

TECHNICAL INSTRUCTIONS ON THE NADIR POINT METHOD*

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GENERAL

THE Army Map Service, Corps of Engineers, U. S. Army in AMS Bulletin No. 12, October 1944 published a method of resecting the aircraft in space given one vertical air photograph of which the principal point and distance are known and in which five points appear whose position and height are known.

The points are required to be placed so that one falls near the centre of the photograph and the other four near the corners. The method only applies to flat ground but it can be used by graphical successive approximation for undulating ground.

From first principles it is clear that there are two redundant data in the problem. It can also be shown that the method used is only true when the centre point lies on the intersection of the diagonals of the figure formed by the other four points. From simple perspective principles, if the four corner points lie in one plane the point of intersection of the diagonals on the ground plane and on the photo plane must correspond. The method can therefore be used with four control points only and the method of construction is then rigorously true for flat ground.

With four points, there is still one redundant datum. It will be seen later that, in the normal case, this is the angle between the diagonals on the map plane, for the correct solution will not be affected by one diagonal being incorrectly oriented with respect to the other on the map plane. Since the original object of the method was to re-establish photo points upon map points even if the latter were incorrect, it is obvious that this last consideration entirely vitiates the original object. Nevertheless, there are cases where the resection of the air station is necessary from ground data in one photograph, and the modified method adapted to points at unequal heights is therefore outlined below.

OUTLINE

The method consists in obtaining graphically the internal perspective pencil of rays from the perspective centre of the lens to three collinear points on the photograph. These points are the two diagonally opposite known points and the intersection of the diagonals.

The external perspective pencil is exactly similar. If this is considered as set up on the plotted map positions of the points and then rotated round the line through the points into the map plane the foot of the perpendicular from the perspective centre will lie along a line at right angles to the line through the points.

If the same procedure is adopted for the other diagonal, another line at right angles to this diagonal is obtained containing the foot of the perpendicular from the perspective centre. The intersection of these two loci is therefore the foot of the perpendicular on the map plane, in other words, the plumb point or Nadir Point.

The following are the steps in the method.

- a. Plotting of the four control points.

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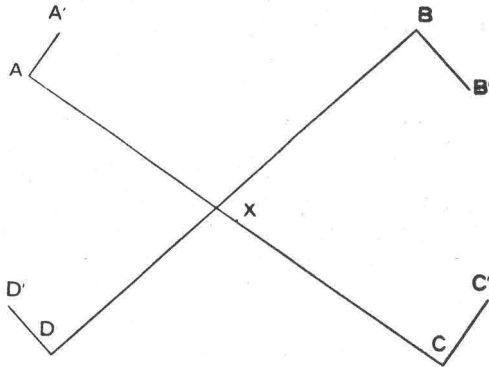


FIG. 1. Plot Sheet.

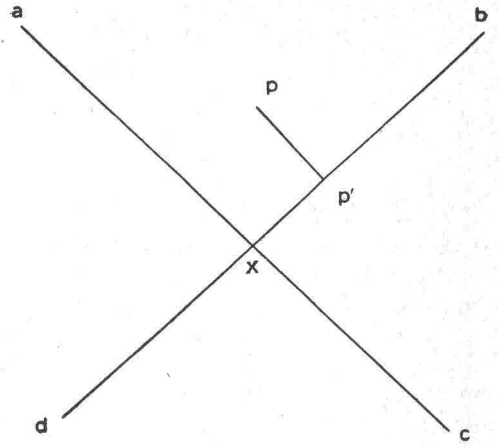


FIG. 2. Photo.

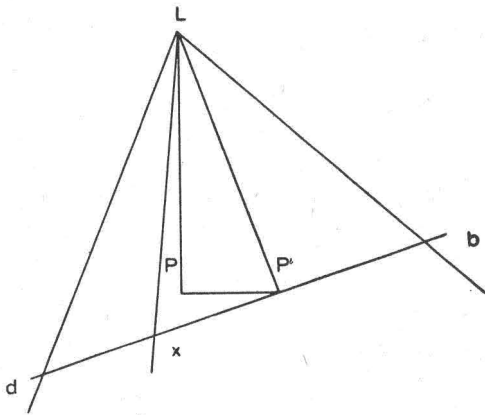


FIG. 3. Kodatrace.

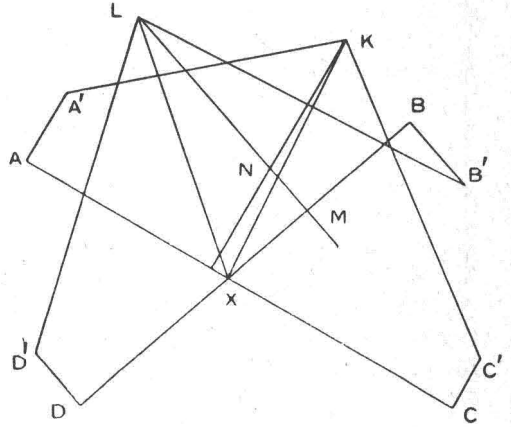


FIG. 4. Resection.

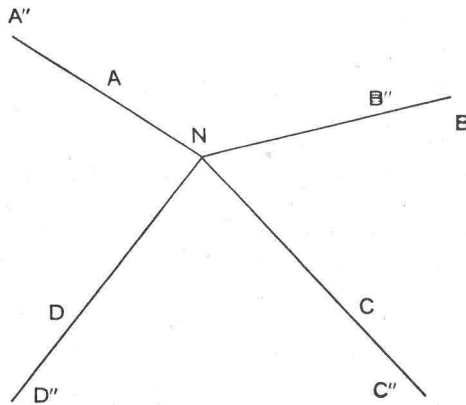


FIG. 5. Height Displacement.

- b. Drawing of the perspective pencils from the photograph.
- c. Resection of the Nadir Point.
- d. Correction of the apparent positions of the control points.
- e. Revised resection of the Nadir point.
- f. Determination of height of aircraft.

PLOTTING (See Fig. 1)

Select a plot scale somewhat larger than the scale of the photograph for preference.

Plot the four control points on the plot. Join the diagonals of the figure, AC and BD . Select a datum height plane as nearly as can be estimated at the height of the ground point falling at the intersection of the diagonals, X .

Decide from which side each pencil will be plotted. Let DXB be plotted with the pencil rotated away, i. e. on to the map plane towards A . Erect DD' and BB' perpendicular to DB and so that DD' , BB' represents the height of points D , B out of the datum height plane, drawn to the plotting scale. DD' is drawn towards A when D is above the datum height. (D is assumed above and B below the datum height in Fig. 1.) Repeat for AA' and CC' .

PERSPECTIVE PENCIL

The perspective pencil is to be taken from the Photograph. It is required from the sloping plane, Lbd .

The construction is as follows (see Fig. 2):

Prick the principal point (p) and the photo images (a, b, c, d) of A, B, C and D on the photograph.

Join ac and bd and mark the intersection x . Draw pp' perpendicular to db with p' on db . On a sheet of kodatrace (see Fig. 3) draw a right angled triangle LPP' with LP equal to the principal distance of the photograph, PP' equal to pp' on the photograph. Draw a line at right angles to the hypotenuse LP' . Place the kodatrace over the photograph with P' over p' and the line db on the kodatrace over the line db on the photograph. L should lie on the side of db selected for plotting in paragraph 3 (viz. the same side as A). Draw the rays Ld, Lx and Lb . Repeat on another piece of kodatrace for Ka, Kx and Kc .

RESECTION (See Fig. 4)

Lay the kodatrace pencil Ld, Lx and Lb on the plot sheet so that Ld, Lx and Lb pass through the points D', X and B' respectively. Prick through L .

Drop a perpendicular LM from L on to DB and extend it.

Repeat for Ka, Kx and Kc . Prick through K and draw KN perpendicular to AC with N on LM .

Then N is the plumb or Nadir point.

CORRECTION OF THE APPARENT POSITIONS OF THE POINTS

If the heights of the four control points are considerably different and N lies well away from the lines BD, AC on the plots, the apparent positions of the control points will have to be obtained on the map plane for the final resection. This is done as follows:

(See Fig. 5.) Draw the lines NA, NB, NC and ND on the plot and measure them. On these lines mark A'', B'' etc., so that AA'', BB'' is the computed height displacement of the points, A, B etc. This is: $AA'' = NAh_a/H$ where h_a is the height of point A above the height datum and H is the aircraft height above datum. A'' is on the same side of A as N if point A is below the height datum, and on the side away from N if A is above the datum.

H may be obtained from the flight data or by the method of paragraph 8. Draw the lines $A''C''$ and $D''B''$ to intersect in X'' .

REVISED RESECTION

Using the same kodatrace perspective pencils, repeat the resection procedure of paragraph 5 using the points A'' , B'' , C'' , D'' and X'' in place of A' , B' , C' , D' and X' . Draw LM and KN at right angles to $A''C''$ and $D''B''$.

AIRCRAFT HEIGHT

The aircraft height above the height datum on the scale of plot is given by the third side of the right angle triangle whose hypotenuse is LM and one side of which is MN .

The construction therefore is:

About M draw a circle of radius MN ;

Draw LG tangent to this circle;

Draw GM at right angles to LG and measure LG on the scale of the plot. This is the aircraft height above the height datum.

ADDENDUM TO ARTICLE ENTITLED, "HISTORY OF STEREOSCOPY"

Revere G. Sanders

The only major American development in photogrammetric equipment is the Brock process. During the First World War, into 1921, the Brock equipment was developed in Philadelphia. It was later successfully used by Brock and Weymouth, Inc. in producing topographic maps to various scales and contour intervals. The accuracy of the method is such that maps showing 1 foot contour intervals may be drawn and large areas have been mapped showing 2 foot contour intervals by the Aero Service Corporation, Philadelphia, present owners and operators of the equipment. A special glass plate camera is used to make the original exposures which are $6\frac{1}{2}'' \times 8\frac{1}{2}''$ in size. Thereafter, glass plate enlargements are made to $13'' \times 17''$ and all subsequent steps are carried forward to glass plates of this size.

The Brock process is so arranged that various steps are completed on separate units of equipment which facilitates production and makes it possible to carry on more than one project at a time. Another feature of the process is that it is the only method which enables the operator to draw contour lines while viewing the stereoscopic model as well as the contour lines that have already been drawn. Greater accuracy and expression are possible because the contours can be directly checked against the stereoscopic model as additional lines are drawn. A complete description and method appears in the "Manual of Photogrammetry," Chapter 11, page 522, or an article in PHOTOGRAMMETRIC ENGINEERING, 1940, Volume VI, No. 2, page 55.