THE RELATION OF SCALE AND HEIGHT ERRORS IN A MULTIPLEX EXTENSION AND A POSSIBLE APPLICATION TO MAPPING

J. A. Eden, Photogrammetric Research, Directorate of Military Survey, War Office, Whitehall, S.W. 1, England

Synopsis: The following article is an examination of certain systematic errors in a multiplex extension, and in particular, the relation of scale and height errors. A possible practical application is suggested which involves obtaining corrected heights over a large area, based upon a minimum amount of vertical control. This bridging of vertical control can be accurately accomplished, the article explains, if the multiplex models are fitted to an adequate system of horizontal control obtained from a templet assembly or from other sources.—*Publications Committee*

INTRODUCTION

THE purpose of this article is to describe a theoretical investigation into the relation of scale and height errors in a multiplex extension.

These errors are occasioned by false orientation of the projectors which may be due either to errors in the setting up of the projectors to form a model in exact correspondence, or to calibration errors in the camera-printer-projector system, film and diapositive distortion etc., which would cause the orientations to be in error even though the rsetituted model was in correspondence.

In the investigation x, y and z refer to the directions of the setting adjustments of individual projectors: x', y' and z' refer to co-ordinates measured in a true plane which is not necessarily a horizontal plane; and X, Y and Z refer to co-ordinates measured in a true horizontal.

If the x', y' and z' co-ordinates of a multiplex model are correct, then the model is a true model without distortion, but not necessarily horizontalized.

It will be shown that along the central section (i.e. flight line) of a multiplex extension, there is a relation between the errors in x' and the errors in z' and that normally the x' errors will be greater than the z' errors. Further, that if x' errors are prevented by fitting the model to XY plan control, z' errors will be prevented along the central section.

In other words, by fitting the multiplex model to plan control, we may obtain a true, but unhorizontalized model along the central section, without resource to any height control.

Practical applications are later explained.

SETTING ERRORS IN MULTIPLEX ADJUSTMENT AND THEIR EFFECTS ON MODEL DEFORMATIONS

Diagrams 1 and 2, together with their explanations, show the effects on the x' and z' positions along the central sections of an extended multiplex model when:—

(i) Successive errors are made in the z settings.

(ii) Successive errors are made in the tip settings.

Errors in the setting of y, tilt, and swing have no effect on x' and z' positions along the central section of the model, except in so far as they influence tip and z settings.

It will be apparent from the diagrams that any errors in z' must be accompanied by:—

(i) Errors in x'.

(ii) Errors in the true heights of the projectors above the model.

z setting errors cause the model to dip down whilst the projectors rise and vice versa, whereas tip setting errors cause the model and the projectors to rise or fall together, although the model falls or rises more steeply than the projectors.

Along the lateral edges of the model, errors in x' will be the same as along the central section, but errors in z' may be caused by setting errors of z, tip, or tilt, and will be greater than the errors along the central section by the amount of error which may be introduced by false tilt settings.

Normally z' errors along the central section will accumulate in the form of a parabola, and the lateral section of the model will be twisted about this parabola as an axis. If the parabola is convex in form, x' errors will make the model too long, and if the parabola is concave in form, x' errors will make the model too short.

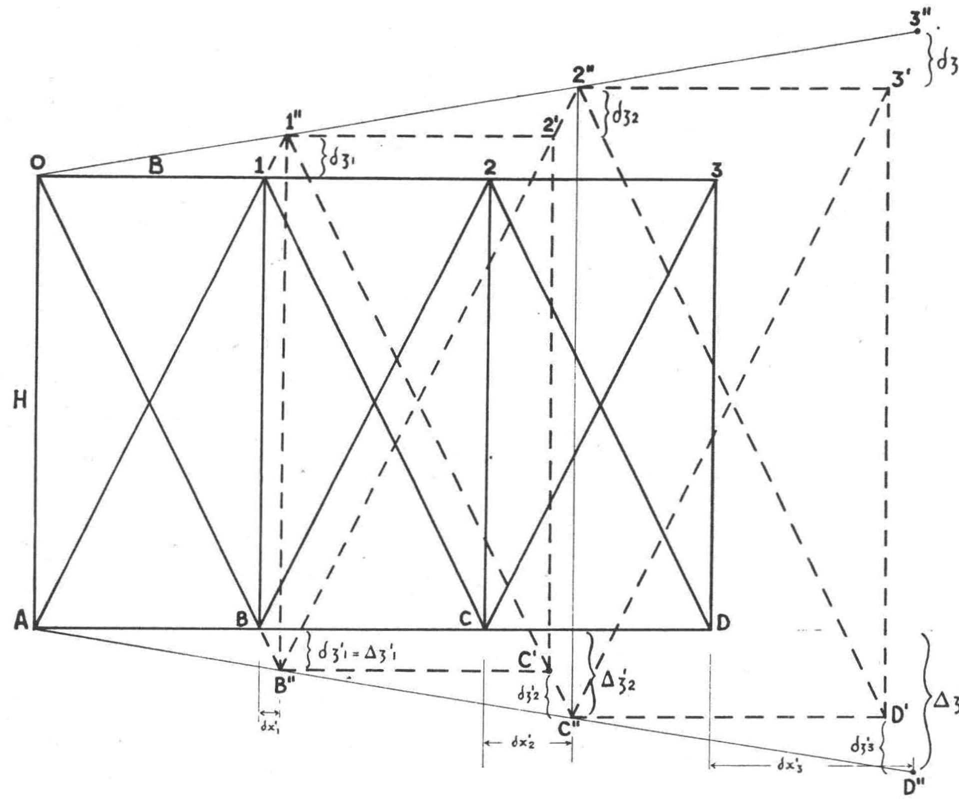

DIAGRAM 1

0.1.2.3 represent 4 projectors set up correctly to control A, B, C, D.

An error of δz_1 is now made in the setting of projector 1 which is raised to position 1'' in a direction to maintain the position of the model at A.

The model at B is then distorted to B''.

Continuing with the extension—to place projector 2 in correspondence with 1'' so that the position B'' fixed by the last model remains unaltered it must be moved to 2'. The model C is then distorted to C'.

If a further error δz_2 is now made in the z setting of 2' the projector is raised to 2'' so that the position of B'' remains the same as in the last model. C' is then further distorted to C''—and so on as the strip is extended.

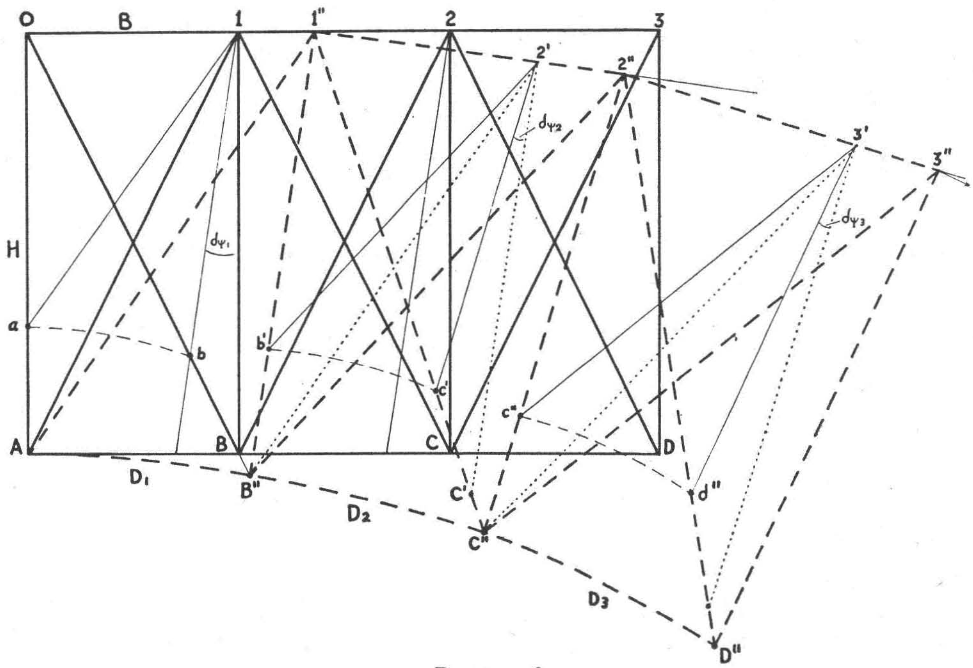

DIAGRAM 2

0.1.2.3. represent 4 projectors set up correctly to control A, B, C, D.

An error $\delta\psi_1$ is now made in the tip setting of projector 1, and the model AB, is raised to ab.

In order to maintain the position of the model at A, projector 1 must now be moved to 1″, thus lowering a to A, and b to B''—continuing with the extension—if no further tip error is made projector 2 must be placed at 2′ to maintain the model position at B''. The model at C is then distorted to C'.

If a further tip error is made at 2′ equal to $\delta\psi_2$ the model B'' C' is raised to $b'c'$. In order to retain the model at B'', projector 2′ must then be moved to 2″ thus lowering b' to B'' and c' to C''—and so on as the strip is extended.

THE EFFECT OF ERRORS IN PLAN POSITION ON Z' ERRORS IN THE MODEL CENTRAL SECTION

z SETTING ERRORS

From diagram 1 showing the effect of z setting errors, the increase in length of the model $\delta x_1'$, at projector 1″ is:—

$$\delta x_1' = \frac{B}{H}\,\delta z_1'$$

and at projector 2″

$$\delta x_2' = \frac{B}{H}\,(\delta z_2' + 2\delta z_1')$$

and at projector 3″

$$\delta x_3' = \frac{B}{H}\left\{\delta z_3' + 2(\delta z_1' + \delta z_2')\right\}.$$

Therefore, at projector n

$$\delta x_n' = \frac{B}{H}\left\{\delta z_n' + 2(\delta z_1' + \delta z_2' + \cdots + \delta z'_{n-1}')\right\}.$$

Further, the z' error $\Delta z_n'$ at the nth projector is given by

$$-\Delta z_n' = \delta z_1' + \delta z_2' + \cdots + \delta z_n'.$$

If we assume $\delta z_1' = \delta z_2' = \delta z_3'$ etc., then

$$\frac{\delta x'}{\Delta z'} = \frac{B}{H}\left(2 - \frac{1}{n}\right).$$

The table below shows values of $\delta x'/\Delta z'$ for extensions of different lengths when $B/H = 0.58$, which is the normal ratio for six-inch photographs overlapping 60% in the fore and aft direction.

2 projectors extension $\delta x'/\Delta z' = 0.58$
3 projectors extension $\delta x'/\Delta z' = 0.97$
5 projectors extension $\delta x'/\Delta z' = 1.04$
10 projectors extension $\delta x'/\Delta z' = 1.10$
15 projectors extension $\delta x'/\Delta z' = 1.12$
Infinite projectors extension $\delta x'/\Delta z' = 1.16$

From which, it is concluded that errors in the z settings of projectors give rise to errors in z' very nearly equal to errors in plan position.

TIP SETTING ERRORS

From diagram 2, showing the effect of tip setting errors: if $\delta\psi_1$, $\delta\psi_2$ etc. are the tip errors applied at projectors 1 and 2 etc., and if D_1, D_2 etc., are the projections of the air bases,

$$D_n = B\left\{1 + \frac{B}{H}(\delta\psi_1 + \delta\psi_2 + \cdots + \delta\psi_n)\right\}$$

and the increase in length $\delta x'$ is given by

$$\delta x_n' = (D_1 + D_2 + \cdots + D_n) - nB.$$

Also the z' error $\Delta z'$ at the nth projector is (to first order)

$$\Delta z_n' = B(\delta\psi_1 + \delta\psi_2 + \cdots + \delta\psi_n).$$

If we assume $\delta\psi_1 = \delta\psi_2 = \delta\psi_3$ etc.

$$\frac{\delta x'}{\Delta z'} = \frac{B}{H}\left(\frac{n+1}{2}\right).$$

The table below shows values of $\delta x'/\Delta z'$ for extensions of different lengths when $B/H = 0.58$.

2 projector extensions $\delta x'/\Delta z' = 0.87$
3 projector extensions $\delta x'/\Delta z' = 1.16$
5 projector extensions $\delta x'/\Delta z' = 1.74$
10 projector extensions $\delta x'/\Delta z' = 3.2$
15 projector extensions $\delta x'/\Delta z' = 4.6$

and so on—increasing.

From which, it is concluded that errors in the tip settings of projectors give rise to errors in plan position which will, in general, be considerably larger than the errors in z'

If an error on the tip setting is partly compensated by a further error in the z setting, the x' and z' errors will be reduced.

ANOTHER SOURCE OF ERROR

Failure to bring one overlap to the height of the preceding overlap, as extension progresses, introduces a height error which remains in the extension. Small errors from this cause will naturally obtain, but they are likely to be random and not systematic, and, as such, they will have no accumulative effect on z' errors in the extended model.

PRACTICAL APPLICATIONS

It has been shown that if a multiplex extension is erected to fit plan control, there will be no model deformation along the central section of the extension. The extension may be of any length.

If a height control point is available at each end of the central section, the section may be horizontalized and true heights may be measured along its entire length.

It has been shown that if the plan control, to which the model has been fitted is in error, then after horizontalization to this control, resultant errors in height along the central section will normally be rather smaller than the errors in plan position.

Thus, if one attempted to fit a model to plan control from a templet assembly which at one point, which was fitted, was 1 mm. in error, one would expect an error of rather less than 1 mm. in the measured height of a nearby point lying in the central section.

The ability to obtain heights along the central section in this way gives rise to a possible practical application.

Suppose that an area to be surveyed is covered with strip photography flown from East to West, and that, in addition, there are a number of tie strips crossing the area from North to South. Suppose that adequate plan control is available from a templet assembly or some other source, and that the only height control consists of four points situated one at each end of the central sections of the most easterly and most westerly of the North-South tie strips.

With these data, we may obtain true heights down the central sections of these two edge tie strips, and, by this means, supply ourselves with height control at each end of all the East-West strips.

In the same way with this supplementary control, we may obtain true heights down the central section of all the East-West strips, and similarly down the central sections of the remaining tie strips.

In this manner, a network of height control points may be obtained all over the area suitable for the erection of individual multiplex models.

Checks would be provided where the central sections of the interior tie strips crossed the central sections of the main East-West strips, and would provide means of adjusting any small discrepancies which might accrue from systematic errors in the plan control.

It should also be clear that since the model is fitted to plan control plotted on the flat surface of the plotting table, the projectors are prevented from taking up a line of curvature corresponding to the curvature of the earth. There is, therefore, no necessity to apply any corrections to heights measured along the central sections to allow for any such curvature.