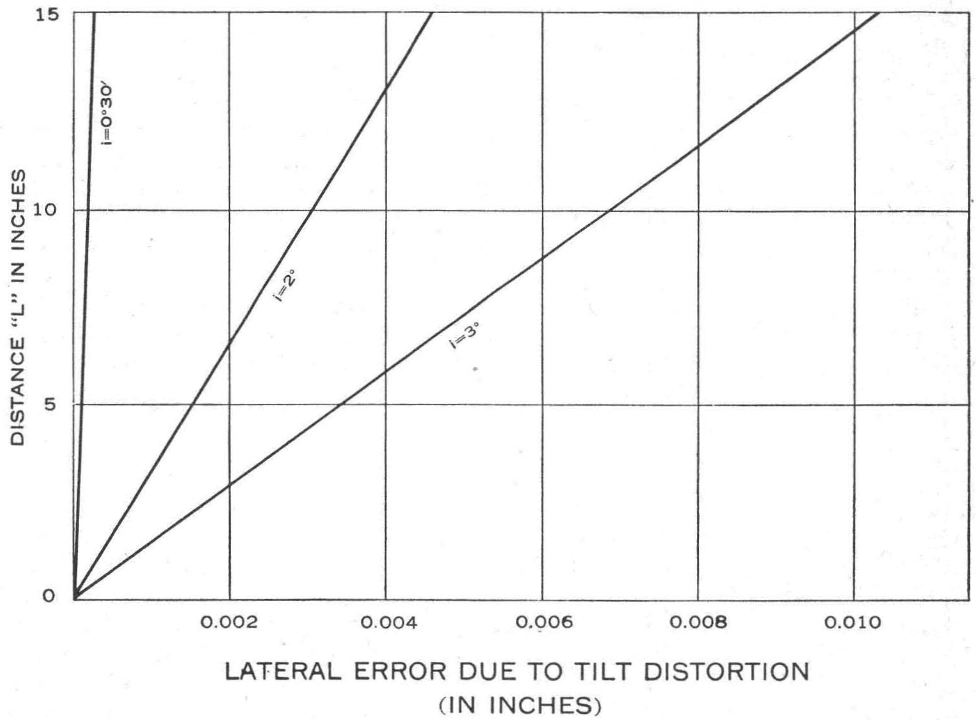

FIG. 4. Curves showing lateral error in inches in radial lines drawn from the principal point due to tilt distortion.

lateral error, and the maximum relief may be determined as before, but this value will be accurate for large tilts, since tilt distortion is allowed for.

An example will make the procedure clearer. It is desired to determine the maximum relief in an area that will be plotted at 1:15,840 with the original 9×9-inch pictures taken at 1:31,680. Under the specifications, the maximum tilt is not to exceed 2°, and it is felt that sufficient accuracy will be obtained if the lateral error in the radial lines does not exceed 0.01 inch. Since the plot is twice the scale of the photographs, L may be assumed to be 13 inches in the worst condition.

Using Figure 4, it is found that for a tilt of 2° the lateral error due to tilt distortion is 0.004 inch. The total acceptable error is 0.01 inch, so the relief distortion in this case must not exceed 0.006 inch. From Figure 2 it is found that for a tilt of 2° and an error of 0.006 inch, the maximum relief is 225 feet.

If it had been decided to have the original photographs at the scale of the plot (1:15,840) the picture size would have been 9×9 inches, and L could safely be assigned a value of 6 inches. Under these conditions, Figure 4 shows that the maximum tilt distortion would be 0.002 inch, leaving 0.008 inch for relief distortion. Using this value in Figure 2, it is found that the relief may be as great as 300 feet without introducing any undesirable error.

It is felt that with these graphs or similar ones made to meet particular situations, it is a simple matter to determine the maximum relief that may be handled with radial line plots without recourse to tilt analysis to determine the vertical point. It should be emphasized that Figures 1, 2, and 3 are approximations only, and should be used with caution. If tilts exceed 2° or if the size of the photographs exceeds 9×9 inches, Figure 4 should always be employed to determine the amount of the tilt distortion. If this is subtracted from the total allowable error, the difference is the value of the relief distortion and may be used in the first three figures to determine accurately the maximum permissible relief.