METHODS OF SUPPLEMENTING GEODETIC CONTROL*

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A VERY detailed network of horizontal and vertical control is necessary to hold in satisfactory position the topographic detail that can be obtained from a series of aerial photographs by the use of stereoscopic map plotting methods. Two strategically located points of known position and three strategically located points of known elevation are theoretically necessary for the absolute orientation of each separate stereoscopic model. (The vertical and horizontal control may or may not be the same points.) In actual practice more control than this is considered essential. Sufficiently accurate control of the required location and density generally is not available at the time a mapping project is started. Therefore, additional control must be obtained by extending the existing network of reliable ground control. The necessary additional control, which is generally of third or fourth order accuracy, may be obtained either by photogrammetric triangulation, or by ground triangulation or traverse. This control, because of its very special distribution, is sometimes called supplementary control. That term is not very descriptive, but it is used herein for lack of a better one.

It is the primary purpose of this article to:

A. Describe briefly the principal methods of obtaining control, other than the well known methods of ground triangulation and traverse.
B. Outline in an elementary fashion the problems of stereo triangulation.
C. Describe briefly methods for controlling errors of stereo triangulation.
D. Describe the practical results that are obtained with stereo triangulation.
E. Describe the accuracy that is required of stereo triangulation.
F. Set forth the author's conclusions.

This article has been subdivided into six main parts corresponding to these objectives. (This article is not concerned with reconnaissance or semi-reconnaissance surveying methods.)

I. Methods of Obtaining Supplementary Control

The orthodox ground survey methods of obtaining the necessary supplementary control for a photogrammetric survey are relatively expensive. For this reason, much time and effort has been devoted to the problem of developing relatively inexpensive photogrammetric triangulation procedures that need very little control established by ground surveys (1).[1] As a result of these efforts the principal methods which have been devised for obtaining the necessary supplementary control are graphic radial triangulation and associated procedures, monocular triangulation, analytical triangulation, and stereo triangulation. (2, 3). Some of these developments have proved quite successful, but in actual practice at least part of the necessary supplementary control generally must still be obtained by the customary field methods.

Graphic Radial Triangulation

Graphic radial triangulation is used to obtain horizontal supplementary

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[1] Numbers in parentheses refer to footnotes and references listed at conclusion of the article, pp. 511–513.

control. In this method, slotted templets are commonly used to extend a network of horizontal ground control. In the United States the principal point is normally used as the center of radiation for graphic radial triangulation. This choice will allow serious errors to occur. For this reason, graphic radial triangulation is generally used to compile maps at scales fairly close to the average scale of the vertical photographs. When accurate contours are needed for maps made by graphic radial triangulation, the necessary vertical control must be obtained by various field methods.

Commander O. S. Reading says, "In some tests (graphic) radial triangulation with the USC&GS nine lens camera has proved more accurate and economical than with wide angle photographs taken with single lens cameras due to the larger field and the smaller number of photographs . . . One advantage of the ultra-wide-angle cameras is that is is economically practical to fly strips with sixty percent side lap. This gives more rigid geometrical conditions for the plot and makes large blunders impossible in careful work. There are so many conditions to be staisfied when the principal points of all adjacent photos appear on any one photograph, that blunders are immediately apparent and the precision of the intersections affords a reliable guide as to the quality of the work."

Monocular Triangulation (4)

Monocular triangulation (involving the use of instruments which measure vertical and horizontal angles) is used to obtain horizontal and vertical supplementary control. In using such instruments as the photoalidade, photogoniometer, Miller Single Eye Piece Plotter, and the Canadian High Oblique Plotter, a graphic radial line plot of the photographs is first made in order to extend the existing network of horizontal control. The elevations of the supplementary control may then be obtained by measuring the vertical angles from the photographs. These vertical angles are converted into elevations by mathematical or mechanical methods involving the use of ground distances measured from the graphic radial line plot. This method is used to best advantage with oblique photographs (either terrestrial or aerial) because at least part of the topography then appears on the photographs more or less in profile. In such cases, vertical photographs may be used in the map plotting if they are available.

The U. S. Forest Service is using terrestrial photographs in connection with monocular triangulation and the publication of maps at a scale of 1:31,680 with a fifty foot contour interval. This method appears to have proved quite successful in obtaining vertical control for stereoscopic plotting from vertical photographs. The method requires a rather large number of exposure stations because of the limited coverage of the terrestrial photographs under normal conditions.

In order to take full advantage of the best features of the U. S. Forest Service method, it would seem desirable to discard the use of terrestrial photographs in favor of low oblique aerial photographs. The compilation procedure used by the Forest Service would remain relatively unchanged.

The camera installations of this proposed monocular triangulation procedure might consist of six cameras similar to the Fairchild F-56, which would be permanently mounted in a single frame. These cameras would be spaced sixty degrees apart in a horizontal plane and each would be tilted sixty degrees from the vertical. A vertical camera could also be used if considered worthwhile. It is assumed that the approximate tilt of each set of exposures would be obtained from a recording gyroscope. Precise tilts would be obtained later as part of the procedure for obtaining vertical control.

This proposed method of taking photographs provides for the adequate coverage of a given area with only a small fraction of the number of photographs required by the present Forest Service method. This would result in a worthwhile saving in the number of photographs that would be required to be processed in establishing the network of supplementary control. It would appear that monocular triangulation is not a suitable method of obtaining vertical control where smaller contour intervals are required.

Analytical Triangulation (5)

Analytical triangulation provides a mathematical method of obtaining vertical and/or horizontal supplementary control. The positions and elevations of any required number of points may be obtained by computations involving the use of a few ground control stations and the coordinates of the images on the photographs. These analytical methods are not at the present time generally used for extending vertical and horizontal control.

The principal work accomplished in the United States in the field of analytical triangulation includes the following:

1. Applications of space analytical geometry to aerial triangulation, concerning which Syracuse University has published several pamphlets by Professor Earl Church.
2. Special cases in the application of space geometry to analytical triangulation by means of oblique photographs, as developed by Mr. O. M. Miller.
3. A method involving the use of intersecting circles proposed by Mr. Everett L. Merritt.
4. A procedure sometimes called the "scale-point method" developed by Mr. Ralph O. Anderson.

Analytical triangulation based upon Mr. R. O. Anderson's scale point theory probably is the most controversial procedure so far developed and publicized. Regarding this form of analytical triangulation Mr. Anderson writes:

"It has long been the popular consensus of opinion that analytical methods (non-stereo) of control extension were too complicated to be of any economical value. This opinion was justifiable, but it has been the writer's contention since 1937 that a semi-graphical method of control extension may be developed that would prove to be practical. The outgrowth of recent research supports the writer's belief quite emphatically. The results of that research appears in the writer's recent publication "Photogrammetric Control Extension." This particular method of control extension serves to establish both horizontal and vertical survey positions of arbitrarily selected image points without using ground control support except at flight ends. In the course of extending control by means of the proposed method both the scale and tilt of the photographs comprising the flight are computed progressively.

"The process is simple. In brief it consists of rectification and ratioing the flying height of the second conjugate to that of the first conjugate. Ratioing and rectification is done in a semi-graphical manner supported by simple arithmetic operations. The two overlapping photographs are termed the conjugate pair; the first in line of flight being the first conjugate, and of course the overlapped or second photograph is the second conjugate.

"The outstanding feature of the proposed method is the triple parallactic differential equation, which is named thusly for want of a better name. It may be a misnomer but a better name may develop with usage. Discussions on this issue are invited. This equation consists of three sets of measured lengths on the two overlapping photographs. When properly processed (rectified and ratioed) they enter into a simple equation of one unknown, namely, the elevation difference.

"The model consists of six points of which three are located in line with the center of the first conjugate (bottom: left, center and right points) and the other three in line with the center of the second conjugate (top: left, center and right points). The right and left points should also fall upon adjacent flights (or side-overlap). These image points should also fall upon three consecutive overlapping photographs in line of flight. By selecting image points in this manner it is only necessary to extend control in alternate flights. This means that there will only be four secondary points (two from each adjacent flight) on each model in each intervening flight.

"The elevation of the upper left point may be obtained from two different bench marks (bottom left and bottom center points). These two elevations will differ when error of any description is present. The foregoing applies with equal force to the upper right point (determined from bench marks: bottom center and bottom right). The top center point elevation may be computed from three bench marks (bottom: left, center and right points). From the foregoing it can be seen that a self checking feature exists. It must be noted that the ideal condition will not be realized until the entire procedure is free of error. This means that the lens distortions must be precisely determined and applied with precision.

"When photographic measurements are used only to the nearest one-thousandth part of an inch, it has been found that control could be established within two feet (both horizontal and vertical) for a given model. There is also further accuracy to be attained by extending the vertical control over the entire flight and thence adjusting the errors of closure over the entire circuit. Following this adjustment, the horizontal ground distances may be computed which in turn are used to compute the horizontal coordinates of the selected image points. It is also possible to adjust all the computed angles in each and every quadrangle but this is not recommended except in very important cases as the work would be excessive.

"It may seem on sight that the proposed method of control extension is lengthy, but it will be found on closer examination that the form work was designed to permit rapid checking; every measurement or computed function that is needed for a later operation is recorded.

"The accuracy of this proposed method of control extension is limited only by the precision of photographic measurements. Steps have been taken to improve the accuracy of these measurements. The ultimate objective of this method is to ratio and rectify every photograph and complete the subsequent topographic and planimetric mapping by some sort of a relatively inexpensive apparatus. This manner of treatment would break the "bottle neck" of production mapping and consequently lower the cost of aerial mapping. While figures are not available, it is quite reasonable to believe that the cost of the proposed method of control extension would be less than ten percent of usual ground control costs of comparable accuracy. Upon these premises. I feel justified in proclaiming the proposed method of control extension, "The Photogrammetry of Tomorrow."

Ir. A. J. van der Weele says, "For analytical radial triangulation we (Netherlands Ministry of Public Works) use a radial triangulator (6) almost without exception. We have three of these instruments—two are Dutch made and the other is Zeiss. These instruments are rather simple but can nevertheless be used to obtain very accurate measurements of picture coordinates. For the most accurate radial triangulation we use the following procedure.

"The strips of vertical photographs are triangulated in a double image apparatus like the Stereoplanigraph or the Autograph A5. (When sufficient vertical control is available, the preliminary aerotriangulation might be executed in a simpler instrument, such as the Wild Stereo Plotter A6.) From this triangulation the tilts and elevations of supplementary control points are calculated. It is then possible to compute the position of the isocenter or nadir point on each photograph. These points are identified on the photographs and then used as centers for the measuring of directions in the radial triangulator. (As to the choice between isocenter and nadir point, see the article of Roelof's in the 1940 *Photogrammetria*, page 41.) When the utmost accuracy is desired, we correct the measurements for the errors caused by tilt or relief (depending upon whether we make the measurements from the isocenter of nadir points). The map coordinates of the points are found by mathematical computations. In this way it is rather easy to use the theory of errors for the final adjustment. In general, I am convinced that the results (x and y coordinates of nine points on each photograph) are more accurate than those which are obtained by the most careful stereo triangulation. The planimetric detail is plotted with the aid of the Wild A6 or Stereometer (a kind of stereocomparagraph). When contours are required, they are also plotted with these instruments.

"We use this method of radial triangulation to compile planimetric maps at scales up to eight times the scale of the photographs. In spite of the elaborate execution, our method might be more efficient (than the American slotted templet method) since it requires the processing of a much smaller number of photographs. The preliminary stereo triangulation, which can be checked by the results of the radial triangulation, provides a network of vertical control which is sufficient for many purposes.

"In the above described radial triangulation procedure we prefer to use diapositives since these are less sensitive to temperature and humidity changes. (We practically never use film or paper prints.)

"Our experience has shown that in Holland, where we have an accurate and dense network of first and second order triangulation points, this method of radial triangulation is economically justified for maps at scales of 1:10,000 and smaller. However, we have even used this method in producing maps at a scale as large as 1:2,500 from photographs having a scale of 1:18,000. Generally, field methods of obtaining horizontal control have proved more satisfactory for scales larger than 1:10,000.

"In general, our work at the Delft Institute can be divided into two parts of different character. One part consists of the mapping program of Holland. This part consists of the compilation of planimetric maps almost without exception, since height differences are very small . . . . The second part consists of our mapping program in the Dutch East Indies. In this part of our work we have very little horizontal control and no vertical control (except the shore lines of lakes and the sea coast, and of the large rivers). In both parts of our work we use the above described method of radial triangulation."

Mr. G. C. Tewinkel offers the following suggestion: "I am convinced that analytic bridging can be performed accurately and economically using photographs taken at relatively high altitudes. . . . The method would require a separate set of photographs at a lower altitude for detailing and contouring. The method for obtaining the positions and elevations of supplementary control from the high altitude photographs would require:

"1. A wide angle surveying camera using glass plates and having a lens of very high resolving power.

"2. A precise comparator for measuring coordinates of images on the glass plates. All the coordinates would then be corrected for lens distortion by the use of a distortion pattern determined by a calibration procedure such as used by the National Bureau of Standards.

"3. The horizontal control might well consist of two stations in each twentieth model. The vertical control would require one elevation in each of the four corners of each model.

"4. A good computing machine. The use of an automatic electronic computor for the solution of simultaneous equations would indeed be helpful.

"It has been shown that positions of objects can be obtained by computation at a relative accuracy equal to that of image identification. At the present time the computations are somewhat tedious but using them will develop simplifications. The computation procedure may be executed by a high school graduate because a knowledge of the theory is not essential.

"The actual map plotting itself would be performed from the low altitude photographs taken on film in the usual manner such as with the multiplex or other device. Each model would be completely controlled by the supplementary points."

Stereo Triangulation

Stereo triangulation provides a stereoscopic method of obtaining a detailed network of vertical and horizontal supplementary control. Stereo triangulation may be defined as the stereoscopic method of precisely integrating several overlapping photographs with respect to each other and to a relatively sparse network of control points of known position and elevation. (According to the author's definition of photogrammetric triangulation (1), *Stereo Triangulation Is Performed Whenever Three or More Photographs are Absolutely Oriented, as an Integrated Unit, to Control that is Inadequate for the Absolute Orientation of the Same photographs When Not Integrated Together.*) Stereo triangulation is accomplished by making a stereoscopic analysis of the aerial photographs. In the case of the Multiplex, stereo triangulation is performed in order to establish the absolute orientation of an entire series of stereoscopic models. After the stereo triangulation of the series has been completed, the actual map plotting may then be immediately undertaken without any further adjustment of the Multiplex projectors. If this is not desirable for any reason, the positions and elevations of the supplementary control may be recorded and the map plotting can then be done later from independent stereoscopic models. In the case of the Autograph and the Stereoplanigraph, stereo triangulation is performed in order to establish supplementary control of sufficient accuracy and density to adequately control the absolute orientation of each of a series of independent stereoscopic models which must be established later for the actual map plotting. After the stereo triangulation of the series of photographs has been completed, then each of these independent stereoscopic models is relatively and absolutely oriented to the control previously established by stereo triangulation.

II. Problems of Stereo Triangulation

The fundamental nature of the stereo triangulation procedure may be most clearly understood by a consideration of Figure 1, Figure 2 and Figure 3. Figure 1 illustrates the results that ought to be obtained from the stereo triangulation of a typical series of vertical aerial photographs P_1, P_2, P_3, P_4 and P_5. The stereoscopic models are represented by the line N_1, N_2, N_3, N_4 and N_5—which has been drawn perfectly straight in order to indicate that all errors of stereo triangulation have been removed. In contrast to Figure 1, Figure 2 illustrates in an exaggerated fashion the general nature of the results that are usually obtained with stereo triangulation. This figure indicates that, in spite of "adequate" ground control within the stereoscopic overlap of the first pair of photographs, the relative and absolute orientation of even the first pair of photographs is not entirely

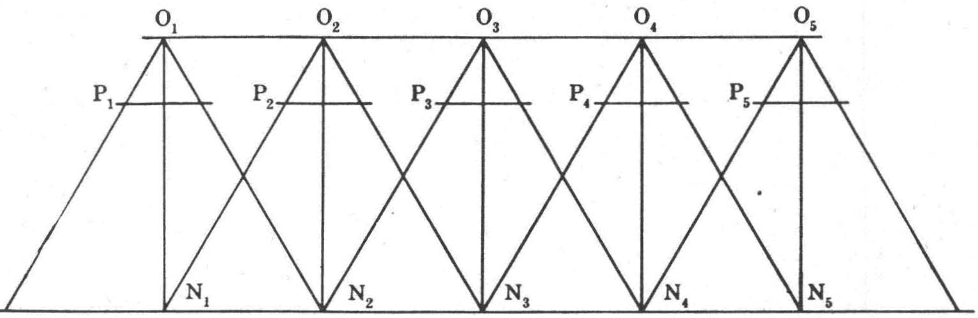

FIG. 1. Schematic diagram of five vertical photographs illustrating the results that ought to be obtained with stereo triangulation.

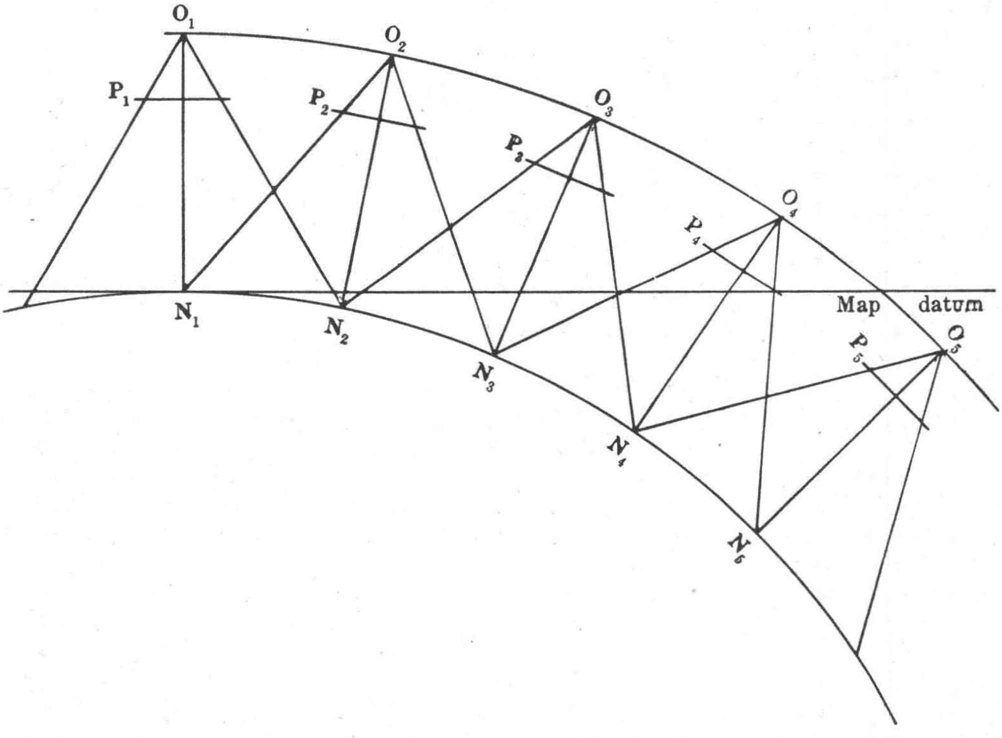

FIG. 2. Schematic diagram illustrating in an exaggerated fashion the general nature of the results that are usually obtained with stereo triangulation.

perfect. In addition, the absolute orientation of each succeeding stereoscopic model becomes progressively worse.

In order to recapture by stereoscopic methods the correct orientation of each of a series of photographs in space, a procedure somewhat similar to the following might be used:

1. The perspective center of each of the series of photographs is precisely located. This is commonly called interior orientation and establishes a geometrically correct bundle of rays representing those light rays which exposed the original negatives.
2. The relative orientation of the first pair of photographs is established by the use of so-called twin point interpolation (7). This procedure requires the removal of y-parallax from at least

five strategically located points in the stereoscopic model. In actual practice, however, y-parallax is removed from all areas of the model as completely as possible. Statoscope readings are used if they are available.

3. The resulting stereoscopic model is absolutely oriented with respect to certain strategically located control points of known position and elevation.
4. The locations and elevations of three strategically located points are precisely determined. These points are used for establishing the scale of the next model. (Individually, these points are commonly called "pass points." Collectively, they form the network of supplementary control.)
5. Interior orientation having been established, the third photograph is relatively oriented with respect to the second by means of twin point interpolation. Statoscope readings are used if they are available. The scale of the resulting model is determined by recapturing the elevations of the pass points selected in the preceding model. Establishing scale must be performed with the utmost possible precision.
6. The location and elevations of three strategically located pass points are precisely determined. These pass points are used for establishing the scale of the next model.
7. Each of the additional photographs of the series is relatively oriented with respect to the last preceding photograph of the series by the use of the procedure described in paragraphs 5 and 6, above.
8. Additional ground control is available in the last model of the series. The positions and elevations of this ground control, as measured from the unadjusted stereo triangulation, are considerably in error. (The sources of these errors will be discussed in considerable detail at the end of this Section.) The entire network of supplementary control must therefore be adjusted in order to properly distribute this error of closure, and to remove at least a part of certain other errors that exist within the stereo triangulation.

Several different methods are used for adjusting errors of stereo triangulation. The general nature of the analytical methods than now seem to be generally used in Europe is graphically illustrated by Figure 3, (8). In this figure, the arc N_1—N_5 represents a portion of the earth's surface reproduced by the relatively oriented stereoscopic models. O_1, O_2, O_3, O_4 and O_5 are the perspective centers of the corresponding photographs; and N_1, N_2, N_3, N_4 and N_5 are the ground nadir points; the center of the arc N_1—N_5 is indicated at C. Now, in establishing the absolute orientation of this imaginary series of aerial photographs, it is necessary to adjust the series of photographs to ground control at the beginning and end of the strip, and to remove all errors of position and elevation as completely as possible. The first model of the series was absolutely oriented to ground control. The first step in the adjustment of the stereo triangulation therefore is to compute the error of closure encountered in the last model. The second step is to reduce the systematic and accidental errors to their minimum values. According to the best modern practice, the reduction of these errors is accomplished by the use of the method of least squares and a mathematical analysis of the network of supplementary control provided by the unadjusted stereo triangulation. According to the method proposed by Prof. Dr. Max Zeller (9) ground control must be located in the middle of the strip of photographs, as well as at both ends. The control in the middle of the strip is used only for estimating the mathematical characteristics of the progression of errors of both the vertical and horizontal supplementary control. (For example systematic and accidental errors generally would cause the arc N_1—N_5 of Figure 3 to be somewhat elliptical in shape.) Dr. Zeller's methods are discussed in more detail in Section III.

As a result of the above described mathematical procedure, new positions are computed for the first and second photographs. This results in a new position for O and C' and moves the perspective centers O_3, O_4 and O_5 to O_3', O_4' and O_5'. Additional computations establish a final adjusted position for the third photograph; thus moving O_3', O_4' and O_5' to new positions at O_3'', O_4'' and O_5''; with C' moving to C''. Additional computations establish the adjusted positions of the

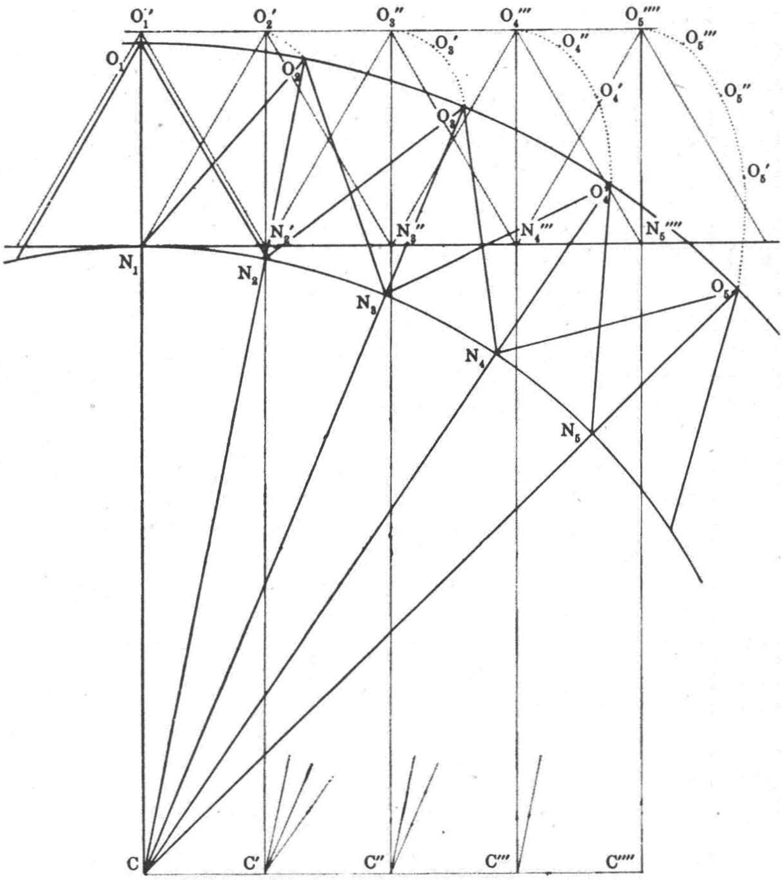

FIG. 3. Schematic diagram illustrating a method of adjusting the errors of stereo triangulation.

fourth and fifth photographs. As a result of this method of adjustment, the curved surface of the earth, represented by the arc N_1—N_5, has been "unrolled" onto the map plane (line N_1—N_5'''' in Figure 3).

The author wishes to have it clearly understood that when the term "stereo triangulation" is used herein it includes the adjustment of systematic and accidental errors and the errors of closure.

The errors of measurement encountered in the stereo triangulation procedure arise from three sources:

1. From imperfections in, or faulty adjustment of, the various instruments and procedures that are involved.
2. From the limitations of human sight and touch.
3. From variations in natural phenomena such as temperature and humidity.

These various reasons for lack of accuracy will be discussed in the next few paragraphs. However, it must be emphasized that these limitations apply more or less equally to all photogrammetric procedures, and not just to stereo triangulation alone.

Base-Altitude Ratio

The base-altitude ratio is defined as the ratio of the air base of a pair of aerial photographs to their flying height. The numerical value of this ratio depends primarily upon the angular field of the surveying camera. (In other words, an ultra-wide-angle camera has a high base-altitude ratio.) Up until the development of the Topogon and Metrogon lenses, the base-altitude ratio imposed a rather severe limitation upon the accuracy of the stereo triangulation procedure. Now that we have the wide angle lenses this limitation is not quite so important. For accurate results of stereo triangulation, the operator must identify with considerable precision the intersections of the conjugate image rays forming the stereoscopic model. It appears to the author that the positions of these intersections may be most exactly located if the conjugate image rays all intersect at angles as close to ninety degrees as possible. In order to attain this condition, the base-altitude ratio should be approximately 2.3:1. Of course this high ratio is not attainable in a single vertical camera. Even the USC&GS Nine Lens Camera has a base-altitude ratio of only 1.7:1 approximately. In addition, except for the Stereoplanigraph, the available map plotting equipment cannot operate at angles of convergence greater than those encountered in the present wide angle photographs. In addition, Ir. A. J. van der Weele points out that "beyond a certain limit it is no longer possible to get a good stereoscopic image because the perspective differences between two overlapping photographs become too great."

The following table gives a general idea of the base-altitude ratio that can be obtained with various representative types of cameras, expressed in terms of an equivalent vertical photograph having a six inch focal length:

$8\frac{1}{4}$ inch focal length vertical, 60 per cent overlap	$\frac{3.6}{8\frac{1}{4}}$	$\frac{2.6}{6}$
6 inch focal length vertical, 60 per cent overlap		$\frac{3.6}{6}$
6 inch focal length vertical, 55 per cent overlap		$\frac{4.0}{6}$
6 inch focal length Spherical Plate Camera vertical, 55 per cent overlap		$\frac{6.6}{6}$
$8\frac{1}{4}$ inch equiv. focal length USC&GS Nine Lens Camera vertical, 60 per cent overlap	$\frac{14.0}{8\frac{1}{4}}$	$\frac{10.2}{6}$
6 inch focal length tandem obliques tilted in line of flight (so-called "split verticals") 60 per cent overlap		$\frac{13.2}{6}$

Aerial Surveying Cameras (10)

In the past, considerable effort has been expended by lens designers in an effort to develop a camera that will cover more terrain from a single exposure station (11). Typical results of these efforts are the nine lens cameras developed by Aschenbrenner, Zeiss, and the U. S. Coast & Geodetic Survey; various tandem cameras such as the trimetrogon assembly and the tandem T-3A five lens cameras; and the various ultra-wide angle cameras such as the Spherical Plate Camera (12) and the German camera using the Pleon lens.

Regarding the nine lens cameras, Mr. Edwin Berchtold says, "Multi-lens cameras have not proved useful in Europe for aerotriangulation . . . The quadruple camera from Zeiss has been abandoned as soon as the Topogon lens has been manufactured." Ir. A. J. van der Weele says, "Multi-lens cameras have not proved to very useful as far as our experience goes. They have advantages with respect to angular coverage and base-altitude ratio, but much of these advantages are lost in the inaccurateness of the composite photographs. We have used many photographs taken with the Aschenbrenner "Panorama" nine lens camera in the radial triangulator and it was shown that the weakest part of the procedure was the formation of an accurate composite of the nine components. Moreover, it was not possible to make complete use of the favorable base-altitude ratio since that gives great difficulties in the stereoscopic examination of the photographs, especially in the case of hilly woodland." It therefore appears that if the component negatives of a multi-lens camera are not precisely oriented on

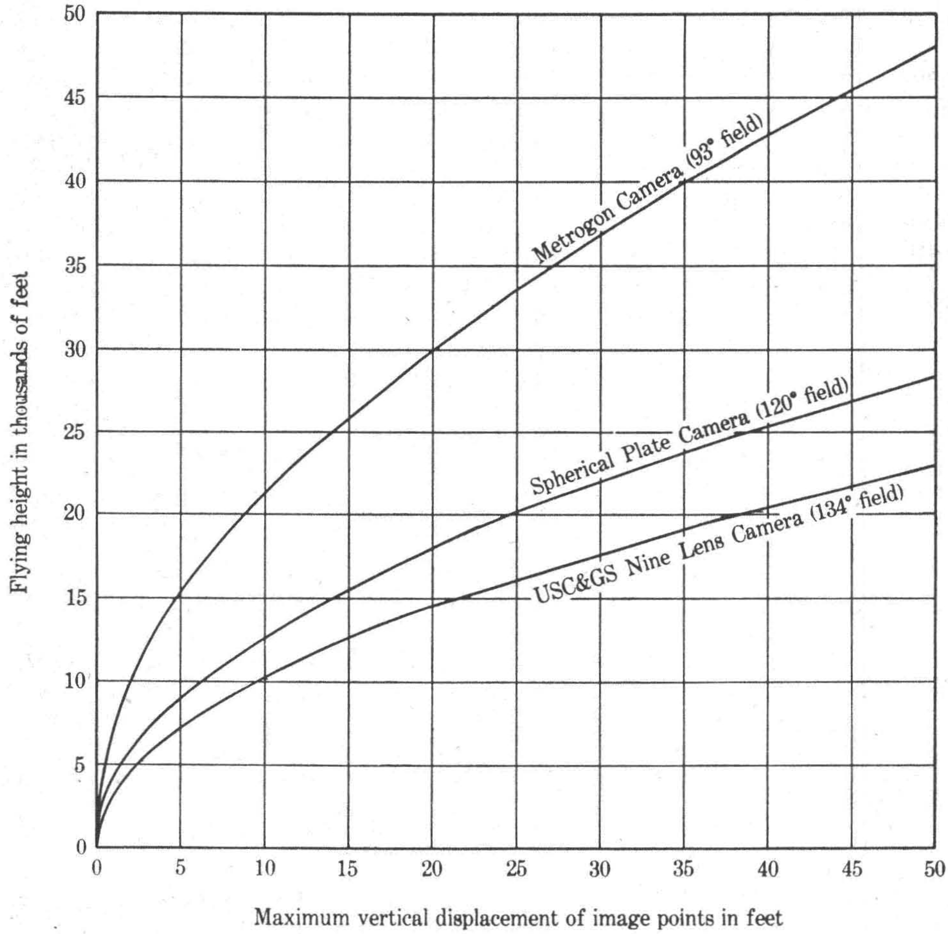

FIG. 4. Curve illustrating an empirical relation between flying height of vertical photographs and maximum vertical error of image points due to the effects of curvature and refraction.

the composite photograph, the camera itself has value only for reconnaissance or semi-reconnaissance mapping. The above line of reasoning is also applicable to various multiple camera installations.

The practical photogrammetric value of ultra-wide angle photographs appears to be quite limited. The most significant photogrammetric characteristics are the high base-altitude ratio, increased parallax differences for a given scale of photograph, and the relatively small scale of the photographic detail for a given flying height. (Severe lens distortion and uneven illumination of the negatives are also apt to exist.) Ultra-wide-angle photographs cannot be used with any comparative advantage in graphic radial triangulation because the base-altitude ratio is not utilized at all. In addition, such photographs must be flown at comparatively low flying heights in order that templets can be made from the photographs with the desired angular precision. Because of the relatively low flying heights that are thus required of the ultra-wide-angle photographs, it is possible to cover a given mapping project with the same number of wide angle photographs possessing the same scale. (13). The practical application of the ultra-wide-angle photographs to stereo triangulation is somewhat limited by the use of suitable map plotting instruments. It is reported that the Stereoplanigraph can be operated at the extreme base-altitude ratio of ultra-wide-angle photographs but that the quality of the stereoscopic image would become poor, especially in rough country, due to the relatively greater differences in perspective (greater parallax differences). The Wild Autograph A5 and the Multiplex cannot use ultra-wide-angle photographs. It appears that analytical triangulation provides a method of making use of the special qualities of ultra-wide-angle photographs in extending control. (See Ir. A. J. van der Weele's comments regarding analytical radial triangulation.) Ir. van der Weele adds, "The precision of analytical triangulation depends largely upon the possibility of stereoscopically identifying corresponding points on successive photographs. When the base-altitude ratio becomes so great that stereo triangulation cannot be executed with the Stereoplanigraph due to the lack of a good stereoscopic image, then the other methods will also prove impractical."

In any event, it seems rather certain that ultra-wide-angle photographs have no special advantage in extending supplementary control unless:

1. Such photographs are taken at altitudes above ten or fifteen thousand feet. (Below these altitudes the practical value of ultra-wide-angle photographs may be largely duplicated by the use of ordinary wide angle photographs.)
2. Precise compilation procedures are used in order to compensate for the comparatively small scale of the photographic imagery and the smaller angular errors of measurement that can be therefore tolerated.

Like all other photogrammetric procedures, stereo triangulation imposes the following well known requirements upon the lens of the surveying camera:

1. High resolving power for sharpness of photographic detail.
2. A relative aperture large enough for the proper exposure of negatives.
3. An angular field of view that permits the best possible compromise between lens distortion and a favorable base-altitude ratio.
4. Freedom from significant lens distortion in order that compilation accuracy may be obtained.

The aerial surveying lenses now in common use are considered to possess the first three requirements in adequate degree. However, they are not of even third order geodetic precision (14) because they seriously distort the angular relations of the light rays that expose the negatives. For example, the Metrogon lens, which is the best wide angle surveying lens now in common use in the United States, is reported (15) to possess angular distortions that may be as great as $4\frac{1}{2}$ minutes of arc. The author has been advised by the National Bureau of Standards that: "The values of distortion given for a Metrogon lens in the Manual of Photogrammetry (16) . . . may be considered typical. Some lenses will show much lower values and some will show appreciably higher values." At a flying height of 20,000 feet two minutes of angular distortion in the survey lens will produce errors of relative elevation in the stereoscopic model that are as large as twenty feet, if a correction for this error is not made in the optical system. Such angular distortions, if not corrected, not only produce significant errors in elevations but also greatly reduce the accuracy of relative orientation. (See paragraph below entitled "Map Plotting Instruments.")

Curvature and Refraction

With the development of ultra-wide-angle photography and photographic airplanes that can fly at altitudes up to 50,000 feet, the curvature of the earth and its atmospheric refraction can cause serious errors in stereo triangulation. Refraction distorts the light rays that expose the photographic negatives and therefore prevent the complete removal of y-parallax from the stereoscopic model. Refraction of the light rays may be caused by:

1. Change of density of the atmosphere with a change in altitude.
2. Air thermals.
3. Exhaust gases from the engines of the photographic airplane.

The curved surface of the earth is reproduced in the stereoscopic model. Figure 4 illustrates an empirical relationship (17) between flying height and errors of vertical elevation caused by curvature and refraction, for three representative types of surveying cameras.

Negatives and Diapositives (18)

Distortions of the medium supporting the emulsion of both the negatives and positive prints can cause serious errors of stereo triangulation. These distortions are due to:

1. The possible lack of flatness of the negative at the moment of exposure.
2. Mechanical tension on the film.
3. Processing errors of film and photographs.
4. Lack of dimensional stability of film base.

In order to partially control these errors, glass diapositives and careful laboratory techniques are commonly used. In addition, glass plates are sometimes used in the surveying camera. The diapositives made from the original negatives should be as large as possible. However, practical limitations prevent them from being larger than 2½ inches square if they are to be used in the Multiplex projectors. The diapositives may be as large as five inches square for a series of Multiplex projectors precise enough to operate at an altitude-contour ratio of 1:1,500. Diapositives can be made either by contact printing or by the use of a reduction printer, depending upon the nature of the surveying camera and the map plotting device.

Due to the fact that diapositives are electrically illuminated, it has been suggested that the resulting variations in temperature might be a source of significant errors. A change in temperature naturally will make a slight change in the size of the diapositive and will therefore make a corresponding change in the focal length. Regarding this problem, Ir. A. J. van der Weele says, "There are a number of measurements, especially those performed by the radial line method, which are not effected by temperature variations. With aerotriangulation temperature correction is applied together with a correction for the uniform shrinkage of the film base of the negatives. In using the Stereoplanigraph or the Autograph A5 the focal distance of each projecting camera is set so that the angle between the fiducial marks measured at the perspective center corresponds with its value at the moment of taking the photograph. The influence of temperature is thus automatically eliminated."

Errors Due to the Chief Ray Assumption

Mr. Ralph O. Anderson points out that "All the mathematical formulas of photogrammetry are based upon the chief ray conception. The use of this conception makes possible a relatively simple mathematical analysis of aerial photographs. Actually, however, this conception is not justified by the wave theory of light. The following quotation explains the nature of this contradiction.

" 'The term "ray," as we have employed it, is a purely geometrical conception but in ordinary usage a ray of light implies generally an exceedingly narrow beam of light such as is supposed to be obtained when sunlight is admitted into a dark room through a pinhole opening in a shutter. But when the experiment is carefully made to try to isolate a so-called ray of light in this fashion, new and unexpected difficulties arise that are contrary to our preconceived notions. We are disconcerted to find that the smaller the opening in the shutter, the more difficult it becomes to realize the geometrical conception which is conveyed by the word "ray." In fact, on consequence of this experiment and others of a similar kind, we begin to perceive that the statement of the law of the rectilinear propagation of light needs to be modified' (19).

"Our practical experience with analytical triangulation has made it clear that, when the utmost possible precision of triangulation is desired, it is necessary to compensate the photographs for the undesirable effects of the wave theory of light. This is accomplished by mathematically computing corrected positions of the supplementary control points used in analytical triangulation."

Physiological Limitations

The keenness of the operator's stereoscopic vision places a human limitation upon the accuracy of the stereo triangulation. This limitation is commonly expressed in terms of the sense of depth perception and the resolving power of the eyes. In stereoscopic instruments, the limit of depth perception has been found to be about ten or fifteen seconds of arc for any given magnification (20). This quantity represents the absolute limit of the operator's ability to stereoscopically estimate elevations from vertical photographs, and to transfer an image point from one photograph to another. If the additional expense could be justified, magnification of the stereoscopic model would reduce the effect of this limitation. This is why the ultra-precise European plotting instruments permit the operator to view the photographs under enlargements as high as ten diameters.

The exact center of the floating mark and any given image point are somewhat indefinite. As a result, the operator will not locate a point in space in exactly the same place twice. Ir. van der Weele reports that in actual practice it has been found that these discrepancies are of negligible importance.

Map Plotting Instruments

The optical design of the surveying camera and the projectors of the map plotting device must be carefully integrated in order to remove the effects of lens distortion. In the Multiplex system this integration has been accomplished by the use of a specially designed reduction printer. In the Wild Autograph A5, this integration may be accomplished by placing the photographs behind glass plates which have been specially ground in order to compensate for the distortion caused by a given lens. In the Zeiss Stereoplanigraph, the photographs are re-projected through lenses possessing optical characteristics similar to those of the taking lens.

Interior Orientation

It is not possible to recover exactly the perspective centers of diapositives, when they are placed in a map plotting instrument. This condition imposes a further limitation of a minor nature upon the precision of stereo triangulation.

Relative Orientation (21)

Twin point interpolation is used with great success for integrating the stereoscopic model. Even under favorable conditions, however, the accuracy of this procedure is limited by the operator's inability to identify and remove small amounts of y-parallax. The resulting relative orientation therefore generally contains small residual errors. These residual errors represent the sum of all the above described shortcomings in the photogrammetric procedure. In a single stereoscopic model these errors generally are not important. A series of consecutive stereoscopic models, however, will tend to accumulate these errors in the manner illustrated in Figure 2.

Establishment of Scale

After relative orientation has been completed, the scale of the model must still be established. This is done by recapturing the position and elevation of each of the three pass points established by the last preceding model. Errors in scale cause errors in both the position and elevation of all subsequent pass points.

III. Controlling Errors of Stereo Triangulation (22)

We have seen from the paragraphs immediately above that the quality of the supplementary control produced by stereo triangulation may be considerably improved by using only the very best of the Metrogon lenses for topographic mapping, by using contact size diapositives, by taking photographs with only fifty-five percent overlap in order to increase the base-altitude ratio, and by using ultra-precise map plotting equipment that permits the operator to view the photographs under considerable magnification. The accuracy of the supplementary control obtained as a result of these improvements would still not be entirely satisfactory. For this reason, many attempts have been made to reduce the significance of these errors of stereo triangulation by the use of:

1. Measurements of flying height obtained by the use of a barometer.
2. Measurements of absolute tilt obtained by the use of a gyroscope, level bubble, or various photogrammetric methods.
3. Various methods of radio triangulation.
4. Various methods of graphical and mathematical adjustment involving the use of the so-called *BZ* curve, tilt curve, and various linear and quadratic formulas.

In the next few paragraphs the various methods of controlling errors of stereo triangulation will be discussed in detail. By way of introduction to this subject, it should be noted that in making accurate measurements of the location, tilt and flying height of the photographic airplane, known standards of reference must be available. For instance, the level bubble gives measurements of tilt referred to the earth's gravity. The barometer gives measurements of relative flying height referred to the density of the air. We know of only a few physical phenomena that provide us with references for measuring tilt, altitude, and geodetic position. These references are generally variable and rather difficult to measure precisely.

Measurement of Flying Height

In the United States the aneroid barometer is the only commonly used instrument for measuring the flying height of the photographic airplane. The

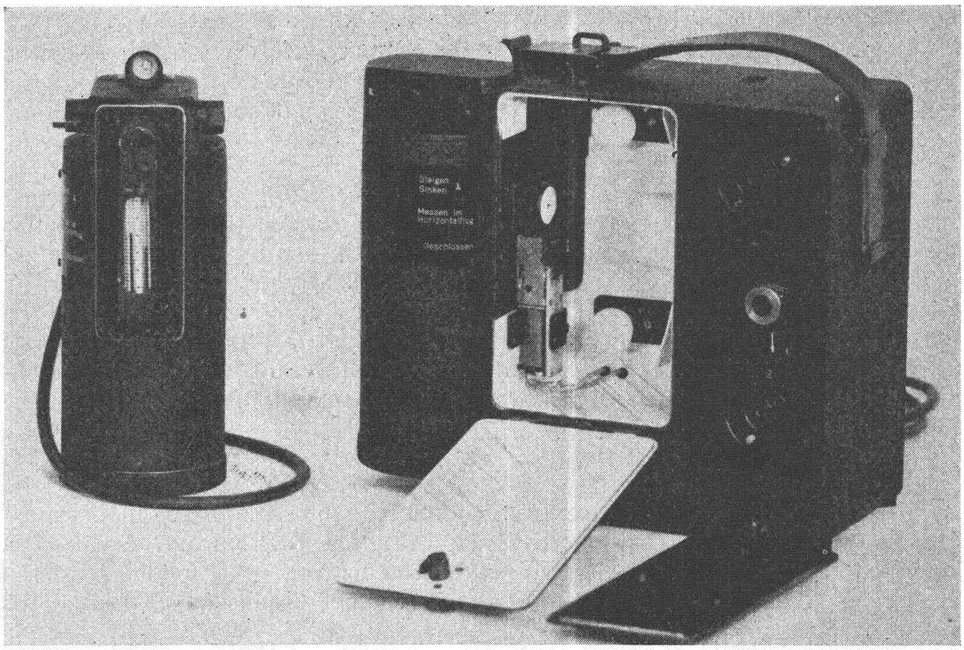

FIG. 5. Wild Recording Statoscope (right), and auxiliary statoscope (left) used by pilot as an aid in maintaining a level flight line.

altitudes thus obtained lack the accuracy that is desirable for large scale topographic mapping. Tests (23) have indicated that, with photographs taken at 20,000 feet, 67 percent of the relative altitudes in one flight will be accurate within plus or minus twenty five feet, and 33 per cent will have larger errors but no greater than seventy five feet. These errors are mainly due to the variable density of the atmosphere along the flight line. The radio altimeter is also used in this country, but information regarding this instrument is not now available to the author.

In Europe the recording statoscope, a form of barometer, appears to be generally used to register differences in flying height of the airplane between the exposure stations. The statoscope was invented by the Finnish physicist Vaisälä (24) and has been made in various forms. The Wild statoscope is illustrated in Figure 5. The construction of this statoscope is illustrated by the schematic diagram of Figure 6. The thermos bottle, A, contains an air chamber, B, which is kept at a constant temperature of 0° Centrigrade by the surrounding mixture of ice and water, W. The manometer tube, M, contains a fluid, F, which indicates changes in the pressure of the surrounding atmosphere. A three way valve, V, is used to adjust the statoscope. In Figure 6, this valve is shown in the operating position that is used when the airplane has reached the desired altitude. The auxiliary views show the valve in the closed position, at V', and in the position for ascent and descent, at V'' (25).

The readings of the statoscope are best evaluated according to the empirical formula:

$$H = h(1 + \alpha \cdot t)\left(0.15 + \frac{1180}{B} c\right) \tag{1}$$

where: H represents the change in elevation from one exposure station to another.

h represents the statoscope reading.

$1+\alpha\cdot t$ represents the temperature coefficient, where α is the heat expansion of the air at 1° Centigrade and t is the air temperature in degrees Centigrade.

B represents the air pressure in millimeters of mercury.

c represents the specific weight of the manometer liquid. (Amyl alcohol ($c=0.82$) is used in the Wild Statoscope.)

Statoscope readings will contain accidental errors (due to the lack of uniformity of the air pressure) and systematic errors (due to the mathematical limitations of the empirical formula, the inertia of the airplane, etc. (25).

Minor changes in the flying height of the airplane, due to air currents and air pockets, have especially undesirable effects upon statoscope readings. Because of its inertia, the airplane does not react immediately to these influences, which are promptly registered by the statoscope. The statoscope therefore registers a change in atmospheric pressure that does not correspond to a change in the actual flying height (26).

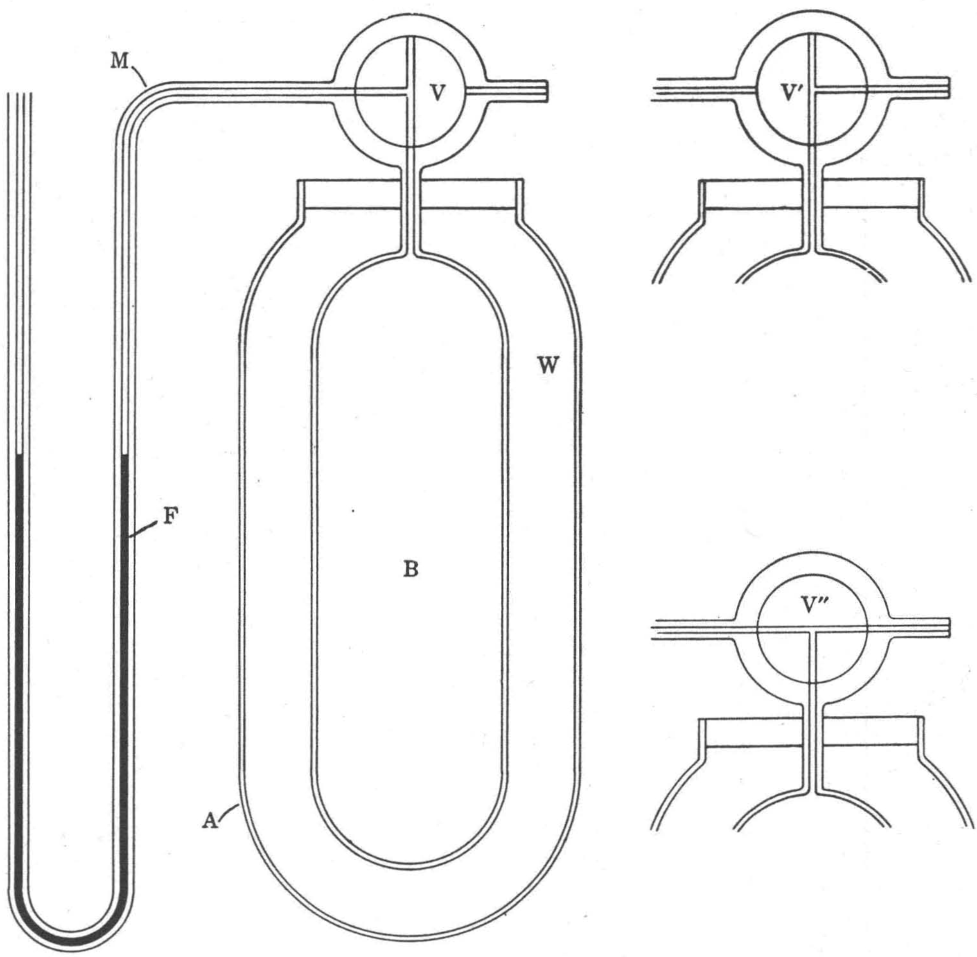

FIG. 6. Schematic diagram illustrating construction of the Wild Statoscope.

Prof. Dr. Max Zeller reports that he has made careful tests of elevations determined by the statoscope, and has determined that median errors of 1.7 meters are to be expected under "normal" meteorological conditions (27).

Measurement of Absolute Tilt (28)

The gyroscope is the most precise device commonly used for measuring the tilt of photographic airplanes. The Sperry Gyroscope Company reports that "tests reveal our Roll and Pitch Signal Transmitter will hold the vertical within one-quarter of a degree. This figure applies to bench tests and we are not prepared to guarantee that accuracy under flight conditions." In actual practice it has been found that this type of gyroscope will continue to rotate in a plane parallel to the plane of its starting position with an error of less than thirty minutes of arc. As the gyroscope is moved horizontally away from its starting position, the plane of its rotation is no longer parallel to the earth's surface. It is therefore clear that gyroscope readings must be adjusted for the distance traveled by the airplane away from its starting point. For many photogrammetric purposes tilt can be measured and recorded with sufficient accuracy by use of the gyroscope, but these measurements have little value in stereo triangulation. Stereo triangulation requires that tilt be known within about thirty seconds of arc.

Level bubbles have sometimes been used to measure the approximate tilt of photographic airplanes. For all practical purposes this method of measuring tilt is worthless because the position of the liquid in the level bubble is greatly affected by unknown horizontal components of acceleration. A vector analysis of the problem clearly shows that gravitational effects due to tilt are exactly equivalent to the inertia effects due to horizontal acceleration. These two effects cannot be separately measured when both are unknown.

The earth's horizon may be photographed in order to provide an indication of tilt. However, mountains may obscure the horizon and clouds and haze will tend to create false horizons, thus giving erroneous values of tilt. Nevertheless, Löfström reports (29) that under favorable conditions tilt may be measured very accurately by the use of horizon cameras. In this country it has been found that tilt can generally be measured directly from the horizons recorded on the obliques of trimetrogon photographs with an error of ± 15 minutes of arc under favorable conditions. The magnitude of these errors of direct measurement has been verified by the photoalidade. Except under rather limited conditions it therefore appears to the author that determining tilt by measuring horizons is not generally sufficiently accurate for the needs of stereo triangulation.

It is reported that Santoni of Italy has developed a sun camera that is useful in measuring tilt. This method is handicapped by the fact that the component of tilt about the line between sun and camera is not measured.

Various graphical and mathematical methods are available for recovering the tilt and flying height of the photographic airplane, when both horizontal and vertical ground control is available. As far as the author has been able to determine, these methods are not used in connection with stereo triangulation.

Radio Triangulation (30)

It does not appear that radio triangulation can ever directly provide the position and elevation of supplementary control with any degree of satisfaction. In establishing such control, radio triangulation can therefore serve only as an auxiliary device in photogrammetric triangulation.

Shoran, the best known of the radio triangulation procedures, is an electronic method of measuring distances in terms of the minute intervals of time that it takes for a radio signal to be transmitted to certain "transponding" stations and returned to the sending station. These intervals of time are measured and recorded by shoran equipment in the survey airplane and later can be converted into the equivalent distances by the use of a rather elaborate mathematical method of computation and adjustment. There are two different methods of using shoran equipment for obtaining control. The geodetic method (31) requires the use of several transponding stations. The position of at least two of these stations must be known with considerable precision. As the shoran equipped airplane flies midway between each pair of transponding stations, it takes a series of shoran observations which can be used to compute the ground distance between that pair of transponding stations. These various ground distances can be used to compute the geodetic position of each of the transponding stations. The accuracy of the resulting computations depends to a considerable extent upon the ability to compensate for errors by taking readings between a large number of transponding stations. The photogrammetric method (32) of using shoran makes it possible to obtain supplementary horizontal control for photogrammetric surveys. In this method, the positions of two transponding stations are known and the distances to the survey airplane are computed for each photographic exposure station. The identification of the exposure stations is accomplished by locating the nadir points of the corresponding vertical photographs. The accuracy of this method partly depends upon the precision with which the nadir point is located on each of the photographs. At this time it is not considered practical to determine the elevations of control stations by the use of shoran.

Other methods of radio triangulation are in various stages of development. These include Distar (33), Decca (34), and Raydist (35). Regarding Raydist, Mr. Andrew Hacskaylo, Liaison Engineer of the Hastings Instrument Company, provides the following information: "A Raydist installation for determining the position and elevation of a photographic airplane would consist of:

1. A continuous wave transmitter in the photographic airplane.
2. One continuous wave transmitter fixed on the ground at some convenient location.
3. Three fixed receivers tuned to receive both transmitted signals. These instruments may be located at convenient positions on the ground, in the general vicinity of the area to be surveyed.
4. A method of transmitting the pair of signals received by each receiver to a computing device consisting of a phase meter and counter. For convenience, the three computing devices are located with the fixed transmitter. (The Raydist system could be arranged in such a way that the computing devices could be located in the photographic airplane.)

"If the transmitter in the photographic airplane is moved in such a manner that the rate of change in distance from two of the receivers remains constant, then no difference will be observed at either receiver in the phase relationship of the two broadcast signals. This type of path is a hyperbola with the two receivers as foci. Therefore, each of the three pairs of receivers are the foci for a family of imaginary hyperboloids (hyperbolic surfaces of revolution). The intersections of the three independent families of surfaces represent three dimensional coordinates of position in space. The positions of the photographic airplane would be known at all times with respect to these coordinates.

"When the photographic airplane is on the ground, the phase difference between the pairs of signals received at the three receivers is fixed. As the airplane moves, however, the signals received by the three receivers go in and out of phase at a rate proportional to the velocity with which the airplane moves with respect to the four fixed ground stations. The three phase meters count and record these changes in phase.

"The Hastings Instrument Company states in its Bulletin R-19 that 'Our goal is to make measurements to 1 inch in a mile, a little better than 1 part in 50,000. We expect to achieve that goal. Apparatus has been built that has consistently repeated to better than one foot in a mile. . . .'

"Raydist is not limited to line-of-sight distances, as is shoran. Many variations of the above described Raydist system have been developed for other special applications."

Adjusting Errors of Stereo Triangulation

It appears that much fundamental research is being performed on methods of adjusting the errors of stereo triangulation—especially in Switzerland. While these investigations are not complete, enough information has been published to give us a fairly good idea of the nature of the methods that are being developed.

Prof. Dr. Mac Zeller reports (36) that errors of stereo triangulation may be classified and corrected in the following manner. (Please allow for difficulties of translation.)

1. x-errors (errors in the direction of the flight line). Here the accidental and systematic errors of the scales are decisive. The former are to be adjusted linearly while the latter increase with x_2. The adjustment formula therefore reads as follows:

$$\Delta x = c_1 + c_2 x^2. \quad (2)$$

2. y-errors (errors at right angles to the flight line). Accidental and systematic errors of tilt as well as errors of the scale effect the errors at right angles to the flight line. Of these, only the systematic errors of tilt show quadratic progression. The formula of adjustment therefore reads:

$$\Delta y = c_3 x + c_4 x^2 + c_5 xy. \quad (3)$$

3. In the case of errors of elevation, relatively large accidental errors must always be expected in the direction of the flight line. These errors directly influence the elevations so that a simple adjustment, as in the case of the x and y errors, is no longer possible. The following formula for the adjustment of elevations allows for the curvature of the earth:

$$\Delta H = c_6 x + c_7 x^2. \quad (4)$$

The above described methods of analyzing and correcting systematic and accidental errors are rather involved. The methods of using the equations (2), (3) and (4) are described in the above cited reference.

Prof. Dr. Zeller reports (37) that the methods of stereo triangulation developed at the Photogrammetric Institute require the use of field control only at the beginning, end, and middle of strips of photographs up to 100 km long. (See Section IV below, for the results obtained.)

The author has not been able to obtain information regarding the methods of adjusting photogrammetric triangulation that are used in this country. It is generally understood, however, that these methods are of a rather simple nature.

IV. Results Obtained with Stereo Triangulation

For all practical purposes the wide angle Multiplex is the only mapping device that is commonly used in the United States for the bridging of control.

This equipment is manufactured by Bausch & Lomb Optical Company according to the designs of the U. S. Army Engineers or the U. S. Geological Survey. Actual tests (38) show that the average error of vertical control obtained by Multiplex bridging of control is about 0.4 mm at compilation scale. This degree of accuracy depends upon the use of the methods of adjustment previously described. Mr. Gilbert G. Lorenz of the Engineer Board says, "The average multiplex error of 0.4 mm at compilation scale as stated in TM 5-244 will hold for bridges up to sixteen models. With 20,000 foot photographs this means a triangulation of 34 miles in length with an average error of 22 feet. The probable error, therefore, is 19 feet which compares very favorably with the 10 to 20 foot accuracy claimed for the Autograph. . . . If the resulting accuracy of Multiplex aerotriangulation will serve the purpose at hand, a strip of 16 models can be oriented and adjusted by one man in approximately 16 hours. This work is not difficult for an experienced operator and the time required certainly is not excessive," It will be seen from Mr. Berchtold's table of ratios given in Section V that this degree of accuracy allows the plotting of 90 foot contour intervals.

In Europe stereo triangulation is commonly used for obtaining supplementary control. Mr. Edwin Berchtold says: "One of the most important advantages of our precise European machines is their qualification for aerotriangulation. The higher the precision of a machine, the longer the distance over which orientation can be carried forward. Strips thirty miles long triangulated in the Wild Autograph A5, for instance would (after adjustment) allow for a probable altimetric error of ten to twenty feet only. The limit of ten feet is possible only under favorable conditions, otherwise a maximum error of as much as ± 20 feet should be tolerated. . . . These figures are based upon several tests made in Switzerland using cameras with angular fields of 55° and 60°. The average flying height was 16,000 feet. In this case thirty five overlaps were necessary for a strip of thirty miles. The probable error in height has been determined to be 5.5 feet in the case of the plate camera and 7.7 feet for the film camera. These were carefully compensated aerotriangulations. The same care will not always be taken in actual practice, and for that reason I have indicated somewhat larger errors (are to be generally expected). It is sure that these errors of aerotriangulation will be reduced when wide angle cameras and a greater flying height are used. Tests of this kind are being made now."

The method of adjusting errors of stereo triangulation developed by the Photogrammetric Institute in Zurich has shown excellent results. The elevations of the test strip Limpach-Burgistein possessed a median error of only ± 1.6 m after adjustment. The strip of photographs taken with the Wild 210 mm focal length aerial surveying camera and published originally by Pastorelli and Blachut showed a median elevation error of ± 3.6 m after adjustment by this new procedure. Finally, stereo triangulation of a strip of photographs 100 km long taken 5,000 meters above the ground with a Wild 165 mm focal length surveying camera resulted in a median error of elevation of ± 4.9 m after adjustment The strip consisted of 72 models and contained field control only at the beginning and end of the strip. Regarding these results Prof. Dr. Zeller reports " . . . they certainly will be improved in the future because . . . (the newly discovered) . . . method of taking into consideration twists of the models has not been used yet"(39).

Ir. A. J. van der Weele says, "Our country (Holland) is so flat that only an accuracy expressed in centimeters has any sense. This vertical accuracy is beyond the reach of economical photogrammetric methods. Nevertheless, we have found that, in country that is fairly rough, stereo triangulation is nearly always cheaper than terrestrial methods of height estimation and gives in most cases sufficient accuracy.

"It is indeed true that European countries commonly use radial and/or stereo triangulation in their mapping. Holland, Belgium, Germany and Switzerland all have a rather good network of trigonometric control. Except for Switzerland, the cadastral surveys of these countries was executed long before the triangulations were completed, so that the density of known points is far less than in Switzerland. This accounts for the fact that, as literature shows, in the first three above named countries photogrammetric methods are used to furnish the supplementary control necessary for the plotting of topographic detail. Naturally, these triangulations in general do not bridge long distances."

These comments by Ir. van der Weele, Mr. Berchtold and Prof. Dr. Zeller seem to sum up the European experience with stereo triangulation. It should be quite clear that the accuracies reported by these authorities depend upon the use of precise equipment, precise operational procedures, and precise methods of adjustment. It is well known that the precision of the European type of instruments depends partly upon extremely precise methods of construction. Additional precision is gained by enabling the operator to view the photographs under enlargements as high as ten diameters. The original negatives may be used in the plotting instrument as well as contact size diapositives. (Ir. van der Weele says, " . . . according to our experience glass diapositives are necessary for accurate work, especially in countries like Holland where changes of temperature and humidity are great and frequent.") A considerable number of the precise European plotting instruments are now in use in the United States.

Mr. Leon T. Eliel, Vice President of Fairchild Aerial Surveys, Inc., makes the following comments regarding this subject: "We ordinarily provide four control points of known position and elevation per stereoscopic model. While, as you say, three points of elevation and two points the positions of which are known, should be adequate to cover a stereoscopic model, as a practical proposition this isn't true. In case there is any mistake in the identification of one point, or in the elevation of one point, there is no very satisfactory way of determining this. In horizontalizing a stereoscopic model, the pattern formed by the horizontal control is almost as important an index as to what is wrong with the model as are the elevations determined by measuring to the vertical control points. If, for example, there are only three known vertical control points, any stereoscopic model can be adjusted to hit any three vertical control points. Thus, if one of them is in error, the plane can be made to pass through these three points in a false position. For this reason, we always desire, if possible, four known elevations per model, and we desire these known points to be substantially in the corners of the models, where each point serves in four models; that is two consecutive models out of each strip. We nearly always find that there is not too much difference in the cost of establishing both horizontal and vertical control over the cost of establishing just one type of control. While this is not universally true, it usually is. We for example, use trigonometric leveling in determining the elevations of our points, and this trigonometric leveling is done by taking vertical angles with our one-second Wild Theodolites at the same time the horizontal control system is being put in. If the terrain is not susceptible to triangulation and the control has to be put in by traverse and level, it is usually just as practical to run the lines over identical courses as it is to try to economize by perhaps doing a little skipping around in the leveling.

"As to aerotriangulation, we have found that in order to meet a given vertical tolerance when attempting to utilize aerotriangulation, it is necessary to fly at a lower elevation; in fact, a very substantially lower elevation. Thus, with the flying at a larger scale, it is necessary to take a great many more pictures and to run a great many more models through the plotting equipment. At this larger scale we have found that you have to establish as much or more ground control per square mile, even when bridging, because of the fact that so much less area is included in each model. There is definitely a point of diminishing return in this sort of business, and we have also found that all of the delay and fuss and bother in plotting the aerotriangulation systems, going back and readjusting it, etc. etc., is an expensive and often not an entirely satisfactory process.

"We have done a great deal of aerotriangulation when it has been necessary. This was expecially true in the tremenduous amount of mapping we did for the Armed Forces during the war. Under these conditions the Army was not able to supply us with an ample amount of control, and we found it necessary in order to draw maps at all to do a great deal of aerotriangulation. Thus, while aerotriangulation must be always considered as a necessary part of any photogrammetric system, to be used judiciously under conditions when the ground cannot be occupied for conventional ground surveys, it is something to be avoided where possible.

"I am not in a position, without considerable search back into our records, to give you the exact figures on aerotriangulation versus control in every model, but we have run these tests in the past, and are so convinced that it is uneconomical that in recent years we have given it no consideration excepting under conditions of military necessity.

"We believe that efficiency in stereoscopic plotting is achieved by having a system that permits flying very high, thus minimizing the amount of control which must be established, As you probably know, we habitually fly 1,250 times the contour interval.

"There is one case I haven't talked about, and that is . . . where the contour interval might, for example, be 50 feet. Obviously we cannot fly 1,250 times 50 feet, and under such specifications there might be some justification for bridging control. I may add, however, that excepting in military mapping there has never been an occasion in our history to draw a map with contour intervals as great as 50 feet."

We can see from the preceding portions of this article that stereo triangulation is a fairly common method of obtaining supplementary control—at least under certain conditions. The potential value of stereo triangulation for use in the United States is not clearly established by the evidence now available to the author. This lack of certainty is partly due to the fact that we do not know for sure the nature of the photography and the plotting equipment that will be available to us in future years.

In view of the fact that production records are not available, even for the mapping activities of the U.S. Federal Government, it appears to the author that the minimum contour-altitude ratio of the various map plotting devices provides about the only practical basis for comparing their potential accuracy and estimating the relative cost of the supplementary control obtained by their use. This potential accuracy can be more exactly stated in terms of the mean square error of elevations. Mr. Edwin Berchtold says, "I should prefer to fix the obtainable precision of elevation by the mean square error which will be for contours plotted with the A5, 1/5000 of the flying height, under favorable conditions. The probable error being about $\frac{2}{3}$ of the mean square error, it is easy to compute the corresponding proportion with regard to the probable error. Or taking the contour intervals as defined in the table of ratios (see Section V) this interval will be 3.30 times the mean square error. Therefore, for the A5 the contour interval will be 3.3 times 1/5000, which is 1/1500 of the flying height. For well defined points (vis. marked by special signals etc.) the mean square error will be only 1/10,000 of the flying height. On the other hand, under unfavorable conditions the mean square error of contour lines may increase up to 1/2500 of the flying height." In other words, the minimum contour-altitude ratio is 1:1,500 under favorable conditions and 1:750 under unfavorable conditions. The minimum contour-altitude ratio for the Multiplex is approximately 1:800 under favorable conditions and approximately 1:500 under unfavorable conditions.

In contrast to these two points of view, Ir. A. J. van der Weele offers the following comments: "As to the most practical basis for a general comparison of various aerotriangulation instruments, I should say that neither the minimum contour-altitude ratio, nor the mean square error in height give a good standard. These quantities give only an idea about the accuracy with which it is possible to measure heights or to draw contours in one stereoscopic pair of photographs. For an evaluation of the accuracy of heights determined by triangulation, there are a number of factors which influence the results and which are not brought into account when using the above mentioned standard. I could mention for instance the partly unknown systematic errors of the instruments and, especially for the Multiplex, the differences in the distortion curves of the lenses by which these systematic errors are different for each projector, the number and location of the supplementary control points, the number of photographs in the triangulation, and other important things.

"A real comparison can, to my view, be accomplished only by the mean or probable error in the observation of both x (horizontal) and y (vertical) parallaxes in each instrument; and by a thorough evaluation of the instrumental and systematic errors of these instruments. The parallax observations are the fundamental elements of the triangulation. Their influence on the point positions can be investigated through the appliance of the law of propagation of errors which will have almost the same consequences for most types of theoretically precise plotting instruments."

V. The Accuracy Required of Stereo Triangulation

In the foregoing discussion we have seen that the accuracy of supplementary control obtained by stereo triangulation varies considerably depending upon the nature of the photographs, plotting equipment and procedures used, and the amount of ground control that is available. The control obtained by stereo triangulation is satisfactory only if it permits the plotting of topographic detail with the required accuracy at a reasonable cost. One of the important objectives in stereo triangulation therefore is to secure measurements that are correct within certain specified limits of error. This section will be devoted to a discussion of the nature of these limits.

In stereo triangulation as in other forms of surveying, every observed or measured quantity contains errors due to a variety of causes and, as a result, a measurement is never absolutely exact. For a single quantity which has been determined by observation, the total error can never be determined exactly, but it can nevertheless be fixed within certain probable limits. These errors are made up of many individual errors, both systematic and accidental in nature.

SYSTEMATIC ERRORS (40) may be instrumental, personal or natural in character. Since they always follow some definite mathematical or physical law, a correction can be determined and applied if the law is known. Consecutive measurements of the same quantity are all assumed to possess the same amount of systematic error. Where the law governing the occurrence of any given kind of systematic error is not known, such errors must be treated as accidental.

ACCIDENTAL ERRORS (40) are due to a combination of causes beyond the ability of the observer to control and for which it is impossible to make correction. For each observation the value of the accidental error is a matter of chance and therefore cannot be computed as can the systematic errors. However, accidental errors taken collectively obey the law of probability. Accidental errors remain after mistakes have been eliminated by checking and systematic errors have been eliminated by correction.

When a series of observations of a single quantity is made under uniform conditions and in such manner as to eliminate systematic errors, the mean of the series is the most probable value of the quantity sought. For the purpose of determining the probable error this mean value is mathematically regarded as being the true value, and the difference between each of the individual measurements and mean value is determined. These differences are termed the residuals or deviations. In order to estimate the probable accuracy of a quantity it is necessary to rely upon the theory of least squares. This theory indicates that the probable error is a function of the square root of the sum of the squares of the residuals. The values of accidental errors are commonly estimated according to one of the two following methods:

1. Probable error. If n observations provide readings $x_1, x_2, x_3 \ldots x_n$, which have been corrected for known errors and whose arithmetical mean is y, then the probable error is defined as:

$$r = 0.6745\sqrt{\frac{\Sigma v^2}{n-1}}, \tag{5}$$

where Σv_2 is the sum of the squares of the residuals, and n is the number of observations.

2. Mean square error. If $x_1, x_2, x_3 \ldots x_n$ are observations of a quantity (all known errors having been eliminated and the residual errors being accidental) the arithmetical mean of which is y, the mean square error is taken as:

$$\epsilon = \sqrt{\frac{\Sigma v^2}{n-1}}. \tag{6}$$

The above formulas are based upon the assumption that a large number of measurements of a single quantity have been taken. The results of experiments indicate, however, that they may be applied to a limited number of observations with good results. It seems doubtful if they can be consistently applied to a series of observations containing less than ten measurements.

In scientific work it is frequently necessary to diminish as far as possible the harmful effects of those small errors which always appear even in the most carefully made measurements. The Method of Least Squares was perfected for this purpose. Its importance is obvious upon reflection that there must always exist a limit of precision beyond which human measurements cannot go. It happens almost always that the degree of precision required in scientific measurements is somewhere near this unattainable limit.

Regarding this subject of errors, Mr. O. M. Miller submits the following pertinent quotation (41): "Whether we should use the mean square error or the probable error in stating the precision is largely a matter of taste. Gauss says:

'The so-called probable error, since it depends on hypothesis, I, for my part, would like to see altogether banished; it may, however, be computed from the mean by multiplying by 0.6744897.' On the other hand, the International Committee of Weights and Measures decided in favor of the probable error: 'It has been thought best in this work that the measure of precision of the values obtained should always be referred to the probable error computed from Gauss' Formula, and not to the mean error.'

"In the United States, in the Naval Observatory, the Coast Survey, the Engineer Corps, and the principal observatories, the probable error is used altogether. So, too, in Great Britain, in the Greenwich Observatory, the Ordnance Survey, etc. In the G. T. Survey of India the mean square error is used, for the reason given by Gauss above. Among German geodeticians and astronomers the mean square error is very generally employed.

"The probable error has a definite meaning, namely, that the chances are even for and against a given error being greater or less than the corresponding probable error. This frequently furnishes a convenient test as to whether the errors of a given series of observations are distributed according to the assumed law of error."

Ir. A. J. van der Weele reminds the author that "It is well known that the probable and mean errors are not real errors for which a correction is possible. They are simply an indication of the limits between which the measurements of a certain quantity will vary with an estimated amount of probability. The PROBABLE VALUE of a quantity therefore is that value for which the sum of the square residuals is a minimum. The accuracy of this value is indicated by the size of the probable or mean error. These errors are therefore no indication of the real amount by which the probable value is in error."

The various measurements made in the stereo triangulation cannot be individually corrected for probable or mean square errors because their signs are not known.

MISTAKE is the unintentional fault of conduct arising from poor judgment or confusion in the mind of the observer. Mistakes have no place in a discussion of the theory of errors. They are detected and eliminated by checking all work. A DESCREPANCY is the difference between various measurements of the same quantity.

Several American photogrammetrists have suggested to the author that a discussion of errors has little significance to photogrammetrists. It is said for example "In extending control by stereo triangulation, the control is never re-extended nor re-observed to determine its probable error. Until photogrammetric equipment and methods approach the refinements of those used in ground surveys, a discussion of probable errors as applied to photogrammetry does not have much significance." In Europe an entirely different viewpoint is commonly accepted. For example, Ir. A. J. van der Weele says, "I object to the remark that the theory of errors has no significance in photogrammetry. It may be true that the observations are of a far less accurate type compared with normal geodetic measurements. This is all the more reason why it is necessary to use all available expedients for adjusting the measurements for which the triangulation was executed. Likewise, it is necessary to form an idea of the propagation of errors and their accumulation. All this can only be done in the right way—by applying the theory of errors and the method of least squares. It is not necessary that these methods be applied at each step in aerotriangulation. It will be sufficient for most purposes that the influence of errors has been theoretically investigated and that the practical measurements take into account the results of the theoretical investigation. This results in a harmonic tuned whole of observations.

"The theory of errors has practical application to photogrammetric triangulation, especially. In these procedures there are always a certain number of superfluous observations. For example, in using the radial triangulator each direction is read four or eight times and moreover there is one superfluous direction in each rhomboid. In adjusting the resulting observations one therefore has an opportunity to compute a trustworthy value for the probable error. In stereo triangulation each setting of a pair of photographs could be performed with parallax observations in five points only. In actual practice, however, six points are always observed. In addition, a careful search is made throughout the entire model to discover any residual *y*-parallax. Although in this case the probable error is not computed, the final adjustment of the stereo triangulation is made after so many superfluous observations that the result is surely not so bad and uncontrolled as some of your American photogrammetrists might believe."

Regarding the subject of errors of stereo triangulation, Prof. Dr. Max Zeller (42) describes a method of measuring elevations of pass points that he recommends. Briefly, this procedure consists of measuring the elevation of each pass point three times, progressing from one point to another in a specified order. After the elevations of the six pass points in a stereoscopic model have been determined in this fashion, then the elevations of each of the six pass points is again determined—using the reverse order of examination. According to this procedure, the elevation of each pass point is determined SIX TIMES.

The accepted United States standard for horizontal map accuracy in peacetime mapping is that 95 percent of all well defined planimetric features shall be shown within 0.02 inch of their true position at publication scale of the map (43). The accepted United States standard for vertical map accuracy in peacetime mapping is that ninety percent of all errors of elevation shall be within one-half the contour interval (43). These compilation requirements represent a maximum limit to the errors allowable in stereo triangulation. It is quite obvious that the average accuracy of stereo triangulation must be considerably better than this. By now it probably has become clear to the reader that the term "stereo triangulation" is rather meaningless unless the degree of precision is indicated in some manner. The following standards of accuracy for terrestrial surveys have been generally accepted in the United States: (44)

Class	Horizontal Accuracy	Vertical Accuracy	Prob. Error in Base Line Accuracy	Angular Precision
First Order	1 in 25,000	$0.017\sqrt{\text{length of line in miles}}$	1 in 1,000,000	1″
Second Order	1 in 10,000	$0.035\sqrt{\text{length of line in miles}}$	1 in 500,000	3″
Third Order	1 in 5,000	$0.050\sqrt{\text{length of line in miles}}$	1 in 250,000	6″

This standard of accuracy does not seem to possess much practical value in discussing errors of photogrammetric triangulation. This is due to the fact that errors are expressed in terms of linear distances. In analytical triangulation and graphic radial triangulation the network of control is established as an integrated unit. It does not appear that the errors are directly proportional to the average distance between control stations. In stereo triangulation the errors are proportional to the number of models involved in the bridge.

Mr. Edwin Berchtold provides the following comments and table of ratios:

"When discussing the precision of aerotriangulation methods, some clear definitions should be given of the different kinds of errors. In some American publications the tolerable limits of errors are defined by saying that ninety percent of all errors should be within one-half of the contour interval. Sometimes this tolerable limit is referred to as the AVERAGE ERROR, but in Europe we prefer the MEAN SQUARE ERROR. The MAXIMUM ERROR cannot be computed, but in Europe we state that the maximum error be equal to three times the mean square error; the law of probabilities shows that only one of 370 accidental errors would reach this "maximum." The PROBABLE ERROR is defined by the value which separates the errors in two equal halves, one half containing all errors which are smaller, the other half all which are greater than the probable error. The relations between these different "tolerable limits of error" are given in the following table.

Average error (arithmetic mean)	*1.00*	1.18	0.80	0.27	0.48	0.24
Probable error (limit containing 50% of all errors)	0.85	*1.00*	0.67	0.22	0.41	0.20
Mean square error (geometric mean)	1.25	1.48	*1.00*	0.33	0.60	0.30
European "Maximum error" (1 of 370 errors)	3.75	4.44	3.00	*1.00*	1.81	0.90
American limit (contains 90% of all errors)	2.07	2.45	1.65	0.55	*1.00*	9.50
American contour interval	4.14	4.90	3.30	1.10	2.00	*1.00*

"In the above tabulation, the same ratio exists between the various figures of each column. The difference between the various columns is that, in each case, a different element has been set equal to unity. Errors of map position may be either accidental or systematic. Strictly speaking, the relationships set forth in the above table are applicable only to accidental errors and are correct only for a great number of observations. It has been found that, for lack of a better rule, we may safely assume that these relations hold for both accidental and systematic errors of the map plotting procedure. We thus have a practical ratio by means of which we may compare the above five kinds of errors."

VI. Conclusions

The accuracy of stereo and analytical triangulation depends upon:

1. The accuracy with which supplementary control points can be identified and transferred from one photograph to another.
2. The accuracy with which photographic distances between supplementary control points can be measured or compared.
3. The accuracy with which accidental and systematic errors may be reduced to their lowest possible values.

Under favorable conditions supplementary control points can be identified and stereoscopically transferred to each of a series of overlapping photographs with a maximum probable error of not more than approximately ±0.001 inch (approximately one minute of arc). This is certainly a low order of geodetic precision. Photographic distances between supplementary control points can be measured or compared with an error not much better than 0.001 inch. This is somewhat less than the second order accuracy of 1:10,000. Stereo triangulation appears to have a definite advantage over analytical triangulation due to the fact that a careful search can be made throughout the entire model for *y*-parallax. On the other hand, analytical triangulation appears to have a definite advantage over stereo triangulation in the reduction of systematic errors due to lens distortion, curvature of the earth, and the refraction of the atmosphere. Otherwise, the reduction of the systematic and accidental errors must be accomplished by approximately the same methods. It would appear quite certain that the accuracy of photogrammetric triangulation will not be of a higher order than the accuracy of the separate component operations after they have been mathematically adjusted. It is therefore likely that second order accuracy of 1:10,000 is about the maximum limit of photogrammetric triangulation unless some supplementary method, such as radio triangulation, is used to maintain a higher degree of accuracy. Apparently this limit of 1:10,000 is already being approached by certain photogrammetric triangulation procedures.

The comparative value of the various methods of obtaining both vertical and horizontal control would seem to depend largely upon the cost of the necessary equipment, upon the availability of qualified personnel, and upon the accuracy of the results. It is well known that the best stereoscopic map plotting equipment is expensive and requires highly skilled operators. Further development of analytical triangulation in this country probably will make it clear that precise equipment and skilled personnel must also be used in this procedure for the best results.

As the precision of our map plotting equipment and procedures increase, the number of photographs that are required to adequately cover a given area decreases. (The flying height of the photographs increases a corresponding amount.) For example, if the accuracy (in terms of the contour-altitude ratio) of the Multiplex were doubled, then only twenty five percent of the customary number of photographs would be required for a given project. *Of course, the amount of supplementary control required would be reduced correspondingly.* However, Ir. A. J. van der Weele makes it clear that the cost of the photogrammetric compilation is not reduced by the same proportion. He says, "When the time required for establishing relative and absolute orientation of a stereoscopic pair of photographs is compared with the time required for the plotting of all the desired topographic detail, it will be found that this relation will vary between 1:1 and 1:10 and more. Let us assume 1:5 as a mean value. It will now be clear that, when the detail in a given area is plotted from two pairs instead of one pair, the time needed for the orientation of the two pairs will be doubled, while the time needed for plotting detail will remain approximately the same. The total cost of these two operations will increase about 17 percent. The cost of the extra photographs will increase about 40 percent, and the stereo triangulation will be twice as expensive. (This rule also applies to other forms of photogrammetric triangulation.) It is obvious that the increase of the total costs will depend on the ratio between those three elements. The increased cost certainly will not be twice as great if twice as many photographs are used to cover a given area. These considerations are in broad outline confirmed by our experience." In order to reduce costs, every reasonable effort should be made to improve our compilation procedures so that higher altitude photographs may be used with the desired accuracy.

In using any method of triangulation, we should know the sources of error, understand the effects of the various errors upon the observed quantities, and be familiar with the procedures necessary to maintain a required precision. In this connection we should establish some generally accepted standards of accuracy. Ir. A. J. van der Weele says, "The normal geodetic principle of extending ground control with known degrees of accuracy (first, second, third and fourth order) should also be followed as a standard practice in photogrammetry. If photogrammetric triangulations were classified according to equivalent standards of accuracy, this practice would result in a maximum of economy and efficiency, and a harmonic accuracy throughout the entire network of control. From a theoretical and economical point of view it is more important that there is a certain decrease in accuracy from first to fourth order than that the exact value of these degrees of accuracy are known. According to this principal the main framework of first order control would be established with the utmost precision. This network then may be filled in with second, third and fourth order triangulation using either field or photogrammetric methods. Second and third order triangulation may be performed with the Wild Autograph A5 or the Stereoplanigraph. The network of control may be completed and the plotting of detail performed with the Wild Stereo Plotter A6, the Multiplex, the Stereocomparagraph, etc. When only horizontal control is needed, mathematical radial triangulation may be used for second order control and graphic radial triangulation may be used for the third order control. The recent development of better methods of evaluating and adjusting errors of photogrammetric triangulation seem to encourage the establishment of definite standards of accuracy for photogrammetric triangulation. On this subject it is impossible to provide all the necessary information without a very thorough study of the various points of view. Therefore, due to the lack of time, I can only outline this idea and hope that it will be of some value."

It seems desirable at this time that we should cooperate in estimating the potential value of the various competing methods of analytical and stereo triangulation.

It is Commander O. S. Reading's conclusion that "The use of aerial triangulation and the esteem in which it is held varies a lot with different authorities and organizations. These differences probably arise from different problems, training, apparatus and techniques. It would appear that more research is needed before we can correctly evaluate these differences of opinion."

In science it has been noted that each significant new development invariably initiates a long series of additional inventions, refinements and modifications. These developments combine to produce a constantly accelerating rate of general technical progress. Photogrammetry appears to be taking part in this general advancement. We have new developments in map plotting equipment and in analytical triangulation. The development of radio triangulation and the high speed electronic computors also may make their contribution to photogrammetry. It is quite possible that a revolution is pending in practical map compilation procedures.

Extending vertical control is now the biggest problem of photogrammetry. Relatively little information on this subject has been published in this country so far. The author has therefore tried to present in this article an elementary review of the general nature of this important subject. Because of the conditions under which it was prepared, this article must of necessity be somewhat incomplete. However, it is hoped that it will serve as a useful introduction to several articles of a more technical and specialized nature which are expected to be published in this magazine by various European and American authorities. It is also hoped that this article will help to bridge the ocean, both literal and figurative, that exists between European and American photogrammetry.

The author is very greatly indebted to many persons for help and information in the preparation of this article. In addition to the specific credits given in the text, the author wishes especially to acknowledge the assistance received from Mr. Edwin Berchtold, Chief Engineer of the Henry Wild Surveying Instruments Supply Company, Heerbrugg, Switzerland; Dr. Walter K. Bachmann, Professor of Geodesy and Photogrammetry, Laussane University, Laussane, Switzerland; Ir. A. J. van der Weele, Chief of the Photogrammetric Section, Netherlands Ministry of Public Works, Delft, Holland; Prof. Dr. Max Zeller of the Photogrammetric Institute, Swiss Federal Institute of Technology, Zurich, Switzerland (Prof. Dr. Zeller has very kindly given the author galley proofs of that portion of his new book dealing particularly with stereo triangulation. See Footnote 14); Mr. Everett L. Merritt and Mr. Arthur C. Lundahl of the Navy Photographic Intelligence Center (P. I. C. provided the author with a translation of the above described portion of Prof. Dr. Zeller's new book); Mr. Leon T. Eliel and Mr. Revere G. Sanders of the Fairchild companies; Col. W. D. Milne, Executive Officer, Army Map Service; Mr. Gilbert G. Lorenz of The Engineer Board; Mr. John I. Davidson, Mr. James G. Lewis and Mr. David Landon of the U.S. Geological Survey; Dr. Duncan E. Macdonald, Director, Boston University Optical Research Laboratory; Mr. O. M. Miller of the American Geographical Society; Com. O. S. Reading and Mr. G. C. Tewinkel of the U.S. Coast and Geodetic Survey; Mr. William T. Pryor of the Public Roads Administration; Prof. Earl Church of Syracuse University; Mr. R. O. Anderson of the Tennessee Valley Authority; and Andrew Hacskaylo, Liaison Engineer of the Hastings Instrument Co.

Where various authorities are directly quoted in this article, their statements have been taken, with their permission, from personal letters to the author.

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1. By the author's definition, photogrammetric triangulation is performed whenever three or more photographs are absolutely oriented, as an integrated unit, to control that is inadequate for the absolute orientation of the same photographs when not integrated together.
2. In both Europe and the United States, the term "aerotriangulation" (or "aerial triangulation") does not appear to have an exact and generally accepted meaning. Because of this confusion, the term "aerotriangulation" is not used herein except when various authorities are quoted directly. It is believed that in these few instances the word "aerotriangulation" means "stereo triangulation."
3. These subdivisions of photogrammetric triangulation are quite descriptive and easily remembered, and have been used for that reason. However, some other method of classification might provide the student with a much better understanding of the various procedures used in photogrammetric triangulation.
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6. The radial triangulator is described in "Photogrammetry" by O. von Gruber, pages 270–275.
7. The terms "twin point interpolation" and "double point interpolation" are sometimes used to describe the process of producing relative orientation by a systematic identification and removal of y-parallax from the stereoscopic model. The term is used in "Photogrammetry" by O. von Gruber on pages 49, 401, etc.
8. Figure 3 is adapted from Figure 194 of Prof. Dr. Max Zeller's "Lehrbuch der Photogrammetrie." Zurich, Switzerland, 1947. The author is pleased to learn that this important book is being translated into English for publication in England.
9. *Ibid.*, pages 245–249.
10. Additional information may be obtained in the following reprints of articles appearing in the Journal of Research of the National Bureau of Standards, for sale by the Superintendent of Documents, Washington, D. C.:

RP 1177 Relation of Camera Error to Photogrammetric Mapping	10¢
RP 1498 Characteristics of Wide Angle Airplane Camera Lenses	5¢
RP 1636 Region of Usable Imagery in Airplane Camera Lenses	10¢

11. A camera possessing a 180° field has been developed by Mr. Frank Benford. (U. S. Patent 2,371,495) of the Research Laboratory, General Electric Company. This camera uses an ordinary camera lens in combination with a convex mirror corrected for spherical aberation. The author has investigated this general type of camera and has found that several different conditions seem to prevent the successful development of a practical ultra-wide-angle surveying camera using a convex reflecting surface.
12. Designed by Dr. James G. Baker, who was at that time Director of the National Defense Research Committee's Optical Research Laboratory at Harvard University. Additional information may be obtained from OSRD Report No. 6016 entitled "Spherically Symmetrical Systems for Wide Angle Aerial Photography." 121 pages. Microfilm copy of this report may be obtained from the Library of Congress, Washington, D. C., at $3.88. (Library of Congress serial number 2926.)
13. A vertical photograph nine inches square taken at 12,000 feet by an aerial surveying camera possessing an angular field of 120° will have exactly the same scale and coverage as a photograph taken with a six inch Metrogon lens at 20,000 feet. At these flying heights, and for a given amount of stereoscopic overlap, it will take the same number of either kind of photographs to cover a given mapping project. Incidentally, it should be noted that, if an ordinary vertical photograph made with the six inch Metrogon lens is enlarged four times, it will have the same size, scale and coverage as a vertical photograph made at one-third the flying height with the USC&GS nine lens camera.
14. Manual of Photogrammetry, page 12.
15. Multiplex Mapping Equipment, War Department Technical Manual TM 5-244, page 73.
16. Manual of Photogrammetry, page 52. These values are given in terms of linear distortion on the photograph. Computations will show that these errors vary between $+1°30'$ and $-1°45'$.
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ANNOUNCEMENT

President Sanders' in his column, page 401, advised of action being taken on the membership classification situation. A mailing made on September 15th forwarded the material to every member. You are urged to read the material, to understand it, and to execute the enclosed ballot according to your own conviction. The Officers and the Board of Direction are most desirous of receiving a liberal response in the form of ballots. By these democratic processes, the affairs and business of the Society are conducted in accordance with the expressed desire of the majority of our membership. The action to be taken on membership classification will be resolved by a majority of the ballots returned, therefore, it behooves every member to send in the ballot in order that the mandate will be truly representative of the majority of our total membership. Please remember—ballots received after November 15, 1947 are not eligible for counting.