TANGENTIAL DISTORTION AND ITS EFFECT ON PHOTOGRAMMETRIC EXTENSION OF CONTROL

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DURING tests conducted by the Corps of Engineers in 1941 to establish proper procedures and probable accuracies of extension of control with the wide angle multiplex equipment several interesting observations were made, but, due to war time conditions, it has been impractical till now to release the results of these investigations for general publication. It is believed, however, that even at this late date, this information will be of considerable interest and value to photogrammetrists.

In these control extension tests it was noted that there was a systematic error present in some of the extensions which was such that with each end of the strip oriented to control, positions in the center of the strip were displaced from ground control check points in a direction normal to the line of flight. That this

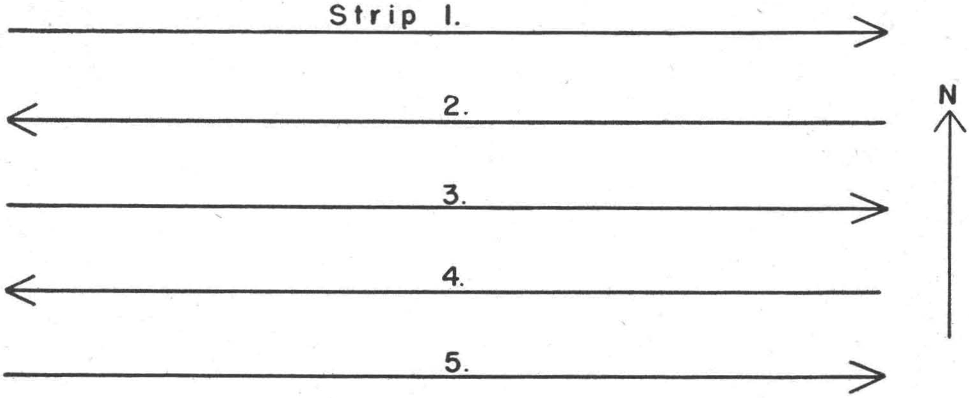

FIG. 1. Flight plan for control extension tests.

was a systematic error was proven by the fact that the direction and amount of this error was consistently the same for any one camera printer combination. For instance in one particular test, in which five parallel overlapping strips of 17 to 18 photographs each were extended independently by four operators, all the strips curved to the left in the direction of flight and missed control check points on the average about 3 mm. in the center of the bridge. In this test the strips were flown in the usual manner, that is with alternate strips in opposite directions as indicated in Figure 1. All strips were extended from west to east using the same projectors in identical relative positions for all extensions and the curvature of the strips was as indicated in Figure 2. It will be noted that as mentioned previously, all strips curved slightly to the left when facing in the direction of flight and as all strips were extended from west to east with the same projectors in the same relative orientations on the multiplex frame, the projectors themselves were eliminated as a possible source for this error.

In some further extension tests two sets of diapositives were made in the reduction printer. One set was made with the negative oriented in the reduction printer in a position 180° opposite to the orientation for the other set. It was noted that in some instances where there was a curvature in azimuth in the extension with one of the sets of diapositives, there was no curvature in azimuth with the other set. In other instances the curvature was in opposite directions with the two sets of diapositives. This indicated that both the reduction printer and the aerial camera were contributing sources to this error.

Systematic errors similar to those noted in these tests have been observed by other investigators within this country and abroad and so far as is known no satisfactory explanation of the exact cause of this type of error has been made, nor is it proposed to fully explain this phenomenon in this article. It is however proposed to advance a theory as to the source of this error and to show factual evidence in support of this theory.

It is possible that this type of error could be either, (1) due to an unsymmetrical distortion in the camera and reduction printer which is such that it is

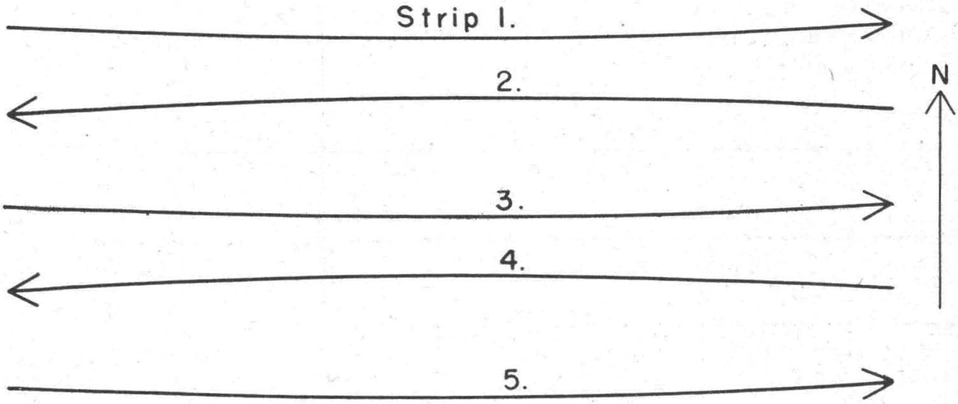

FIG. 2. Characteristic curvature of extended strips.

impossible to completely remove the parallax in the entire stereoscopic model or, (2) due to a tangential distortion in the camera and reduction printer, this distortion being such that an image of a straight line through the center of the field would project as a curve in the model or, (3) due to a combination of both.

The first condition mentioned above was excluded as a possible source of error in these tests as the cameras and reduction printers had been accurately calibrated, and it was known such a condition did not exist. That this could be a source of error will be apparent to those familiar with the normal control extension procedures with stereoscopic instruments.

In these procedures the relative orientation of successive projections of a strip is consummated by accurately eliminating the parallax at five positions in a stereoscopic model with a sixth position as a check. It is apparent that an unsymmetrical distortion could result in unconsciously averaging out the residual parallax which would inevitably occur at this sixth position, thus possibly causing a false swing to be introduced at each projector.

The second condition mentioned above was considered the more likely contributing source of error and as will be shown later considerable factual evidence has been obtained in support of the postulate that such a distortion is present, although the exact cause of the distortion is at present not fully known.

As mentioned above, the term tangential distortion refers to a distortion that is such that the image of a straight line through the exact center of the field of an optical system is a curve. In other words, the distortion in the optical

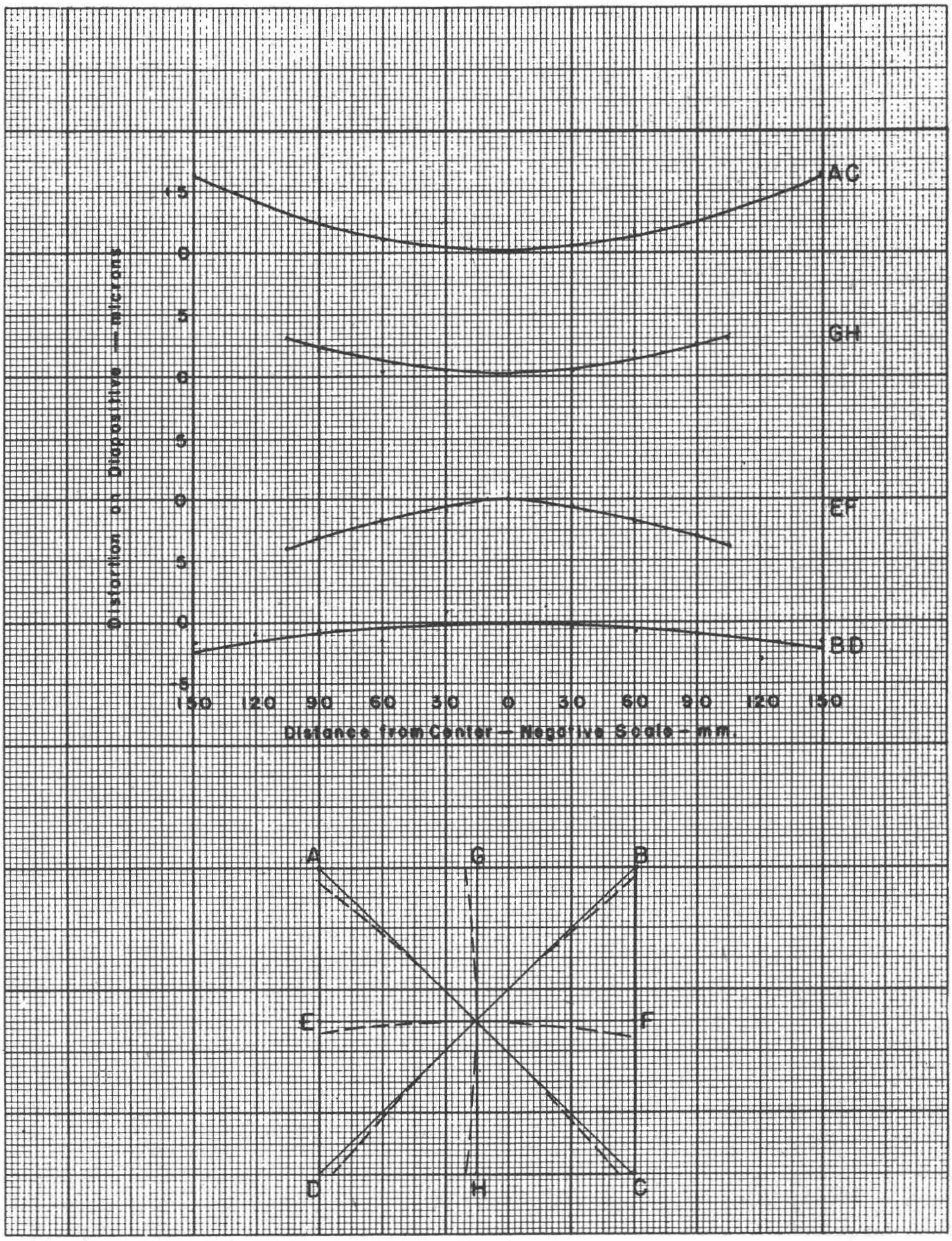

FIG. 3. Tangential distortion on a Multiplex Reduction Printer.

system is assumed to have two components, one radial from the optical axis and the other normal thereto or tangential.

It was apparent that a very small amount of tangential distortion would be sufficient to cause an azimuth curvature of the order found in these extensions and that very precise methods would need be used to actually measure it. Expressing the curvature in terms of the middle ordinate to the chord across the axis of the format it was determined that in the focal plane of the camera the

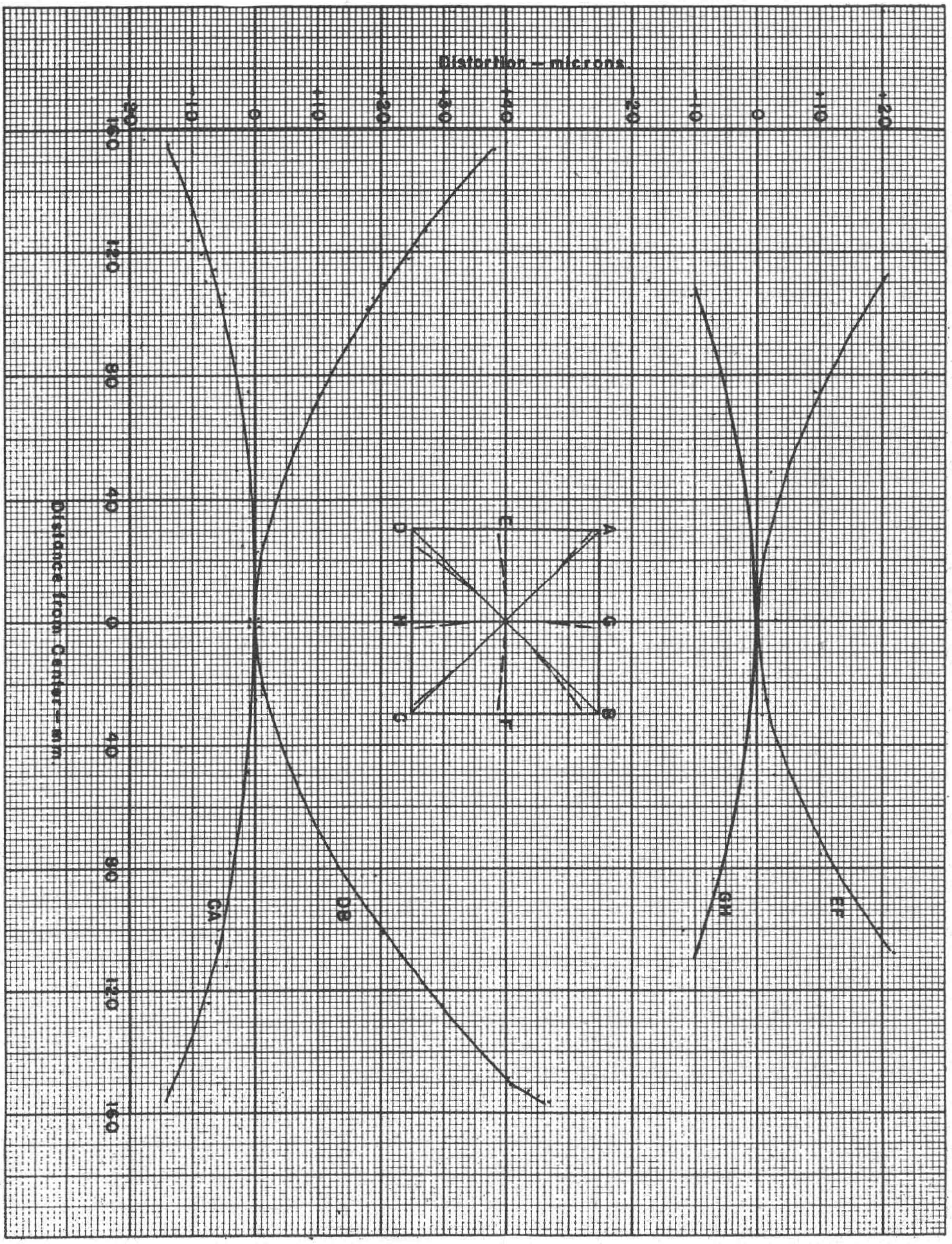

FIG. 4. Tangential distortion in a Wide Angle Camera.

middle ordinate of about .016 mm. and in the diapositive a middle ordinate of about .003 mm. could cause the curvature found.

The deviation of any line from straight may be determined by making two identical copies of the line on glass and superimposing these copies emulsion to emulsion so that corresponding ends of the lines are coincident and check whether the two lines coincide throughout their length. This is basically the procedure followed in these investigations, except that instead of attempting to register the lines in absolute coincidence, they were separated slightly and the distance between the two lines was accurately measured at intervals along the lines. This data was then adjusted and reduced to obtain the absolute deviation of the lines in question from straight. If a truly straight line is photographed

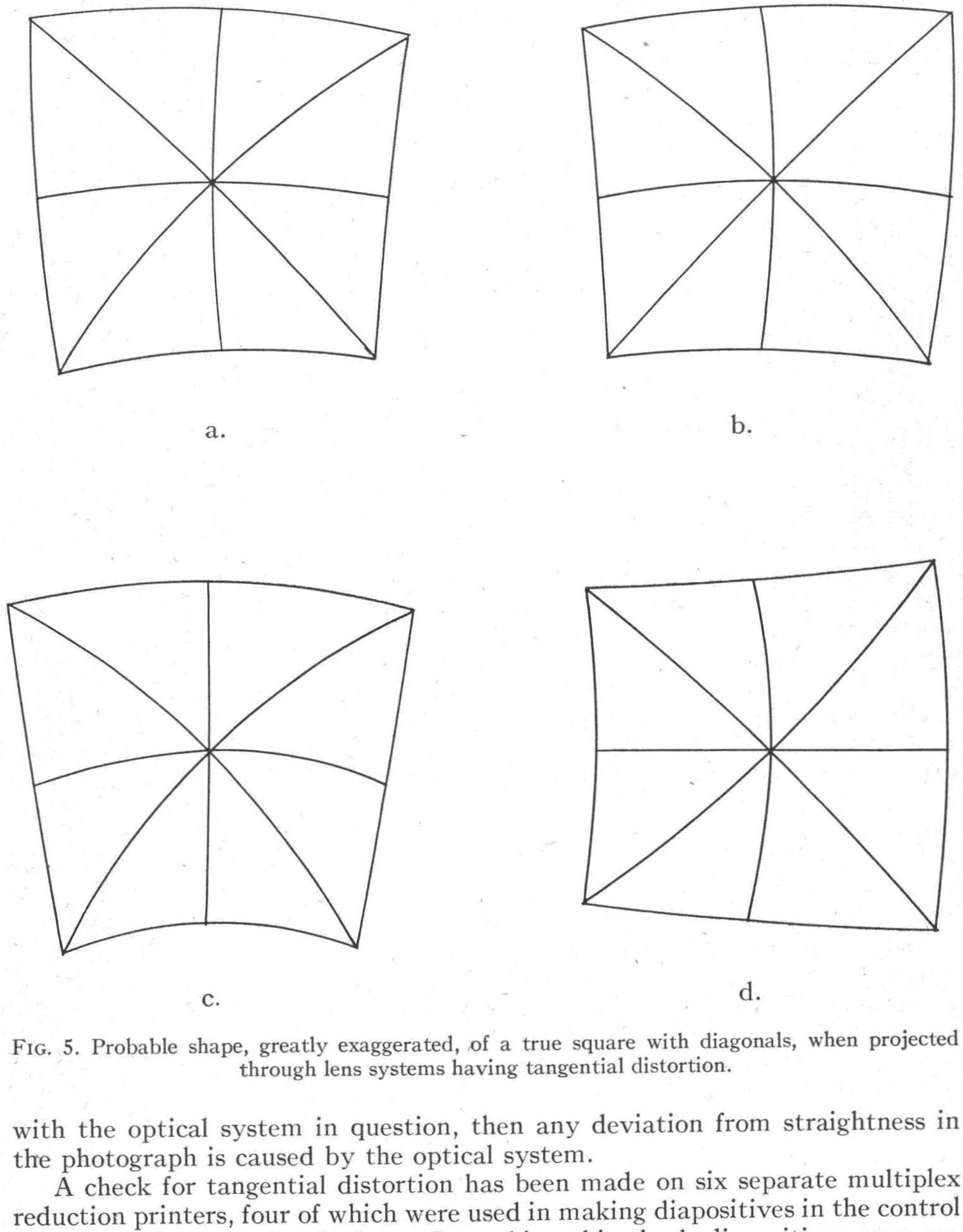

FIG. 5. Probable shape, greatly exaggerated, of a true square with diagonals, when projected through lens systems having tangential distortion.

with the optical system in question, then any deviation from straightness in the photograph is caused by the optical system.

A check for tangential distortion has been made on six separate multiplex reduction printers, four of which were used in making diapositives in the control extension tests mentioned above. In making this check diapositives were exposed in each printer with a master plate in the negative plane on which is etched two perpendicular lines intersecting at the center of the plate. Diapositives were made with the master lines oriented on the axes of the diapositive format as well as on the diagonals. Figure 3 shows the tangential distortion found in one of these reduction printers, which is more or less typical. The small chart in the figure shows the relative orientation of the diagonals checked and the dashed lines indicate the direction of distortion.

A similar check was also made on three aerial cameras, two of which were used in obtaining the aerial photography for these control extension tests. A level guard rail along a highway about 600 ft. distant from the camera station was used as a target for these measurements. The guard rail was photographed with the camera oriented so that the image would fall on each of the diagonals and each axis of the format. From each exposure, two contact prints were made on glass for measurement. To determine the true profile of the guard rail so that appropriate corrections could be applied, two negatives were exposed for each diagonal in one of which the camera was revolved about its axis to a position exactly 180 degrees opposite from the other. An average of the guard rail profiles measured from each of these exposures thus eliminated any distortion caused by the lens and gave the true profile. The difference between this average and the profile determined from a single negative gave the tangential distortion of the lens.

Figure 4 shows the results obtained in one of the cameras checked. The small chart in the figure indicates the relative orientation of the axes checked and the dashed lines indicate the direction of curvature. The other cameras gave similar results.

It is interesting to note here that the distortion data shown in Figures 3 and 4 indicates that in each case there is an axis of zero distortion and that the axis where it appears the maximum distortion occurs is at right angles to this axis. In Figure 3 the axis of zero distortion falls somewhere between diagonal *BD* and axis *GH* with the maximum distortion in a line somewhere between *AC* and *EF*. In Figure 4 the zero appears to fall between *AC* and *GH* with the maximum somewhere between *BD* and *EF*. This will be discussed more fully later.

Figure 5 illustrates, greatly exaggerated, the probable shape of a photograph of a true square with diagonals exposed through each of the optical systems for which tangential distortion data is given in Figures 3 and 4. Figure 5a is for the camera, figure 5b, the reduction printer, figure 5c the combination of camera and reduction printer with the camera negative oriented normally in the reduction printer, and figure 5d the combination of camera and reduction printer with the negative reversed or rotated 180° from the normal case.

With the distortion data similar to Figures 3 and 4 for the aerial camera and reduction printer used in a control extension, it is possible, without making a strictly formal mathematical analysis, to make a fair estimate of the resultant curvature in azimuth in a control extension when this particular combination is used. This estimate can be made by assuming that the azimuth would follow an arc or curvature equal to the mean resultant arc along the axis of the photograph corresponding to the axis of the strip.

To determine whether there was any correlation between the measured tangential distortion and the azimuth curvature of various control extensions such an estimate has been made for 41 separate extensions in which photography from four different flights has been used. In these extensions two cameras and four reduction printers were involved in various combinations. Table I shows the correlation found by this procedure. In the last two columns of this tabulation it will be noticed that the agreement between estimated curvature and actual curvature is on the whole quite close. It will also be noted that there is

TABLE I

Test No.	Camera	Printer	Neg. Normal or Reversed	Length of Extension	Estimated Middle Ordinate			Multiplex Middle Ordinate
					Camera	Printer	Camera and Printer	
				mm.	mm.	mm.	mm.	mm.
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+3.0
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+2.8
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+3.4
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+2.8
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+2.8
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+4.1
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+2.4
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+2.2
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+0.5
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+1.6
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+3.8
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+2.4
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+3.8
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+2.8
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+1.5
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+2.4
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+2.0
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+2.6
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+5.2
62	A	A	Normal	3200	−0.3	+3.4	+3.1	+3.6
62	A	B	Normal	2900	−0.3	−1.3	−1.6	−3.5
62	A	B	Normal	2900	−0.3	−1.3	−1.6	−1.6
62	A	B	Reversed	2900	−0.3	+1.3	+1.0	−0.7
62	A	B	Reversed	2900	−0.3	+1.3	+1.0	+1.2
62	A	C	Normal	2900	−0.3	+1.8	+1.5	+1.0
62	A	C	Normal	2900	−0.3	+1.8	+1.5	+1.0
77	B	A	Normal	2400	+1.0	+2.2	+3.2	+3.0
77	B	A	Normal	2600	+1.1	+2.6	+3.7	+2.8
77	B	A	Normal	2600	+1.1	+2.6	+3.7	+2.0
77	B	A	Reversed	2600	+1.1	−2.6	−1.5	0
77	B	A	Reversed	2600	+1.1	−2.6	−1.5	−1.5
85	B	A	Reversed	2600	+1.1	−2.6	−1.5	0
85	B	A	Reversed	2600	+1.1	−2.6	−1.5	−1.0
85	B	D	Reversed	2600	−1.1	−0.4	−1.5	0
85	B	D	Reversed	2600	−1.1	−0.4	−1.5	0
85	B	B	Reversed	2650	−1.2	−1.3	−2.5	−0.5
85	B	B	Reversed	2650	−1.2	−1.3	−2.5	−1.0
85	B	B	Normal	2650	−1.2	+1.3	+0.1	+0.5
85	B	C	Reversed	2650	−1.2	+1.5	+0.3	+0.8
85	B	C	Reversed	2650	−1.2	+1.5	+0.3	0
89	B	C	Normal	2800	−1.3	−1.7	−3.0	−3.7

considerable accidental error indicated but that this error on the whole is small in comparison to the systematic or instrumental error.

Another rather consistent tendency was noted in these control extension tests which should be mentioned here although no detail data in this respect will be given. This was the tendency for the datum of the extended strips to twist consistently in the same direction for any particular camera printer combination and further that this twist in datum appeared in general to be greatest in extensions in which the azimuth curvature was largest.

The exact cause of a distortion of the type noted in this investigation is not known; however, it could possibly be caused either by slight decentrations of the components of the lens system or by a weak prismatic effect in the filter on the aerial camera and in some of the optical parts of the multiplex reduction printer, which are so slight that they cause no noticeable deterioration in photographic image quality. It is well known that relatively few compound lenses are constructed such that the optical axis is a truly straight line through the lens system. There is nearly always a certain amount of deflection of the optical axis and it has been noted that its effect can be approximated by treating the lens system as an ideal lens plus a weak prism.[1] In the calibration of aerial cameras for the precise location of the principal point by the National Bureau of Standards and the Corps of Engineers, this is considered in its effect on the component of distortion radial from the optical axis. It will be noted that there is a marked similarity between the tangential distortions measured in this investigation and the distortion that would theoretically be produced by adding a weak prism to the lens system. In all cases there was found one axis of zero distortion with the axis of maximum distortion perpendicular thereto. This would seem to substantiate the validity of making such an approximation, although no attempt has been made to determine whether there is any correlation between the actual deflection of the optical axis and the tangential distortion measured.

One operational method of eliminating to a great extent the effect of this type of distortion in the reduction printer is obvious. That is, to make alternate diapositives with the negative oriented normally in the reduction printer and reverse the film for the remaining diapositive. This, however, will afford no compensation if the distortion occurs in the aerial camera. For compensation the camera would likewise need to be rotated between exposures, which is not very practical.

Whether it is reasonable to expect that such a distortion can be eliminated completely from the optical systems can not be determined until more is known of the actual cause of the distortion. Even then it may be found that the tolerances required to eliminate it completely would place unreasonable demands on the optical manufacturer, who is even now working beyond the limits of measurement in some instances, in manufacturing optical parts for use in aerial photogrammetry.

Further investigation of this phenomenon may very well show that distortion of the type noted in these studies are the exception more than the rule. It should be noted here that in recent control extension made at the Engineer Board in connection with other investigations, no appreciable systematic error of this nature has been noted and in these extensions no extra precautions have been taken in anticipation of such an error.

[1] Bennet, A. H., J. Opt. Sos. Am. & Rev. Sei. Inst., 14, 235 (1927).