EFFECT OF EARTH CURVATURE IN SCALING FLIGHTS BY MULTIPLEX

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IN MULTIPLEX work one frequently hears the expression, "scaling a flight." By scaling a flight we mean the orientation of a series of successive photographs in multiplex projectors so that each model maintains a preassigned scale, both horizontally and vertically.

We assume that the datum for measuring elevations is sea level. For very

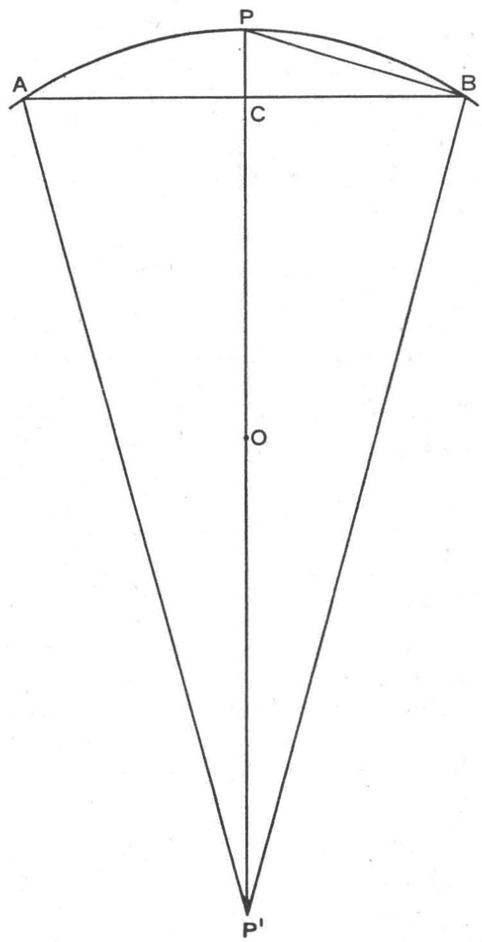

FIG. 1.

small areas the datum is sensibly a plane, but for larger areas the datum must be considered as curved since the earth's surface is curved. In scaling flights on the multiplex it is the practice to use a flat table top as a reference plane, so that the datum plane is a certain distance from the table top and parallel to it. We are thus interposing a flat datum plane for an area in which the datum surface is curved. Thus, when a flight has been scaled and levelled across the ends, a line in the direction of the flight and at a fixed elevation above sea level is not a straight line, but a curve corresponding to an arc on the earth's surface. In this paper we shall calculate the distance of this arc from its chord at the middle ordinate. Let *APB* represent the arc on the earth's surface in Figure 1. *PO* is the radius and *POP'* is the diameter.

We assume the earth to be a sphere with radius equal to the mean of the equatorial radius and the polar semi-axis, according to the Clarke spheroid of 1866. The radius, *R*, is thus taken to be 3,956 miles.

From the diagram we see that triangle *PBP'* is a right triangle since it is inscribed in a circle and one side is a diameter. Since *BC* is perpendicular to *PP'* we have, from geometry:

$$PC:BC::BC:CP'. \tag{1}$$

Designating the length of the arc *APB* by x we see that the length of *BC* is very nearly equal to $x/2$ and the length of *CP'* is very nearly equal to $2R$. Substituting these values in (1) we get:

$$PC = \frac{x^2}{8R}. \tag{2}$$

Using this formula the table below is readily computed

Arc in Mi.	*Dist. Arc to Chord*
0	0 ft.
10	17 "
20	67 "
30	150 "
40	267 "
50	417 "
60	600 "
70	817 "
80	1,068 "
90	1,351 "
100	1,668 "

From the above table we see that if the flight exceeds 8 miles, vertical control established in the center of the flight will be in error by .3 mm. on a 1:10,000 scale. Thus in establishing vertical control for such flights it is necessary to adjust the elevations read from the plotting table. As a concrete example let us consider a flight of 20 miles and a scale of 1:10,000. The table shows that an elevation point in the center of the flight will read too high by an amount equivalent to 67 feet or 2.0 mm. on the map scale. Thus if a point reads 58.3 it is adjusted to an elevation corresponding to 56.3 mm. The adjustment for points not in the center of the flight vary as the square root of the distance from the nearest end of the flight. For a point midway between the center of the flight and one end the adjustment is

$$\sqrt{.5} \times 2 \text{ mm.} = 1.4 \text{ mm.}$$