Abrams Award 12/16

AUTOMATIC MAP PLOTTING INSTRUMENTS*

THE DEVILLE TYPE OF EQUIPMENT

By Duane Lyon
Aeronautical Chart Service

MOST of the map plotting devices developed in the United States resemble certain instruments previously proposed by investigators in other countries. This is not surprising in view of the large amount of research work that has been done on plotting instruments during the last fifty years. Each of these "new" American devices possesses the basic theory of the original prototype although its construction may be quite different due to an improved optical and mechanical design. The gradual improvement in the design of map plotting devices is typical of the development of almost all scientific instruments. This evolutionary process is clearly demonstrated in the development of the Multiplex Aeroprojector.

In 1898 Scheimpflug first proposed the theory of double projection[1] which was utilized by Gasser in 1915 in the development of a crude sort of plotting instrument[2] somewhat resembling the Multiplex. Gasser's device used the principal of anaglyphic projection proposed earlier by Smith and d'Almeida for somewhat different purposes.[2] Practical use of twin point interpolation for the establishment of relative orientation apparently was not made until 1923.[3] Shortly after this time Zeiss began the manufacture of the Multiplex Aeroprojector. These instruments probably were the first projectors to possess an adequate depth of sharp focus.[4] Photogrammetrists in this country have contributed major improvements to the optical and mechanical design of this instrument. For example, the oblique Multiplex was developed about 1935, the improved wide-angle Multiplex in 1937. The re-designing of the Multiplex reduction printer in 1939 permitted the establishment of stereoscopic models relatively free of lens distortions.

The principal of the Deville plotter has followed a course of development that is comparable to that described for the Multiplex, although it has not been carried to such a state of perfection. Instruments such as the K.E.K. Plotter, the Mahan Plotter, the Stereotopograph, and the Multiscope are all similar to Deville's plotting instrument and have contributed to the knowledge and technical background required for the understanding and proper evaluation of the new instrument designs to be described in this article.

It is interesting to note that the fundamental designs of Scheimpflug and Deville received very little consideration until the last fifteen years or so. Possibly this was because no one was able to discover a relatively simple combination of optical and mechanical elements that would satisfy the theoretical requirements of these particular designs. The theory, construction and operation of the Multiplex Aeroprojector are generally understood. This article is therefore mainly concerned with a discussion of those plotting instruments developed in the United States which resemble Deville's original proposal.

* Published by permission of the Commanding Officer, Aeronautical Chart Service.

[1] Von Gruber, O., *Photogrammetry*, American Photographic Pub. Co., page 160.

[2] Ibid, page 198.

[3] Ibid., pp. 201, 434

[4] *Manual of Photogrammetry*, Pitman Publishing Corp., pages 53, 67.

DEVILLE PLOTTER

Figure 1 illustrates Dr. E. G. Deville's original map plotting device.[5] The instrument consists of a viewing stand *A*, possessing two apertures *D* and *D* which serve as eye pieces, and two half-silvered mirrors *F* and *F*; of two frames *B* and *B* for supporting the photographs; and of a tracing stand *C* which supports a tracing pencil *N* and a floating mark mounted on the viewing screen *L*. The virtual images of the photographs are corrected for tilt and azimuth by the rotation of the two half-silvered mirrors *F* and *F* about their respective vertical and horizontal axes. The mechanisms for making these adjustments are not shown in Figure 1.

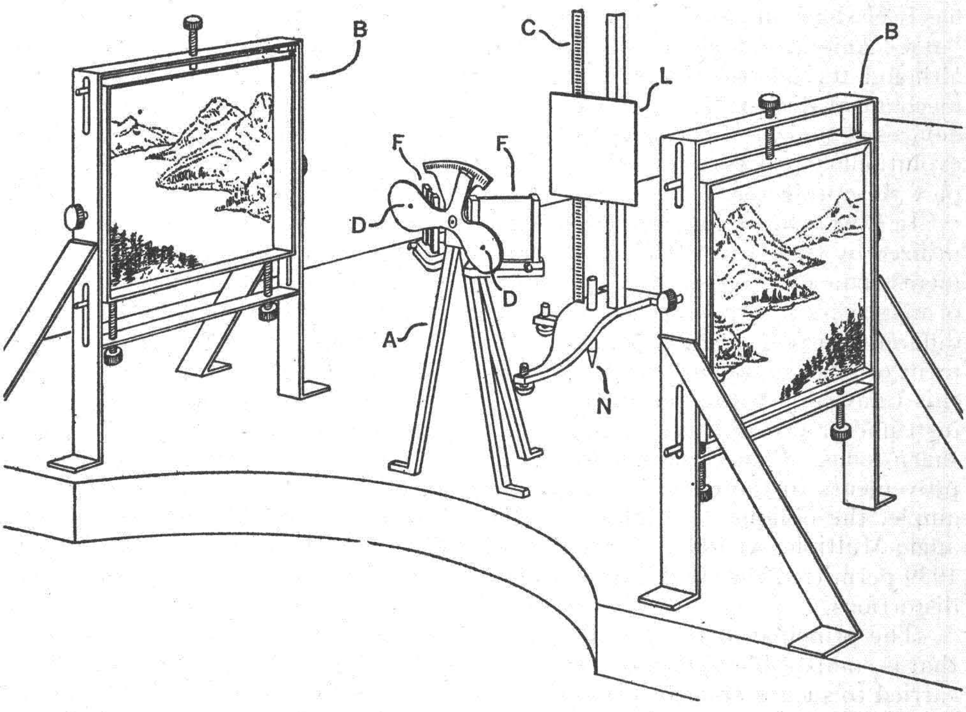

FIG. 1. The Deville Plotting Instrument. (Courtesy, Royal Society of Canada).

In connection with this device it will be noted that:

1. It was obviously intended by Dr. Deville that the apertures, which serve as the eye pieces of the device, should be placed at the true perspective centers of the virtual images of the corresponding photographs. However, the method which has been provided for making changes in tilt introduces small errors in the location of the perspective centers. These errors might be of significance in the plotting of oblique or terrestrial photographs.
2. The proper dimension of the apertures was not specified by Dr. Deville. However, the diagram (Figure 1) that accompanied his original proposal indicated that these apertures were to be very small. It is not known whether or not Dr. Deville was aware of the important optical effects that would be produced by the use of these small apertures.

[5] Deville, E., "On The Use of Wheatstone Stereoscope in Photographic Surveying," From *The Transactions of the Royal Society of Canada*, Vol. VIII, Section III, 1902. (Read May 27, 1902.)

3. No provision was made for changing the plotting scale.
4. Special photographs, printed in reverse, were to be used in order to correct for the reversed field of view caused by the use of a single mirror in each field of view.
5. The instrument was designed to use terrestrial (horizontal) photographs.
6. The device was relatively simple, both optically and mechanically.

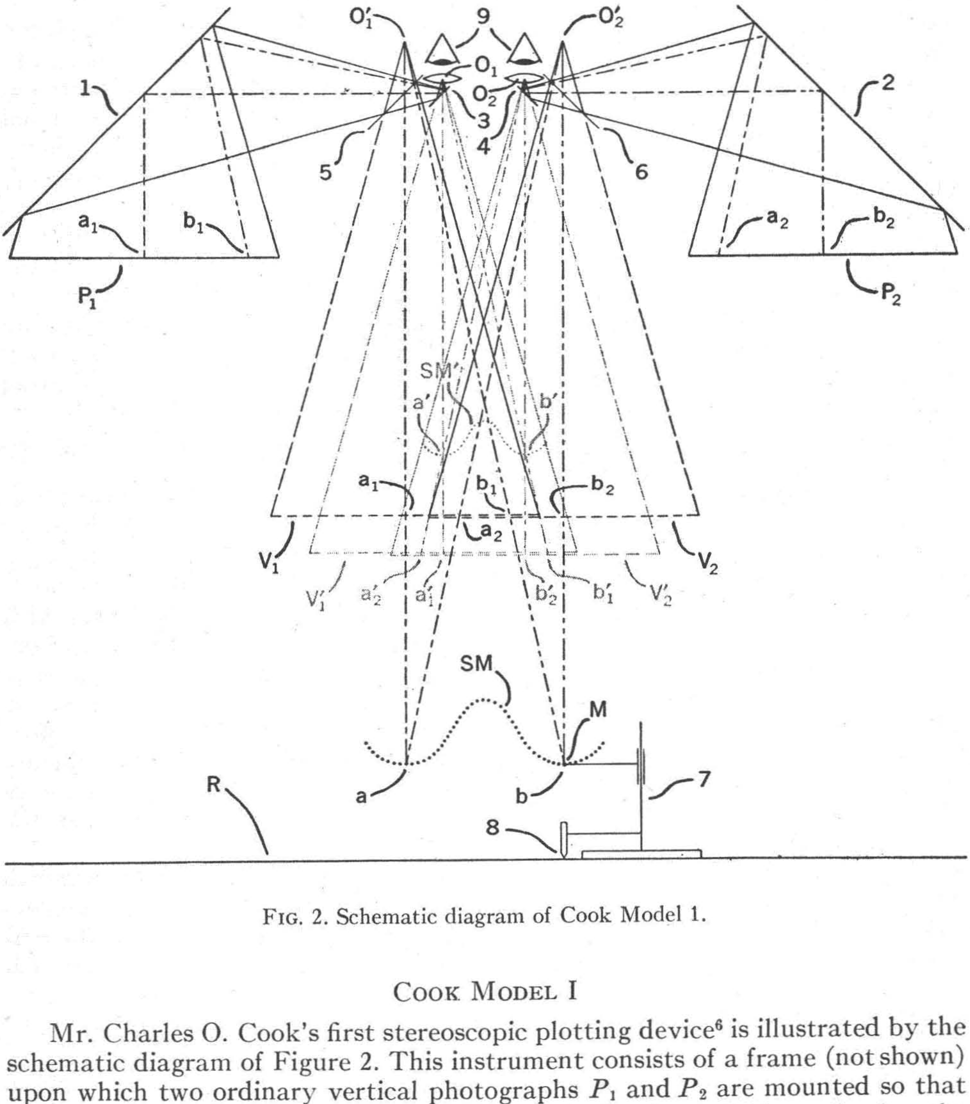

FIG. 2. Schematic diagram of Cook Model 1.

COOK MODEL I

Mr. Charles O. Cook's first stereoscopic plotting device[6] is illustrated by the schematic diagram of Figure 2. This instrument consists of a frame (not shown) upon which two ordinary vertical photographs P_1 and P_2 are mounted so that they may be tilted about their respective principal points, which are fixed on the frame of the instrument. The full-surfaced mirrors 1, 2, 3 and 4 form an ordinary mirror stereoscope. The operator's eyes 9 are placed immediately behind the simple magnifying lenses O_1 and O_2 which serve as the eye pieces of the instrument and locate in space substitute perspective centers for the photographs. The

[6] U.S. Patent 2,363,643.

half-silvered mirrors 5 and 6 produce virtual images of the perspective centers at O_1' and O_2' and of the photographs at V_1 and V_2. It is assumed that the virtual images V_1 and V_2 are relatively and absolutely oriented.

When the operator stereoscopically examines the photographs P_1 and P_2, an imaginary three-dimensional image of the terrain appearing in the stereoscopic overlap appears to the operator to be formed at SM' by the intersections of an infinitely large number of pairs of conjugate image rays that seem to enter the eyes from the virtual images V_1' and V_2'. This three-dimensional surface is commonly called a stereoscopic image. In a similar manner, another imaginary three-dimensional surface, which is commonly called a stereoscopic model, appears to be formed in space at SM. It should be clearly understood, however, that the stereoscopic model SM, the stereoscopic image SM', and the virtual images V_1, V_2, V_1' and V_2' are entirely imaginary. Typical pairs of the imaginary rays forming the virtual images V_1 and V_2, which are commonly called conjugate image rays, are represented in Figure 2 as intersecting at a and b. These rays originate from the conjugate image points a_1 and b_1 on photograph P_1 and from the conjugate image points a_2 and b_2 on photograph P_2. The total effect of the infinitely large number of intersections of pairs of conjugate image rays is to form an imaginary three-dimensional surface SM which is an exact duplicate at a small scale (except for considerable exaggeration of the vertical dimension due to the use of substitute perspective centers) of the original terrain appearing in the stereoscopic overlap of the corresponding photographs P_1 and P_2. The stereoscopic model is located entirely within the useful plotting field created by the stereoscopic overlap of the two photographs.

In order to identify and locate in space above the reference plane R the intersection of any given pair of conjugate image rays, a so-called floating mark or pointer M is mounted on a movable carriage, indicated diagrammatically at 7. (In actual practice, the tracing stand of the Multiplex is used.) When the floating mark is placed in space so that it appears to be in perfect coincidence with any given point on the imaginary surface SM, such as b, those light rays from the floating mark that enter the eyes of the operator coincide with the light rays that originated from the corresponding conjugate image points located on the photographs P_1 and P_2 (b_1 and b_2 in this instance). The floating mark therefore can be used to identify and locate the intersection of any pair of conjugate image rays. Each such intersection represents the true horizontal position above the reference plane R of the corresponding object point on the surface of the original terrain, if vertical photographs are being used.

Established below the floating mark M, on the carriage 7, is a pencil 8 which traces on the map manuscript at R an orthographic projection of the movements of the floating mark. The floating mark M is vertically adjustable on the carriage 7—always remaining directly above the pencil 8. The floating mark M is a brightly illuminated aperture about 0.1 mm. in diameter.

Relative and absolute orientation of the virtual images V_1 and V_2 can be established by the use of twin-point interpolation[7] in a manner that is similar to that used in the operation of the Multiplex. After relative orientation has been established, the distance between the perspective centers O_1' and O_2' represents the distance between the corresponding exposure stations of the original negatives. The ratio of these two distances is the plotting scale of the instrument, and may be varied from approximately 1.25 to 2.00 times the scale of the vertical photographs. The plotting scale can be made larger than that illustrated in Figure 2 simply by increasing the distance between the two half-silvered mirrors

[7] Von Gruber, O., *Photogrammetry*, pp. 401–402. Also pp. 20–32.

5 and 6. This movement of the half-silvered mirrors causes a corresponding change in the horizontal distance between the perspective centers O_1' and O_2'. Incidentally, differences in flying height of the two photographs may be represented by moving the half-silvered mirrors 5 and 6 in such a manner that the virtual images O_1' and O_2' are formed at proportionately different elevations.

Under certain limited conditions, Cook Model I is a very simple and practical device. It should be noted in particular that, while the vertical and horizontal plotting scales are not the same, they are related at all times by a fixed mathematical ratio.

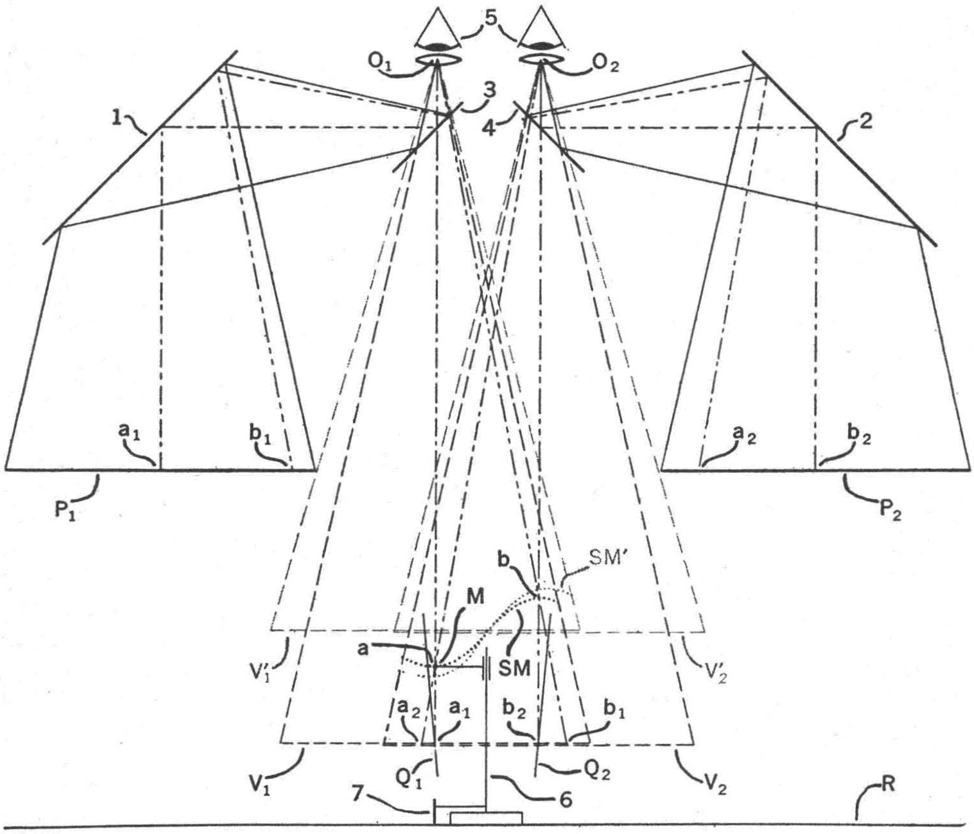

FIG. 3. Schematic diagram of stereotopograph.

STEREOTOPOGRAPH

Mr. Charles O. Cook's second plotting device, the Stereotopograph, is illustrated by the schematic diagram of Figure 3. This instrument consists of a frame (not shown) upon which are mounted two ordinary vertical photographs P_1 and P_2. The mirrors 1, 2, 3 and 4 comprise a mirror stereoscope—mirrors 3 and 4 being half-silvered. The operator's eyes 5 are placed immediately behind the simple magnifying lenses O_1 and O_2 which serve as the eye pieces of the instrument and locate in space substitute perspective centers. The virtual images of the photographs P_1 and P_2 are represented at V_1 and V_2. It is assumed that the virtual images V_1 and V_2 are relatively and absolutely oriented. The stereoscopic model is formed in space at SM by the intersection of an infinitely large number of pairs of conjugate image rays. In Figure 3, representative pairs of these rays are represented as intersecting at a and b. A tracing stand 6, similar to that used with the Multiplex, supports the floating mark M and the drawing pencil 7. In order to establish relative orientation, the photographs are tilted about their principal points. The instrument and the tracing stand are supported by the top of the table R, which serves as the reference plane upon which the map manuscript is placed. In its present form this device is limited to the use of vertical photographs at plotting scales ranging from about that of the photographs to one-half of that scale. If the floating mark were mounted on a pantograph attachment, similar to that used in the K.E.K. Plotter, greater changes in plotting scale could be obtained. The plotting scale of this instrument is changed by moving the virtual images V_1 and V_2 along the respective lines Q_1 and Q_2 in such a manner that they continue to remain parallel to their original positions and at equal distances from the respective eye pieces O_1 and O_2. This method of changing scale is illustrated in Figure 3, where SM' identifies the stereoscopic model that results from moving the virtual images V_1 and V_2 to V_1' and V_2' respectively. It will be noted that the new stereoscopic model SM' is somewhat larger than the original model SM. A number of different ways exist for producing the above described movement of the virtual images. In the Stereotopograph, this movement is produced by a relative movement of the mirrors 3 and 4 and of the photographs P_1 and P_2. The mechanism for making these adjustments is not shown in Figure 3.

The theoretical basis for this method of making changes in plotting scale is illustrated in Figure 4. A pair of relatively oriented vertical photographs, corresponding to the virtual images V_1 and V_2 of Figure 3, is assumed to be located in the plane P. The principal points of these photographs are located at p_1 and p_2 and the true perspective centers are located at O_1 and O_2. Representative pairs of conjugate image rays have been drawn from the conjugate image points a_1, b_1, c_1 and d_1 and from a_2, b_2, c_2 and d_2 to locate in space the stereoscopic model a-b-c-d, which is subdivided into four equal quadrants. This stereoscopic model has been created under theoretically precise conditions and therefore contains no distortions. In order to illustrate the above described method of changing plotting scale, Figure 4 also represents the photographs moved to a new position in plane P'. The principal points of these photographs are now located at p_1' and p_2', the lines Q_1 and Q_2 representing the direction of movement of these photographs. The new positions of the conjugate image points are at a_1', b_1', c_1' and d_1', and at a_2', b_2', c_2' and d_2'. Conjugate image rays have been drawn to these new positions, thus locating in space a new stereoscopic model a'-b'-c'-d'. It should be noted that the center point e of the original model has not been moved as a result of this change of scale. Point e lies in a horizontal plane whose elevation is fixed for any given stereoscopic pair. Since the elevation of this plane varies from model to model, because of changes in flying height and stereoscopic overlap, it has no particular significance and cannot be used as a reference plane for making measurements of relative elevation.

An inspection of Figure 4 will make it quite clear that this method of changing scale introduces certain distortions into the shape of the stereoscopic model. For example, the upper two quadrants of the new stereoscopic model are considerably larger than the bottom two quadrants, and therefore are somewhat distorted in shape. The relative proportions of the stereoscopic model cannot be improved, at this reduced scale, by moving the photographs to some position other than that illustrated in Figure 4. For example, if the photographs had been moved closer together in the plane P so that their principal points were located

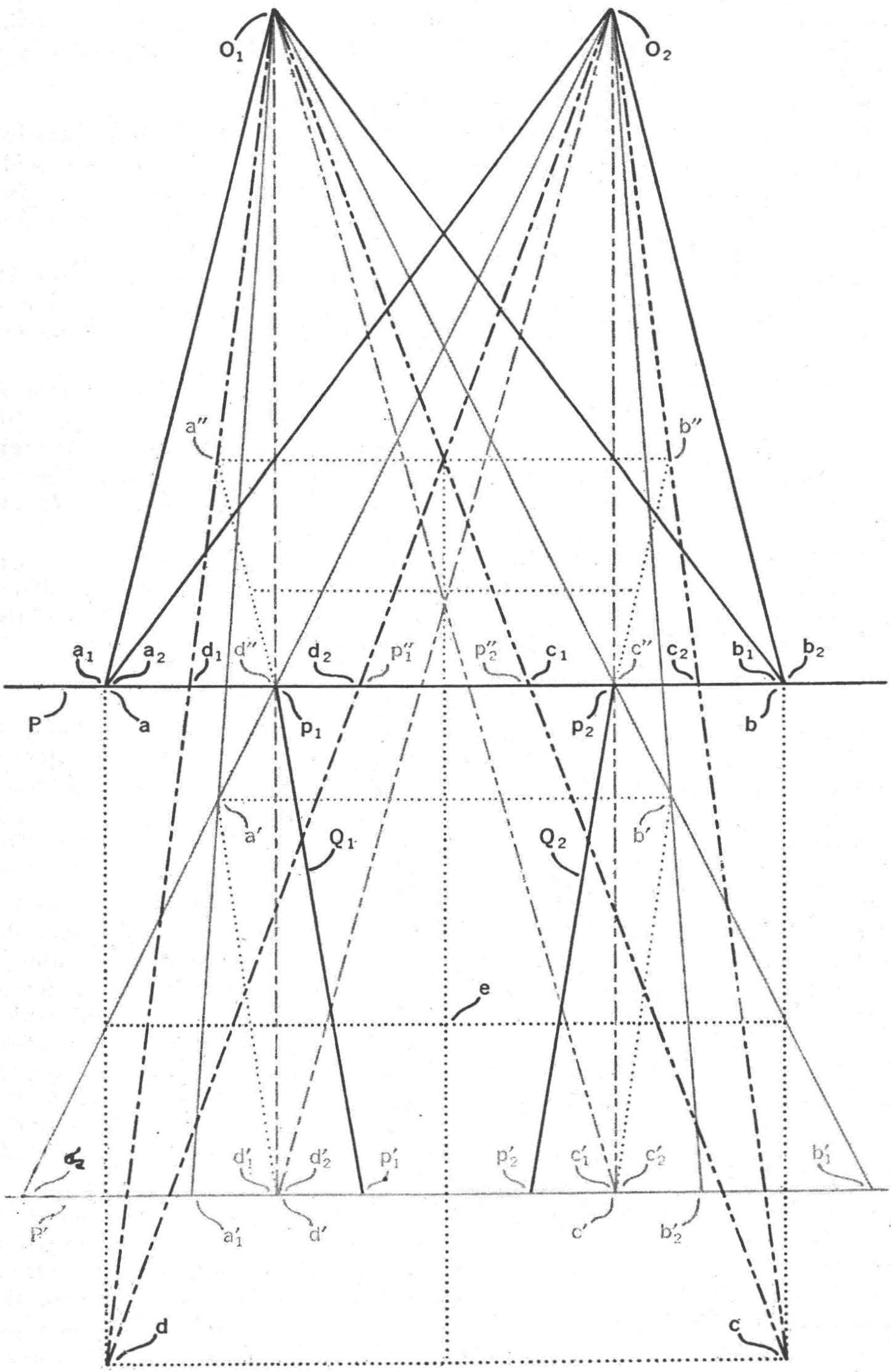

FIG. 4. Diagram illustrating method of changing plotting scale used in stereotopograph.

at p_1'' and p_2'', directly above p_1' and p_2', then a stereoscopic model would have been formed at a''-b''-c''-d''. The plan view of this model, as projected on the mapping plane R, would be identical with that of the stereoscopic model a'-b'-c'-d'.

In connection with this device it will be noted that:

1. It is restricted to the use of vertical photographs, preferably at plotting scales approximately equal to the theoretically precise plotting scale established by the interpupillary distance of the operator's eyes. (The theoretically precise plotting scale is equal to approximately seventy-five per cent of the scale of the photographs.)
2. When errors in planimetric position occur, due to the presence of considerable relief, they will be greatest around the outside edges of the stereoscopic model, and probably will never exceed .02 inch at practical plotting scales. Normally they should not be this large.
3. The vertical plotting scale is variable and is equal to the horizontal plotting scale only at one elevation within the stereoscopic overlap. (This elevation may not be anywhere near the surface of the stereoscopic model.) Therefore, the elevations obtained must be corrected by the use of mathematical or graphical computations in order to remove the effects of an erroneous vertical plotting scale.
4. The eye pieces identify and locate in space entirely new substitute perspective centers with each change of tilt, plotting scale, or interpupillary distance. This results in a corresponding change in the proportions of the stereoscopic model, and in its location in space.

K.E.K. Plotter

The K.E.K. Plotter[8] is illustrated by the schematic diagram of Figure 5. This instrument consists of a cabinet (not shown) within which the two ordinary vertical photographs P_1 and P_2 may be moved up or down by the use of a supporting mechanical linkage incompletely represented at L. (In Figure 5 the photographs are shown at the top of their possible vertical movement.) The mirrors 1, 2, 3 and 4 comprise an ordinary mirror stereoscope which forms virtual images of the photographs at V_1 and V_2. It is assumed that these virtual images are relatively and absolutely oriented. The operator's eyes 5 are placed at the eye pieces of the instrument O_1 and O_2. The use of simple magnifying lenses in the eye pieces is optional. The two half-marks M_1 and M_2, which fuse stereoscopically to form the floating mark M, are attached to a pantograph which also supports the tracing pencil (not shown). Virtual images of the half-marks appear at M_1' and M_2'. In order to establish relative orientation, the photographs are tilted about imaginary centers of rotation which are located on the principal lines of the respective photographs and in the general vicinity of the respective eye pieces. After absolute orientation of the photographs has been established, the distance between the two half-marks is fixed and their movement, together with that of the floating mark M, is restricted to one horizontal plane. Because of this restriction the stereoscopic model itself must be raised or lowered in space so that the floating mark can be placed upon points of various elevations within the stereoscopic model. These vertical movements of the stereoscopic model, which are caused by corresponding vertical movements of the mechanical linkage supporting the photographs, provide a means of measuring the differences of elevation to be found within the stereoscopic model. In

[8] King, J. E. "The K.E.K. Streoscopic Plotter," PHOTOGRAMMETRIC ENGINEERING, Vol. X, No. 4, pp. 252–263. Also U.S. Patent 2,263,971.

its newest form this device is able to plot at scales ranging from 0.5 to 2.5 times that of the vertical photographs.

The theoretical nature of this device is also illustrated by Figure 5. Representative pairs of conjugate image rays have been drawn from the conjugate image points a_1, b_1, c_1, and d_1, and from a_2, b_2, c_2 and d_2, in order to locate in space the stereoscopic model a-b-c-d, which is subdivided into four equal seg-

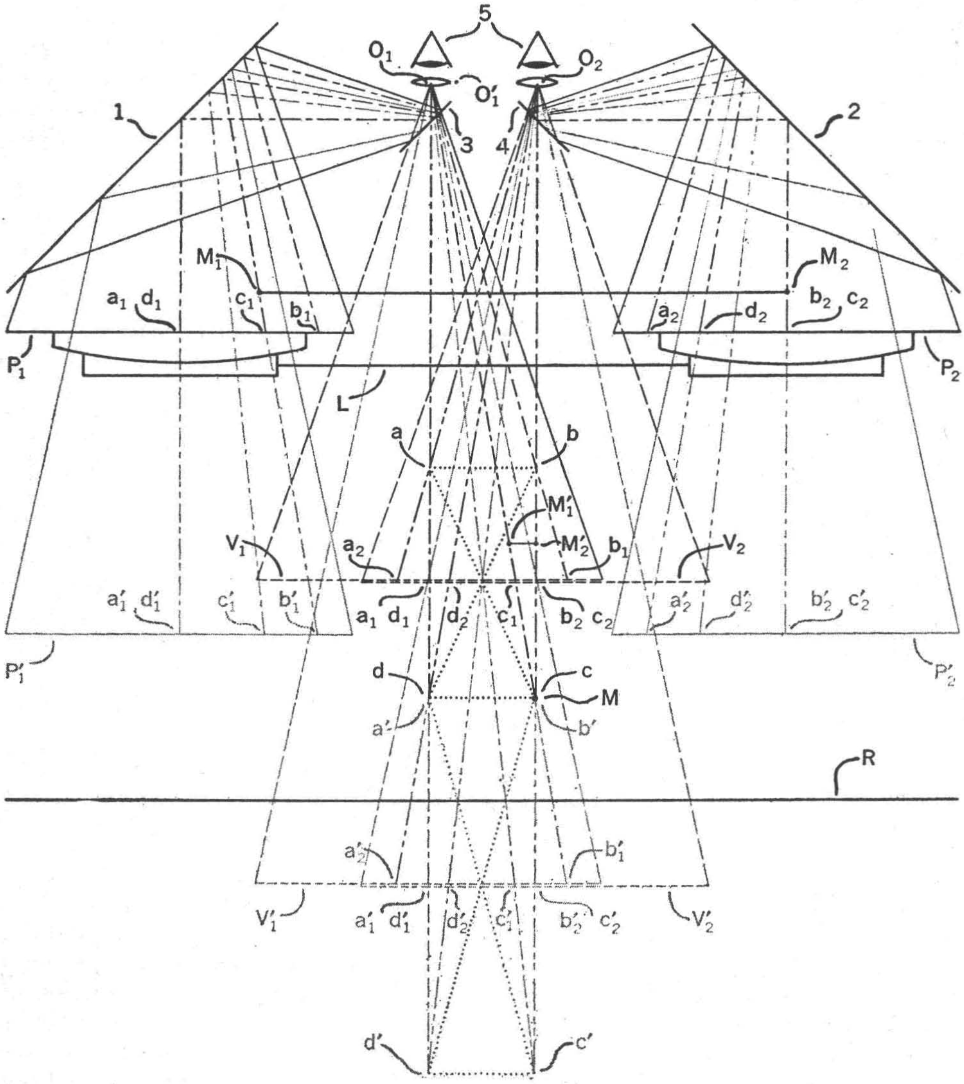

FIG. 5. Schematic diagram of K.E.K. Plotter.

ments. (The vertical dimensions of this stereoscopic model are somewhat exaggerated because of the necessity of using substitute perspective centers. The stereoscopic model would consist of four equal triangular segments, if this vertical exaggeration did not exist.) The perspective centers O_1 and O_2 together with the half-marks M_1' and M_2' form a Zeiss Parallelogram.[9] This parallelogram,

[9] Von Gruber, O., *Photogrammetry*, American Photographic Pub. Co., pp. 284 (footnote).

located at O_1-M_1'-M_2'-O_1', is an imaginary construction used in explaining the performance of the half-marks. Briefly, it may be proved that the triangle O_1'-O_2-M_2' is similar to the triangle O_1-O_2-M and that the motions of the half-marks are therefore directly proportional to the movements of the floating mark M.

When the stereoscopic model is raised or lowered in order that the floating mark may be placed upon any given image point, a corresponding change occurs in the vertical dimensions of the stereoscopic model. (It should be noted that the horizontal dimensions of the model are not changed.) This change in the vertical dimensions of the stereoscopic model is illustrated by Figure 5, where the photographs are represented in new locations at P_1' and P_2'—the corresponding virtual images appearing at V_1' and V_2'. The corresponding new positions

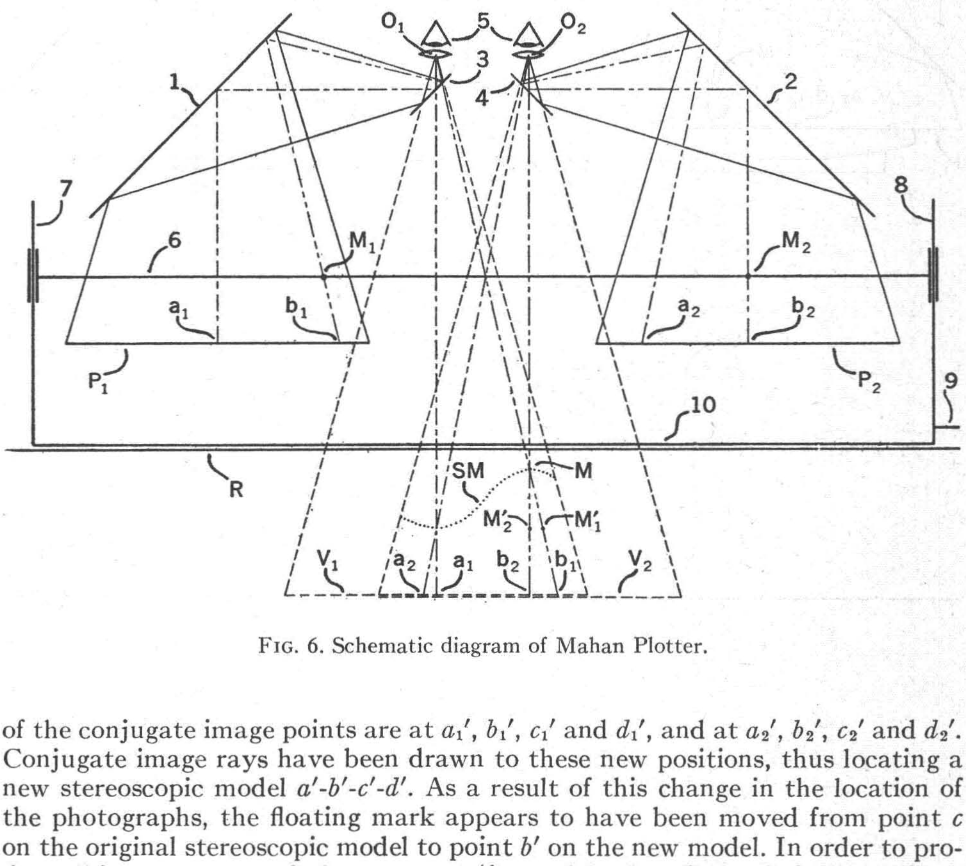

FIG. 6. Schematic diagram of Mahan Plotter.

of the conjugate image points are at a_1', b_1', c_1' and d_1', and at a_2', b_2', c_2' and d_2'. Conjugate image rays have been drawn to these new positions, thus locating a new stereoscopic model a'-b'-c'-d'. As a result of this change in the location of the photographs, the floating mark appears to have been moved from point c on the original stereoscopic model to point b' on the new model. In order to produce this movement of the stereoscopic model, the photographs have been moved a vertical distance that is greater than the original distance from c to b. This illustrates the fact that the vertical plotting scale of the instrument varies with respect to any given fixed horizontal plotting scale. Elevations of the floating mark are obtained by direct readings from a parabolic scale located within the instrument. The setting of this scale must be changed for each new stereoscopic model, due to the effects of change of flying height and air base of the photographs. In the most recent designs of the K.E.K. Plotter, the distance from the eye pieces to the table top is approximately 14 inches.

Mahan Plotter

The Mahan Plotter[10] is illustrated by the schematic diagram of Figure 6. This instrument consists of a frame (not shown) upon which are mounted two ordinary vertical photographs P_1 and P_2. The distance between the two photographs is fixed by the construction of the instrument. The mirrors 1, 2, 3 and 4 comprise an ordinary mirror stereoscope. The operator's eyes 5 are placed immediately behind the simple magnifying lenses O_1 and O_2 which serve as the eye pieces of the instrument, and locate in space substitute perspective centers for the photographs P_1 and P_2. The virtual images of the photographs are represented at V_1 and V_2, and are assumed to be relatively and absolutely oriented. The stereoscopic model is represented at SM. Two half-marks M_1 and M_2, which fuse stereoscopically to form the floating mark M, are supported by a mechanical linkage. The principal parts of this linkage consist of the horizontal slide 6 which supports a pair of simple non-magnifying lenses (not shown) and the accompanying half-marks M_1 and M_2; and a frame consisting of the uprights 7 and 8 rigidly attached to a supporting carriage incompletely represented at 10. Virtual images of the half-marks appear at M_1' and M_2' and thus form a Zeiss Parallelogram. In order to establish relative orientation, the photographs are tilted about their respective nadir points which are fixed on the frame of the machine. After an approximate orientation of the photographs has been established, the distance between the two half-marks is fixed. Because of this restriction, the horizontal slide 6 must be raised or lowered within the uprights 7 and 8 so that the floating mark M can be moved from one elevation to another within the stereoscopic model. These vertical movements of the slide 6 and the accompanying half-marks provide a means for measuring differences of elevation to be found within the stereoscopic model. The vertical plotting scale varies only with changes in flying height and overlap of the photographs. The mechanical linkage supporting the half-marks M_1 and M_2 is free to move in a horizontal direction, and therefore provides a means for moving the floating mark horizontally around over the stereoscopic model. A pantograph, which supports a tracing pencil, is attached to the mechanical linkage at 9. In its present form this device is able to plot at scales ranging from nearly twice that of the photographs down to about one-third of that scale. In actual operation, the above described mechanical linkage supporting the half-marks is used in a manner that is very similar to that followed in manipulating the Multiplex tracing stand.

The theoretical nature of the Mahan Plotter is quite similar to that of the K.E.K. Plotter, although the mechanization of the two instruments is somewhat different. It is reported that, in actual use, the nadir point is located on each photograph by the use of a mathematical tilt analysis procedure. In establishing relative orientation the photographs are tilted about these nadir points.

The Multiscope

The Multiscope,[11] when used in the form that is best adapted to the plotting of topography, theoretically resembles the Stereotopograph,[12] previously described. As in the Stereotopograph, the horizontal plotting scale is changed in the Multiscope by making a corresponding change in the distance between the virtual images of the two photographs. This change is produced optically by the

[10] Van Camp, C. P., "Mahan Plotter," Photogrammetric Engineering, Vol. XI, No. 4, pp. 336–339.

[11] Spurr, Stephen H., and Brown, C. T., Jr., "The Multiscope: A Simple Stereoscopic Plotter," Photogrammetric Engineering, Vol. XI, No. 3, pp. 171–178.

[12] *Ibid.*, page 177 (last three paragraphs).

use of prisms which results in a distorted stereoscopic model of the type illustrated at *a″-b″-c″-d″* of Figure 4. Each change of scale destroys the relative orientation of the stereoscopic model, which therefore must be re-established before a further attempt can be made to establish absolute orientation.

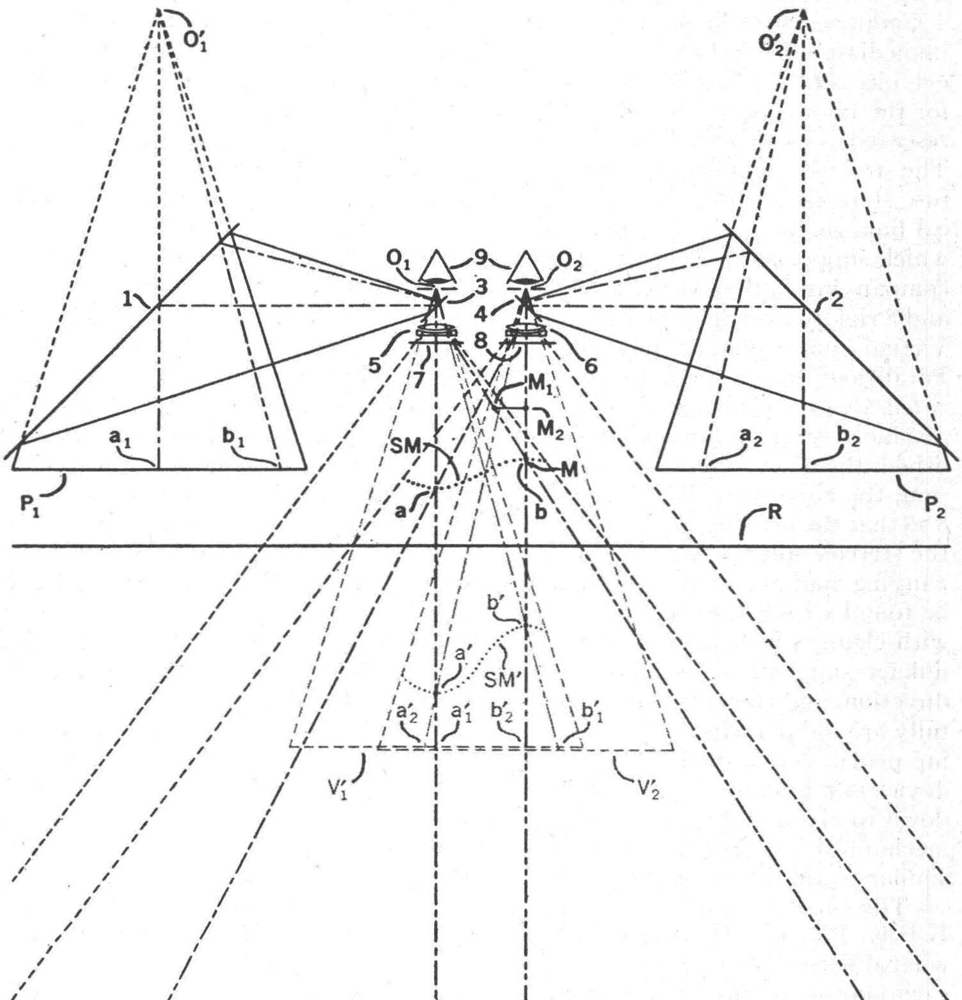

FIG. 7. Schematic diagram of Lyon Model II.

LYON MODEL II

The instruments so far described are limited to the use of vertical photographs. Lyon Model II[13] is similar to these devices in many ways, however, it can use either oblique or vertical photographs. Figure 7 is a schematic diagram of Model II. The plane of the paper is considered as being vertical. This instru-

[13] It is reported that Mr. J. L. Buckmaster has designed a somewhat similar map plotting device.

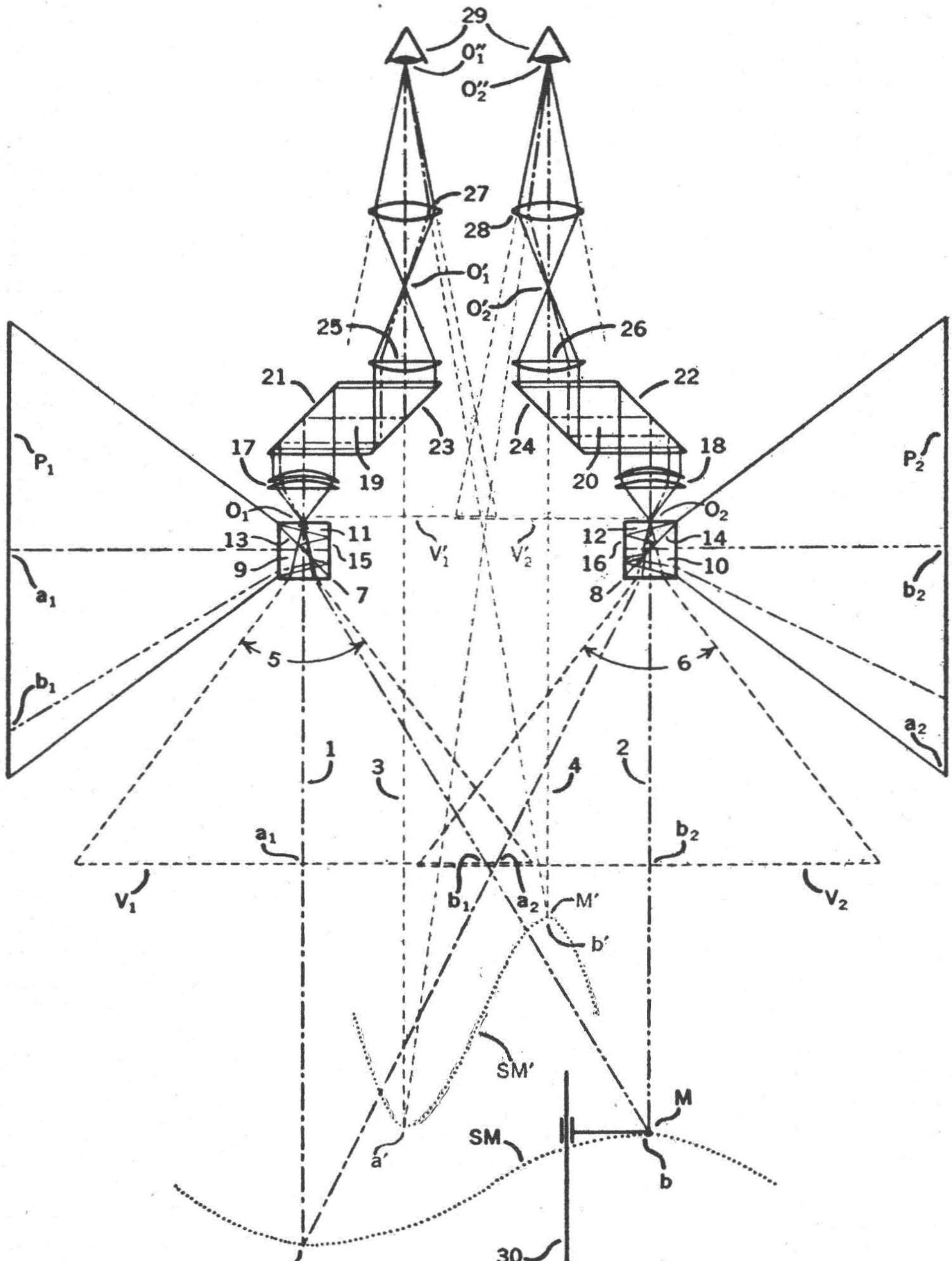

Fig. 8. Schematic diagram of Lyon Model III.

ment consists of a frame (not shown) upon which two ordinary photographs P_1 and P_2 are mounted so that they may be tilted about their respective substitute perspective centers O_1' and O_2'. (Points O_1' and O_2' are virtual images of the substitute perspective centers O_1 and O_2.) The mirrors 1, 2, 3 and 4 comprise a mirror stereoscope—mirrors 3 and 4 being half-silvered. The operator's eyes are placed immediately behind the small apertures O_1 and O_2, which serve as the eye pieces of the instrument and locate in space the perspective centers of the optical system. The virtual images of the photographs are represented at V_1 and V_2, and are assumed to be relatively absolutely oriented. The stereoscopic image is formed in space at SM' by the intersection of an infinitely large number of conjugate image rays. In Figure 8, representative pairs of these rays are represented as intersecting at points a' and b'. Immediately below the half-silvered mirrors are located plano-convex lenses 5 and 6. These lenses are of such nature that if enlarged copies of the photographs were placed at V_1' and V_2', they would appear to the operator to be located at V_1 and V_2. This illustrative example makes it clear that V_1 and V_2 are virtual images of the photographs P_1 and P_2. The conjugate image rays coming from the virtual images V_1 and V_2 would intersect in space to form a stereoscopic model at SM. Any point on the stereoscopic model SM can be identified and located in space by the use of a floating mark in the customary manner. However, in actual practice, it is believed that illuminated half-marks M_1 and M_2 should be used. These half-marks ought to be mounted on a pantograph attachment similar to that used in the K.E.K. Plotter. Light filters using complimentary colors are provided at 7 and 8 so that the operator can fuse the half-marks according to the anaglyphic method.

Lyon Model III

Lyon Model III is illustrated by Figures 8, 9, 10 and 11. This device is intended for use with ordinary photographs, either verticals or obliques, where sufficient geographic control is available for controlling the absolute orientation of each stereoscopic model. Similar reference characters indicate corresponding parts in each of the four drawings.

Figure 8 is a schematic diagram of Model III. The plane of the paper is considered as being vertical. At P_1 and P_2 are shown two ordinary photographs. These are assumed to be verticals possessing about sixty per cent overlap. At numerals 7 and 8 are shown compound prisms[14] each composed of two prisms cemented together at the common surface 13 and 14 respectively. These surfaces 13 and 14 both transmit and reflect light, while surfaces 15 and 16 are totally reflecting. Aperture stops about .05 inch in diameter are located at O_1 and O_2. The optical elements O_1 and 17, O_2 and 18, form modified telecentric systems.[15] Such systems are commonly used to obtain great depth of sharp focus and to eliminate parallactic displacement in the making of accurate measurements. The optical system composed of elements 17, 19, 25 and 27 creates a real image of the aperture stop O_1 and O_1'' and a virtual image of the photograph P_1 at V_1'. Likewise, the optical system composed of elements 18, 20, 26 and 28 creates a real image of the aperture stop O_2 at O_2'' and a virtual image of photograph P_2 at V_2'. The eyes, 29, of the operator, located at O_1'' and O_2'', can stereoscopically examine the virtual images V_1' and V_2'. In Figure 8 these vir-

[14] This type of prism was developed by Lt. G. W. Brainard, Office of Research and Inventions, Navy Department. It is understood that an application for patent has been filed covering this prism. A somewhat similar compound prism has been developed by Mr. J. L. Buckmaster and is described in U.S. Patent 2,370,143.

[15] Hardy, Arthur C., and Perrin, Fred H., *Principals of Optics*, McGraw-Hill Book Co., page 74.

tual images are assumed to be relatively oriented. The infinitely large number of pairs of conjugate image rays that seem to enter the eyes from the virtual images V_1' and V_2' intersect in space to form the stereoscopic image SM' that is seen by the operator.

If all the light rays which enter the eyes of the operator from the surfaces of the two photographs P_1 and P_2 are traced backwards through their respective optical systems and out of the compound prisms in the direction of the respective axes 1 and 2, it will be demonstrated that real images of the diapositives P_1 and P_2 are formed at V_1 and V_2. This illustration makes it clear that virtual images of the diapositives P_1 and P_2 are normally formed at V_1 and V_2, according to well known laws of image reflection.

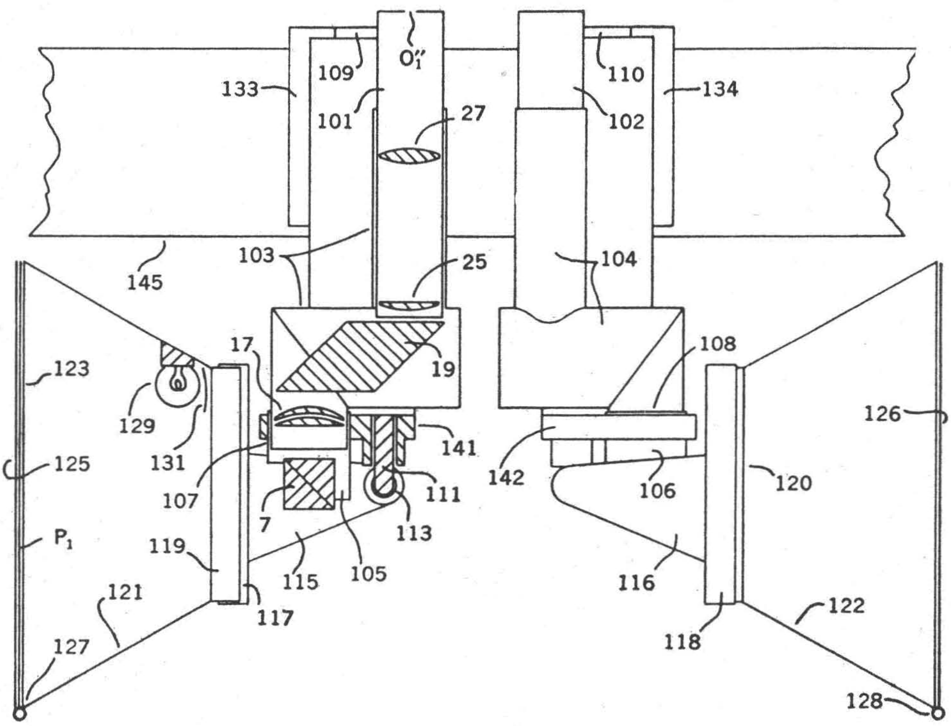

FIG. 9

It is assumed that the virtual images V_1 and V_2 are relatively and absolutely oriented. Therefore, the imaginary rays forming these two virtual images possess exactly the same geometrical relationships with respect to each other and to the reference plane R that were possessed by those light rays which entered the camera to expose the two original negatives. Typical pairs of these imaginary rays, which are commonly called conjugate image rays, are represented in Figure 8 as intersecting at a and b. The total effect of the infinitely large number of intersections of pairs of conjugate image rays is to form an imaginary three-dimensional surface SM which is an exact duplicate, to a small scale, of the original terrain appearing in the stereoscopic overlap of the corresponding photographs P_1 and P_2. This imaginary surface SM is commonly referred to as the stereoscopic model, and is located entirely within the useful plotting field created by the stereoscopic overlap of the two photographs. In order to identify and locate in space above the reference plane R the intersection of any given pair of conjugate image rays, a floating mark M is mounted on a movable carriage, indicated diagrammatically at 30. (In actual practice, the tracing stand of the Multiplex would be used.) When the floating mark is placed in perfect coincidence with any point on the imaginary surface SM, such as b, those light rays from the floating mark that pass through the aperture stops O_1 and O_2 will exactly coincide throughout the rest of the optical system with the principal light rays that originated from the corresponding conjugate image points located on the photographs P_1 and P_2: (b_1 and b_2 in this instance.) The floating mark therefore can be used accurately to identify and locate the intersection of any pair of conjugate image rays. Each such intersection represents the true position

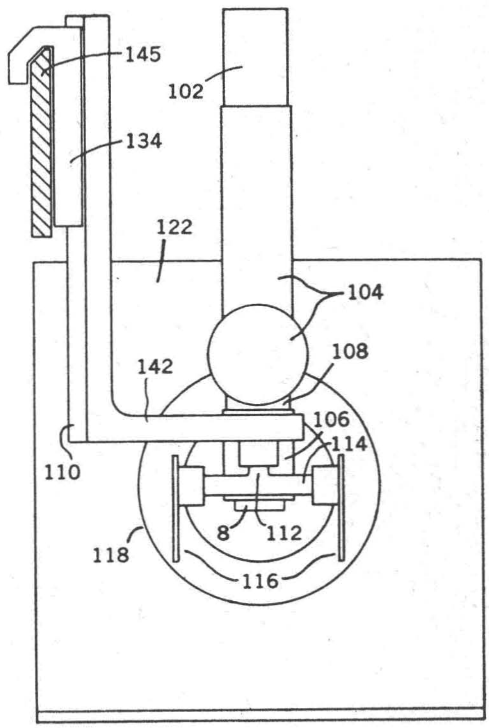

FIG. 10.

above the reference plane R of the corresponding object-point on the surface of the original terrain.

Established below the floating mark M, on the carriage 30, is a pencil 31 which traces on the map manuscript at R an orthographic projection of the movements of the floating mark. The floating mark M is vertically adjustable on carriage 30—always remaining directly above the pencil 31. The floating mark M is a brightly illuminated aperture about 0.1 mm. in diameter.

Relative and absolute orientation of the virtual images V_1 and V_2 can be established by the use of twin-point interpolation in a manner that is identical with that used in the operation of the Multiplex. After relative orientation has been established, the distance between the perspective centers O_1 and O_2 represents the distance between the corresponding exposure stations or the original negatives. The ratio of these two distances is the plotting scale of the instrument. This plotting scale can be made smaller than that illustrated in Figure 8 simply by decreasing the distance between the two perspective centers O_1 and O_2 by the desired amount. The resulting change in plotting scale is produced according to a well known law. The axes 3 and 4 are rotated about the new position of axes 1 and 2, while remaining parallel and at a constant distance from said axes 1 and 2, in order to re-establish the desired interocular distance between the virtual images O_1'' and O_2''.

A consideration of the above descriptive material will make it clear that after relative orientation has been established, the light rays which enter the two eyes 29 will cause the operator to receive a mental impression of a three-dimensional image of the original object. This image is commonly called a stereoscopic image and its shape and position in space is indicated at SM'. Each point on the stereoscopic image appears to the operator to be located at a unique position and elevation in space. The reason for this will become apparent by considering, for example, conjugate image points b_1 and b_2 in the photographs P_1 and P_2, and the corresponding pair of conjugate image rays. The paths of these rays through the optical system, and their projections from the apertures O_1 and O_2 to the intersection at b, are represented by dash-dot-dot lines. Likewise, conjugate image points a_1 and a_2 are indicated in the photographs P_1 and P_2, together with the conjugate image rays from them through the optical system. These rays are represened by dot-dash lines. The projections of these lines intersect at a point a on the stereoscopic model SM. This point a appears to the operator to be located at an entirely different position and elevation in the plotting field than that occupied by any other point, such as b. Points a and b are representative of the infinitely large number of intersections which form the stereoscopic model SM. Likewise, points a' and b' are representative of the infinitely large number of intersections which form the corresponding stereoscopic image SM'.

It is to be understood that, before contours and other topographic features can be successfully plotted, both relative and absolute orientation must be established for the stereoscopic pair. Relative orientation is the re-establishment of the same geometrical relations between a pair of photographs that existed when the original negatives were exposed. Following relative orientation which creates the stereoscopic model, absolute orientation is established, thus fixing the scale, position and orientation of the model with respect to the horizontal and vertical control plotted on the reference plane. Relative orientation requires provision for moving the two telescopic projectors laterally and perpendicularly, and rotating them about their respective apertures O_1 and O_2. The mechanism for making these and other necessary adjustments is illustrated in Figures 9, 10 and 11.

Figure 9 is a front view of the instrument, and illustrates its general appearance. Figure 10 and Figure 11 are side and bottom views, respectively, of the right hand telescopic projector appearing in Figure 9, The telescopic projectors consist of the three main parts illustrated in Figure 9: The camera bodies 121 and 122, the viewing telescopes 103 and 104, and the supporting mechanisms. Each of the camera bodies is mounted so that it may be adjusted for x-tilt, y-tilt and swing motions. Horizontal slides 133 and 134 can be moved along the supporting bar 145 and thus provide for changes in plotting scale (x motion). Vertical slides 109 and 110 can be moved up and down within the horizontal slides in order to provide changes in flying height (z motion). The brackets 141 and 142 support the corresponding telescopic projectors and camera bodies in the manner illustrated. The camera bodies can be oriented for x-tilt and y-tilt by rotation about the primary axes 111 and 112, and the secondary axes 113 and 114.

Concentric sliding sleeves 117 and 119, 118 and 120, provide for swing and changes in focal length.

The practical value of the proposed optical system can be determined by making the following simple experiment: Place pieces of cardboard over the eye pieces of an ordinary mirror stereoscope. (All lenses should be removed.) Punch small holes in the cardboard, at the correct interocular distance, to represent the aperture stops O_1 and O_2 of Figure 8. Correctly orient a stereoscopic pair of photographs in such a way that the principal lines of their virtual images pass through the corresponding aperture stops. A stereoscopic examination of the photographs will disclose that:

1. An excellent stereoscopic model can be seen.
2. Very great depth of sharp focus is obtained.

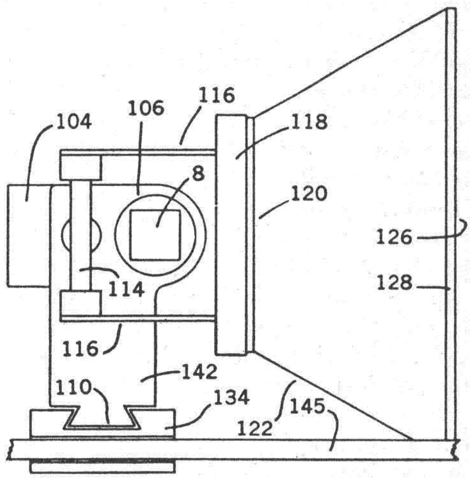

FIG. 11.

3. Any portion of the entire stereoscopic model can be seen without eye strain. This is even true when high oblique photographs are used, although the interocular distance must be varied slightly in order to inspect the corners of the model. (This motion has been provided for by the use of the binocular telescopes.)

When Model III is used for the plotting of high oblique photographs, slight errors of elevation exist in the stereoscopic model due to the effects of the curvature of the earth and the refraction of light by the atmosphere. These errors are indirectly proportional to the horizontal distance from one of the nadir points. Therefore, these errors may be automatically eliminated by the use of a cam that is actuated by a rod or string pivoted about the map nadir point of one of the obliques. The cam will automatically make the necessary corrections in the height of the floating mark and thus correct for the effects of curvature and refraction.

LYON MODEL I

Model I, a somewhat more precise type of automatic map plotting instrument, is illustrated by Figures 12, 13 and 14. This device is intended for use with diapositives, either verticals or obliques, and is able to manufacture part of its own geographic control. This device is similar in many ways to Model III. For this reason, similar reference characters have been used to indicate corresponding parts on the drawings illustrating these two devices.

Figure 12 illustrates the general nature of the two telescopic projectors. Part of the frame of the left telescopic projector has been cut away to show the arrangement of certain optical elements. The viewing telescopes 103 and 104 are identical with those illustrated more fully in Figures 8 and 9, and are attached to the supporting bar 145 by means of gimbal rings 45 and 46 and the supporting brackets 141 and 142. The camera bodies 121 and 122 are rigidly attached to the telescopes. The plane of the paper is considered as being vertical.

At P_1 is shown a standard size diapositive, which is supported within the

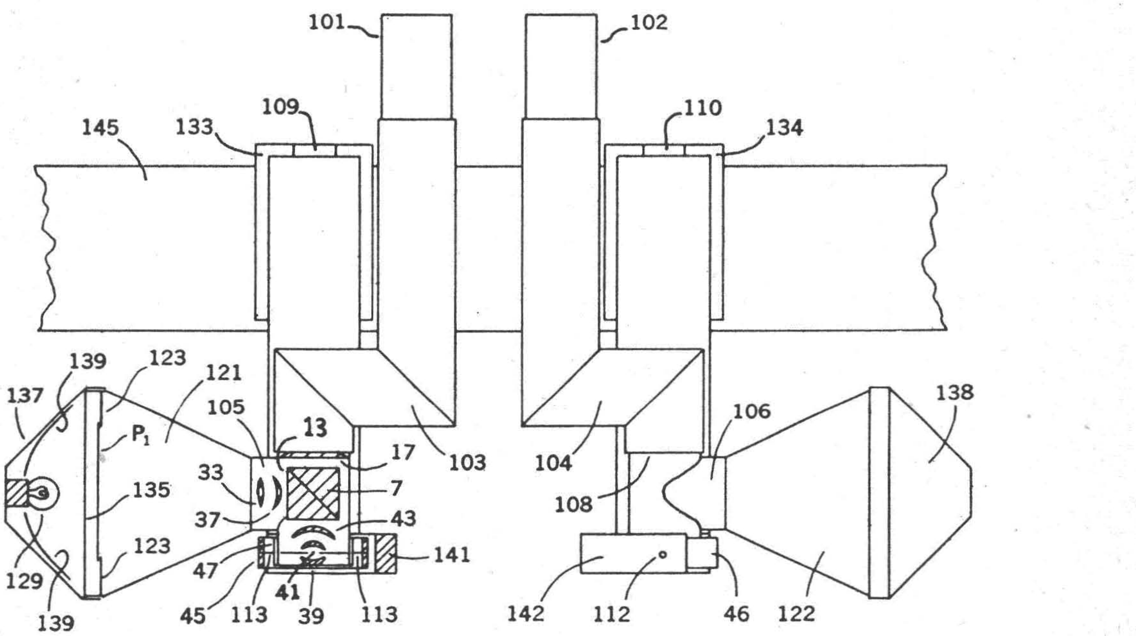

FIG. 12.

camera body 121 by the stage 123. This diapositive is illuminated by light obtained from the diffusion plate 135, which in turn is illuminated by the light source 129 and the reflector 139. The first objective lens 33 and the corresponding condensing lens 37 are equivalent to the objective lens of an ordinary telescopic optical system, but are located at 90° to their usual position in such systems. At 90° to the optical axis of lenses 33 and 37 is located the second objective lens 39 and the corresponding condensing lens 43. At the optical center of the second objective lens is placed a small aperture stop 41, which is near the so-called pin hole size. The exact size of this aperture stop depends upon the depth of sharp focus that is desired. At the intersection of the optical axes of the first and second objective lenses is located the half-silvered surface 13 of the compound prism 7. This surface will both reflect and transmit light. Behind and above the compound prism 7 is located the condensing lens 17 of the observing telescope. The other optical elements of this telescope are similar to those illustrated in Figure 8, and therefore do not need further explanation.

Figure 14 provides a more detailed illustration of the above described optical elements. The paths that various representative light rays take through the system have been shown. The diapositive is shown at P_1, the principal point being at p_1. Point 35′ is the virtual image of the optical center of the first objective lens 35. Point 39′ is the virtual image of the optical center of the second objective lens 39. The light rays from the diapositive P_1 which pass through the optical center 35 of the first objective lens 33 are reflected from the half-silvered reflecting surface 13 of the compound prism in such a manner that, after reflection, they appear to have originated from the diapositive's virtual image V_1 and to have passed through the aperture stop 41. It is therfore clear that a virtual image of the first objective 33 is located at the virtual image 39′ of the second objective lens 39. For purposes of illustration, it is assumed that the front and rear nodal points of the second objective lens are located at its optical center.

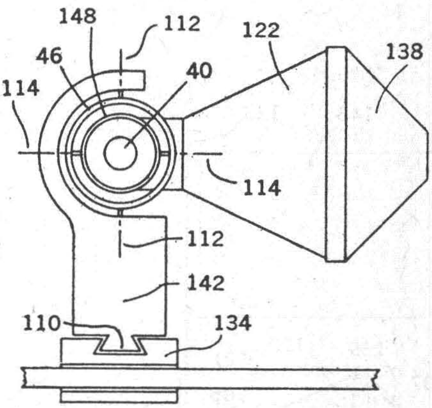

FIG. 13.

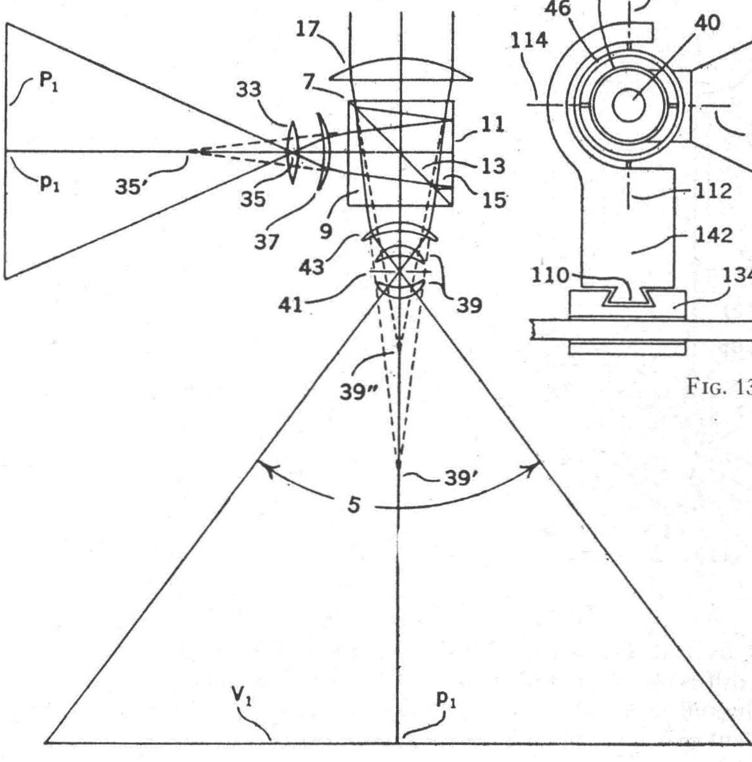

FIG. 14.

The aperture stop 41 therefore becomes the perspective center of the virtual image V_1. The size of the virtual image V_1 and its distance from the corresponding perspective center (aperture stop 41) depends upon the focal length of the second objective lens 39 and of the condensing lens 43. The focal lengths of these lenses are chosen to provide the most desirable depth of sharp focus and, at the same time, establish the correct geometrical relationship between the size of the virtual image V_1 and its distance from the aperture stop 41. The principal light rays, upon leaving the compound prism 7, are refracted so that they appear to have passed through the optical center of an objective lens placed at 39″. The field of the second objective lens 39 is shown at 5. This field is much wider than that of the first objective lens 33. The construction of the observing telescopes is similar to that illustrated in Figure 8. The optical elements of the right telescopic projector are similar to those described above for the left-hand telescopic projector.

Figure 13, a bottom view of the right telescopic projector represented in Figure 12, illustrates the method of supporting the telescopic projectors. The bottom end of the telescopic projector is represented at 148—the second objective lens appearing at 40. The primary axis 112 and the secondary axis 114 of the gimbal ring 46 permits the rotating of the telescopic projector 148 about the optical center of the second objective lens 40. The vertical bracket 142 may be raised or lowered by means of the vertical slide 110, and may be moved laterally by the use of the horizontal bracket 134 and the supporting bar.

For purposes of illustration, the three basic Lyon proposals have been represented and described here in their simplest possible forms. Various desirable optical and mechanical improvements in the basic design of these three devices can be made quite easily. However, it is believed that a detailed description of these modifications is outside the scope of this present article. None of these instruments possesses all of the desirable characteristics that a plotting instrument should have.[16] Perhaps the most serious inherent shortcoming is the lack of magnification. It is true that, if the additional expense and mechanical complexity could be justified, high magnification could be obtained by the use of an optical and mechanical system first proposed by Mr. O. M. Miller.[17] The construction of Mr. Miller's viewing system is very ingenious and would introduce no errors into the plotting system of the resulting device. In this more complicated form, the Lyon I proposal might very well attain the extreme precision of the European plotting instruments. However, the author feels that, for the time being at least, the development of Model I and Model III should tend towards extreme simplicity of construction and operation. Model II probably possesses relatively little practical value. It has been described here only as a matter of interest. No working models have been built for any of these three devices.

Conclusions

It will be clear to those skilled in the art of designing plotting equipment that improvements and modifications can be made in most of the instruments described herein.

In connection with these devices it will be noted that

1. In each case four mirrors are used to form a mirror stereoscope—such mirrors being fixed with respect to each other after absolute orientation has been established.
2. Except in the Mahan Plotter, the photographs are viewed from substitute perspective centers that are from about twelve to eighteen inches from the respective photographs. As a result, any lack of flatness in the photographs produces a relatively small amount of angular displacement of the photographic images (except in Lyon Model III). Also, when vertical photographs are used, this longer viewing distance creates a considerable exaggeration of the differences of relief seen by the operator. Contours thus may be plotted with perhaps unexpected accuracy. When wide angle oblique photographs are used, the perspective relationships existing between two such photographs are modified so that the angles of con-

[16] Lyon, Duane, "Automatic Map Plotting Instruments—Classification and Analysis," Photogrammetric Engineering, Vol. XI, No. 4, pp. 292–298.

[17] Miller, O. M., "An Optical Device to Aid in Mapping from Photographs," *Journal Optical Society of America*, Vol. 25, 1935, pp. 185–189.

vergence required of the operator's eyes are no greater than those normally encountered in nature.

3. In each case the distance from the eye pieces to the top of the table supporting the instrument is rather large and therefore places the operator in a rather uncomfortable working position. This shortcoming could be very easily corrected in Lyon Model I and Lyon Model III.
4. Cook Model I, Stereotopograph, K.E.K. Plotter, Mahan Plotter and the Multiscope are limited to the use of vertical photographs containing less than three or four degrees of tilt.[18] Parallactic displacement exceeding .01 inch is normally encountered in their operation, except when the floating mark (or half-marks) is within one inch of the plane of the photographs (or of their virtual images). Parallactic displacement is controlled in the K.E.K. Plotter by the use of so-called gun sights within the eye pieces. It could be prevented in these instruments by the use of small apertures, which would also provide an adequate depth of sharp focus.
5. Cook Model I, Stereotopograph, Multiscope and the Lyon proposals make use of a very small electrically illuminated aperture as a floating mark, which requires that the instruments be operated in semi-darkness. (The use of a new light source developed by Wetsern Union probably would eliminate this shortcoming.) The K.E.K. and Mahan plotters use small black dots for the half-marks and therefore can be operated in day light. The relatively large size of the resulting floating mark is not altogether satisfactory.

An understanding of the theoretical nature of the plotting instruments described herein provides a means of estimating their limitations and relative advantages but it does not give us a true measure of their practical value. In any peace time civil mapping program the practical value of a compiling procedure depends upon the cost and quality of the resulting map. (In a military mapping program the practical value of the compilation procedure also depends upon its ability to make satisfactory use of existing photographs and geographic control.) Unfortunately, a comparative breakdown of the costs and quality of various photogrammetric compilation procedures is not available. For this reason, it is possible only to estimate the probable value of the various available compilation procedures.

A fairly useful preliminary evaluation of the devices described in this article can be obtained by comparing their probable performance in the following manner:

Amount of geographic control required

The cost of obtaining a complete network of vertical and horizontal control is the most expensive part of any accurate map compilation procedure. Theoretically, four strategically located control points of known position and elevation are sufficient to control the absolute orientation of any stereoscopic model. All of these instruments are able to produce acceptable work with this amount of control. Lyon I and Lyon III possess the advantage of being able to manufacture some part of their own control. However, it should be noted that aerial cameras at present are surveying instruments of only fourth order geodetic precision.[19]

[18] Tewinkel, G. C., "Vertical Scale Correction For The Multiplex Projector." No. 5 in a series of special bulletins on aerial photogrammetry published by Syracuse University, Syracuse, N. Y.

[19] Manual of Photogrammetry, Pitman Publishing Co., p. 13.

Plotting accuracy and speed

The plotting accuracy that can be obtained with any plotting device depends upon its theoretical limitations, of course, but it also depends upon the following practical considerations:

1. Design and construction of the instrument.
2. Type of positives used. Paper prints are cheaper but diapositives give more accurate results. Lyon I and Lyon III can both make use of contact size diapositves. Lyon Model I can also use the small size (64 mm by 64 mm) diapositives made with the Multiplex reduction printer. It should be understood that the making of diapositives with the reduction printer is relatively quick and inexpensive. Normally two diapositives are made from each negative at a cost of about fifty cents a pair. It is reported that the average operator of a reduction printer can produce about 100 pairs of diapositives per eight hour shift.
3. Quality of the stereoscopic model. This depends upon the precision of the survey camera, upon the precision and resolving power of the optical system, upon the quality and correct orientation of the virtual images presented to the eyes of the operator, and upon the magnification and illumination of the photographs.
4. Illumination and size of the floating mark, and the precision of the associated mechanical linkage.
5. Physical comfort of the operator. This depends upon the amount of eye strain involved in the operation of the device, and upon the posture required. The Multiplex permits by far the best posture, which very definitely is of practical advantage.
6. Amount of unremovable y-parallax that exists in the stereoscopic model.

The above factors also govern the speed of compilation which tends, naturally enough, to vary inversely with accuracy. It is reported[20] that the compilation produced with the K.E.K. Plotter is almost as accurate as that which would normally be produced with the Multiplex under the same conditions.

All of the devices described herein make use of similar procedures for establishing relative and absolute orientation and for the tracing of contours and planimetry. It therefore seems likely that they possess approximately the same rates of compilation.

Required contour interval

In topographic mapping, accuracy requirements for vertical heights are normally more stringent than those for horizontal positions. For example, a 1:20,000 scale map with 20 foot contour intervals usually allows a vertical error of 10 feet and a horizontal error of 33 feet.[21] In stereoscopic map plotting instruments, the intersection angles of conjugate image rays are such that vertical locations are less definite than horizontal locations. For these reasons, when the customary vertical accuracy is attained, the horizontal accuracy usually presents no problem. In order to obtain the customary vertical accuracy with automatic map plotting instruments, theoretical limitations require that the flying height should be no greater than about two thousand times the desired contour interval, and the stereoscopic overlap should not be greater than sixty

[20] King, J. E., "K.E.K. Stereoscopic Plotter," PHOTOGRAMMETRIC ENGINEERING, Vol. X, No. 4, pp. 262–263.

[21] Multiplex Mapping Equipment, War Dept. Technical Manual TM 5–244, p. 71.

per cent.[22] In actual practice the shortcomings of our aerial cameras and photogrammetric methods require that the ratio of contour interval to flying height be not greater than 1:1000. In the Multiplex compilation procedure the ratio of contour interval to flying height is about 1:750. It is reported that some of the ultra-precise plotting instruments manufactured in Europe can be operated at ratios somewhat more favourable than 1:1000.

Cost of compilation

For any given mapping procedure the cost of mapping an area depends a good deal, of course, upon the nature of the topography, the desired quality of the compilation, the contour interval, and the amount of reliable geographic control that is already available. The cost of mapping a typical project by the use of the plane table probably provides the most worthwhile standard for comparing the costs of other compilation procedures. It seems to be a commonly accepted opinion among those qualified to judge that many areas can be mapped with the Multiplex, under favorable conditions, at one-half the cost of similar plane table compilation. Even better cost figures are reported by users of the K.E.K. Plotter.

After all due consideration, it seems to the author that the various map plotting instruments described herein must compete with each other principally upon the basis of their precision and their purchase price. Also, that further reduction in costs of topographic surveys using photogrammetric methods probably depends upon the development of:

1. Cheaper methods of obtaining geographic control. (The development of ultra-precise methods of aerotriangulation and radio triangulation should be seriously considered by photogrammetrists in this country.)
2. More precise equipment and methods so that fewer photographs may be used for a given compilation. This requires the development of a simple, very precise map plotting device and of a very precise wide-angle survey camera. The satisfactory development of ultra-wide-angle photography or of tandem photography would give an equivalent result.
3. Mechanical or mathematical methods of precisely determining the tilt of every photograph.

[22] Hardy, Arthur C., and Perrin, Fred H., The Principles of Optics, McGraw-Hill Book Co., pp. 521 and 533.