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RECONNAISSANCE MAPPING FROM OBLIQUE AERIAL PHOTOGRAPHS WITHOUT GROUND CONTROL

John Lyon Rich, University of Cincinnati

IN THE course of reconnaissance from the air many occasions arise when it would be desirable to be able to make a map—even though it were not entirely accurate—that would show the spacial relations of objects on the ground more definitely than they could be judged from photographs alone. The geologist, for instance, might desire to show the locations and relative trends of features such as contacts or faults, or he might wish to determine approximately the dips and strikes of strata in rough terrain.

All of these things could be done fairly simply if overlapping vertical photographs could be made, but in ordinary reconnaissance work this is not likely to be feasible. A method for accomplishing the purpose by the use of oblique photographs would be useful.

About two years ago the writer hit upon a method for making a reconnaissance map from oblique photographs which does not require ground control and which, unlike the Canadian grid method or the spot-location method recently described by Desjardins,[1] does not require that the entire area be essentially flat, but is applicable to rough or mountainous terrain provided that two starting points that are at essentially the same elevation can be found. The writer recently had opportunity to test the method on two separate sets of photographs[2] and found that it is capable of yielding surprisingly good results.

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[1] Desjardins, Louis, "Useful Graphical Constructions on Aerial Photographs," PHOTOGRAMMETRIC ENGINEERING, Vol. XI, No. 3, 1945, pp. 194–229.

[2] Grateful acknowledgement is made to Dr. Edward S. Wood, Jr., of Harvard Institute of Geographical Exploration who kindly supplied a set of four photographs for the test.

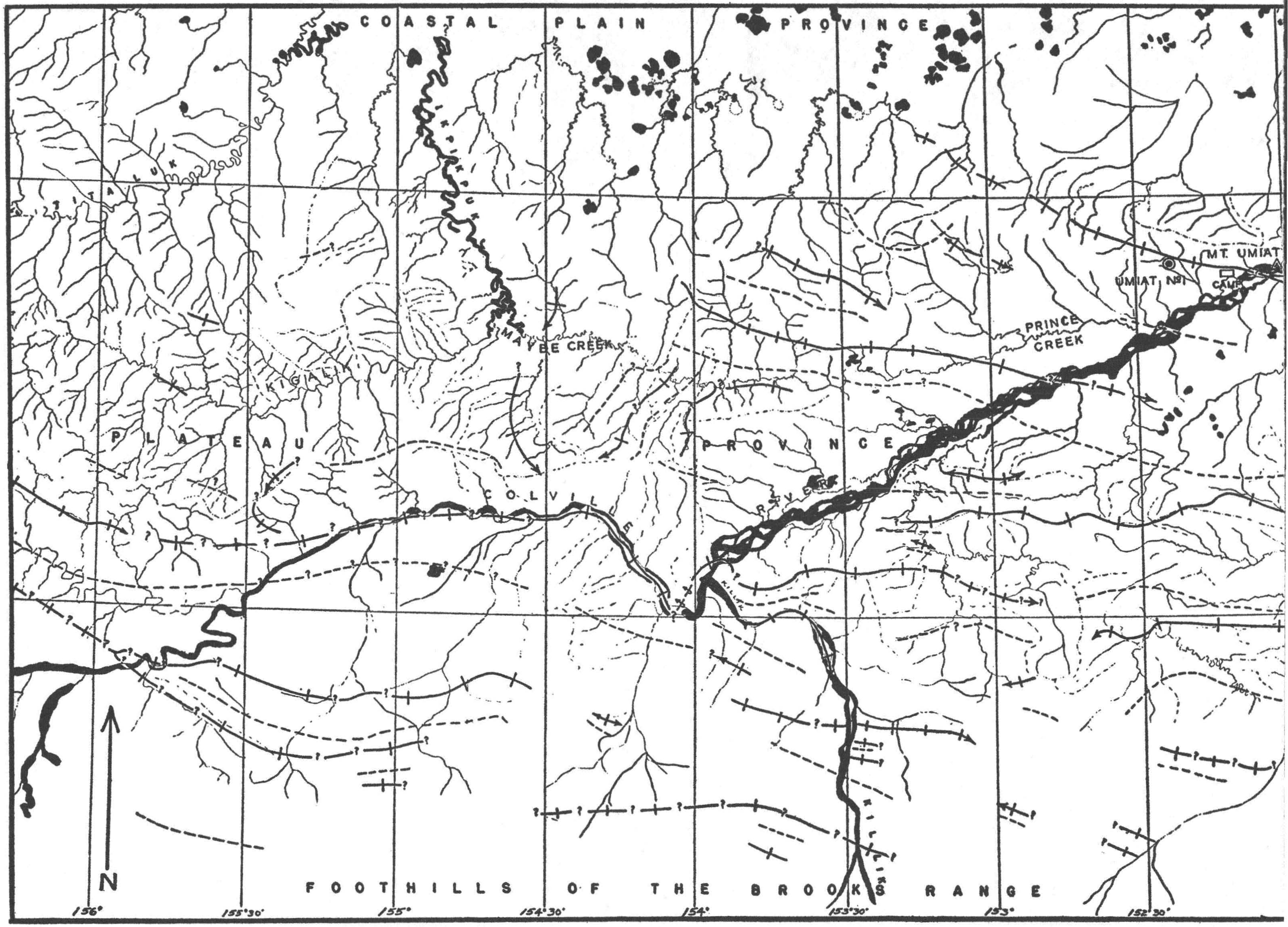
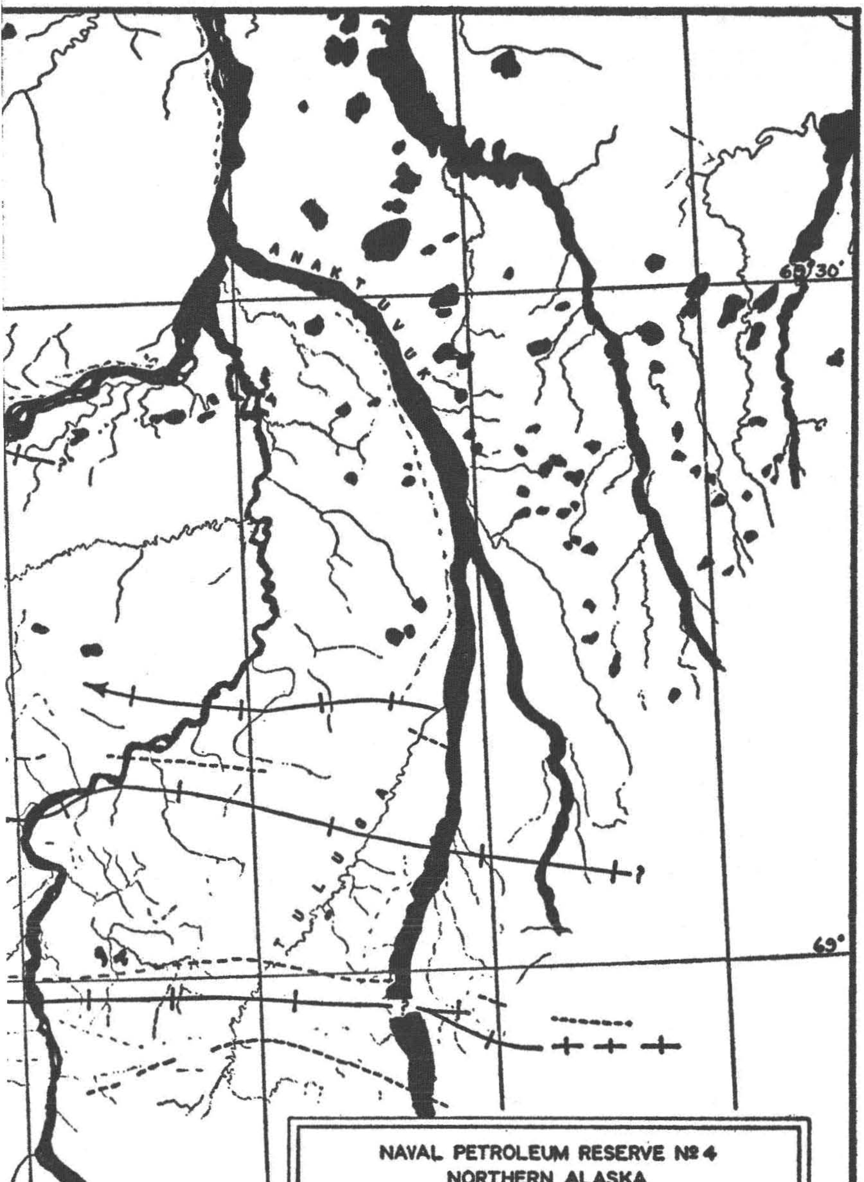


FIG. 8. Reconnaissance photogeologic map—eastern part of the plateau province, northern Alaska. Symbols of interpreted strike and dip were placed on the 1:200,000 scale base map photogrammetricall drafted to eliminate strike and dip symbols. Arrows indicate recognizable plunge of the axes. Parts of several structures are not shown owing to unfo



y before delineating the axes of the anticlines and synclines. The map was then re-
rtunate gaps in photography.

The method is so simple and it has so many possible applications in geological reconnaissance that its publication in a symposium on geological applications of photogrammetry seems especially appropriate.

The method involves three distinct steps. The first is to determine on each of two overlapping oblique photographs the spot location of a near-by point shown on the photograph[3] and the true direction from it to another point lying at essentially the same level as the first, and visible on both photographs.

The second step is to make transparent templates corresponding to each of the photographs and showing, on each:—(a) the principal line; (b) the point on the principal line representing the projected map position of the camera; (c) the spot-location above mentioned; and (d) the direction-line from it toward the second of the two selected points on the photographs. These templates are then laid over each other with the already-determined spot locations and direction lines to the second station superposed in exact coincidence. This step orients the two photographs with respect to each other and fixes the relative map positions of the camera stations.

If the two starting stations are shown on a third photograph, the accuracy of the mapping can be increased because three-point intersections then become possible.

The third step is to cut in by intersection, by any of the methods commonly used for oblique photographs, as many points as may be desired within the area of overlap of the original two or three photographs. Relative elevations of these points can then be determined in the regular way.

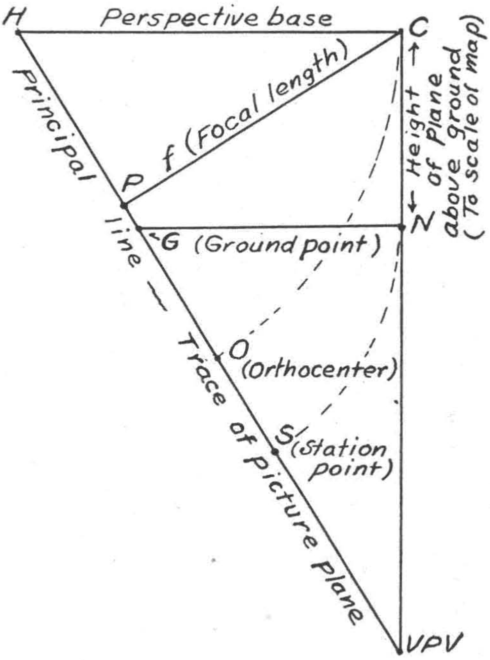


FIG. 1. The fundamental triangle of an oblique photograph. A vertical cross section in the principal plane.

Before attempting to master the technique of the method here described it will be necessary for the reader to understand certain of the fundamental principles of oblique perspective. In the following paragraphs an attempt is made to explain them in the simplest possible terms.

The various properties of an oblique photograph and the terms used in the following discussion are defined and are illustrated on the accompanying diagram, Figure 1, which represents the principal plane of the photograph (i.e. a vertical plane passing through the camera station and the axis of the lens system of the camera) enlarged to the scale of the map to be made by its use. The terms used are defined as follows:

> H represents the true *horizon*. Its position is found to lie above the apparent horizon by an angle (measured in minutes) at C equal to

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[3] The methods used for determining a spot-location and for determining the true direction of a line on an oblique photograph relative to that of the principal line are the same as those worked out independently by Desjardins (*loc. cit.*).

.98 $\sqrt{h}$, where h is the altitude, in feet, of the plane above the general level of the terrain.

C is the perspective center. It represents the position of the camera.

f is the *focal length* of the camera.

P is the *principal point*—the point where the optical axis of the camera intersects the plane of the picture.

PP—the *picture plane* is the plane of the photograph extended. Its trace on a vertical plane shows as a line in this drawing.

VPV is the *vanishing point for verticals*. It is the vanishing point in the picture plane for all vertical lines in nature. It lies at the point where a perpendicular dropped from the camera pierces the picture plane.

The line VPV-H is the *principal line* (PL) of the photograph.

O is the *orthocenter*[4]—a point in the principal line of the photograph lying at a distance equal to HC from H. Its position is found by swinging an arc of radius HC from H.

N is the *nadir point*, lying vertically below C at a distance equal to the altitude of the plane above the ground, as measured at the scale of the map.

G is the *ground point*, where the principal line in the picture plane cuts the ground. A line in the picture plane passing through G and parallel to CH is known as the *ground line*.

S is the *station point*. It lies along the principal line a distance equal to GN from G in the direction away from H. Its position is readily found by swinging an arc from G having a radius GN. The station point represents the map position of the vertical projection of the camera station.

V, V′, V″ etc. are *vanishing points* on the horizon line for rays parallel to those represented by various lines on the picture.

For the actual use of the properties and definitions treated in the preceding paragraphs we turn to a diagram, Figure 2, representing the picture plane rotated up into the horizon plane, using the horizon line as a hinge. In practice, the construction shown in Figure 2 is drawn on a sheet of transparent tracing acetate or other relatively transparent material.

Let Figure 2 represent a transparent construction-sheet which has been superposed on a photograph on which the true horizon line has been drawn and which shows two points, A and B, of which we wish to determine the true map position of point A and the true direction of B from A. At this stage we do not require the true map position of B.

As a first step, the ground line is drawn through G (see Fig. 1) parallel to the horizon line. The ground line, since it is drawn along the line where the picture plane, extended, would cut the ground, has the property that all points along it are true to the scale of the map provided that the line CN of the fundamental 12 triangle (Fig. 1) of the photograph is drawn to the scale of the map. The ground line is an *isoscale* line (Desjardins, Louis, *loc. cit.*).

Next locate the orthocenter, O, along the principal line a distance equal to HC (Fig. 1) from H. Also locate the station point S, on the principal line a distance equal to GN (Fig. 1) from the ground line.

A basic principle of perspective is that from the observer's station all parallel horizontal lines seem to vanish at a point on the horizon known as the vanishing point. Another principle of perspective is that from the observer's

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[4] **Desjardins, Louis, *loc. cit.***

station (equivalent in photogrammetry to the camera station C), the true direction of a system of parallel lines is determined by looking along one of those lines to the vanishing point of that system. In the problem before us this means that at C (Fig. 1) the true direction of a line with respect to the direction CH of the principal line of the photograph is measured by the angle at C between H and the vanishing point on the horizon line for lines of that system.

By rotating the picture plane up into the horizontal plane of CH the point C falls at O on the picture plane. Any line on the picture plane (photograph) extended to the horizon line locates there the vanishing point of all lines having

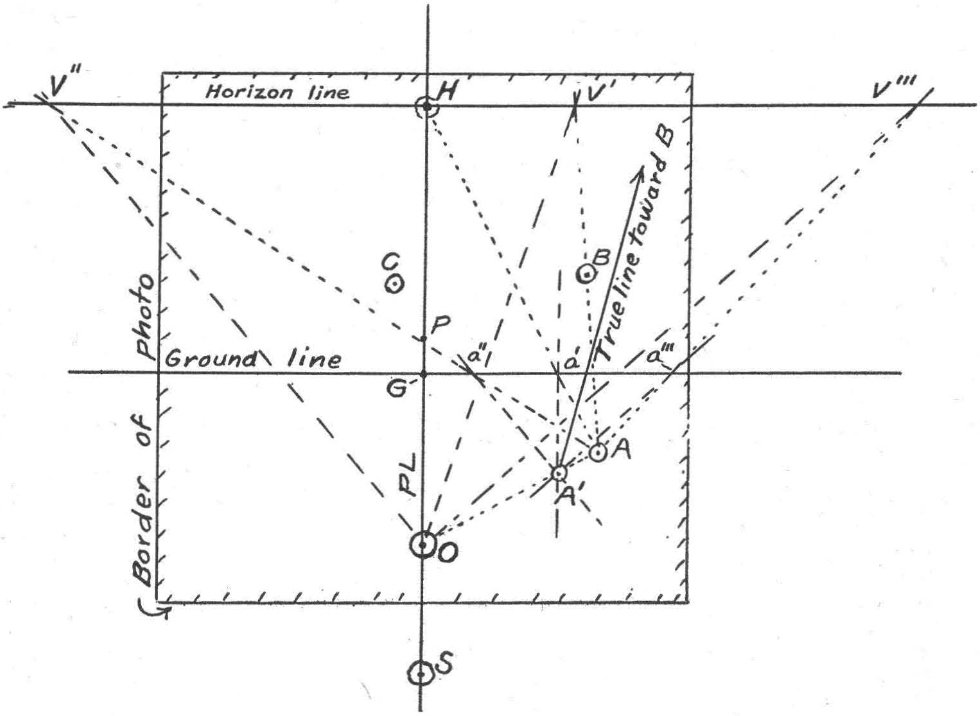

FIG. 2. Constructions in the picture plane of an oblique photograph for determining the true directions of lines shown on the picture and for determining the map position of the camera station and the spot map-locations of points appearing in the photograph. These constructions require that the ground be essentially flat.

the same direction as that line, and a line drawn from O to that vanishing point has the true direction of the line in question with respect to the principal line of the photograph. Obviously any system of lines vanishing at H would be parallel to the principal line.

Bearing in mind the foregoing discussion of principles, we now proceed to determine the true map position of point A (Fig. 2) and the true direction of the line between A and B.

The procedure is as follows: From A, draw a line to any convenient point on the horizon line V'', for example, cutting the ground line at a''. V'' then becomes the vanishing point of all lines parallel to AV'', including a line from O to V''. But a line from the camera station to a vanishing point fixes the true direction of all parallel lines of the system vanishing at that point, consequently the true direction of the imaginary line AV'' with respect to the principal line is that of

OV″. Since all points of the ground line of a photograph are true to scale, the point a″, where AV″ cuts the ground line, must be in its true map position, consequently the true map position A′, of the point imaged on the photo at A must be somewhere along a line passing through a″ in a direction parallel to V″O.

By choosing any other random point such as V‴ on the horizon line and drawing a line cutting the ground line at a‴ we can pass a line through a‴ parallel to V‴ O, and, according to the reasoning used in the preceding paragraph, the true map position A′ of the photo point A must lie somewhere along this line also. Consequently the true map position A′ is located at the intersection of lines through a″ and a‴ parallel respectively to V″ O and V‴ O.

Actually the simplest construction, usable in all instances except where the photopoint in question is too close to the principal line, is to draw a line from H to A cutting the ground line at a′. Such a line, by the rules of perspective must have a true direction parallel to the principal line of the photograph, consequently the true map position A′ will lie somewhere along a line through a′ parallel to the principal line. Also, as Desjardins[5] has shown, O on the photograph is an orthocenter from which lines to points on the photograph radiate in directions that are true, consequently a line from O to A will also have true direction and the true map position A′, must lie somewhere along that line. Actually all lines drawn as above described should meet at a point—the true map position of the spot imaged at A on the photo.

In Figure 2, point A′ has been located by four intersecting lines, of which any two would have been sufficient. If, however, the greatest possible accuracy in the location of a point is desired, at least three intersections are naturally better than two.

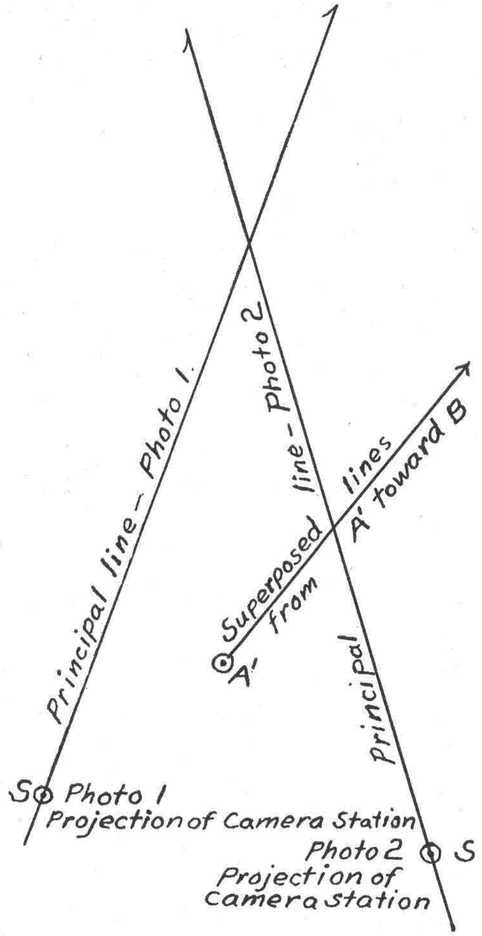

FIG. 3. The superposition of templates made from two overlapping oblique photographs so as to orient the two photographs with respect to each other and to determine the map positions of the two camera stations.

Having determined the map location A′ of photo-point A we now require a line extending from A′ in the true map direction toward the map position of B. To do this we draw a line from A to B on the tracing overlying the photograph and prolong it until it intersects the horizon at V′. A line from O to V′ then gives

[5] Desjardins, Louis, *loc. cit.*

the true map direction of the photo-line AB. A line drawn through A′ parallel to OV′ provides the tie-line we are seeking.

The next step in the mapping procedure is to locate by a similar procedure on another acetate overlay the map position of A′ and the true direction of a line from A to B as shown on the second overlapping photograph made from a different air station, but showing points A and B.

Having prepared the two acetate overlays as described above, these are superposed with A′ and the line of direction from A to B, as shown on the two overlays, in exact coincidence (see Fig. 3). We thus orient the two pictures and determine the map positions of the two camera stations from which the two pictures were taken. Those stations are represented by the points S on the two overlays. (That S represents the projection of the camera station will be apparent from the fact that the map locations of points, as made by the perspective method described above, are made with respect to the ground point, and that the map projection point of the camera station is situated a distance GN (Fig. 1) along the principal line of the picture from the ground point.)

The next step in mapping is to transfer onto a base map (which may well be another larger sheet of tracing acetate) the positions of the camera stations and the directions of the principal lines of the two pictures, as well as the position of the point A′ and the direction-line from A to B.

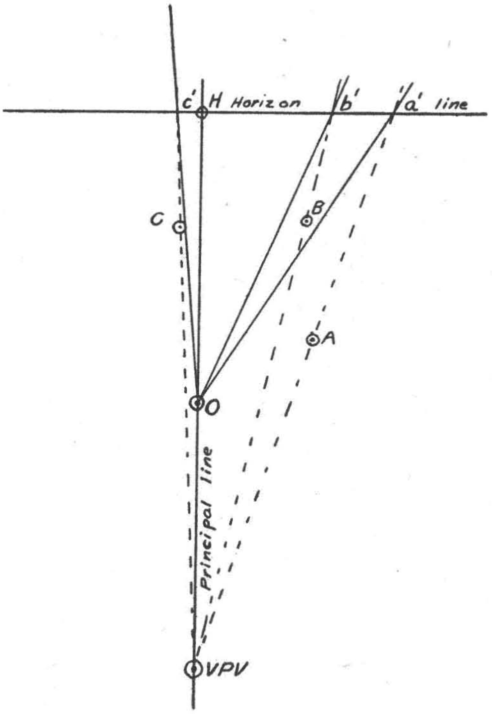

FIG. 4. The Army Air Force Method for drawing true-direction rays on the picture plane of an oblique photograph.

The remainder of the mapping is done by the methods of intersection regularly used in oblique photography. These methods are well known, but since feasible means of using them without the use of more or less complicated mechanical equipment may not be so well known, the method devised and used by the writer will be described briefly.

For each of the two photographs we prepare an acetate overlay showing true-direction rays to various points to be intersected. These rays may be found either by the Army Air Force Method[6] (Fig. 4), or by the Crone Method,[7] (Fig. 5).

The Army Air Force Method is the simpler if the camera is tilted down steeply enough so that the line HVPV of Figure 1 is not inconveniently long. Otherwise the Crone Method is probably the most practicable.

Using the Army Air Force Method, we prepare an overlay (Fig. 4) on tracing

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[6] *Manual of Photogrammetry,* pp. 678–683.

[7] Trorey, Capt. L. G. "Survey by High Obliques: The Canadian Plotter and Crone's Graphical Solution," *Geogr. Jour.*, Vol. 100, August 1942, pp. 57–64

acetate representing the picture plane and showing the following elements derived from the fundamental triangle: principal line, horizon line, the orthocenter C, and the plumb point VPV, which is the vanishing point for all vertical lines.

For each point, such as A on the photograph, a line is drawn on the acetate from VPV through the point, and continued to the horizon line. From the point where such a line cuts the horizon line, a line to O has true bearing with respect to the principal line. Similar lines are drawn for B and C and as many other points as it may be desired to intersect at one operation. Next, a similar acetate template (which, to save material and make for easier handling, may take the form of a triangle of suitable size) is prepared for the second photograph, and any others that may be used in that particular project.

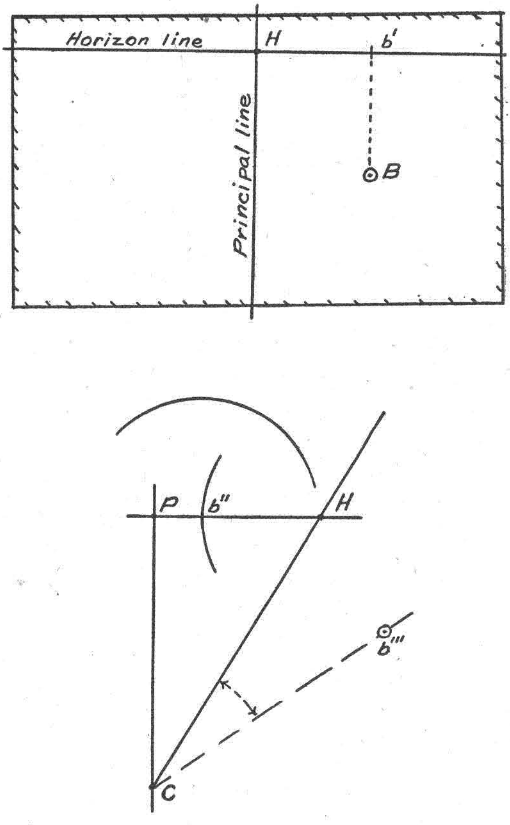

FIG. 5-A and 5-B. The Crone Method for drawing true-direction rays on the picture plane of an oblique photograph.

When these triangular acetate templates have been completed, each is laid down over the base map showing the camera stations as determined in Figure 3 with the point O of the template corresponding to the station point S of the previous work. The principal lines are brought into coincidence, and the templates are taped fast. We then have at the intersections of lines from the two camera stations the true locations of all points to which rays have been drawn. These locations are true irrespective of relief because the lines drawn through A, B, C, etc. from VPV to the horizon are the traces of imaginary vertical lines in nature, and hence their positions are unaffected by relief.

The use of transparent acetate triangular template in the manner suggested has a distinct advantage over the mechanical photoalidade because templates from as many as 6 wide-angle camera stations can be laid down at once and the accuracy or inaccuracy of the intersections of rays to a given point from the several stations can be seen at a glance. This gives a good check on the accuracy of the work as it proceeds.

When the Crone Method is used, the procedure is the same in all respects except that of determining the true directions of rays from the camera station. The Crone Method is also independent of relief.

Figure 5 shows the procedure in using the Crone Method. The method makes use of the part CPH of the fundamental triangle of Figure 1. This is prepared on a sheet of tracing acetate of suitable size. To determine the true bearing at the camera station of a line to point B, for example, with respect to the bearing of the principal line of the photo, we set a pair of dividers for the distance from B to the horizon line at B′ of the photo, and from H of the template swing an arc of that radius to cut line HP at B″. From B″ we then draw an arc of like radius on the acetate. We next place the acetate over the photograph with the line CH in coincidence with the principal line of the photograph, and slide the template up or down along that line until the arc last drawn is tangent to the horizon line of the photograph. With the template in that position we trace onto the template the picture image of B and call it B‴. The line CB‴ then makes with CH the same true angle as the map angle at the station point makes with the principal line of the photograph.

Having thus, by either the Air Force or the Crone method, obtained the correct map positions of points in the area to be mapped, their elevations with respect to the known or assumed elevation of the camera can be obtained with fair accuracy by a simple graphical method.[8]

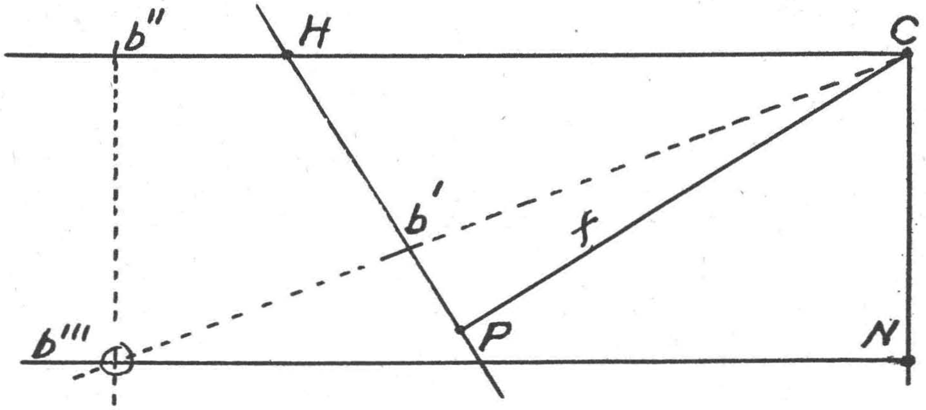

FIG. 6. Method of determining altitudes from oblique photographs.

The method depends on the fact that on level ground, the true elevation of any point along a line parallel to the horizon line of a photograph can be obtained by projecting that point perpendicularly to the principal line of the photograph, and then multiplying the map distance from the projection of the camera station to that point on the principal line by the tangent of the angle of depression, then correcting for curvature and refraction (if necessary) by use of the formula—C & R (in feet) = .574 × square of distance in miles, or, if more convenient, the formula—C & R (in feet) = .02059 × square of the distance in thousand-foot units. In using this formula we employ the true map distance to the point in question—not the map distance to its projection onto the principal line.

The graphic solution for altitude makes use of an acetate template, illustrated in Figure 6. Since various elements of the template will vary with each photograph, while the focal length for pictures of a single batch will generally remain the same, the templates are most conveniently made by inking or scratching the lines PC and PH extended onto the acetate so that they remain permanently while point H varies with each photograph. The line HC is drawn with erasable pencil and CN is similarly drawn at right angles to it.

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[8] Landen, David, "A Principal Plane Photoalidade for Oblique Photographs," PHOTOGRAMMETRIC ENGINEERING, Vol. XI, 1945, pp. 245–254.

To determine the elevation of a point such as B (Fig. 2), transfer the distance between B and the horizon line to point b′ on line HP of the template, Figure 6. Draw line Cb′ and extend it as far as necessary.

Next, on the map, project point B perpendicularly to the principal line and then transfer the distance between the station point and the projected position of B onto the line CH, extended if necessary, of the template, where in this instance it falls at b″. Now drop a perpendicular line from b″ to its intersection with the line Cb′, projected. The difference in level between b″ and this intersection at b‴, can be scaled, and the resulting figure plus the correction for curvature and refraction gives the difference in elevation between the camera station and point B on the ground. The sea-level altitude of the plane less this figure would give the sea-level altitude of point B. The elevations of other points can be determined in the same way.

In the particular example illustrated by Figure 6 the elevation of B turns out to be the same as that of the ground line (except that it would actually be lower by the amount of the correction for curvature and refraction). If the scale of Figure 6 is such that CN is 8,000 feet and Cb″ is 23,000 feet the correction for curvature and refraction would be only 10 feet—an amount too small to be significant when graphic methods such as this are being used.

On preceding pages a method has been described whereby good reconnaissance maps can be made from the air without ground control. It remains to discuss the requisities and limitations of the method and to point out briefly some of its possible applications.

The prime requisites for the accurate application of the method here described are that reasonably clear horizon lines be shown on the photographs and that somewhere in the area common to two of them two points having essentially the same altitude can be recognized. These might be along the shores of a lake or sea, around a playa flat, along a river whose gradient is low or could be estimated with fair accuracy, or even a formerly peneplaned upland surface such as occurs in much of central United States.

If the *scale* of the original map is to be accurate, the height of the plane above the two starting points must be known. If the altitude is not known the map and elevations determined from the mapping will be consistent within themselves, but their exact values will not be known. They can be determined and corrected later, however, if subsequently the true map distance between two or more points is determined on the ground or from previous surveys.

It should be possible to carry the mapping for a considerable distance along a line of flight, because after intersections and altitude determinations have been made from two or three overlapping pictures, these should permit the survey to be carried along from picture to picture. If the locations and elevations of two points are known, those points can be used for tying successive photographs together in the way described, even if those points are not at the same altitude because the position, on the photograph, of the farther one of them can readily be corrected to what it would be if it were at the same elevation as the nearer point.

The method permits reconnaissance mapping in regions of high relief where the Canadian grid method and the method of determining spot elevations from single pictures would not be applicable.

The method requires no equipment except a camera with a good lens (surprisingly good results can be had with any one of the better miniature cameras such as the Leica); a pair of triangles and a scale; and a supply of tracing acetate or similar material.

If no ground control is available and the elevation of the ground is not known, the use of a stop watch to time each exposure while flying on a straight course would furnish a means of determining very closely the distances between exposure stations, and hence the scale, because ordinarily the speed of an airplane is surprisingly constant, and the relation between recorded airspeed and ground speed could be determined by re-flying the course between recognizable points.

As to bearings, in the total absence of any ground indicators such as section lines, the pilot might determine the compass bearing of a line between two recognizable points by pointing the plane along the line between them and reading the plane's compass. Even shadows could be used for determining bearings if time of exposure and approximate latitude were known.

THE MULTIPLEX COMPILATION OF GEOLOGIC MAPS*

John D. Strobell, Jr., U. S. Geological Survey

MAPPING in the field on air photos offers the geologist easy, speedy, accurate plotting without instrument work. The lack of uniform scale, true azimuth, and elevations with respect to datum, however, makes impossible the accurate construction of an areal map, profiles, and structure sections, the projection of subsurface geology to the surface (and vice versa), and the calculation of ore reserves. There are no quantitative data until the geology is on a map, or as one geologist succinctly remarked, "No map—no geology." When, for lack of a base map or for convenience, geology is plotted on air photos, the compilation of that information into a map or its transfer to a base map is thus of crucial importance both to the geologist's analysis and to those who would utilize the data he has gathered and analyzed. It is my purpose to outline briefly one method of accomplishing this transfer or compilation.

There are several photo-mapping instruments which are able to solve this problem for the geologist, among them the Mahan, and K.E.K. plotters, the Multiplex Aeroprojector, and more elaborate instruments such as the Stereoplanigraph and Stereoautograph. These instruments use stereoscopic pairs of overlapping aerial photographs oriented so as to produce a true representation of the terrain in the area of overlap. To do so, the light rays that entered the aerial camera may be represented in the instrument by actual light rays or by mechanical levers, but they must be reproduced with their relationship to each other the same as when the photos were taken. This requires close integration in design between the plotting instrument and the taking camera, and each instrument is designed around photography of a specific kind. The specifications of the aerial camera thus are basic, and in mapping operations photography is planned to meet the requirements of the plotting instrument to be used, but when geologists take advantage of available photography their choice of a plotting instrument is immediately somewhat limited by the characteristics of that photography. The Mahan and K.E.K. plotters, for example, are designed primarily for photos taken with cameras of 8¼-inch nominal focal length. Some multiplex equipment is available for use with cameras of 8¼- and 4-inch focal lengths, but most is designed for cameras of 6-inch and 5.2-inch focal lengths.

The practical mechanics of the design impose other limits on the range of the plotting instrument. In the Mahan and K.E.K. plotters, standard contact

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