SLOTTED TEMPLET ERROR

A formula for the calculation of mean error as a function of control density.

Lyle G. Trorey, Chairman, Aerographic Surveys Ltd.
London, England

THEORY OF ERRORS OF A SLOTTED TEMPLET BLOCK

CONSIDER a slotted templet block laid to ground control, and let e be the arithmetic mean error of a number of well defined image points located by templet intersections, and checked by ground survey. From the theory of errors we might expect e to vary inversely as the square root of the number of control points to which the block is laid—assuming a reasonable distribution of control.

The number of control points remaining unchanged, we would expect e to be larger for a greater number of templets in the block. Experience leads to the belief that it is the lineal distance between controls which is the determinant, thus e may also be expected to vary as the square root of the number of templets. Based upon the foregoing assumptions, we would then have:

$$e = k(t/c)^{1/2}$$

where e is the arithmetic mean error in millimeters, k a constant, t the number of templets (or overlaps) covering the area, and c the number of control points. Note that t/c is *control density* expressed as number of templets per control point. From *empirical data*, the value of the constant k is 0.16—to include a reasonable "factor of safety."

Such a formula, if its truth be established, is of obvious practical value since it will permit the specification, with exactitude, of the number of control points we must have in a given area in order to map to a specified precision at publication scale. Consider the following typical applications.

Area 100,000 square miles, required a planimetric map at 1:250,000, specify the ground control.

Consider an aircraft of ceiling 30,000 ft. above the mean level of the ground, and a 6-in., wide angle lens. Net gain per overlap, 23 sq. mi. Number of templets $100,000/23 = 4,800 = t$. A specification for ordinary good mapping is that mean position error at publication scale shall not exceed 0.5 mm. Hence $e = 0.5 \times (250,000/60,000) = 2.1$ mm., the grid scale being contact scale of 1:60,000.

$$c = (0.16/e)^2 t = (0.16/2.1)^2 \times 4,800 = 28$$

Thus we site the ground control, if we can, some 60 mi. apart giving one point per 3,600 sq. mi.

If stations of a primary triangulation net average 25 miles apart, can good planimetric mapping at 1:100,000 be performed without the necessity of running additional ground control?

H 30,000 ft., net gain per overlap 23 sq. mi.

Hence $t/c = 25^2/23 = 27$.

Therefore $e = 0.16 \times 27^{1/2} = 0.83$ mm.

The error at the compilation scale of 1:60,000 is 0.83 mm., and at publication scale is $0.83 \times 60,000/100,000 = 0.5$ mm.

If half a millimeter mean error can be tolerated at 1:100,000, no additional control need be run.

U. S. Department of Agriculture Tests

Extensive templet tests have been made by various authorities, one of the more comprehensive of these is the series by the U. S. Dept. of Agriculture, and is reported by H. T. Kelsh in the Department's Miscellaneous Publication No. 404.

The Beltsville, Md. test area is 155 sq. mi. and in the area were 273 points of known position. Relief 200 ft., photo scale 1:12,000. Prints 9 in.×9 in. There were 12 flights of about 19 photographs each, and 233 templets were laid. The tests were made with various amounts of control and with different control distributions. In each case the photogrammetric positions of those known points which were not used for control were checked against their survey positions, and the error measured. Number of control points in the several tests varied from a minimum of 4 to a maximum of 31.

Further tests were then made by the same Authority, this time in the Bird and Caney watersheds in Kansas and Oklahoma. 4,400 sq. mi. covered by 2,700 templets—compilation (grid) scale 1:15,840. Certain of the above tests are summarized in Table I following.

Critical examination of the results of such tests led to the belief that the relation between error, scale or number of templets, and number of control points, could be expressed by the formula given above. Column 6 of Table I gives values of error calculated from this formula, taking k as 0.16. It is seen that the agreement between the calculated and the observed arithmetic mean error is excellent excepting only in the case of the Test 3.

The factor of the last column of the table is obtained by taking the ratio, *probable error of a single observation/arithmetic mean error*, as 0.845 and by the application of *Chauvenet's principle* to determine the ratio *max e/arithmetic mean e.* In Test 3 the ratio 5.1 of maximum to mean is far too large, indicating abnormal error distribution. Note that the ratio, *measured max e/calc mean* $e=2.3/0.7\frac{1}{2}=3.1$ which, not only is more reasonable, but also brings this test into agreement with all the other tests. From the data presently available to the writer, the reason for the Test 3 discrepancy cannot be stated.

Table I. Comparison Between Calculated and Observed Error in Slotted Templet Block Plots

Calculated error from $e=.16(t/c)^{1/2}$ Observed error from U. S. Dept. of Agriculture templet tests

No.	t	c	Distribution of control	Actual mean e in mm.	Calc mean e in mm.	Max. e	Max./mean	Number points checked	Ratio max/mean Chauvenet
1	233	4	One at each corner	1.1	1.2	3.2	3.0	237	3.7
2	233	5	As above with an added 5th point in the centre	0.9	1.1	3.1	3.5	236	3.7
3	233	8	Uniform	$0.4\frac{1}{2}$	$0.7\frac{1}{2}$	*2.3*	*5.1*	232	3.7
4	233	31	4 on each edge flight, 1 at each end of each intermediate	0.4	$0.4\frac{1}{2}$	1.3	3.3	206	3.6
5	2,700	87	Around edges, substantially as 4 above	0.9	0.9	2.5	3.1	192	3.6

Effect of Geographic Position of Control

In Tests 4 and 5 of Table I, the azimuth of the outside flights is held—these flights being strongly controlled. In Tests 1 and 2, this is not the case. The very close agreement between Tests 1, 2, 4, and 5, and the calculated errors—in spite of large differences in number and in position of control, and in extent of area—seems to establish the basic principle that (within reason) error is not a function of distribution of control. That is to say error seems to vary inversely as the root of c whether we distribute it more or less uniformly, or whether we concentrate it at the edges of the block. Where the control spacing is more or less uniform, the geographic distribution of error may, too, be expected to be uniform. Where it is concentrated at the edges, the larger position errors may be expected to be in the centre of the block. In each case the arithmetic mean error is of the same magnitude, provided there is no undue control concentration.

Constant k Will Change under Different Conditions

The value of k, 0.16, is when metal studs are used. Plastic studs are not (at present) manufactured to the same limits. Hence a plastic stud assembly is looser and, accordingly, a larger value of k is indicated. There should be no variation in k over rough terrain, provided tilts are within ordinary limits. With inexperienced operators we would expect k to show an increase.

Where the plotting is not at photo scale the effect is roughly analogous to direct photographic enlargement of the photo scale plot, and k therefore increases, or decreases in direct proportion, as the plot is enlarged or reduced—that is, if the enlarging or reducing means do not change the angular precision of the templet. In Test 5 above, plotting was at 1:15,840 from 1:20,000 contact prints. This has been taken into consideration.

Closer agreement would be obtained with k 0.15 or 0.14, but the value 0.16 may be considered as including a factor of safety.

NEWS NOTES

The American Congress on Surveying and Mapping has announced the Seventh Annual Meeting to be held on August 14–15, 1947 at the Hotel Statler, Washington, D. C. Details regarding the program are not yet available.

The American Polar Society, American Museum of Natural History, Central Park West at 77th Street, New York 24, N. Y., has extended an invitation to our membership to become associated with their organization.

Word has been received of the passing of one of our outstanding Australian members. Edwin Thomas Holdaway, Head of the Cartographic Branch of the Survey Office died at St. Martin Hospital, Brisbane on February 4, 1947. Mr. Holdaway, a member of our Society since 1942, was a man of many interests, the chief of which was the promotion of adequate surveys for the Commonwealth of Australia, with particular emphasis on the photogrammetric aspects of the work.