# SPACE RESECTION

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## Introduction

As THE science of aerial photogrammetry approaches maturity, the difference between the degree of accuracy obtainable by aerial triangulation and the degree of accuracy obtainable by ground triangulation will diminish. The causes of the discrepancies between the two methods in the final precision of extended ground control might be summarized in the following statements: (1) the difference between the empirical resultant resolution of the image on the aerial film and the empirical resultant resolution of the image in the eyepiece of the surveying theodolite, and (2) the fact that thus far there is no way to make reciprocal observations between space stations, or reciprocal observations between the objects and the corresponding images of a single space station. The accuracy of a space station fixed by resection exclusive of the first stated difference is analogous to the accuracy of a point fixed on the ground by three-point resection when the latter is compared with the accuracy of a point fixed by reciprocal observations in a triangulation scheme. Currently the bulk of the mapping in the United States consists of a combination of ground surveying and graphic photogrammetry; the former is expensive and time-consuming, while the latter is lacking in accuracy. The author believes that the time is close at hand when persistent demands of the photogrammetrist will stimulate manufacturers to allocate more and more research to improving the emulsions of aerial film, the eventual results of which will be the removal of empirical resultant resolution as a limiting factor. When the foregoing consideration is achieved, space triangulation will become a highly specialized division of aerial surveying in the composite science of aerial mapping. The problems of the aerial triangulator will then parallel those of the geodetic surveyor in that the aerial triangulator will be concerned with infinitesimal errors introduced by the fluctuation of physical phenomena and with the geometric strength of not only the separate photograph pyramids, but also the triangular geometric strength of space stations with respect to each other. In view of the foregoing, it is believed that methods of space resection have a greater utility to the professional photogrammetrist than has been generally evident in the past ten years.

## Purpose

The purpose of this paper is to describe several methods (conceived by the author), of determining the exterior orientation of a space station. The methods described are based on the determination of the length of the perspective rays as a prerequisite to the computation of the $X$, $Y$, $Z$ elements of exterior orientation, which is in turn a prerequisite to the computation of the tilt and swing elements of exterior orientation. In the calculation of the linear values of the chief rays two solutions are given: One solution is a general case requiring three control points without restricting conditions, while the other solution is a special case requiring four control points which are coplanar. Greater emphasis is placed on the general case—hereafter referred to as the "three-point method," since the subsequent determination of space coordinates and space orientation requires only three points.

Any method of space resection hinges on satisfying a selected unique relationship between the geometry of the point images on the photograph and the geometry of the corresponding point objects on the ground with respect to the perspective center. For example, in one of the methods of space analytics derived by Professor E. Church,[1] the unique relationship depends upon finding the point in space where the angles at the perspective center between pairs of perspective rays are equal to the angles at the same perspective center subtended by the corresponding ground lengths. Similarly, in the method devised by Professor P. H. Underwood,[2] the unique relationship depends upon finding the correction required to make the common perspective rays equal to each other. In the new method here proposed, the unique solution depends upon finding the point where a perspective ray cuts a circle, the radius of which is the corresponding altitude of sloping triangle $A'B'C'$. See Figure 1.

In addition to a glossary of photogrammetric terms and symbols used throughout this discussion, a comparative table of absolute and computed values for the elements of exterior orientation is included at the end of the paper.

## Scope

The underlying principles, geometric theorems, and methematical formulas accompanying each operation are given in the chronological sequence of computation. The methods are accurate and require no mathematics beyond geometry and trigonometry.

## Space Resection: Three-Point Method

### *Analytical Solution*

Photograph pyramid $A'B'C'L$ is shown in Figures 1 and 10. $A'$, $B'$, and $C$ are the three visible ground points and $A$, $B$, and $C$ are their corresponding positions on the datum plane; $Z_A$, $Z_B$, and $Z_C$ are the elevations of the point objects above the datum plane; $L$ is the perspective center of the exposure station; $LA'$, $LB'$, and $LC'$ are the perspective rays concurrent at $L$; $a$, $b$, $c$ are the point images of the point objects; $P$ is the photograph perpendicular; $X$, $Y$, and $Z$ are the coordinates of $L$; $x$, $y$, with appropriate subscripts are the photo coordinates of the point images; $N$ and $n$ are the ground and photo nadir, respectively; $f$ is the principal distance; $\alpha$, $\beta$, and $\gamma$ are the angles at $L$ between pairs of perspective rays; $t$ and $s$ are the tilt and swing of the photograph; $M$ and $m$ with appropriate subscripts are the angles between the perspective rays and the plumb line and between the perspective rays and optical axis, respectively; and $\Delta$ is the tilt of the sloping reference plane. All other symbols are either self evident or defined in the glossary of terms and symbols. The sloped distances are computed from the given survey data.

$$\begin{aligned} \overline{A'B'} &= \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2 + (Z_A - Z_B)^2} \\ \overline{B'C'} &= \sqrt{(X_B - X_C)^2 + (Y_B - Y_C)^2 + (Z_B - Z_C)^2} \qquad (1) \\ \overline{C'A'} &= \sqrt{(X_C - X_A)^2 + (Y_C - Y_A)^2 + (Z_C - Z_A)^2} \end{aligned}$$

The interior angles of triangle $A'B'C'$ are computed.

$$\cos A'B'C' = \frac{(X_B - X_C)(X_B - X_A) + (Y_B - Y_C)(Y_B - Y_A) + (Z_B - Z_C)(Z_B - Z_A)}{(\overline{A'B'})(\overline{B'C'})}$$

[1] Church, Earl. Revised Geometry of the Aerial Photograph, Syracuse University Bulletin, No. 15, 1945.

[2] Underwood, P. H. "Space Resection Problems in Photogrammetry," American Society of Civil Engineers *Proceedings*, vol. 72, no. 7, Sept. 1946, pp. 939–962.

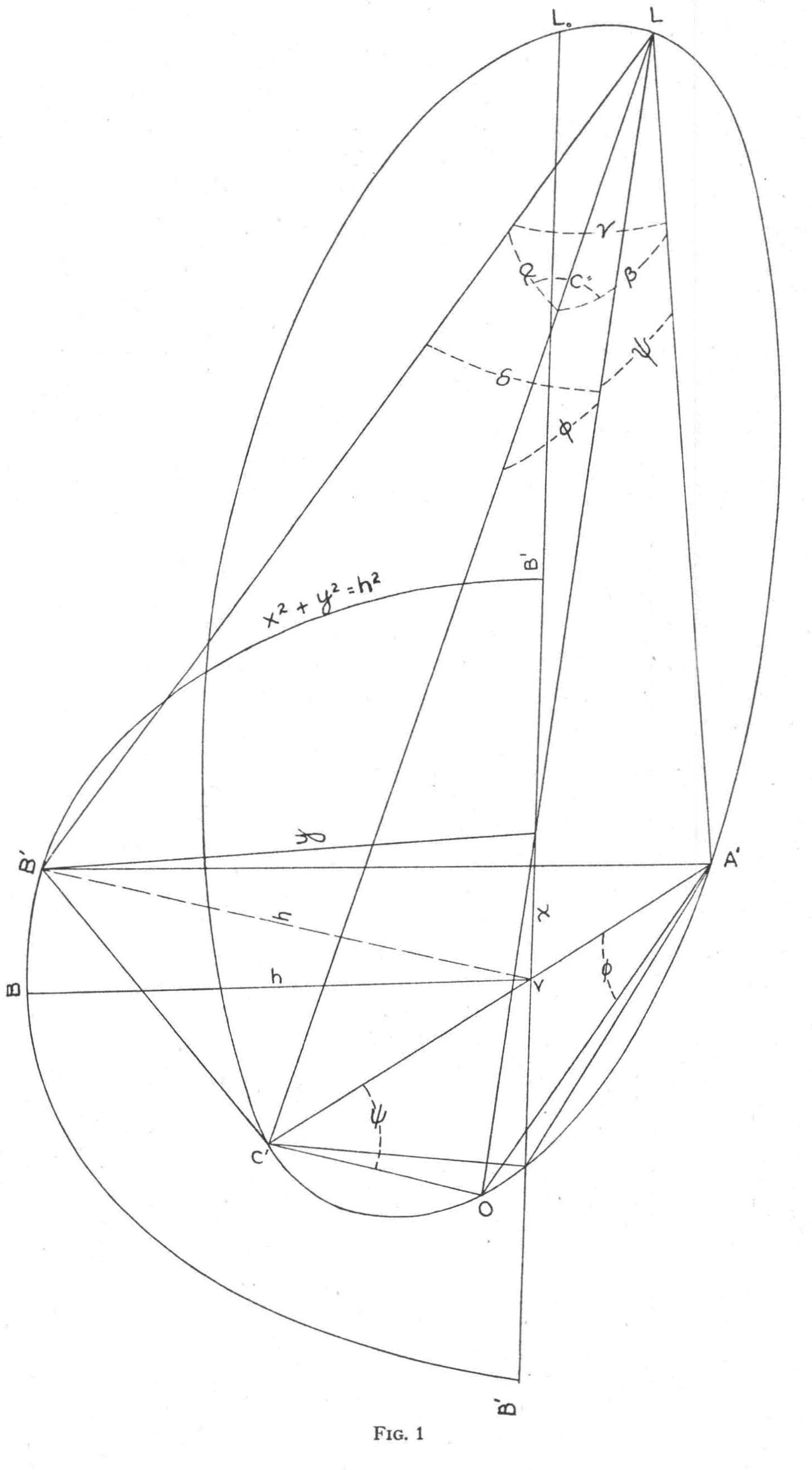


FIG. 1

$$(2)\ \cos B'C'A' = \frac{(X_C - X_A)(X_C - X_B) + (Y_C - Y_A)(Y_C - Y_B) + (Z_C - Z_A)(Z_C - Z_B)}{(\overline{B'C'})(\overline{C'A'})}$$

$$\cos C'A'B' = \frac{(X_A - X_B)(X_A - X_C) + (Y_A - Y_B)(Y_A - Y_C) + (Z_A - Z_B)(Z_A - Z_C)}{(\overline{C'A'})(\overline{A'B'})}.$$

The perpendicular $h$ from any vertex to the side opposite of the sloping triangle $A'B'C'$ is computed.

$$(3)\qquad h = \sin B'A'C' \cdot \overline{A'B'} = \sin B'C'A' \cdot \overline{C'A'}.$$

The face angles at $L$ are computed.

$$(4)\qquad \cos\alpha = \frac{x_b x_c + y_b y_c + f^2}{(Lb)(Lc)}$$

$$\cos\beta = \frac{x_c x_a + y_c y_a + f^2}{(Lc)(La)}$$

$$\cos\gamma = \frac{x_a x_b + y_a y_b + f^2}{(La)(Lb)}$$

and

$$(5)\qquad La = \sqrt{x_a^2 + y_a^2 + f^2}$$

$$Lb = \sqrt{x_b^2 + y_b^2 + f^2}$$

$$Lc = \sqrt{x_c^2 + y_c^2 + f^2}.$$

The foregoing formulas may be found in any analytic geometry text.

Angle $\delta$ at $L$ in a plane including $LB'$, perpendicular to plane $A'LC'$, and defined by $B'LO$ is computed.

$$(6)\qquad \sin\delta = \sin\gamma \sin B'A''C' = \sin\alpha \sin B'C''A'$$

and

$$(7)\qquad \cos B'A''C' = \frac{\cos\alpha - \cos\gamma\cos\beta}{\sin\gamma\sin\beta}$$

$$\cos B'C''A' = \frac{\cos\gamma - \cos\beta\cos\alpha}{\sin\beta\sin\alpha}$$

$LO$ in Figures 1 and 2 is the trace of a vertical plane through $LB'$ in plane $A'LC'$. $L$, $A'$, and $C'$ are three points on a circle, and side $\overline{A'C'}$ is chord of that circle. Angles $\phi$ and $\psi$, whose common side is $LO$ in plane $A'LC'$, are computed.

$$(8)\qquad \cos\phi = \frac{\cos\alpha}{\cos\delta}, \qquad \cos\psi = \frac{\text{cs}\,\gamma}{\cos\delta}.$$

The chords of $\phi$ and $\psi$ are computed.

$$(9)\qquad D = \frac{\overline{C'A'}}{\sin\beta} \quad \text{and} \quad \overline{A'O} = \sin\psi \cdot D, \qquad \overline{C'O} = \sin\phi \cdot D.$$

The magnitude of angle $\delta$ ($B'LO$) is a combination of the functions of the deviation of $LO$ from $B'VB'$ produced and the rotation of $h$ about axis $A'C'$. Determining the length of $LB'$ is a problem of determining where the terminal side $LB'$ of angle $\delta$ cuts the circumference of the circle whose radius is $h$. As

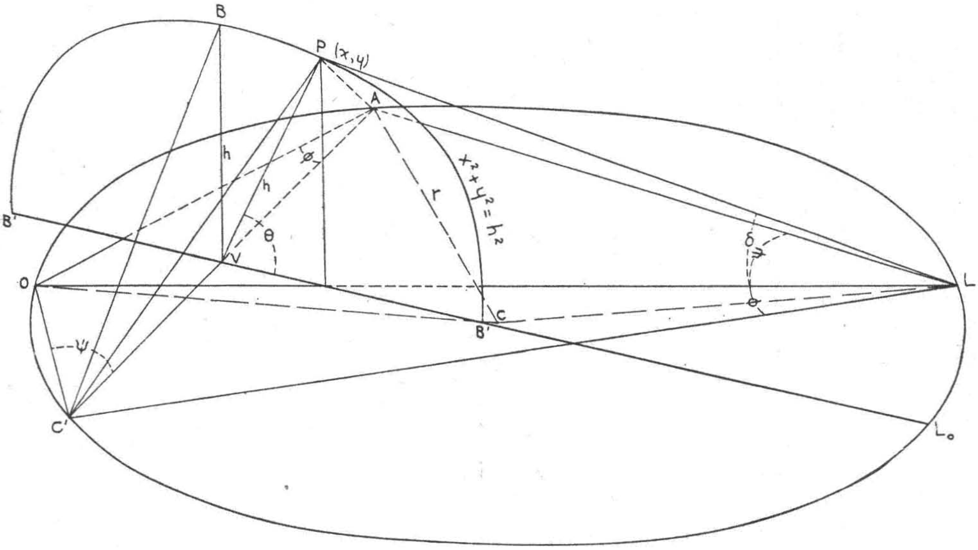


FIG. 2

shown in Figure 3, chords from $O$ are made to cut line $B'VB'$ at $x_1=0$, $x_2=h/2$, and $x_3=h$, respectively. The chords of circle $A'LC'$ are computed as indicated.

(10a) $$\overline{A'V}=\cos B'A'C'\cdot\overline{A'B'}$$

(10b) $$\overline{A'X} = \sqrt{X^2 + (\overline{A'V})^2}$$

$$\sigma = \phi \pm \sigma'$$

(10c) $$\tan \sigma' = \frac{x}{\overline{A'V}}$$

(10d) $$OX = \sqrt{(\overline{A'X})^2 + (\overline{A'O})^2 - 2(\overline{A'X})(\overline{A'O}) \cos \sigma}$$

(10e) $$\sin A'OL = \frac{(\overline{A'X}) \cdot \sin \sigma}{\overline{OX}}$$

then

(10f) $$LO = \frac{\sin\,[180° - (A'OL + \psi)]\cdot\overline{A'O}}{\sin \psi}.$$

From three such chords and the corresponding $OX$ values the ordinates $y_1$, $y_2$, and $y_3$ corresponding to $x_1$, $x_2$, and $x_3$ are computed.

(11) $$y = \tan \delta(LO - OX).$$

If $O$ falls on the side toward $C'$ in circle $A'LC'$, $\overline{C'V}$ replaces $\overline{A'V}$, $\overline{C'O}$ replaces $A'O$, $\phi$ replaces $\psi$, and $\psi$ replaces $\phi$ in the above equations. It must be understood that the coordinates of the three points $P_1(x_1y_1)$, $P_2(x_2y_2)$, and $P_3(x_3y_3)$ have no connection with the given ground coordinates. These coordinates are values in an arbitrary plane fixed by the rotation of $h$ and are not referred to again after the length of $LB'$ has been computed. $P_1$, $P_2$, and $P_3$ are three points on the loci of the curve in the plane of circle $h$, whose coordinates are known. The path of this curve is defined by the rotation of $LO$ about $O$ while $L$ is kept constantly on the circumference of circle $A'LC'$. Although $P_1P_2P_3$ is not an arc of a true circle, it may be treated as a circle because the deviation of so small a sector of the curve from a true circle is negligible when the radius of the circle is very

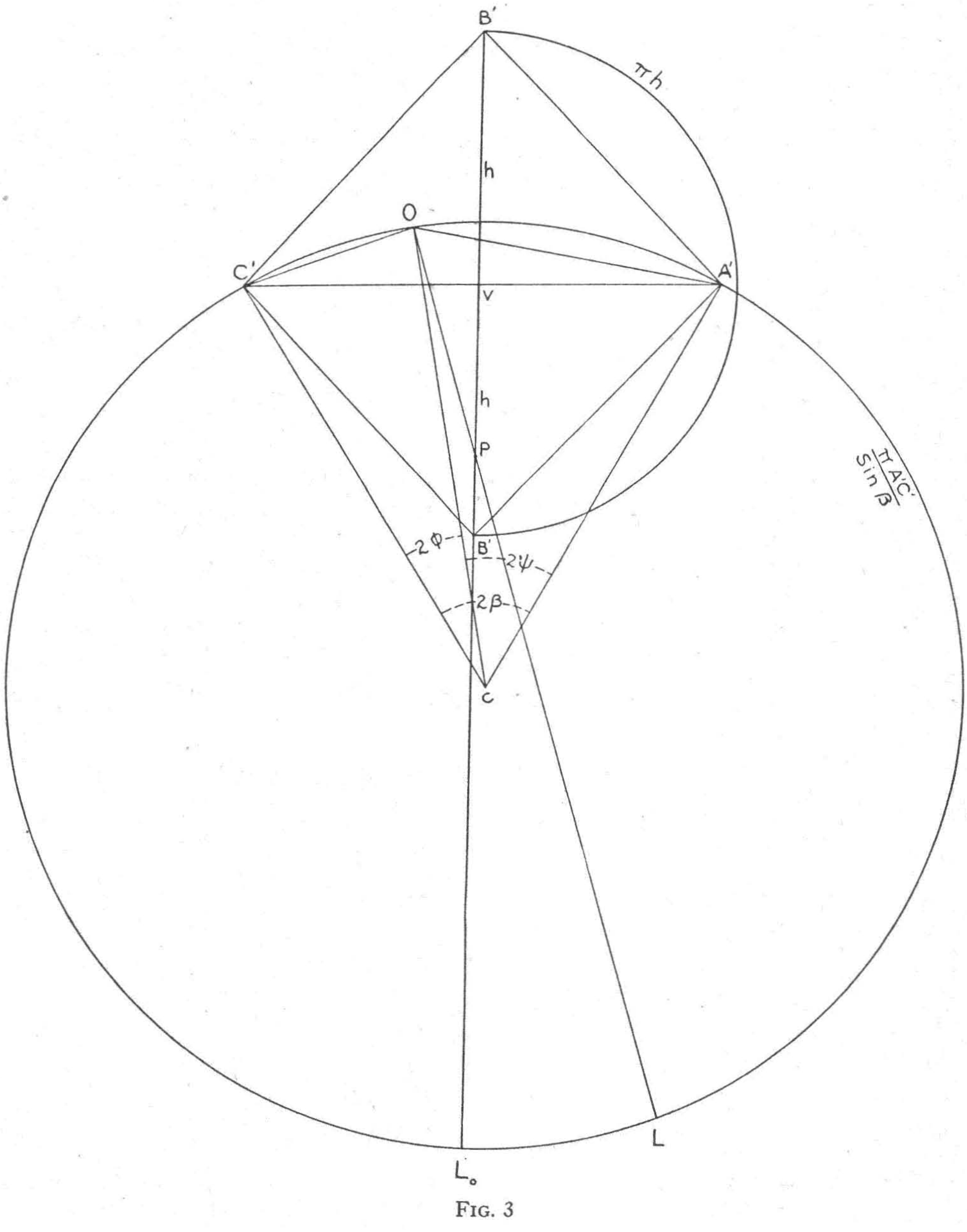


FIG. 3

large. The coordinates of the center $(h_1k_1)$ and the radius $(r_1)$ of the circle whose equation is satisfied by the numerical values of $x_1y_1$, $x_2y_2$, and $x_3y_3$ may be determined by substituting the given coordinates in three general equations of a circle and by solving simultaneously for $D$, $E$, and $F$ when $D=-2h$, $E=-2k$, and $F=h_1^2+k_1^2-r_1^2$.

$$
\begin{aligned}
&x_1^2 + y_1^2 + Dx_1 + Ey_1 + F = 0\\
(12a)\qquad &x_2^2 + y_2^2 + Dx_2 + Ey_2 + F = 0\\
&x_3^2 + y_3^2 + Dx_3 + Ey_3 + F = 0.
\end{aligned}
$$

Collecting and transposing

$$
\begin{aligned}
Dx_1 + Ey_1 + F &= -Q_1 \quad \text{when} \quad Q_1 = (x_1^2 + y_1^2)\\
Dx_2 + Ey_2 + F &= -Q_2 \quad \text{when} \quad Q_2 = (x_2^2 + y_2^2)\\
Dx_3 + Ey_3 + F &= -Q_3 \quad \text{when} \quad Q_3 = (x_3^2 + y_3^2).
\end{aligned}
$$

Then by determinants

$$D = \frac{Q_1(y_2 - y_3) - Q_2(y_1 - y_3) + Q_3(y_1 - y_2)}{x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2) = K}$$

$$E = \frac{x_1(Q_2 - Q_3) - x_2(Q_1 - Q_3) + x_3(Q_1 - Q_2)}{K}$$

$$F = \frac{x_1(y_2Q_3 - y_3Q_2) - x_2(y_1Q_3 - y_3Q_1) + x_3(y_1Q_2 - y_2Q_1)}{K}.$$

The resulting coordinates and the radius of the larger circle are deduced.

$$h_1 = -\frac{D}{2}, \qquad k_1 = -\frac{E}{2}, \qquad r_1 = \sqrt{h_1^2 + k_1^2 - F}$$

or

$$r_1 = \sqrt{\frac{D^2 + E^2}{4} - F}$$

$h_1$ and $k_1$ will always be in either the third or fourth quadrant. The center of the circle whose radius is $h$ is designated $V$ while the center of the circle whose radius is $r_1$ is designated $V_1$. The separation of $V$ and $V_1$ is equal to

$$\text{(12b)} \qquad VV_1 = \sqrt{\frac{D^2 + E^2}{4}}.$$

The radii of the two circles form the vertex of triangle $VV_1P_4$ only at the point where the two circles intersect; therefore

$$\text{(13a)} \qquad \cos\theta_2 = \frac{h^2 + (\overline{VV_1})^2 - r_1^2}{2(h)(\overline{VV_1})}$$

$$\text{(13b)} \qquad \tan\theta_1 = \frac{E}{D}$$

$$\theta_2 - \theta_1 = \theta$$

and $x_4=\cos\theta\cdot h$, $y_4=\sin\theta\cdot h$ which are the coordinates of $B'$ in the plane of $h$. Then

$$LB' = \sqrt{y_4^2 + (\cot^2 \delta y_4^2)}.$$

The length of one perspective ray with slope lengths $\overline{A'B'}$, $\overline{B'C'}$, $\overline{C'A'}$ and vertex angles $\gamma$, $\alpha$, $\beta$ is sufficient data to compute the length of the two remaining perspective rays and the pyramid base angles.

If a higher degree of accuracy is required, the value of $y_4$ is verified by producing chord $LO$ through $O$ and $x_4$, respectively, and then computing the actual $y_4$. The difference $\Delta y=(y_4-y_4)$ is noted and another $x_5$ is selected so that the resulting equation of a line will cut circle $h$. In a similar fashion chord $LO$ is produced through $O$ and $x_5$ and the value of $y_5$ determined. An equation of a line is set up from the values of $P_4(x_4y_4)$ and $P_5(x_5y_5)$ which, when solved simultaneously with the equation of circle $h$, yields the coordinates of $B'$ to a high degree of accuracy.

(14a)
$$\frac{y_6 - y_5}{x_6 - x_5} = \frac{y_5 - y_4}{x_5 - x_4} = m$$

$$y_6 - y_5 = m(x_6 - x_5)$$

$$y_6 = mx_6 + (y_5 - mx_5) \qquad \text{Let } (y_5 - mx_5) = l$$

(14b)
$$y_6 = mx_6 + l.$$

Substitute in equation $x_6^2+y_6^2=h^2$.

$$x_6^2 + (mx_6 + l)^2 - h^2 = 0$$

$$x_6^2 + m^2x_6^2 + 2mlx_6 + l^2 - h^2 = 0.$$

Let $A=(1+m^2)$, $B=2ml$, and $C=(l^2-h^2)$.
Then

$$x_6 = \frac{- B \pm \sqrt{B^2 - 4AC}}{2A}$$

and

$$y_6 = \sqrt{h^2 - x_6^2}$$

$LB'$ is computed from the coordinates of $P_6$ as indicated with $P_4$. A chord is used after the preliminary intersection of the circles $V$ and $V_1$ is determined, because henceforth the chord is sufficiently short to be equal to its arc in length for all practical purposes. Finally,

$$LB' = \sqrt{y_6^2 + (\cot \delta y_6)^2},$$

after which the remaining perspective rays and base angles are computed.

$$\sin LA'B' = \frac{LB'\cdot\sin\gamma}{\overline{A'B'}}, \qquad LA' = \frac{\sin 180^\circ - (\gamma + LA'B')\cdot\overline{A'B'}}{\sin\gamma},$$

$$\sin LC'B = \frac{LB'\cdot\sin\alpha}{\overline{B'C'}}, \qquad LC' = \frac{\sin 180^\circ - (\alpha + LC'B')\cdot\overline{B'C'}}{\sin\alpha}.$$

Check

$$\overline{A'C'} = \sqrt{(LA')^2 + (LC')^2 - 2(LA')(LC') \cos \beta}.$$

The method described is particularly useful in that the computer may determine the accuracy of the final value of $LB'$ by merely noting $\Delta y = (y^1 - y)$. It is also interesting to note that excessive tilts cause no direct limitation on the accuracy of a computed perspective ray. In fact, the accuracy increases with the increase in tilt because the intersection of the two circles becomes more nearly normal with increase in tilt and increase in the eccentricity of the coordinates of $N$ with respect to the center of triangle $ABC$.

*Graphic Solution:* (Figures 3 and 4)

The three-point method of space resection can be accomplished graphically. A graphic solution is considered useful for two reasons: (1) as a substitute for the preliminary simultaneous equation of circle computation, (2) as a value sufficiently accurate for locating the photo nadir on templates of badly tilted photographs in a slotted template assembly. The graphic solution is briefly described.

Draw a circle whose diameter is $\overline{A'C'}/\sin \beta$. Lay off chord $\overline{A'C'}$ on this circle. Strike arcs $\overline{A'B'}$ and $\overline{C'B'}$ from $A'$ and $C'$, respectively, so that they intersect both inside and outside the circle. The intersection of these arcs is $B'$ in the plane of $A'LC'$ when triangle $A'B'C'$ is rotated about axis $\overline{A'C'}$. A straight line drawn through the pair of arc intersections is equal to $2h$ and is perpendicular to $\overline{A'C'}$. The point of intersection of $\overline{A'C'}$ and $2h$ is the center $(V)$ of circle $h$. Strike an arc whose radius is $A'O$ from $A'$ and another arc whose radius is $\overline{C'O'}$ from $C'$. These arcs should intersect on the circumference of the circle. The intersection of arcs $\overline{A'O}$ and $\overline{C'O}$ is designated $O$. Presumably the plotting to date has been done on a plotting sheet at the largest convenient scale. Draw on a piece of acetate a semicircle whose radius is $h$. Construct on a second piece of acetate a graphic model of angle $\delta$. Since the exact magnitude of angle $\delta$ is critical, it is best constructed by a combination of polar and rectangular coordinates. The adjacent side of angle $\delta$ is drawn slightly greater in length than $D$. The terminals of $D$ are designated $O$ and $L$. The side opposite and the hypotenuse are computed as follows.

$$\text{When} \quad b = D, \; a = \tan \delta \cdot D, \quad \text{and} \quad c = \sqrt{D^2 + (\tan \delta \cdot D)^2}.$$

Then arcs with radii $a$ and $c$ are described from $O$ and $L$. An accurate graphic model of angle $\delta$ is completed by connected $O$ and $L$ with the point of intersection of arcs $a$ and $c$. It is known that the perspective ray $LB'$ has been reconstructed graphically when a perpendicular from $LO$ at the intersection of $B'VB'$ and $LO$ is concurrent with the terminal side of angle $\delta$ ($LB'$ produced) and the circumference of circle $V$. The graphic solution consists of manipulating the templates of circle $V$ and angle $\delta$ over the original drawn circle until the above condition is satisfied. This is accomplished by making $LO$ pass through the plotted position $O$ while holding the vertex $L$ of angle $\delta$ on the circumference of the circle. At the same time the diameter of circle $V$ is aligned with $B'VB'$. Then circle $V$ is rotated about the intersection of $LO$ and $B'VB'$ until the diameter of circle $V$ is aligned with $LO$. A perpendicular from $LO$ at the intersection of $LO$ and $B'VB'$ is erected. If this perpendicular is concurrent with $LB'$ and the circumference of circle $V$, the geometry of the pyramid has been reconstructed at the scale and accuracy of the plot.

The manipulation of the two templates over the plot of the circle is demonstrated in Figure 4. The preceding operation is not difficult to accomplish if a drafting triangle is oriented so as to be the perpendicular. Once the orientation is executed, lengths $LA'$, $LB'$, and $LC'$ may be scaled off directly.

The angles computed from the data obtained from the calibrated principal distance and coordinate measurement of the photo images can also be derived

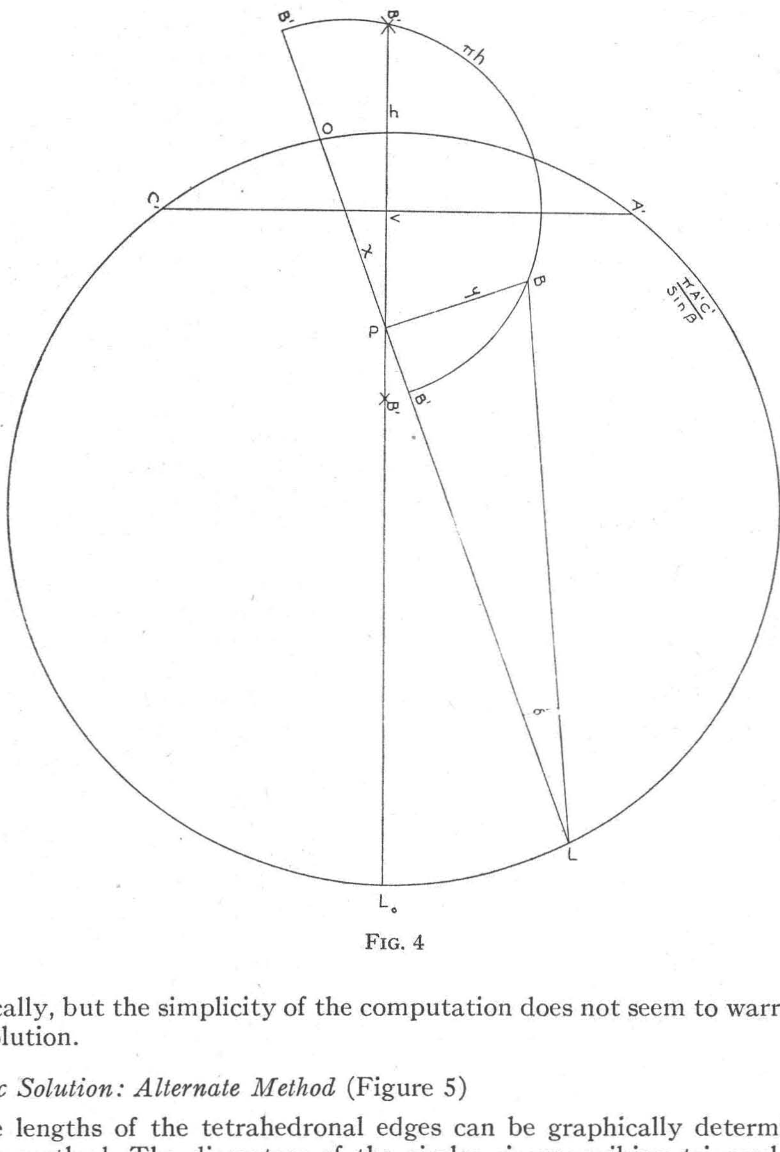

FIG. 4

graphically, but the simplicity of the computation does not seem to warrant any such solution.

*Graphic Solution: Alternate Method* (Figure 5)

The lengths of the tetrahedronal edges can be graphically determined by another method. The diameters of the circles circumscribing triangular faces $A'LB'$, $B'LC'$, and $C'LA'$ are computed.

$$D_1 = \frac{\overline{A'B'}}{\sin \gamma}, \qquad D_2 = \frac{\overline{B'C'}}{\sin \alpha}, \qquad D_3 = \frac{\overline{C'A'}}{\sin \beta}.$$

Each circle with its corresponding chord is drawn on a piece of acetate. $B'$ of circle $A'LB'$ and $C'$ of circle $C'LA'$ are superimposed carefully over $B'$ and $C'$, respectively, of circle $B'LC'$. Pins are driven through $B'$ and $C'$, causing $B'LC'$ to be immobile, while $A'LB'$ and $C'LA'$ are rotable about $B'$ and $C'$, respectively. The geometry of the pyramid is reconstructed in a plane when the three circles are concurrent at the point where chord $LA'=LA'_1$, $L$ being designated the point of concurrency. The outer circles are rotated so that they intersect on the circumference of the middle circle at a point where the two outer chords are equal.

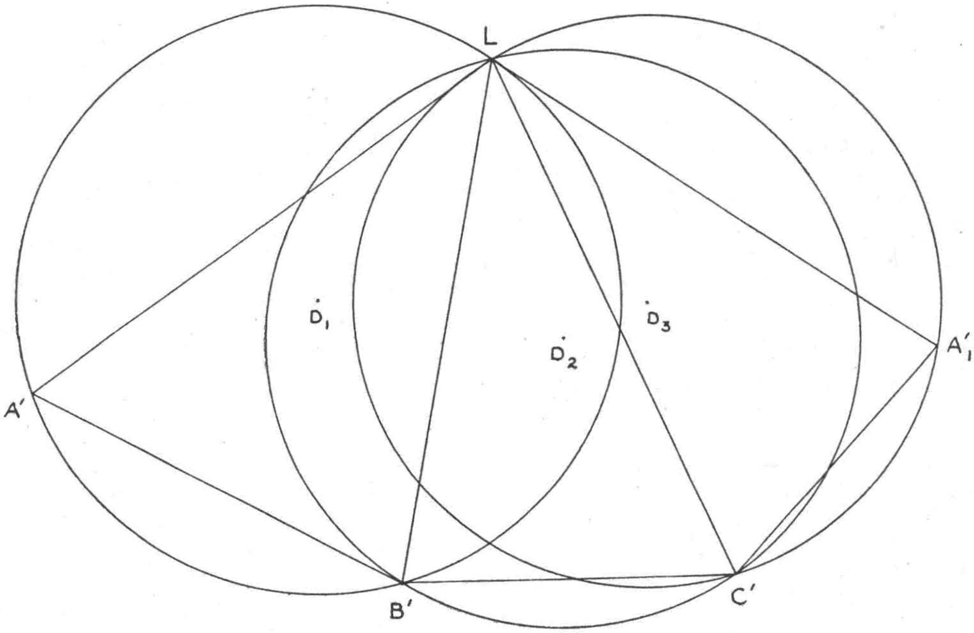


FIG. 5. Three circles concurrent at $L$ and $LA'=LA'$.

This will be true at one, and only one, point. When this point is found, the lengths $LA'$, $LB'$, and $LC'$ are scaled directly from the plot.

If the operator is careful, the plotting error should not exceed $\pm.01''$, which at a scale of 1:500 would be $\pm 1'$ approximately.

## SPACE RESECTION FOUR-POINT METHOD

*Analytical Solution* (Figure 6)

It has been stated that the four-point method is a special case in that it requires four ground points which are coplanar. It is felt, however, that the four-point method has a special application to aerial hydrographic surveys of harbors, low shorelines, and low relief islands where four sea-level elevations or nearly equal elevations are easily obtained. This useful adaptation combined with the fact that a direct solution is possible would appear to justify the space allotted to it in this paper.

$A$, $B$, $C$, and $D$ are the four point objects which are coplanar in a ground plane. $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ are the known sides; $\overline{AC}$, $\overline{BD}$ the known diagonals; $Z_A$, $Z_B$, $Z_C$, and $Z_D$ the known elevations; $O$ the intersection point of diagonals $\overline{AC}$, $\overline{BD}$; and $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, $\overline{OD}$ the known lengths determined by the intersection of $\overline{AC}$ and $\overline{BD}$. $L$ is the perspective center, as before, through which perspective rays *LA*, *LB*, *LC*, *LD*, and *LO* are concurrent. *a*, *b*, *c*, and *d* are the point images; *f* is the principal distance; *p* is the photograph perpendicular; *o* is the picture plane trace of line *LO;* and *ab*, *bc*, *cd*, *da*, *ac*, and *bd* are picture plane traces of the known ground lengths. Angles $\alpha$, $\beta$, $\gamma$, $\delta$ . . . are angles indicated by the drawn arcs.

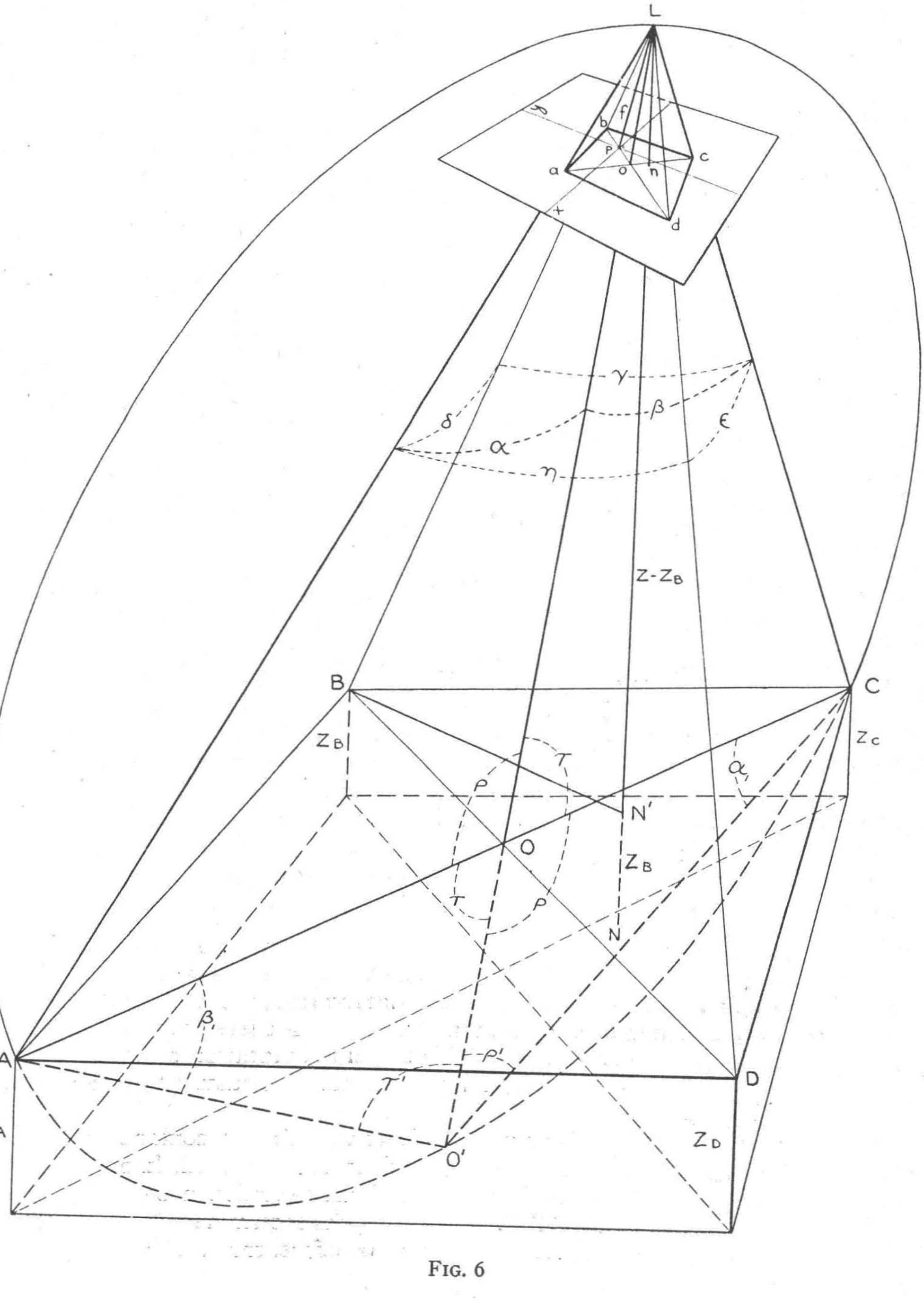


FIG. 6

It is emphasized that $O$ is not a point object; nor is $o$ a point image of a point object. $O$ is the intersection of $\overline{AC}$ and $\overline{BD}$ just as $o$ is the intersection of $ac$ and $bd$. $LAC$ and $LBD$ are two three-sided planes that intersect along line $LoO$. The intersection of two planes defines a straight line; therefore $L$, $o$, and $O$ are colinear. Since three points determine one, and only one, circle, let either $L$, $A$, and $C$ or $L$, $B$, and $D$ be three points on a circle. In practice, whichever plane evidences the best geometric conditions is used except when a check computation is required, in which case both planes might be used. All inscribed angles subtending the same arc are equal; therefore $\alpha=\alpha_1$, and $\beta=\beta_1$. Chord $\overline{AC}$ is in the ground plane, and inscribed angles $\alpha_1$ and $\beta_1$ are in that part of the arc of circle $LAC$ extending below chord $\overline{AC}$. Angles $\alpha$ and $\beta$ are derived from the photograph.

By equations (4), (5), and (9)

$$\cos\alpha = \frac{x_a x_0 + y_a y_0 + f^2}{(La)(Lo)}$$

$$\cos\beta = \frac{x_c x_0 + y_c y_0 + f^2}{(Lc)(Lo)}$$

$$La = \sqrt{x_a^2 + y_a^2 + f^2}$$

$$Lc = \sqrt{x_c^2 + y_a^2 + f^2}$$

$$Lo = \sqrt{x_0 + y_0 + f^2}.$$

The diameter $D$ of circle $ALC$ is

$$D = \frac{\overline{AC}}{\sin(\alpha+\beta)}.$$

And chords $\overline{AO'}$ and $\overline{CO'}$

$$\overline{AO'} = \sin\alpha\cdot D, \qquad \overline{CO'} = \sin\beta\cdot D.$$

Then

(15)
$$\tan\frac{(\rho-\rho')}{2} = \frac{\overline{CO'}-\overline{CO}}{CO'+CO}\cdot\tan\frac{(180^\circ-\alpha)}{2}$$

$$\rho = \frac{(\rho+\rho')}{2} + \frac{(\rho-\rho')}{2}$$

$$\rho' = \frac{(\rho+\rho')}{2} - \frac{(\rho-\rho')}{2}$$

$$LOC = \alpha_1 + \rho' = \tau$$

$$LCA = 180^\circ - (\tau+\beta)$$

$$LAC = 180^\circ - (\rho+\alpha)$$

and

$$LA = \frac{\sin LCA\cdot\overline{AC}}{\sin(\alpha+\beta)}\cdot \qquad LC = \frac{\sin LAC\cdot\overline{AC}}{\sin(\alpha+\beta)}$$

*LB* and *LD* are similarly computed with the vertex angles and *LA* and *LC* as given data.

With reference to Figure 7, it is emphasized that plane *ABCD* does not have to be level but it must be coplanar; otherwise $\overline{AC}$ and $\overline{BD}$ will not intersect at a point. When points *A*, *B*, *C*, and *D* are coplanar, the horizontal slope lengths $\overline{CO}$ and $\overline{AO}$ are easily computed.

## Graphic Solution

A plot of the rectangle formed by the slope distances $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ is drawn on a piece of acetate at a suitable scale, as shown in Figure 8. The

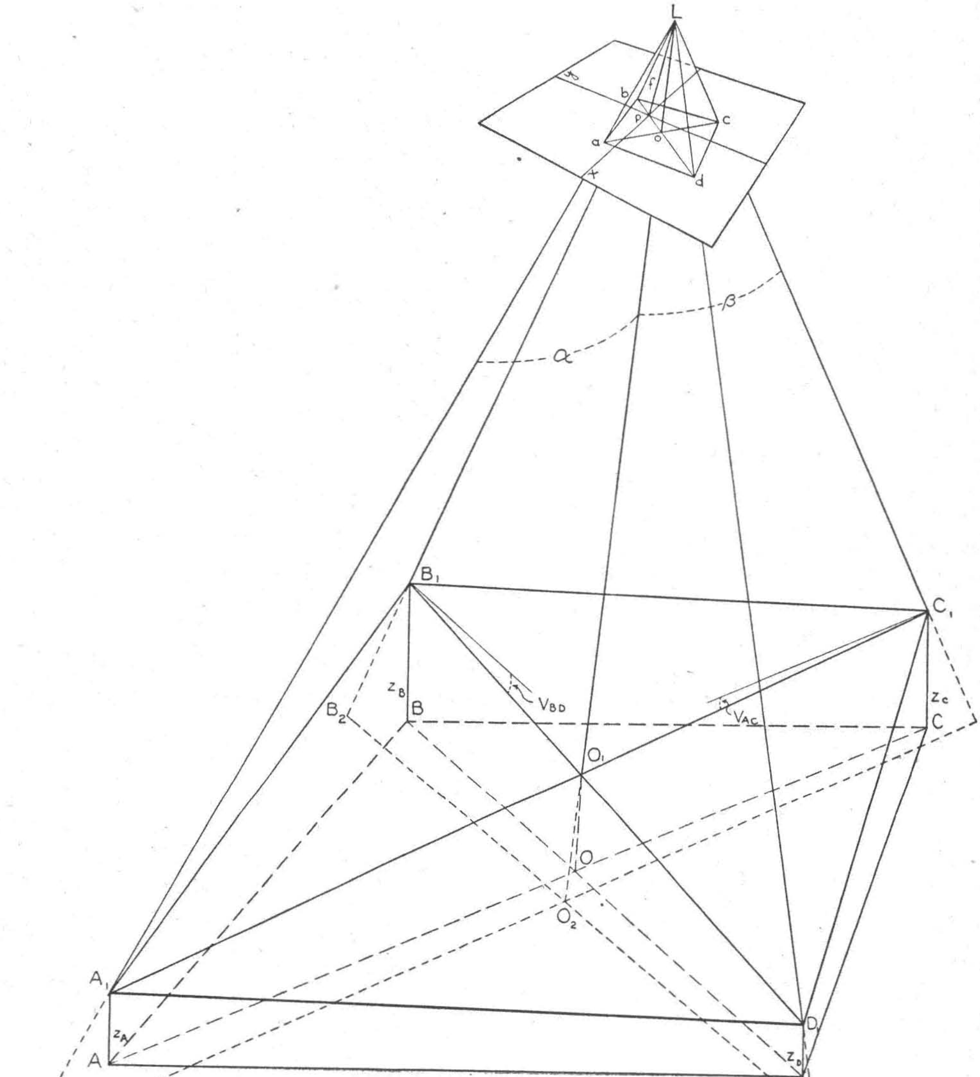


FIG. 7

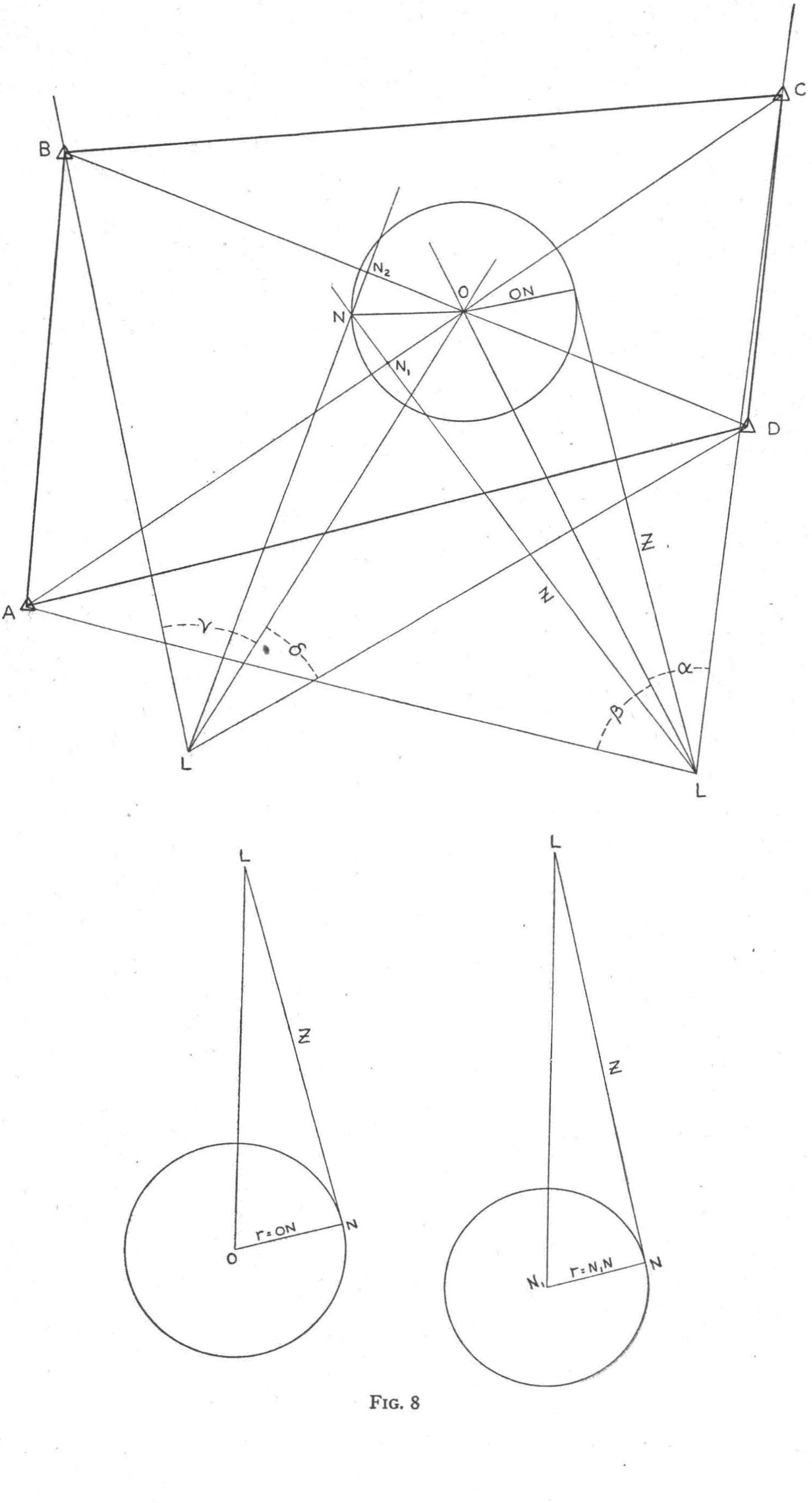


FIG. 8

opposite corners are connected by diagonals, their intersection being labeled $O$. Angles $\alpha$ and $\beta$ are laid off with a protractor on another piece of acetate. This second piece of acetate is manipulated over the first until the rays $LA$, $LO$, and $LC$ extended pass through the plotted positions of $A$, $O$, and $C$. When the resection is complete, a perpendicular is dropped from the vertex $L$ to line $\overline{AC}$. The foot of the perpendicular is designated $N_1$. The above procedure is repeated with angles $\gamma$ and $\delta$ in plane $BLD$, and the second perpendicular is designated $N_2$. The intersection of the perpendicular from $\overline{AC}$ at $N_1$ and from $\overline{BD}$ at $N_2$ is the position of the foot of a perpendicular from $L$ to plane $ABCD$. The length of the perspective rays can be scaled directly from the plot. $LN$ is graphically determined from either the scaled values $LN_1$ and $N_1N$ or $LN_2$ and $N_2N$. The graphic determination of $LN$ is illustrated in Figure 8.

Since only three points are required, the method of determining space coordinates to be described has no reason for being classified as a four-point method or a three-point method. In the event a four-point method is used, the fourth point is omitted in the determination of space coordinates unless a check is desired.

## Determination of the Space Coordinates $(X, Y, Z)$ of the Exposure Station

*Analytical Solution*

Tetrahedron $A'B'C'L$ is shown in Figure 9. A perpendicular $(LN')$ from $L$ to plane $A'B'C'$ is required. This may be accomplished by several methods. $A'LC'$ is considered a spherical triangle whose vertex is $B'$ and whose spherical sides are angles $LB'A'$, $LB'C'$, and $A'B'C'$. The additional required values can be computed from these angles by the law of cosines in spherical trigonometry.

$$(16)\qquad \begin{aligned} \cos LC''A' &= \frac{\cos LB'A' - \cos A'B'C' \cdot \cos LB'C'}{\sin A'B'C' \cdot \sin LB'C'} \\ \cos LA''C' &= \frac{\cos LB'C' - \cos A'B'C' \cdot \cos LB'A'}{\sin A'B'C' \cdot \sin LB'A'} . \end{aligned}$$

And by Napier's Analogies

$$(17)\qquad \sin \nu' = \sin LC''A' \cdot \sin LB'C' = \sin LA''C' \cdot \sin LB'A'.$$

Then

$$(18)\qquad LN' = \sin \nu' \cdot LB'.$$

$LN'$ can also be computed with derived linear elements. It can be seen in Figure 2 that angle $\theta$ is a dihedral angle between sloping reference plane $A'B'C'$ and triangular face $LA'C'$. A perpendicular is dropped from $L$ to line $\overline{A'C'}$.

$$LN_1' = \sin LC'A' \cdot LC'.$$

The sides of the dihedral angle $\theta$ are bound by lines that are mutually perpendicular to line $\overline{A'C'}$, which is formed by the intersection of planes $A'B'C'$ and $LA'C'$. $LN_1'$ is made perpendicular to line $\overline{A'C'}$ or $\overline{A'C'}$ produced; therefore the angle between $LN_1'$ in plane $LA'C'$ and a line perpendicular to $\overline{A'C'}$ in plane $A'B'C'$ is $\theta$. Thus

$$LN' = \sin \theta \cdot LN_1'$$

or

(19) $$LN' = \sin\theta \cdot \sin LC'A' \cdot LC'.$$

If $A'B'C'$ were a level reference plane, $LN'$ would equal $Z$. It is assumed in Figure 10 that $A'B'C'$ is a sloping reference plane, in which case $LN'$ is not perpendicular to the datum plane. A ground isometric parallel $A'I$ is produced from the intermediate elevation $Z_A$ to line $\overline{B'C'}$.

(20) $$\overline{B'I} = \frac{(Z_A - Z_B)\cdot\overline{B'C'}}{(Z_C - Z_B)}$$

$$\overline{A'I} = \sqrt{(\overline{B'I})^2 + (\overline{A'B'})^2 - 2(\overline{B'I})(\overline{A'B'}) \cos A'B'C'}.$$

A ground principal line parallel is produced from the highest elevation $Z_C$ to the ground isometric parallel.

$$\cos IA'B' = \frac{(\overline{A'B'})^2 + (\overline{A'I})^2 - (\overline{B'I})^2}{2(\overline{A'B'})(\overline{A'I})}$$

$$\angle C'A'I = \angle C'A'B' - \angle I'A'B'$$

$$\overline{C'I} = \sin \angle C'A'I \cdot \overline{A'C'}.$$

The tilt $\Delta$ of the sloping reference plane with respect to the datum plane is computed.

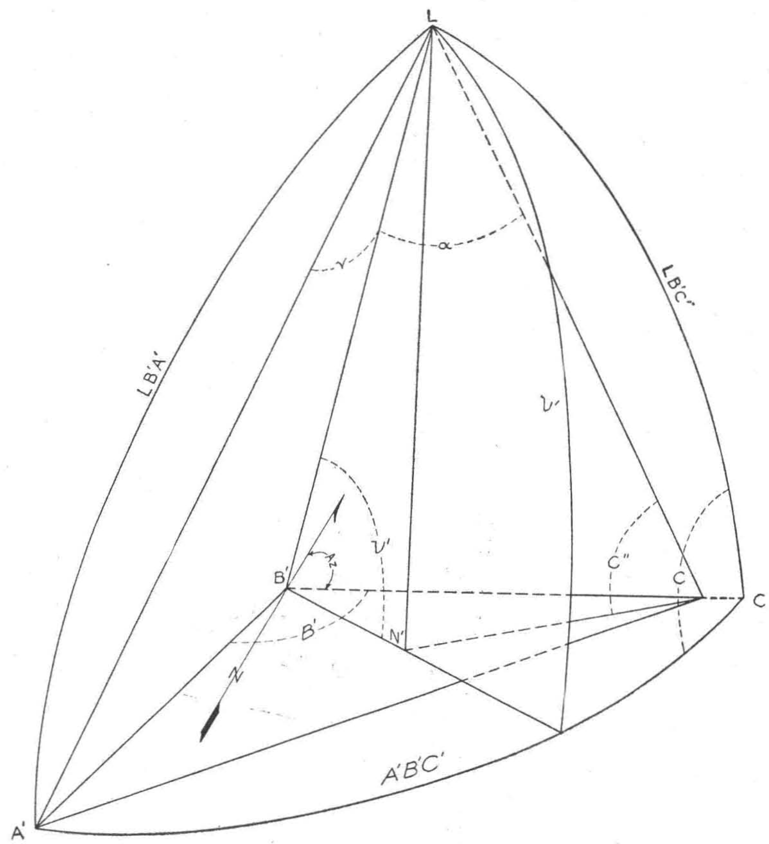


FIG. 9

(21) $$\sin \Delta = \frac{(Z_A - Z_C)}{\overline{C'I'}}.$$

Angle $LI_NN'$ ($v''$) in the ground principal plane perpendicular to the ground isometric parallel is required. Preliminary data is computed.

$$\overline{A'N'} = \sqrt{(LA')^2 - (LN')^2}$$

$$\overline{CN'} = \sqrt{(LC')^2 - (LN')^2}$$

$$\cos N'A'C' = \frac{(\overline{A'C'})^2 + (\overline{A'N'})^2 - (\overline{C'N'})^2}{2(\overline{A'C'})(\overline{A'N'})}$$

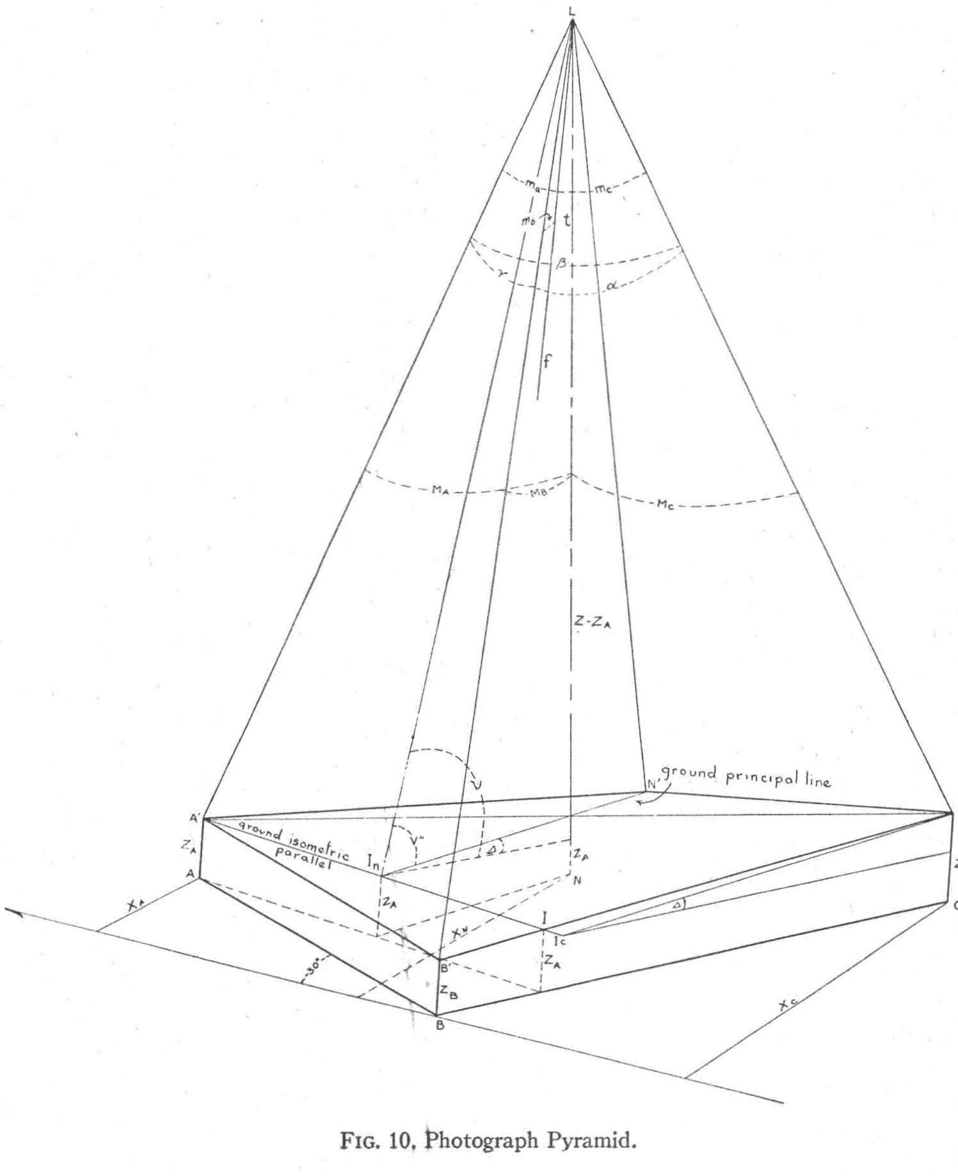


FIG. 10. Photograph Pyramid.

$$N'I_N = \sin(C'A'I + N'A'C')\cdot\overline{A'N'}$$

$$LI_N = \sqrt{(\overline{N'I_N})^2 + (LN')^2}$$

$$\cos\nu'' = \frac{N'I_N}{LI_N}$$

$$\nu = \nu'' + \Delta$$

(22) $$Z = (\sin\nu\cdot LI_N) + Z_A \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

The horizontal ground radials from $N$ to each point object are computed.

(23)
$$R_A = \sqrt{(LA')^2 - (Z - Z_A)^2}$$
$$R_B = \sqrt{(LB')^2 - (Z - Z_B)^2}$$
$$R_C = \sqrt{(LC')^2 - (Z - Z_C)^2}.$$

From these values $X$ and $Y$ can be computed.

$$\cos NBA = \frac{(\overline{AB}) + (R_B)^2 - (R_A)^2}{2(AB)(R_B)}$$

(24a) $$X = X_B \pm \sin(NBA \pm AZ_{BA})\cdot R_B \quad \ldots\ldots\ldots\ldots\ldots 2$$

(24b) $$Y = Y_B \pm \cos(NBA \pm AZ_{BA})\cdot R_B \quad \ldots\ldots\ldots\ldots\ldots 3$$

## Determination of the Space Orientation ($t$, $s$) of the Exposure Station

*Spherical Angle Method*

Spherical trigonometry[3] can be used to compute the tilt and swing of the photograph by considering $L$ the center of the sphere whose radius is $f$. The spherical angle solution is demonstrated in Figure 11. Two pairs of angles, $M$ and $m$, with respect to any pair of perspective rays are required.

(25a) $$\cos M_A = \frac{Z - Z_A}{LA'}, \qquad \cos M_B = \frac{Z - Z_B}{LB'}, \qquad \cos M_C = \frac{Z - Z_C}{LC'}$$

(25b) $$\cos m_a = \frac{f}{La}, \qquad \cos m_b = \frac{f}{Lb}, \qquad \cos m_c = \frac{f}{Lc}.$$

Angles $\alpha$, $\beta$, and $\gamma$ have been previously computed. Perspective rays $La$ and $Lb$ are chosen.

From spherical trigonometry

(26a) $$\cos b_1 = \frac{\cos M_A - \cos\gamma\cdot\cos M_B}{\sin\gamma\,\sin M_B}$$

(26b) $$\cos b_2 = \frac{\cos m_a - \cos\gamma\cdot\cos m_b}{\sin\gamma\,\sin m_b}$$

[3] At this point it may be emphasized that the preceding derivations were made without the benefit of directly related photogrammetric references. Subsequent examination of the literature reveals that P. H. Underwood has stated, on pp. 952–3 of the reference in Footnote no. 2—"by spherical trigonometry. . . . Then in the triangle *avo*, for instance, the sides *av* and *ao* are known; the angle *vao* is the difference between angles *bav* and *bao*. A solution may then be made for the length *vo* or *i* and for the angles at points *v* and *o*."

and

$$b = b_1 - b_2.$$

Then

(26c) $$\cos t = \cos m_b \cos M_B + \sin m_b \sin M_B \cos b \quad \ldots\ldots\ldots\ldots\ldots 4$$

and

(26d) $$\sin bpn = \frac{\sin M_B \cdot \sin b}{\sin t}, \qquad \tan \alpha_{pb} = \frac{x_b}{y_b}$$

(26e) $$s = 180^\circ - (\alpha_{pb} + bpn) \quad \ldots\ldots\ldots\ldots\ldots 5$$

The photo coordinates of the nadir are easily computed.

(27a) $$pn = \tan t \cdot f$$

(27b) $$xn = \sin s \cdot pn \quad \ldots\ldots\ldots\ldots\ldots 6$$

(27c) $$yn = \cos s \cdot pn \quad \ldots\ldots\ldots\ldots\ldots 7$$

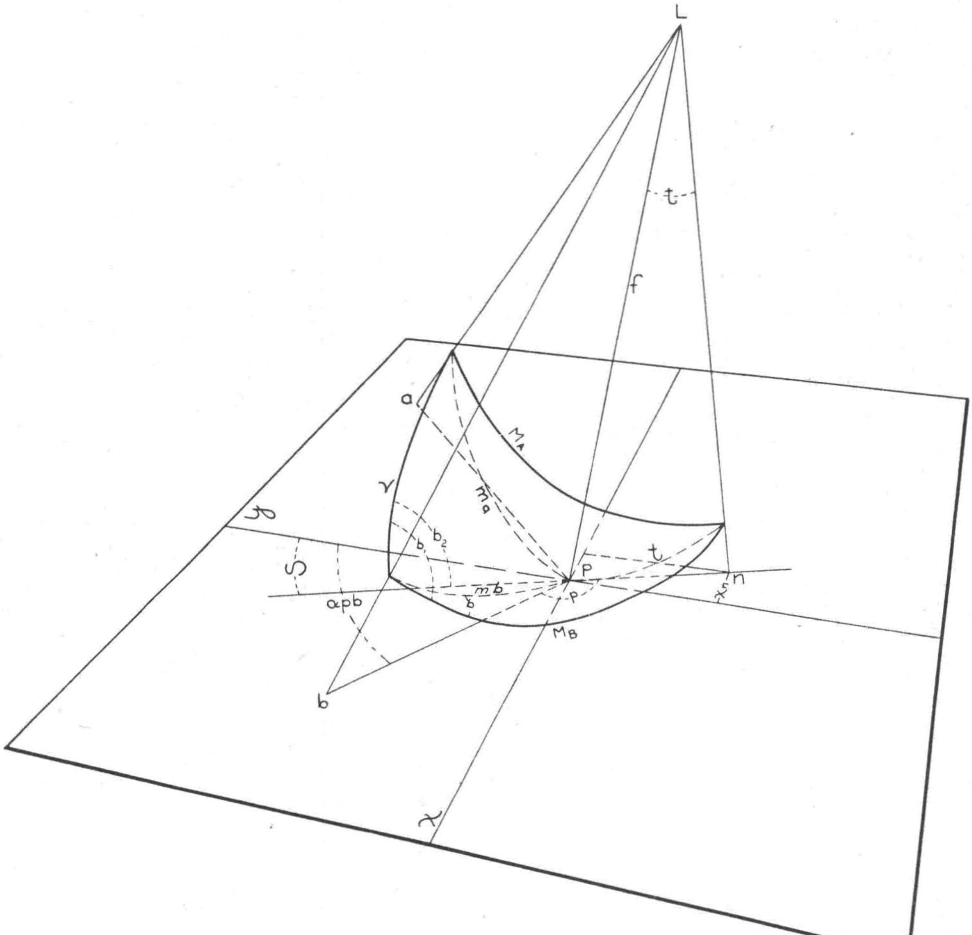


FIG. 11

The $x$ and $y$ photo coordinates of the various photo images can be translated and rotated in the conventional manner, using the angle of swing and $pn$ as constants in the formula.

### INTERSECTING SIDES

The tilt and swing can also be determined from linear quantities. See Figure 12. The elements of an equivalent vertical are computed from the previously calculated data. It is known that the equivalent vertical containing triangle $a'b'c'$ will trace the isoline on the photo plane containing triangle $abc$. After the isoline has been located on the photo plane, the tilt and swing can readily be computed.

Thus

$$La' = \frac{f}{\cos M_A}, \qquad Lb' = \frac{f}{\cos M_B}, \qquad Lc' = \frac{f}{\cos M_C} \tag{28a}$$

$$a'b' = \sqrt{(La')^2 + (Lb')^2 - 2(La')(Lb')\cos\gamma}$$

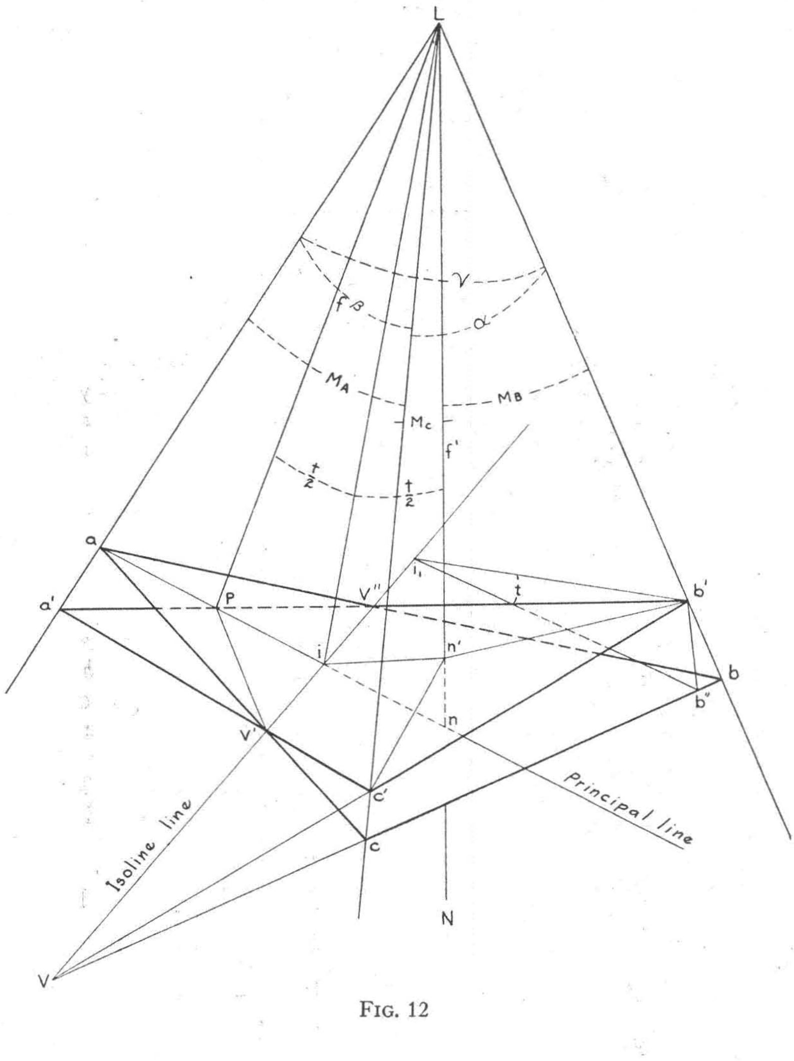


FIG. 12

(28b)
$$b'c' = \sqrt{(Lb')^2 + (Lc')^2 - 2(Lb')(Lc') \cos \alpha}$$
$$c'a' = \sqrt{(Lc')^2 + (La')^2 - 2(Lc')(La') \cos \beta}$$

Then

(28c)
$$\sin Lab = \frac{Lb \cdot \sin \gamma}{ab}, \qquad \sin La'b' = \frac{Lb' \cdot \sin \gamma}{a'b'}$$
$$\sin Lbc = \frac{Lc \cdot \sin \alpha}{bc}, \qquad \sin Lb'c' = \frac{Lc \cdot \sin \alpha}{b'c'}$$
$$\sin Lca = \frac{La \cdot \sin \beta}{ca}, \qquad \sin Lc'a' = \frac{La' \cdot \sin \beta}{c'a'}.$$

The intersections of $ab$ and $a'b'$, $bc$ and $b'c'$, and $ca$ and $c'a'$ are three points on the isoline. The points of intersection are computed as follows.

(29a)
$$La - La' = aa', \qquad Lb - Lb' = bb', \qquad Lc - Lc' = cc'$$

and

(29b)
$$av'' = \frac{\sin La'b' \cdot \overline{aa'}}{\sin (La'b' - Lab)}$$
$$bv = \frac{\sin Lb'c' \cdot \overline{bb'}}{\sin (Lb'c' - Lbc)}$$
$$cv' = \frac{\sin Lc'a' \cdot \overline{cc'}}{\sin (Lc'a' - Lca)}.$$

At this point tilt and swing can be determined graphically by laying off $va''$ from $a$ on $ab$, $bv$ from $b$ on $bc$, and $cv'$ from $c$ on $ca$. A line drawn through $v''$, $v$, and $v'$ is the isoline. A perpendicular to this line from $p$ is a segment of the principal line. The foot of this perpendicular is designated $i$.
Then

$$\tan \frac{t}{2} = \frac{pi}{f}, \qquad pn = \tan t \cdot f.$$

The swing angle $s$ is the angle at $p$ between the intersection of $pn$ produced and the line connecting the leading and trailing fiducial marks. Since $abc$ and $a'b'c'$ are triangles, they can intersect in only two points; therefore one of the three lines, $av''$, $bv$, and $cv'$, will intersect the corresponding line on the equivalent vertical off the photograph, and none of them can intersect in the area of the triangle if the area of the triangle is above or below the isoline. The line intersecting outside the triangle $abc$ is neglected. The graphic solution is illustrated in Figure 13.

It may be desirable to compute tilt and swing without recourse to graphics. For the explanation it will be assumed that $av''$ and $cv'$ fall on the lines $ab$ and $ca$ respectively, within the limits of the photograph.

(30a)
$$N'V'' = \sqrt{(av')^2 + (av'')^2 - 2(av')(av'') \cdot \cos cab}$$

(30b)
$$pv' = \sqrt{(pa)^2 + (av')^2 - 2(pa)(av') \cos pac}$$

(30c) $$pv'' = \sqrt{(pa)^2 + (av'')^2 - 2(pa)(cv'') \cos pab}$$

(30d) $$\cos pv'v'' = \frac{(pv')^2 + (V'V'')^2 - (pv'')^2}{2(pv')(v'v'')}$$

$$pi = \sin pv'v'' \cdot pv', \qquad \tan \frac{t}{2} = \frac{pi}{f}$$

$$\tan t \cdot f = pn$$

$$\cos v'pa = \frac{(pv')^2 + (pa)^2 - (av')^2}{2(pv')(pa)}$$

$$s = (90° + pv'v'') - (v'pa - \alpha_{pa})$$

$$\tan \alpha_{pa} = \frac{x_a}{y_a}.$$

Tilt and swing can also be computed by plane analytics.

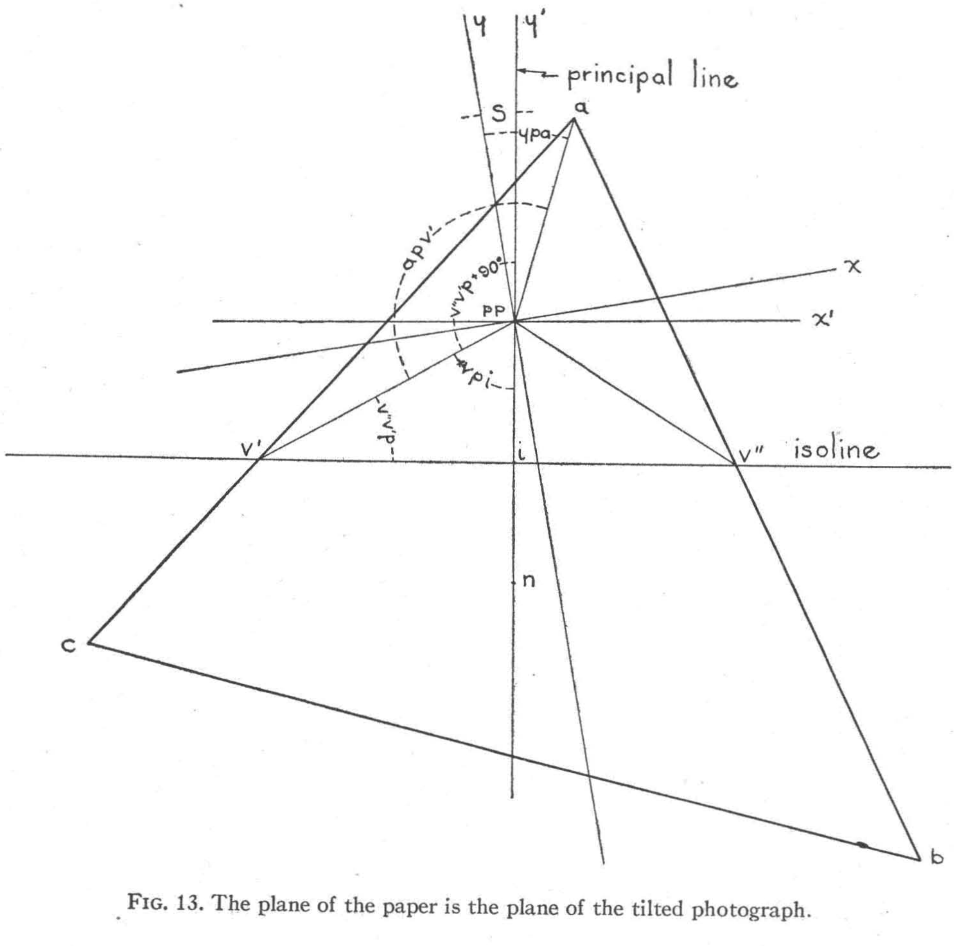


FIG. 13. The plane of the paper is the plane of the tilted photograph.

$$\text{(31a)} \qquad \tan \alpha_{ac} = \frac{x_a - x_c}{y_a - y_c}, \qquad \tan \alpha_{ab} = \frac{x_a - x_b}{y_a - y_c}$$

$$cab = \alpha_{ac} + \alpha_{ab}$$

and

$$\text{(31b)} \qquad \begin{aligned} xv' &= x_a \pm (\sin \alpha_{ac} \cdot av') \\ xv'' &= x_a \pm (\sin \alpha_{ab} \cdot av'') \end{aligned}$$

$$\text{(31c)} \qquad \begin{aligned} yv' &= y_a \pm (\cos \alpha_{ac} \cdot av') \\ yv'' &= y_a \pm (\cos \alpha_{ab} \cdot av'). \end{aligned}$$

From these photo coordinates swing is computed with a slope formula.

$$\text{(31d)} \qquad \tan s = \frac{yv' - yv''}{xv' - xv''} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 8$$

$$pv' = \sqrt{(xv')^2 + (yv')^2}$$

$$\tan \alpha_{pv'} = \frac{yv'}{xv'}$$

$$pv'v'' = \alpha_{pv'} + s$$

$$pi' = \sin pv'v'' \cdot pv'$$

$$\tan \frac{t}{2} = \frac{pi}{f}.$$

If the tilt is small, it is suggested that a horizontal plane be passed through an image the greatest distance from $p$ instead of the equivalent vertical plane. The resulting values $aa'$, $bb'$, and $cc'$ would be somewhat larger and would therefore yield a better tilt value.

## Scale of Point Images

Tilt and swing can be computed by determining the scale of each of the three point images and subsequently locating the isoline. The scales $Sa$, $Sb$, $Sc$, and $Si$ are then determined by the following equations.

$$\text{(32)} \quad Sa = \frac{Z}{La \cdot \cos M_A}, \quad Sb = \frac{Z}{Lb \cdot \cos M_B}, \quad Sc = \frac{Z}{Lc \cdot \cos M_C}, \quad Si = \frac{Z}{f}.$$

These values combined with the length of the sides are used to locate the isoline.

$$\text{(33)} \qquad \begin{aligned} av' &= \frac{(Sa - Si) \cdot ac}{(Sa - Sc)} \\ av'' &= \frac{(Sa - Si) \cdot ab}{(Sa - Sb)} \\ bv &= \frac{(Sb - Si) \cdot bc}{(Sb - Sc)}. \end{aligned}$$

The isoline having been located, the tilt and swing are computed as before.

## Vertical Differences of Point Images

Tilt and swing can also be computed by treating the picture plane with respect to the equivalent vertical in a manner similar to the treatment of the sloping reference plane with respect to the datum plane in space coordinate determination.

The vertical differences between the point image in the picture plane and the equivalent vertical are computed. The equivalent vertical and the picture plane in space orientation correspond to the level datum plane and sloping reference plane in space resection, and the vertical differences $za$, $zb$, and $zc$ correspond to the elevations $Z_A$, $Z_B$, and $Z_C$.

$$z_a = f - La \cdot \cos M_A, \qquad z_b = f - Lb \cdot \cos M_B, \qquad z_c = f - Lc \cdot \cos M_C. \tag{34}$$

Then

$$av' = \frac{za \cdot ac}{(z_a - z_c)}, \qquad av'' = \frac{za \cdot ab}{(z_a - z_b)}, \qquad bv = \frac{zb \cdot bc}{(z_b - z_c)} . \tag{35}$$

And again, the isoline is the line passing through the terminals of $av'$, $av''$, and $bv$, from which location the tilt and swing can easily be computed.

## Numerical Example

Insofar as it was desirable to know the absolute error of each element of exterior orientation determined by the described three-point method, a synthetic pyramid, the components of which were derived by calculation, was used for the computation of the numerical example. Lack of space does not permit the inclusion of all the intermediate computations and values of the numerical example. Only the given survey and photo data, along with the absolute and derived elements of exterior orientation, are shown in the table below:

### Given Data

| | Ground Coordinates | | | | Photo Coordinates | | |
|---|---|---|---|---|---|---|---|
| | $X$ | $Y$ | $Z$ | | $x$ | $y$ | $z$ |
| A | 12,464.476 | 23,444.453 | 90.00 | a | −83.243 | −60.712 | 0 |
| B | 10,354.000 | 19,789.000 | 70.00 | b | 6.270 | −106.512 | 0 |
| C | 15,605.451 | 18,957.158 | 182.00 | c | 21.780 | 19.293 | 0 |
| | | | | p | 0.00 | 0.00 | 210.00 |

### Exterior Orientation

| | Absolute value | Computed value | Error |
|---|---|---|---|
| $LA'$ | 9,765.112′ | 9,764.700′ | −0.412′ |
| $LB'$ | 9,930.979′ | 9,931.239′ | 0.260′ |
| $LC'$ | 8,544.987′ | 8,547.357′ | 2.379′ |
| $X$ | 15,298.440′ | 15,295.572′ | −2.868′ |
| $Y$ | 19,771.108′ | 19,774.554′ | 3.446′ |
| $Z$ | 8,682.580′ | 8,684.539′ | 1.959′ |
| $t$ | 3°0′0″ | 2°58′37.4″ | −0°1′22.6″ |
| $s$ | 10°0′0″ | 9°41′22.2″ | −0°18′37.8″ |

The above values are based on a single approximation. The interested reader may obtain a step-by-step procedure along with a detailed numerical computation by writing E. L. Merritt, U. S. Naval Photographic Interpretation Center, Receiving Station, Washington 25, D. C.

## Conclusion

It is believed that the greatest value of the described three-point method lies in the fact that the required degree of accuracy can be obtained in the calculation of the perspective rays by carefully noting the difference of $y'-y$ in the plane of circle $h$ and thereafter executing a direct computation of the five elements of exterior orientation. The various graphics described should be most useful in quickly determining preliminary values.

However, the methods set forth, like all other methods, are not wholly satisfactory. The U. S. Naval Photographic Interpretation Center has made a systematic investigation of space analytics; tentatively it appears that a final streamlined solution having the greatest combination of accuracy and speed of execution will necessarily consist of the best elements of several methods.

## Glossary of Terms and Symbols

*Colinear:* points on the same line.

*Concurrent:* rays passing through the same point.

*Coplanar:* lying in the same plane.

*Emergent nodal point* ($L$):* perspective center ($L$) of the camera lens at which the perspective rays may be considered to be concurrent.

*Exterior orientation:** a set of quantities which fixes the position of the camera station by five elements: three elements, $X$, $Y$, $Z$, referred to as space coordinates, and two elements, tilt and swing, referred to as space orientation.

*Ground isometric parallel:* any line parallel to the line defined by the intersection of a sloping reference plane and a level datum plane.

*Ground principal plane:* a vertical plane perpendicular to the isometric ground parallel and including a perpendicular from $L$ to the sloping reference plane.

*Isocenter* ($i$):* point on photograph intersected by the bisector of the angle between a vertical line through $L$ and the photo perpendicular.

*Isometric parallel:** a line passing through the isocenter parallel to the line defined by the intersection of the sloping picture plane and the horizontal ground plane.

*Photograph perpendicular:* the perpendicular to the plane of the photograph from the interior perspective center. In this paper both the fiducial axes intersection and the principal point will be considered the photograph perpendicular.

*Photograph pyramid:* a pyramid whose base is a triangle formed by three point images on a photograph and whose apex $L$ is the interior perspective center of the photograph. The definition is extended to include the triangle formed by three point objects on the ground.

*Picture plane:* photograph.

*Principal line:** the trace of the principal plane on the picture plane.

*Principal plane:** a vertical plane that includes the optical axis and a vertical line concurrent with optical axis at $L$.

*Swing* ($s$): the angle at $p$ in the picture plane between the photo $Y$ axis and the principal line.

*Tilt* ($t$): the angle at $L$ in the principal plane between the plate perpendicular and photo nadir.

| | |
|---|---|
| $A, B, C, D$ | Ground control points. |
| $a, b, c, d$ | Corresponding image points on picture plane. |
| $X, Y, Z$ | Space coordinates of $L$ and with the appropriate subscripts the space coordinates of any point. |

* Definitions are in accordance with those given in the American Society of Photogrammetry's *Manual of Photogrammetry*, pp. 774–810.

| | |
|---|---|
| $x, y, f$ | Photo coordinates of corresponding images with appropriate subscripts. |
| $N, n$ | Ground and photo nadir, respectively. |
| $t, s$ | Tilt and swing of photograph. |
| $\Delta$ | Tilt of the sloping reference plane with respect to datum plane. |
| $LA', LB', LC'$ | Perspective rays from $L$ to point object. |
| $La, Lb, Lc$ | Perspective rays from $L$ to point image. |
| $\overline{AB}, \overline{BC}, \overline{CA}$ | Horizontal ground lengths. |
| $\overline{A'B'}, \overline{B'C'}, \overline{C'A'}$ | Slope ground lengths. |
| $\alpha, \beta, \gamma, \cdots$ | Any angle. |
| $M$ | With appropriate subscripts any vertical angle between a vertical line through $L$ and a perspective ray. |
| $m$ | With appropriate subscripts any angle between the camera's optical axis and a perspective ray. |
| $R, r$ | Ground and photo radial from $N$ and $n$ to ground object and corresponding image, respectively. |

ACKNOWLEDGMENT

The writer is grateful to the following employees of the U. S. Naval Photographic Interpretation Center: James W. Hissey for his excellent drafting of the figures, Sylvia E. Adams and James A. Kowalski for the computation of the numerical example, Arthur C. Lundahl for his numerous suggestions in the composition of the paper, and Elaine D. Merritt for the preparation of the manuscript.

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Although it has been known for several months that an International Congress of Photogrammetry would be held at Amsterdam in 1948, an announcement to this effect has been withheld due to an uncertainty of dates. The dates have now been fixed and the formal announcement is carried in Resolving Power column. The dates, September 1 to 10, 1948, inclusive, were selected because the International Geodetic and Geophysical Conference at Oslo will be held just prior to September 1st, and the International Geographers Conference will be held at Lisbon just after September 10th. The schedule was arranged to permit attendance at the three conferences, if desired.

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