NOTES ON STEREOSCOPY

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THE HUMAN EYE AS A CENTER OF PERSPECTIVE

IN PLOTTERS such as the Mahan and KEK it is desirable to know the location within the eye of that point which forms the virtual center of perspective. In the KEK Plotter the half-marks must be located about 14 inches beneath this point. In the Mahan Plotter, whose wide-angle stereoscope permits positioning the photographs at a principal distance equal to the focal length of the taking lens;[1] the correct location of this point is even more important since it determines how closely the eyes may be positioned over the eye-mirror assembly.

What point within the eye serves as the center of perspective? The answer does not, to my knowledge, appear in the available photogrammetric texts, which sometime incorrectly use the node of the eye-lens as perspective center. The matter has been the subject of questioning by Van Camp, Buckmaster, and others concerned with the design of the Mahan Plotter. The author, whose company is now building a plotter based on the Mahan design, has found the answer in standard works on physiological optics.

The data presented herein are based on Gullstrand's Schematic Eye as described by J. P. C. Southall.[2] The Schematic Eye is an ideal or hypothetical eye whose various dimensions and indices of refraction have been determined by years of experimentation and research.

Figure 1 is a sketch diagram of the eye. No attempt is made to correctly depict all anatomic detail. The purpose is to indicate clearly the relative positions of the various cardinal points of interest to photogrammetrists. *A* is the anterior vertex of the cornea. *N* is the anterior node of the eye-lens, and *N'* is the posterior node.[3] These nodes are actually only about $\frac{1}{4}$ millimeter apart and for all practical purposes the nodal point of the eye is considered to lie on the optic axis at its junction with the posterior vertex of the eye-lens. *AB* is this axis and *B* is the posterior pole of the eye, that is, the point where the optic axis intersects the retina *R*.

The eye is not truly spherical. Southall calls it a "bulbus." But the swivelling action of the eye in its socket closely resembles that of a sphere rotating within a close-fitting spherical shell. The three pairs of muscles which swivel the eye act so as to rotate it around a fixed center of rotation, *Z*.[4] The center of rotation *Z* is the true perspective center.[5]

Figure 1 also shows a spectacle lens in front of the eye. Cardinal distances are as follows:

DISTANCES FROM ANTERIOR VERTEX OF THE CORNEA, A

To anterior node of eye-lens, *N*	7.078 mm.
To posterior node of eye-lens, *N'*	7.332 mm.
To posterior Pole *B* (Polar distance)	24.0 mm.
To Center of Rotation *Z*	13.0 mm.

[1] Van Camp, C. P., *Mahan Plotter*, PHOTOGRAMMETRIC ENGINEERING, Vol. XI, No. 4, pp. 337–338.

[2] Southall, J. P. C., *Introduction to Physiological Optics*, Oxford Univ. Press, 1937, pp. 56–58; 164–174.

[3] Southall, J. P. C., *Mirrors, Prisms and Lenses*, Macmillan, 1923, p. 432.

[4] Southall, J. P. C., *Introduction to Physiological Optics*, Oxford Univ. Press, 1937, pp. 56–58; 164–174.

[5] *Ibid.*

The inside surface of a spectacle lens is from 6 to 12 mm. in front of the corneal vertex, depending on length of eye-lashes, etc. This dimension is far from constant. It will be seen therefore that the distance from the front surface of a spectacle lens to the center of rotation Z will vary, say, between 20 and 28 mm. This is the closest the eye's virtual perspective center can be positioned over the eye-mirror-lens assembly of a stereoscope, when the operator wears

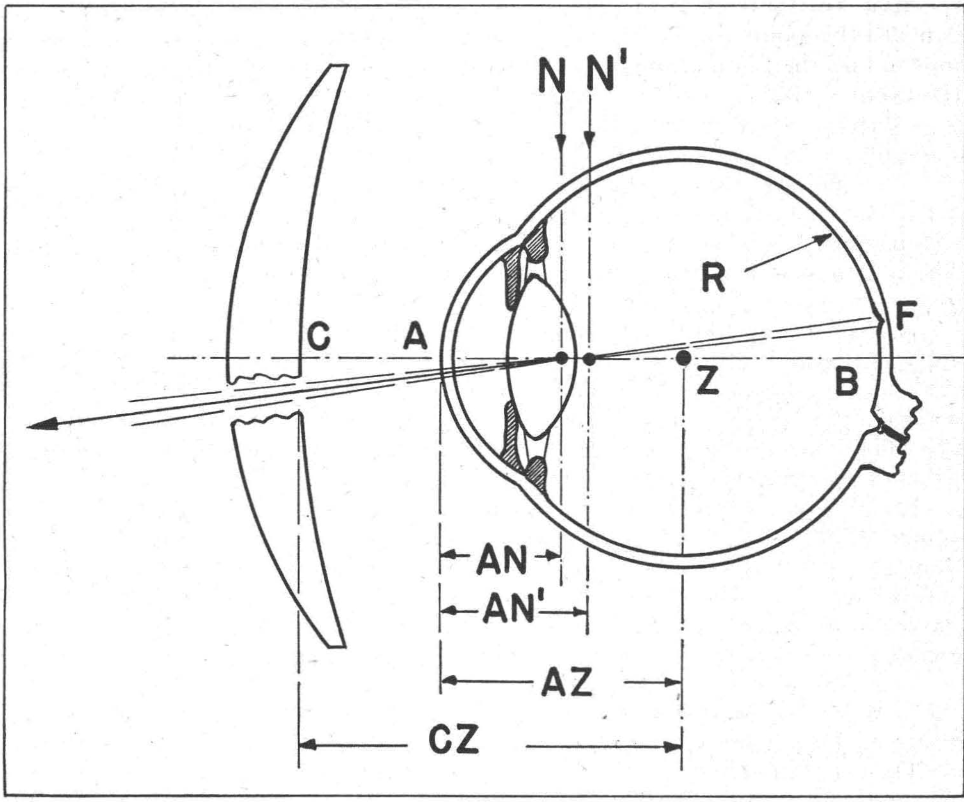


FIG. 1

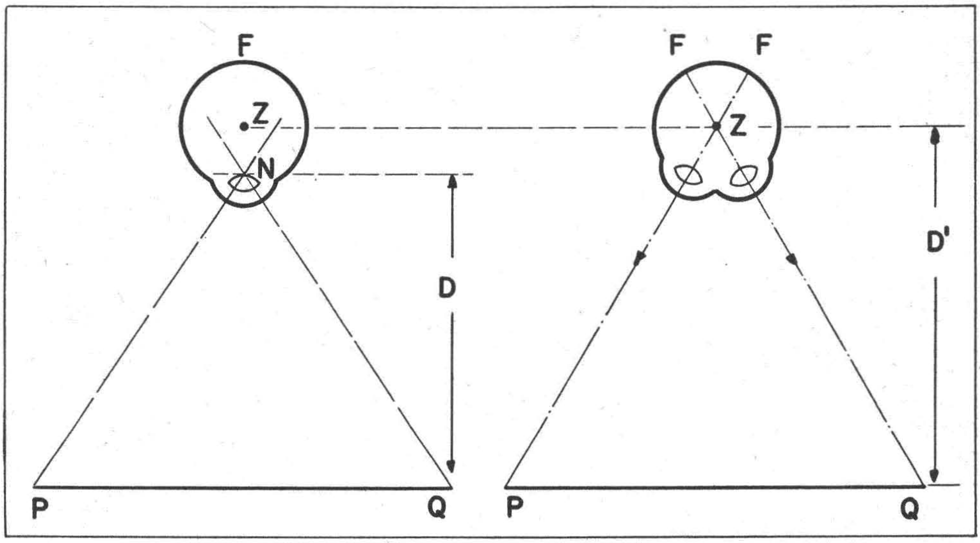
glasses. Without glasses this distance can be reduced about 13 mm. This means that about ½ inch of principal distance can be saved by not wearing spectacles. Since many plotter operators must wear spectacles, the solution would seem to be: to provide an eye-mirror-lens assembly which would permit quick insertion of lenses of varying powers, set in some sort of standard mount. In such an assembly the operator could have his spectacle prescription made up to fit a pair of supplementary lens cells, which would be inserted in the stereoscope's eyepiece.

In re-designing the stereoscope for our new plotter we have tried to increase its field so that 9×9 inch photos exposed with lenses of focal lengths down to 6 inches, could be oriented with the eyes at the true center of perspective. S. H. Spurr[6] has implied that this is not possible; but I think this statement is too broad. We have succeeded in designing a stereoscope based on the original

[6] Spurr, Stephen H., *Reply to Comments on the Multiscope*, PHOTOGRAMMETRIC ENGINEERING, Vol. XII, No. 3, p. 313.

Mahan Plotter's distance between principal points (10 inches) which covers a $5\frac{1}{2}$ inch overlap area on a pair of 9×9 inch photos; and in which the centers of rotation of the eyes can be positioned just a little more than 6 inches from the pictures. This extremely close positioning of the pictures is achieved by lowering the stereoscope so that the lower edges of its wing mirrors almost touch the photo-tables, and by setting the mirrors at angles of 50 degrees from the horizontal, rather than the usual 45 degrees.

Whether or not the plotter will work in this position is another question!



A B

FIG. 2

But certainly it seems possible to construct a reflecting stereoscope with a workable principal distance of 6 inches either by decreasing the separation of the principal points to less than 10 inches, or by somewhat decreasing the field of view in the X direction; or by a combination of both.

It can easily be shown that the center of rotation Z must be used as the center of perspective rather than the eye-lens node (Fig. 1). The misconception occurs if the eye is considered as working like a photographic camera.

In Figure 1, F is the *fovea centralis*, or area of distinct vision. The visual axis used when looking at any object is the line from F through the nodes, and indicated by the arrow. When we look at an object our eyes "see" it by scanning swiftly across it from point to point, with the visual axis FN directed toward the point monentarily observed. This line FN is called the *line of fixation*[7] The angle of distinct vision which is defined by the size of the *fovea centralis* is extremely small, being given by D. H. Jacobs as less than one degree.[8]

Figure 2-A shows the conception of the eye's perspective center as given in some photogrammetric texts. As photograph PQ is viewed, light rays from points P and Q pass through the eye-lens node N, presumably to form an image on

[7] Southall, J. P. C., *Introduction to Physiological Optics*, Oxford Univ. Press, 1937, pp. 56–58; 164–174.

[8] Jacobs, Donald H., *Fundamentals of Optical Engineering*, McGraw-Hill, 1943, p. 72.

the retina. This conception assumes that the eye is similar to a photographic camera. The nodal point N would then be the perspective center and the principal distance would equal D.

Figure 2-B illustrates the correct conception. The eye does not work like a photographic camera, in which the camera lens defines the entire field sharply on the focal plane. The eye cannot see sharply the entire field at once. To "see" the picture PQ the eye must swivel in its socket around its center of rotation Z so that its visual axis is successively directed toward all of the various points observed. To see point P it swings to the left: to see point Q, toward the right. In each case the image is brought to a sharp focus on the *fovea centralis* F. It is clear from this conception that the true center of perspective is coincident with the center of rotation Z; and thus the actual principal distance in the figure is increased to equal D', that is, the original distance D increased by the distance from the eye-lens node to its center of rotation (about 6 mm.).

Reduction of Head-Movement Parallax by Use of Small Eye-Apertures

D. Lyon has stated that parallactic displacement due to movement of the head could be eliminated in plotters such as the Mahan and KEK by placing small apertures between the eyes and the stereoscope eye-mirrors.[9] This was also investigated by the original designers of the Mahan plotter. In our Model L-1 Vertical Sketchmaster an optional "peep-sight" eyepiece is provided, with a hole or eye-aperture about $\frac{3}{16}$ inch in diameter. This substantially reduces head-movement parallax; but as J. L. Buckmaster has stated[10] it is necessary, in order to completely eliminate such parallax in the Sketchmaster, to exactly equalize the two foci between eye-and-photo and eye-and-map-base.

Figure 3 illustrates an interesting experiment using an ordinary calling card A in which has been punched a clean round hole not over about 3/32 inch in diameter. C is a table-top or other flat surface on which rests a piece of paper. B is the edge of a transparent triangle or other flat surface held in fixed position vertically above C. Two sighting points P and Q are marked on the edge of B. The distance between A and B should be at least 10 inches; and the marks P and Q should be as far apart as possible consistent with ability of the eye to line up both of them. The head must be completely immobilized, with the eye positioned as shown, and with the cornea practically touching the upper surface of the card A.

With the hole in card A centered on the eye pupil and held in place as shown sight downward past P and Q and mark on surface C two points R and S which appear to be coincident with P and Q respectively. Then, while holding the head motionless, withdraw the card from in front of the eye. P and Q will then appear to lie *inside* R and S, instead of coincident with them. This is due to a change in location of the perspective center.

When a card with a hole somewhat less in diameter than the eye pupil is held before the eye, the hole itself becomes a substitute perspective center. The eye can only see P and Q by rolling the head from side to side while the center of rotation Z swings through the arc $Z-Z$. When the card is removed the perspective center shifts to the center of rotation Z', and the lines of fixation from the

[9] Lyon, Duane, *Automatic Map Plotting Instruments*, PHOTOGRAMMETRIC ENGINEERING, Vol. XII, No. 3, p. 336.

[10] Buckmaster, J. L., *The Camera Lucida for Aero-Mapping*, PHOTOGRAMMETRIC ENGINEERING, Vol. XII, No. 2, pp. 239–240.

the eye to points R and S then lie outside the lines of fixation to P and Q, resulting in displacement of the images.

From the above it is evident that by use of small eye-apertures: 1. Since the eye-apertures themselves can be made to serve as substitute perspective centers it will be possible to increase the field of view and decrease the principal distance of the stereoscope system accordingly. But for this purpose the apertures must be quite small, less in diameter than the pupil; and this is subject to the objection that in order to see the entire field of view the operator must roll his head around. Most operators would probably find this tiring and objectionable. 2. Parallax due to head-motion relative to the stereoscope is effectively eliminated.

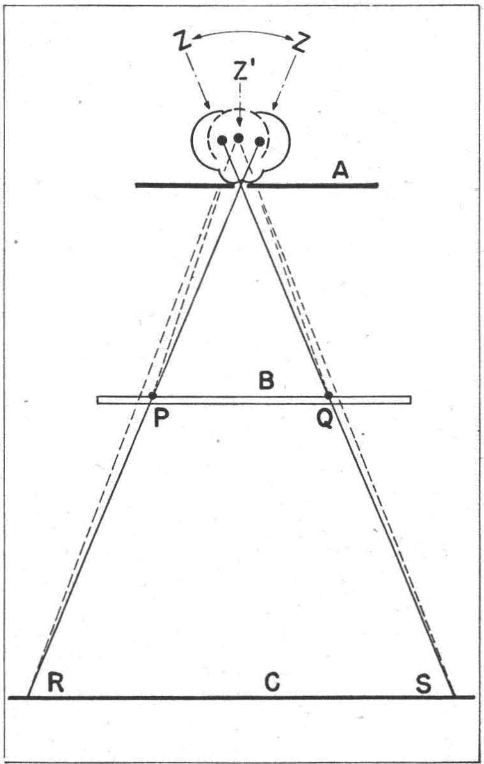


FIG. 3

3. By using an aperture of somewhat larger diameter, similar to the "peep-sight" eyepiece of the L-1 Sketchmaster, parallax can be substantially decreased while allowing the operator free vision and ability to see the entire field of view with little or no rolling of the head. The larger aperture, being usually just a little greater in diameter than the pupil, tends to keep the eyes centered in the stereoscope, since every time the head moves to an eccentric position one edge of the eye aperture is sensed as fouling the line of vision. The difficulty with the "gunsight" type of marker as used on the KEK Plotter is that the eyes must move over a considerable angular range during the act of alignment. This is more or less tiring. The eye apertures of moderately small diameter line up the eyes almost equally as well, without fatigue. The "gunsight" marker can always be included as standard equipment on these plotters for optional use.

It should be noted that these larger apertures, greater in diameter than the eye pupil, cannot be used as substitute perspective centers.