A RECONSIDERATION OF TERRESTRIAL PHOTOGRAMMETRY*

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PART II

INVESTIGATION OF THE HEYDE PHOTOGONIOMETER AND COMPARISON WITH THE WILSON PHOTOALIDADE

FOLLOWING the outline laid down in Part I of this paper which appeared in the June 1947 issue of PHOTOGRAMMETRIC ENGINEERING,[1] the authors have concentrated their attention on terrestrial space resection methods based on the use of the Heyde Photogoniometer[2] and the Wilson Photoalidade.[3] Part II includes a detailed description of the characteristics, operational procedure and adjustments of the Heyde Photogoniometer along with tabulations of survey values and errors derived from the application of the Photogoniometer and Wilson Photoalidade to glass plate photographs of the same field test area in the south corner of Washington, D. C.

DESCRIPTION OF THE HEYDE PHOTOGONIOMETER

A. GENERAL:

The Hyde Photogoniometer, illustrated in figures 1(A), 1(B) and 1(C), is an instrument for measuring horizontal and vertical angles from an exposure station to ground points imaged on a photograph, diapositive or negative. The instrument consists fundamentally of a base (1) on which are mounted the easel frame (2) and the theodolite (3). The true angular relations of perspective rays at the camera's perspective center are observed by reconstructing the camera's orientation at the moment of exposure. The camera orientation is simulated by integrating the geometric relations of the theodolite and easel frame on the common base with that of the camera and landscape seen from the exposure station.

B. DETAILS

1. *Base:* The base consists of a homogeneous iron casting 54 cm. long, 34 cm. wide, and 8 cm. high. The standards (19) located at both ends of the base are a part of the base casting and support the *U* bearing (45) of the easel horizontal axis. The standards are 16 cm. high and 14 cm. wide where they join the base.
2. *Azimuth Circle:* The aximuth circle (5) is 28 cm. in diameter and is located in the center of the base in such a manner that the upper surface of the aximuth circle is flush with the upper surface of the base. The azimuth circle is graduated in single degrees and has a micrometer drum (11) that reads to single minutes with interpolations to ten seconds. The azimuth circle is rotated by turning wheel (9) and is clamped by pushing level (10) toward the micrometer drum.

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[1] PHOTOGRAMMETRIC ENGINEERING, Vol. XIII, No. 2, p. 295.

[2] Wilson, A. M., "Observations on Captured German Mapping Equipment," PHOTOGRAMMETRIC ENGINEERING, Vol. XII, No. 3, p. 294.

[3] Wilson, R. M., "Oblique Photographs and the Photoalidade," PHOTOGRAMMETRIC ENGINEERING, Vol. 4, No. 2, pp. 64–74.

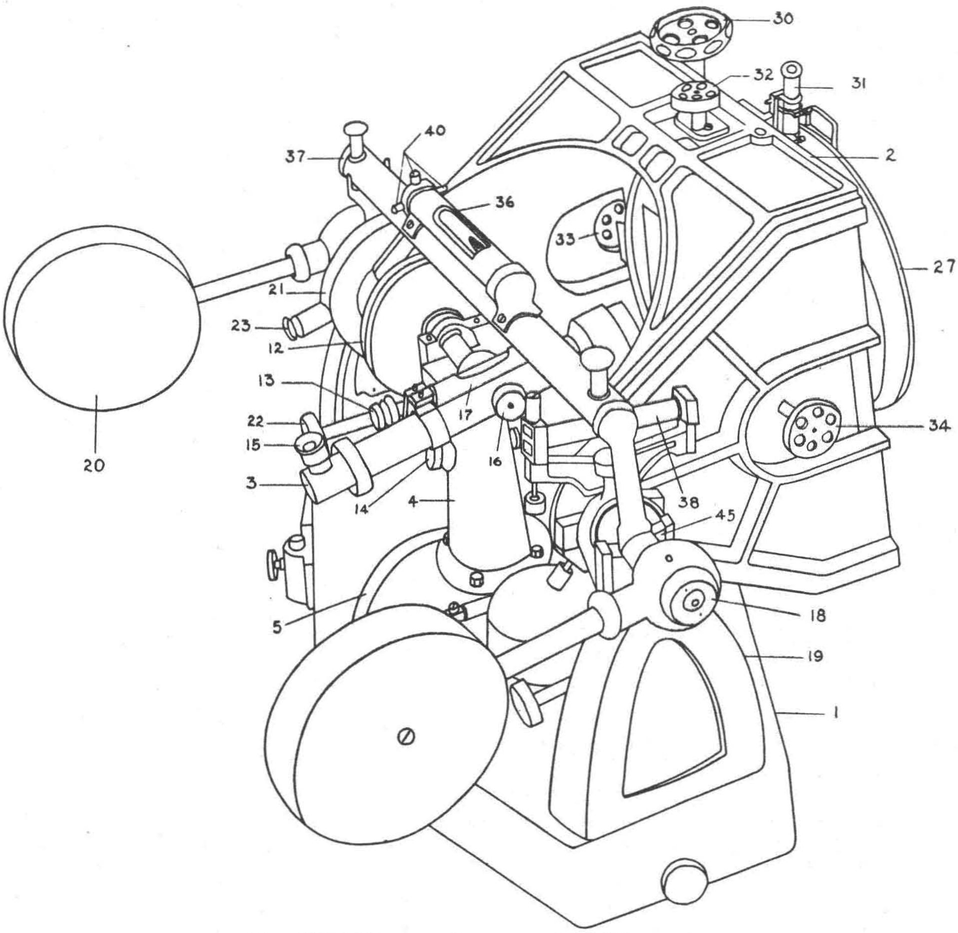


FIG. 1(A). Photogoniometer—side oblique view.

3. *Theodolite:* The theodolite, consisting of a telescope (3) and a vertical circle (12), is supported by an iron stem (4) that is mounted eccentrically on the azimuth circle so that when the theodolite is properly oriented, the telescope horizontal axis, the telescope optical axis, and azimuth circle vertical axis are concurrent.
4. *Theodolite Balance Weight:* This mass is mounted on the opposite side of the azimuth circle and consists of a metal disc (8) so located as to balance the weight inequalities caused by the eccentricity of the theodolite stem.
5. *Plate Levels.* Two levels (7) mounted perpendicular to each other are located on the azimuth circle between the theodolite balance weight and the theodolite stem. The plate level bubbles are centered by working the foot-screws (6).
6. *Telescope:* The telescope (3) and vertical circle (12) are rotated in parallel vertical planes on opposite ends of a common horizontal axis which is supported by a rectangular bearing surface that is centered over the theodolite stem. The telescope of the theodolite is 28 cm. long and has an objective lens 28.5 mm. in diameter. The eyepiece (15) is rotatable in a plane perpendicular to the geometric axis of the telescope tube. This

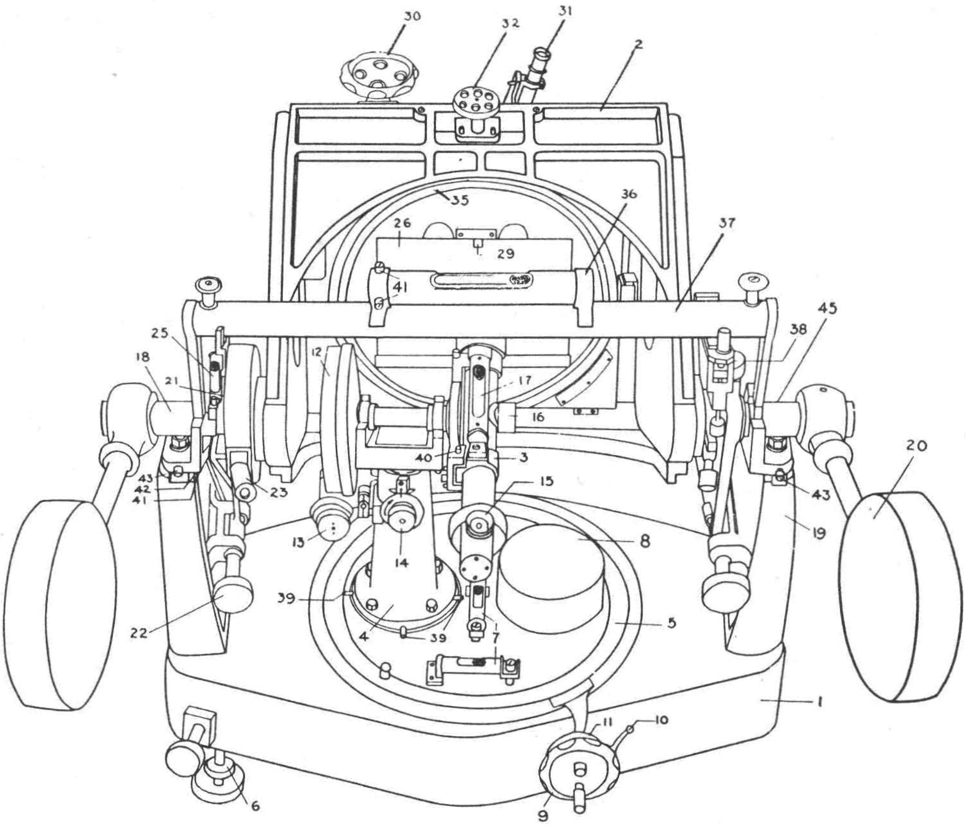


Fig. 1(B). Photogoniometer—front view.

feature allows the observer to view the negative plate by merely looking downward into the telescope eyepiece. The negative plate is brought into sharp focus by working focus screw (16). The telescope level (17) is located on the top of the telescope, parallel to the telescope tube.

7. *Vertical Circle of Theodolite:* The vertical circle (12), supported on the opposite end of the horizontal telescope axis, is 13 cm. in diameter and 2.25 cm. thick. It is graduated to single degrees and has two micrometer drums (13) 180° apart that read to single minutes and may be interpolated to ten seconds. The vertical motion is clamped by rotating screw (14) which is mounted on the upper front side of the theodolite stem. The micrometer drums located on opposite sides of the vertical circle provide equal facility in making both direct and reverse observations.
8. *Easel frame:* The easel frame (2) is a homogeneous iron casting, 36 cm. wide and 33 cm. high. It is counter-balanced on its horizontal axis (18) by two circular iron discs (20) each 15 cm. in diameter. The purpose of the easel frame is to support the negative plate in the geometric relation existing at the moment of exposure. In order to obtain this relation, the horizontal axes of the theodolite and easel must be made colinear; the distance from rear nodal point of the camera lens to the plate perpendicular must be duplicated; the vertical circle (21) reading of the easel must coincide with the vertical circle reading of the theodolite when the theodolite is bisecting the plate perpendicular of the negative plate; and

the horizon line of the negative plate must be parallel to a line defined by the trace of the telescope cross-hair intersection when the telescope is rotated in azimuth. The easel is perhaps best described by a brief statement on the part it plays chronologically in the operational procedure. First, the common base is leveled up, after which the theodolite is oriented with respect to the base and the easel is oriented with respect to the theodolite. Once the orientation sequence is accomplished, the vertical circle level (25), located on the left side just above the easel's horizontal axis, is made to be centered when the easel's vertical circle (21) reads 0°.

9. *Easel Vertical Circle:* The easel vertical circle is graduated to single degrees and has a vernier that may be read to thirty seconds. The vernier is observed through one of the twin oculars (23) attached symmetrically opposite to each other on the front and rear edge of the easel vertical circle case. In practice, the complement of tilt of the camera is set on the easel vertical arc by rotation of screw (22) located directly below and in front of the easel vertical arc. This angular relation is then retained by adjusting set screw (24), located on the back upper surface of the base, until the set screw's upper point contacts and supports the easel frame.
10. *Rear Ground Glass Window:* The easel has a circular opening 23.5 cm. in diameter in the rear through which the negative plate may be passed to the plate holder (26). Once the negative plate is installed, a circular ground glass window (27) is fitted into the circular opening. The light

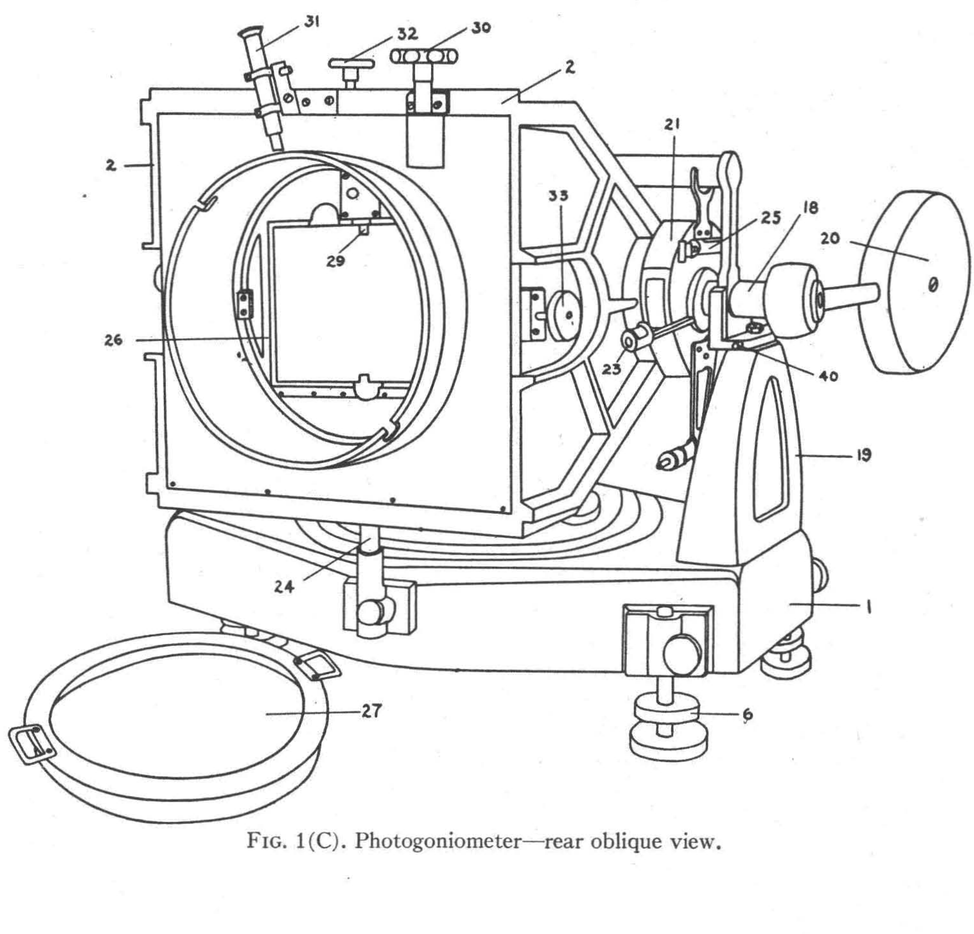


FIG. 1(C). Photogoniometer—rear oblique view.

source of the negative plate is admitted through this window.

11. *Plate Holder:* The plate holder will accommodate a 180 mm. ×130 mm. negative plate. The plate is held secure by spring pressure of the photo plate clamp (29) located on the upper edge of the plate holder. The plate holder has two half-circle depressions on the upper edge and one half-circle depression on the lower edge to accommodate the fingers when installing the negative plate. The photo spring clamp forces the negative plate to bear against three small supports projecting from the edge of the plate holder in the plane of the installed negative plate. The surfaces of these supports are coplanar on the side toward the negative plate.
12. *f Motion:* After the negative plate is installed, the focal distance of the taking camera is set on the instrument by working the *f* scale setting screw (30) located on the upper left surface of the easel. The setting is read in the *f* scale ocular (31) located on the upper right rear surface of the easel.
13. *X Motion:* The line connecting upper and lower fiducial marks is made to pass through the cross-hair intersection of the telescope by working *X* screw (33) located on the middle back left side of the easel.
14. *Y Motion:* The line connecting left and right fiducial marks is made to pass through the cross-hair intersection of the telescope by working *Y* screw (32) located on the middle back upper surface of the easel.
15. *Swing Motion:* The horizon line is made parallel with the trace of the telescope cross-hair intersection when rotated in azimuth by working *S* (swing) Screw (34) located on the middle back right side of the easel. The gear of the *S* screw engages the rack of the swing circle (35) located on the inside back face of the easel frame. The swing circle is graduated to single degrees and has a vernier that may be read to single minutes.
16. *Striding Level:* The striding level (37) is used to make the horizontal axis of the easel parallel in any horizontal plane with the theodolite horizontal axis.
17. *Auxiliary Easel Level:* The auxiliary easel level (38) is used to check the vertical orientation of the easel after it has been established and clamped.

OPERATIONAL PROCEDURE FOR THE HEYDE PHOTOGONIOMETER

As described in paragraphs 10 and 11, the negative plate is installed in the plate holder and the ground glass window fit in the rear access frame. The base is leveled by manipulating the three foot screws. The appropriate focal distance of the taking camera is set on the focal scale drum by turning the *f* scale screw. At this point, the complement of the angle of tilt of the taking camera is set on the vertical circles of the easel and the theodolite. The azimuth circle is then made to read 0°. To make the principal point of the negative plate coincide with the crosshair intersection of the telescope, the *X* and *Y* screws are alternately and appropriately turned while the original settings of the azimuth circle and respective vertical circles are maintained. A line connecting the left and right fiducial marks is then made to coincide with a line defined by the trace of the horizontal rotation of the telescope crosshair intersection. This is accomplished by making appropriate rotation of the plate holder. The vertical crosshair should coincide with the line connecting the upper and lower fiducial marks when the telescope is rotated in a vertical plane, if the easel, theodolite and base are correctly oriented with respect to each other. The horizontal and vertical angles of image points may be read directly if and when the negative plate is oriented in *f*, *x*, *y*, and *s* respectively.

Selected images are bisected with the telescope crosshairs, and the vertical and horizontal angles are read and recorded. The vertical angles represent the true vertical angles above or below the horizontal plane passing through the camera lens. Horizontal angles represent the true horizontal angles left and right of the principal plane. Inasmuch as the principal line was made to read zero azimuth, there is no need to record the sign of the angles as the minus values will be near 360°, or in the fourth quadrant, and the plus values will be less than 90° and in the first quadrant. The horizontal and vertical angles of each image are measured at least two times direct and an equal number of times reverse to cancel out collimation error. The plus horizontal angles are unchanged whereas the minus horizontal angles are deduced by substracting the mean of the left readings on a single frame from 360°.

The operational procedure for the Heyde Photogoniometer is simple and fast, once the instrument is in correct adjustment. In a recent test, no more than twenty minutes total time were required to observe two direct and two reverse readings on a negative plate having eight ground objects imaged. Somewhat less than twenty minutes were required to orient each negative plate with respect to f, x, y, and s settings. However, the adjustments of the theodolite with respect to the base and the easel with respect to the theodolite are more involved and therefore require a considerably longer period of time.

ADJUSTMENTS OF THE HEYDE PHOTOGONIOMETER

A. GENERAL:

Certain geometric conditions must be satisfied before the observed horizontal and vertical angles duplicate the angles made by the perspective rays at the perspective center. These conditions are listed as follows in the order in which they are normally corrected.

1. The top of the base must coincide with a horizontal plane passing through the foot of the theodolite stem.
2. Horizontal and vertical axes of the telescope and the vertical axis of the azimuth circle must be concurrent.
3. The horizontal axis of the telescope must be colinear with the horizontal axis of the easel, and these colinear axes must be parallel with the horizontal plane passing through the foot of the theodolite stem. Likewise, the vertical arc of both easel and telescope must have the same reading when the telescope is bisecting the principal point of the negative plate.
4. The optical axis of the telescope must be perpendicular to the negative plate when the telescope crosshairs are bisecting the fiducial axes intersection and the perpendicular distance from the fiducial axes intersection to the colinear easel and telescope axes must be equal to the focal length of the taking camera.

Inasmuch as several separate mechanical adjustments are required in most instances to satisfy a specific geometric condition, a second list is given which consists of the actual mechanical adjustments in the chronological order in which they are made.

1a. Plate Levels
2a. Eccentricity of the telescope
3a. Horizontal Crosshair (Index Error)
4a. Telescope Level
5a. Leveling Easel Horizontal Axis (Striding Level)
6a. Making X axis of Negative Plate perpendicular to the Optical Axis of the Telescope

7a. Making Y axis of Negative Plate perpendicular to the Optical Axis of the Telescope
8a. Making the Level Easel Horizontal Axis to be in the same horizontal plane as the Telescope Horizontal Axis.
9a. Making the Horizontal Easel and Telescope Axes to be in the same vertical plane when they are in the same horizontal plane (Focal distance adjustment).

B. PROCEDURE:

1. *Plate Level Adjustment.* The plate level bubbles are centered by turning the three foot screws—two on the left and one on the right. After the bubbles have been centered, the azimuth circle is rotated through 180°. If the bubbles are no longer centered, the ground glass arcs of the bubbles are not parallel to the top of the base and the plate levels themselves must be adjusted. Half the bubble error is corrected by altering the inclination of the plate levels with an adjusting pin and the remaining half by turning the foot screws. The azimuth circle is then rotated back to its original position and the bubbles are again checked for residual error. If the bubbles are still not exactly centered, the adjusting procedure is repeated until the bubbles are centered in both positions. The whole procedure is identical with the plate level adjustment on any field transit.
2. *Eccentricity of the Intersection of Telescope Horizontal and Optical Axes with Respect to Axis of Rotation of the Azimuth Circle.* Fig. 2. The telescope is leveled and the projection of the crosshair intersection is marked on a distant target. Both azimuth circle and vertical circle are rotated through 180°, and the projection of the crosshair intersection again marked on the distant target. The horizontal separation of the two marks on the distant

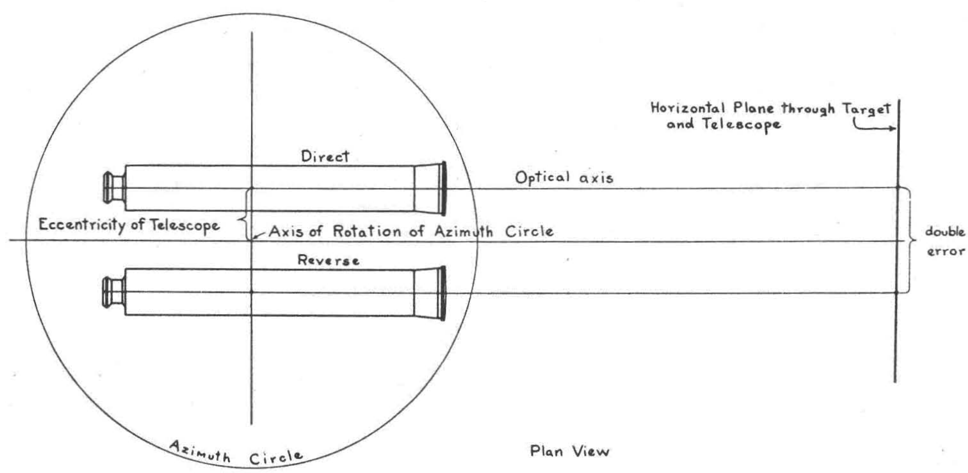


FIG. 2. Eccentricity of telescope.

target is equal to double the horizontal eccentricity of the telescope. With reference to figure 1(B), there are four adjusting pins (39) perpendicular to and in the base of the theodolite stem. A mark is noted midway between the two marks on the distant target. By manipulating adjusting pins (39), the vertical crosshair of the telescope is made to bisect the midmark noted. This operation is repeated, if necessary, until no eccentricity exists. The adjusting pins located in the telescope tube carriage, and bearing against the tube, may be used to remove small residual eccentricities.

3. *Adjusting the Horizontal Crosshairs.* Figs. 3(A) and 3(B). A telescope sight is made on a distant target (a blackboard was used in this test) with the theodolite vertical arc reading 0° 0′ 0″ and the projection of the crosshair intersection on the target is noted (a). The azimuth and vertical circles of the telescope are rotated through 180° and the projection of the crosshair intersection on the target is again noted (b). The theodolite vertical arc is then clamped and the appropriate adjusting pins are manipulated until the horizontal crosshair bisects *c*, a mark placed half way between *a* and *b* along a vertical line on the target. When the direct and reverse bisections of *c* are 180° apart, the adjustment is complete.

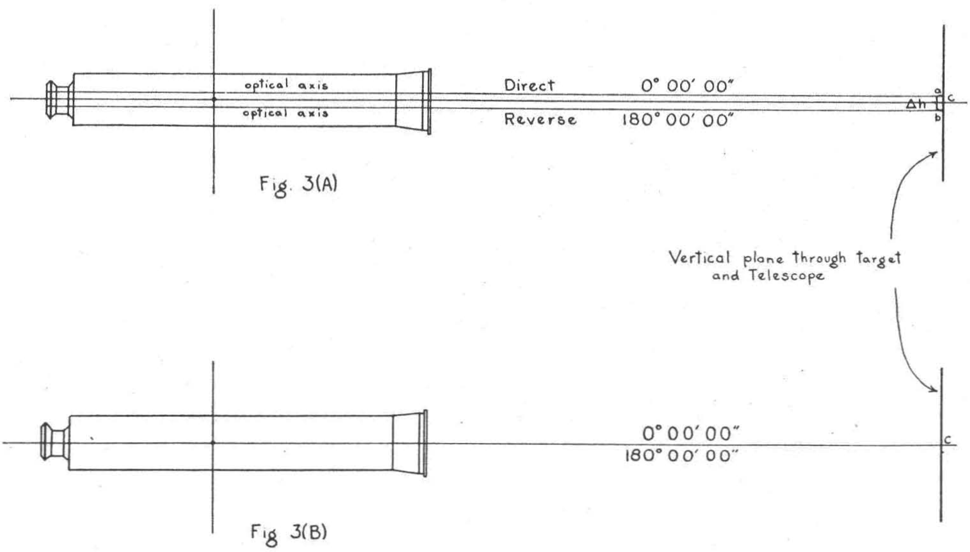


FIG. 3(A) (B). (Index error.)

4. *Adjustment of the Telescope Level.* Fig. 4. To this point, adjustments have been so made that a line, defined by the crosshair intersection and the projected point of crosshair intersection on the target, passes through a line defined by the horizontal axis of the telescope. However, the former line is not necessarily parallel to a plane tangent to the arc of the centered bubble of the telescope level. The telescope bubble is centered when the vertical arc of the theodolite reads 0°. To make the necessary adjustment, a target on a Philadelphia Level Rod is bisected at a known distance and the vertical arc reading of the horizontal crosshair on the target is recorded. The horizontal and vertical circles are transited through 180°, and a second reading on the target is recorded. The level error is equal to $\alpha/2$ and $\tan \alpha/2 = \Delta h/2D$ where Δh is the difference between the two level target readings and D is the distance from the target to the intersection of horizontal and vertical axes of the telescope. The vertical circle may be adjusted for $\alpha/2$, but the simplest solution is to treat it as an index error of $\alpha/2$, which is applied algebraically to all vertical angles.

5. *Leveling the Easel Horizontal Axis.* Fig. 5. Up to now, the easel has been detached from the standards because in the foregoing adjustments it was

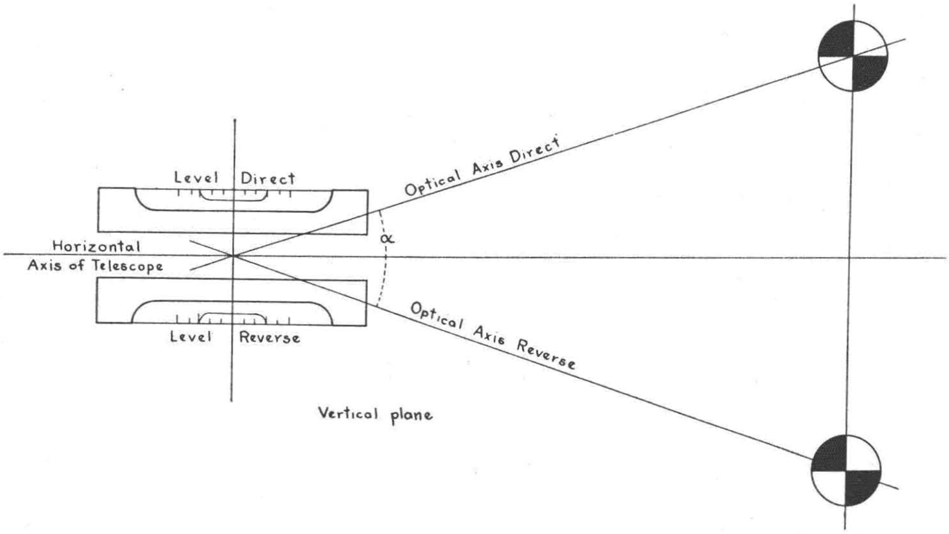
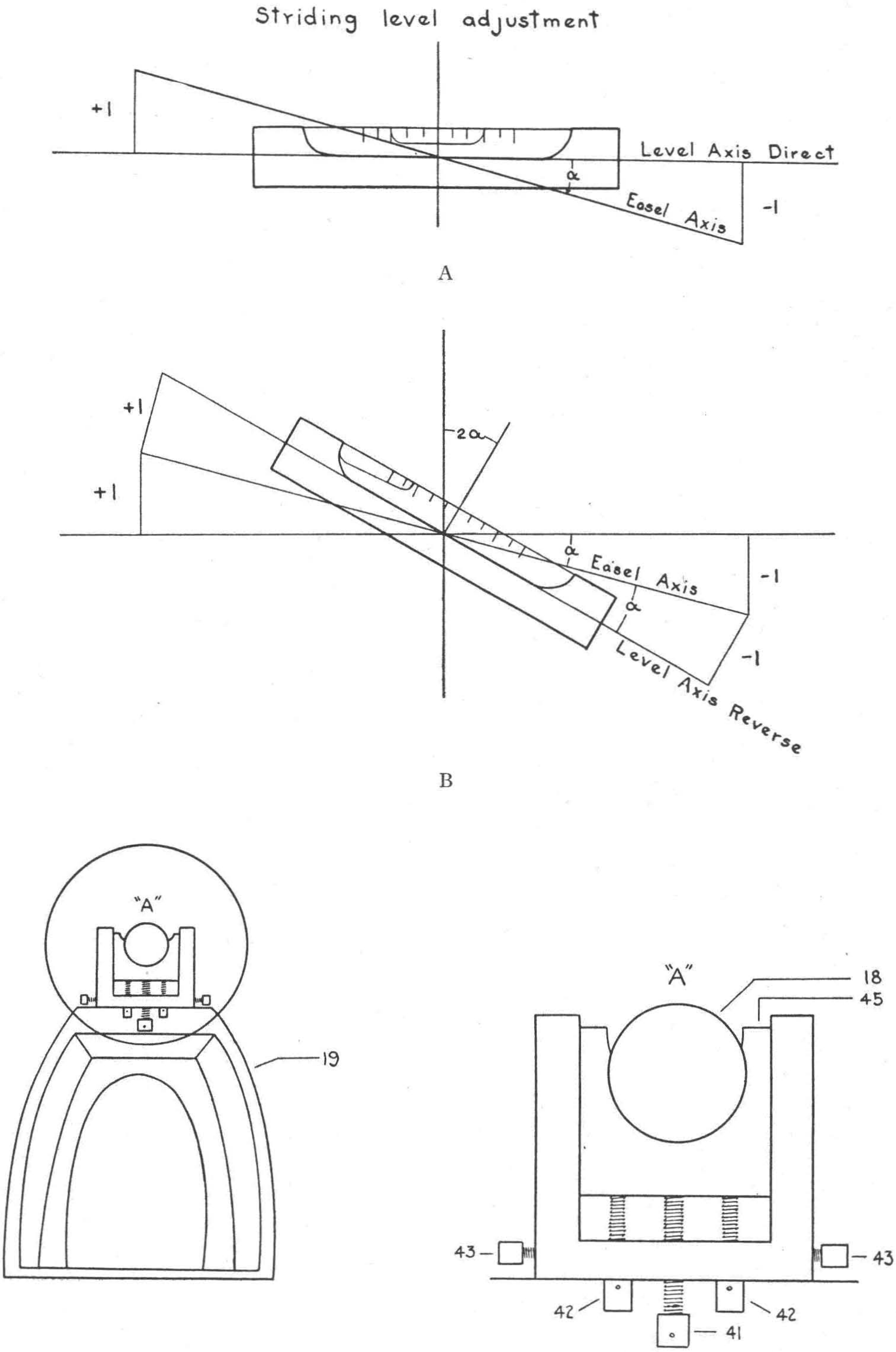


FIG. 4. Telescope level adjustment.

not required. The easel horizontal axis is carefully placed in the "*U*" bearing (45) Fig. 1A, located on the top of standards (19). The striding level (37) is carefully placed on the exposed easel axis. The bubble is centered by manipulating the appropriate adjusting pins (40) on the striding level tube. The position of the striding level is then reversed on the easel axis. The mean reading of the two ends of the bubble in the reverse position, subtracted from the mean of the two ends of the bubble when centered, times the value of one bubble division in arc is equal to twice the deviation of the easel axis from the horizontal. Obviously, then, one-half the error must be taken up by adjusting the easel axis and the other half by adjusting the striding level bubble. The error doubling is illustrated in figures 5(A) and 5(B). Beneath the top of both standards are three vertical adjusting pins shown in figure 5(C). Half the bubble error is removed by working the adjusting pins (42) and (43) in the opposite direction, in a vertical plane; one set is located beneath the top of the left standard, while a duplicate set is located beneath the top of the right standard, each working so as to bear vertically upward against opposite ends of the easel horizontal axis. The remaining half of the bubble error is removed by adjusting pins (40) on the striding level proper. The striding level is then placed in the initial direct position, and the residual error, if any, is noted. The procedure is repeated until the bubble is centered in both positions. At this point the horizontal axes of the telescope and the easel are both in a horizontal plane but not necessarily in the same plane.

6. *Making the X axis of the Easel Perpendicular to the Optical Axis of the Telescope.* Fig. 6. A calibration plate having two lines intersecting at a 90° angle is installed in easel plate holder. The terminals of the horizontal line are designated x and x', while the terminals of the vertical line are designated y and y' as shown in figure 6. The intersection O is equidistant from y and y', x and x'. The horizontal and vertical circles are each made to read 0°. By manipulating the x, y, and z screws, the horizontal crosshair of the telescope coincides with xx', the vertical crosshair coincides with



C

FIG. 5(A) (B) (C). Striding level adjustment.

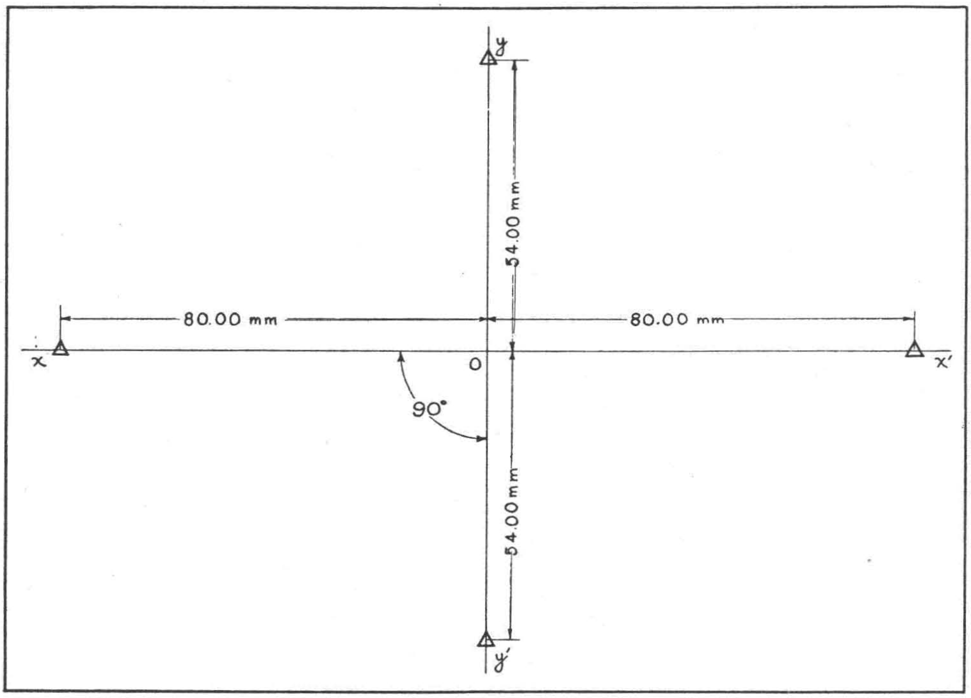


FIG. 6. Calibration plate.

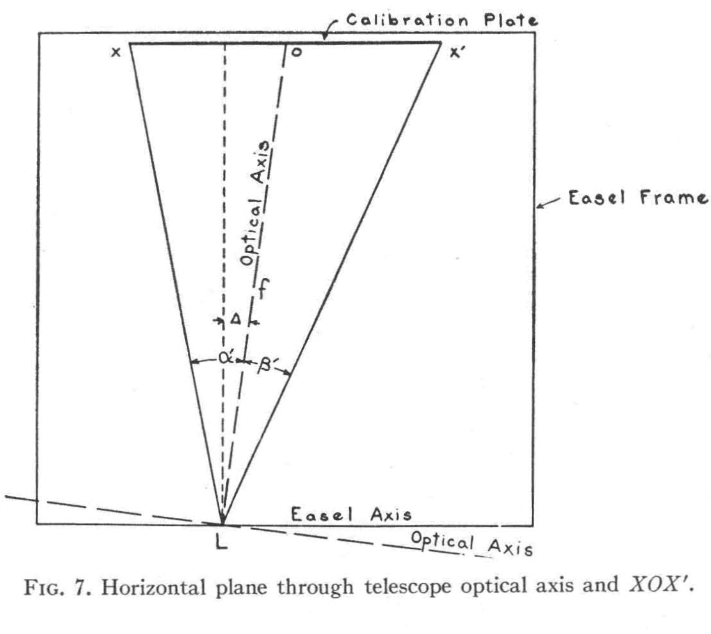


FIG. 7. Horizontal plane through telescope optical axis and XOX'.

yy', and the crosshair intersection is at o. Since $ox=ox'$, α will equal β when a vertical plane passing through the easel horizontal axis is parallel to a vertical plane passing through the telescope horizontal axis. Angles xLo and oLx' are observed and found to be α' and β', respectively, because the optical axis of the telescope fails to be perpendicular to xox' by an angular increment equal $\Delta°$. The failure of the optical axis to be perpendicular to xox' is illustrated in figure 7. The angle Δ is computed as illustrated in figure 8.

$$xo' = 2\sin\alpha'\cdot R \quad \text{and} \quad x'o' = 2\sin\beta'\cdot R$$

$$R = \frac{xx'}{2\sin(\alpha' + \beta')}$$

$$(oo')^2 = (xo')^2 + (xo)^2 - 2(xo')(xo)\cdot\cos\beta$$

$$\sin\rho = \frac{xo\cdot\sin\beta'}{oo'} \quad \text{where} \quad xoL = \beta' + \rho$$

$$= 90° - (\beta' + \rho).$$

The azimuth circle is adjusted to the angular correction of $\Delta°$. The observation is repeated until $\alpha=\beta$, when angles are measured between x and o and o and x' respectively, at L.

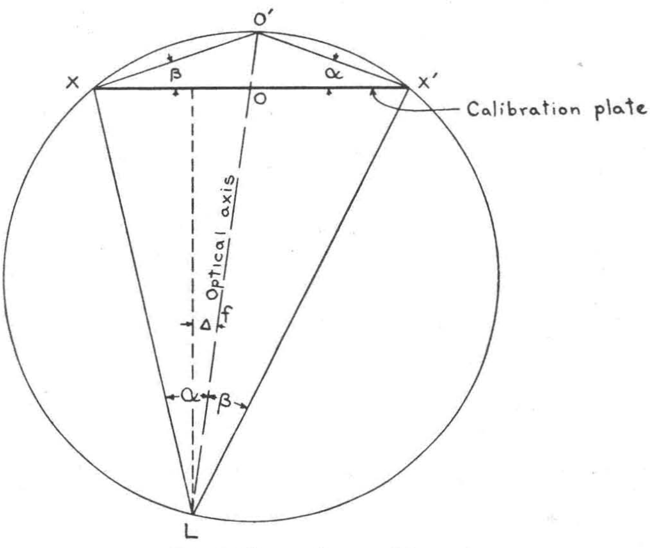


FIG. 8. Same plane as Figure 7.

7. *Making the Y Axis of the Negative Plate Perpendicular to the Optical Axis of the Telescope.* Fig. 9. The adjustment is identical to the foregoing adjustment except that it is done in a vertical plane $Lyoy'$. After angle Δ in the vertical plane has been computed from measurements made on y and y', the easel is rotated about its horizontal axis by working the easel setting screw (24—fig. 1(C)) an amount equal to $\Delta°$. The principal point o is again made to be pierced by the optical axis by working the y screw. The procedure is repeated until $\alpha=\beta$ in a vertical plane when α is yLo and β is oLy'. At this point, the easel horizontal axis and the telescope hori-

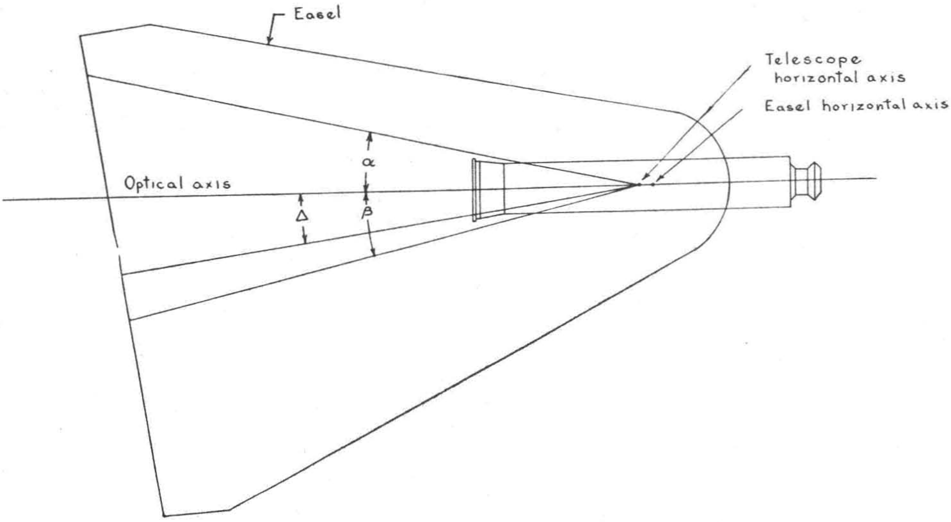


FIG. 9. Vertical plane through telescope optical axis.

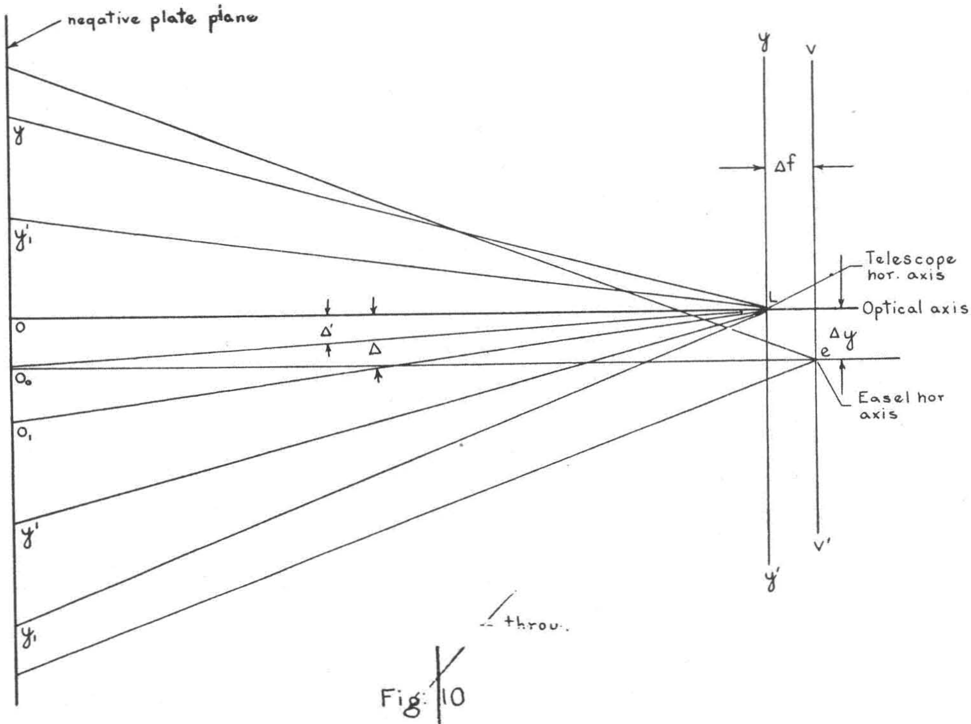


FIG. 10. Principal plane passing through optical axis.

zontal axis have been made to be in parallel horizontal planes and parallel vertical planes, but these are not necessarily identical horizontal and vertical planes.

8. *Making the Easel Horizontal Axis and the Telescope Horizontal Axis Lie in the Same Horizontal Plane.* In figure 10 oL is parallel to o_oe and yy' is parallel to vv'. The telescope horizontal axis and easel horizontal axis are displaced in the horizontal plane by an amount Δf and in the vertical plane by an amount Δy. To make $\Delta y = o$, the following adjustments are made: The leveled telescope is made to bisect o on the calibration plate. The calibration plate is then rotated through 180° by turning screw S (34). If $\Delta y = o$, the principal point will not move, as it will represent the axis of rotation. If $\Delta y \neq o$, the principal point will describe a half circle whose radius will be Δy.

$$2\Delta y = f \cdot \tan \Delta' \quad \text{and} \quad f = \frac{y_0}{\tan \alpha_V} \quad \text{or} \quad \frac{x_0}{\tan \alpha_H}$$

$$\Delta y = \frac{2\Delta y}{2} \qquad \tan \Delta' = \frac{\Delta y}{f}.$$

Angle Δ' is set on the telescope vertical arc. The easel is then elevated or lowered, as the case may be, by working the easel setting screw and the adjusting pins beneath the top of both easel standards. Each of the three screws is turned an equal amount in the same direction so that the easel will be corrected in a plane parallel to that previously held, an amount

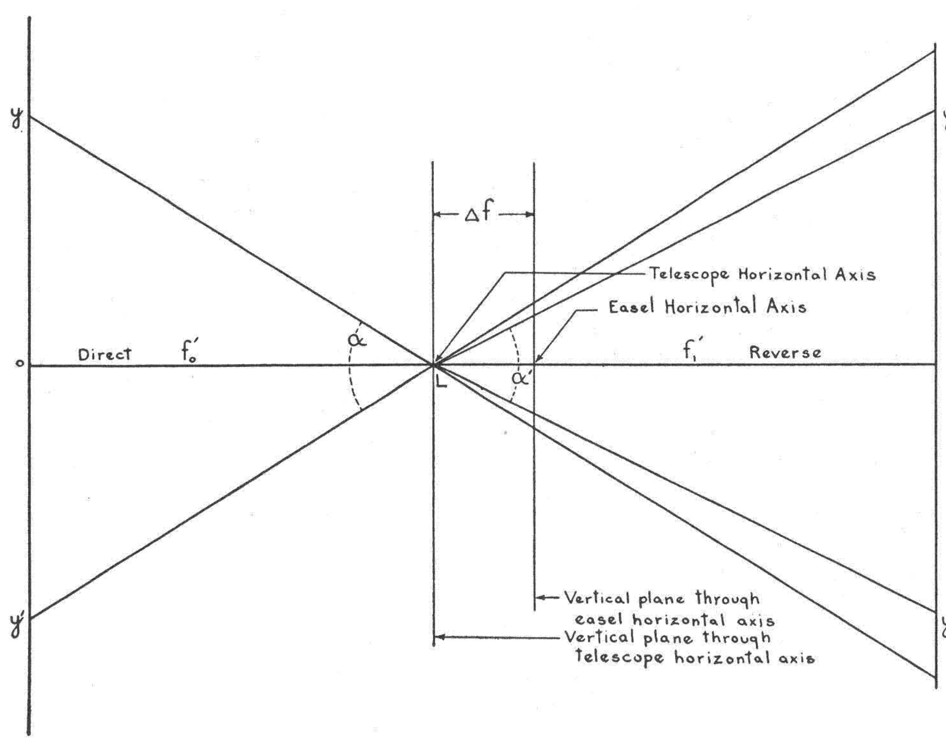


FIG. 11. Vertical plane through $Ly'y$.

vertically equal to Δy. The easel is moved vertically until the image o is again bisected by the telescope vertical arc. This adjustment is most difficult and may necessitate repeating some of the previous adjustments as it is almost impossible to elevate the easel to each of three points an exactly equal amount.

9. *Focal Length Setting Calibration.* Fig. 11. The purpose of this last adjustment is twofold: First, to determine the error, if any, of the focal length scale. Second, to make the easel horizontal axis and the telescope horizontal axis lie in the same vertical plane. The telescope is leveled and is made to bisect o with line yy' perpendicular to Lo in the principal plane. The vertical angle, yLy', is measured with the easel in the direct position and re-observed with the easel in the reverse position. If the easel horizontal axis and the telescope horizontal axis were in the same vertical plane, angle α in the direct position would equal α' in the reverse position, since both axes are in the same horizontal plane and $yy'=y'y$. However, this condition is rarely true. The horizontal difference of the two vertical planes is equal to $\Delta f=f_o'-f_1'$ when

$$f_o' = \frac{yy'}{2}\cot\frac{\alpha}{2}, \qquad f_1' = \frac{yy'}{2}\cot\frac{\alpha'}{2}$$

$$\Delta f = \frac{yy'}{2}\cot\frac{\alpha}{2} - \frac{yy'}{2}\cot\frac{\alpha'}{2} = \frac{yy'}{2}\left(\cot\frac{\alpha}{2} - \cot\frac{\alpha'}{2}\right)$$

$$= yo\left(\cot\frac{\alpha}{2} - \cot\frac{\alpha'}{2}\right).$$

The easel is shifted in a horizontal plane so that the horizontal axis of the telescope and horizontal axis of the easel are in the same vertical plane. This is accomplished by turning adjusting pins, (41 and 42) shown in figure 1. The angles α and α' are observed alternately with the shifting of the easel in the horizontal plane until $\alpha=\alpha'$ and $\Delta f=O$. When $\Delta f=O$, $yy'/2\cdot\cot\alpha/2$ should equal the reading on the focal length drum. If the computed f in millimeters does not equal the value in millimeters read in the f ocular, the f scale drum must be adjusted for the difference, or the difference applied as a correction to the focal length indicated on the f scale drum. The latter method is preferred. As has been mentioned previously, the adjustments of the photogoniometer are so interrelated that many of the adjustments must be repeated because they are in turn altered by adjustments that follow. These tedious time-consuming adjustments, the excessive weight of the instrument, and the near necessity of a stable (concrete) base are the three paramount objections to the Heyde Photogoniometer.

COMPARISON OF ACCURACY OF THE HEYDE PHOTOGONIOMETER AND THE WILSON PHOTOALIDADE

A. GENERAL:

The Heyde Photogoniometer was tested for its accuracy in establishing the horizontal and vertical coordinates of ground objects by triangulation methods. As detailed in Part I,[4] two stations, A and B, 19,303.89 feet apart at the south corner of Washington, D. C. were occupied by a Navy field party using a Fair-

[4] PHOTOGRAMMETRIC ENGINEERING, Vol. XIII, No. 2, p. 308.

child Camera Transit.[5] Three horizontal and three inclined photographs were so exposed at each station that the Washington metropolitan area was imaged in the overlap common to both sets of photographs. Conspicuous landmarks such as steeples, chimneys, towers, etc. have been designated by letters from C to S inclusive. All test results described in Part I were based on linear values while the Heyde Photogoniometer accuracy tests are based on angular values. The authors were interested not only in the absolute accuracy of the Photogoniometer, but also in the relative accuracy when compared with some standard American instrument. For this reason the same plates were observed with the Wilson Photoalidade. It must be emphasized at the start that an unfair advantage is taken of the Wilson Photoalidade in applying it to a different problem

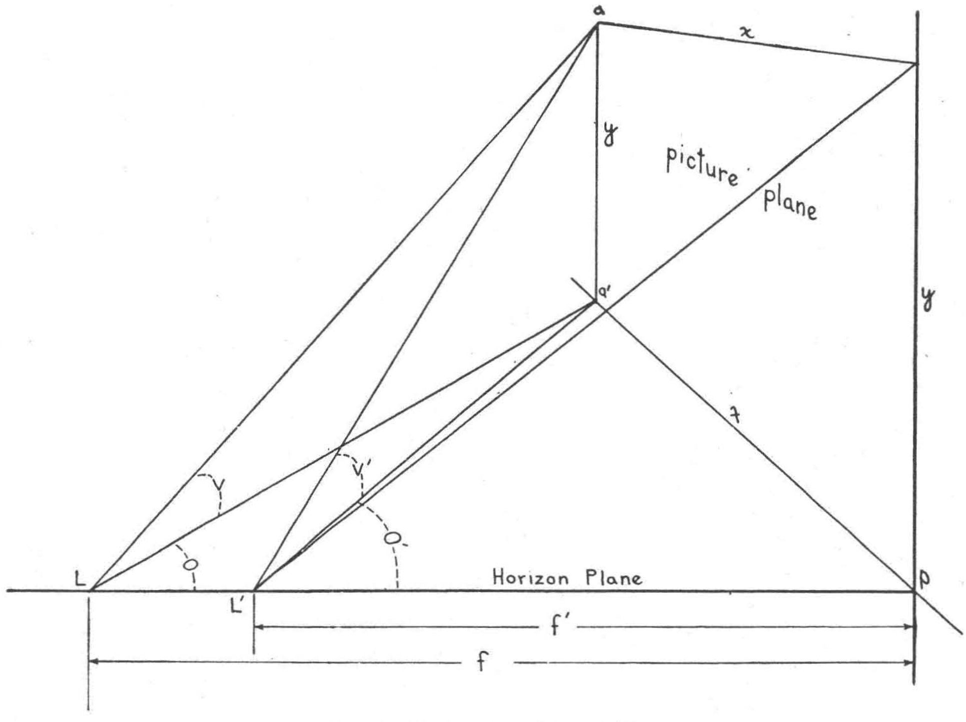


FIG. 12. Derivation of k and k'.

Symbol definition:

- a — image observed
- a' — projection of observed image on horizon $v'v$
- o — true horizontal angle
- o' — observed horizontal angle
- f — focal length of taking camera
- f′ — focal distance of photogoniometer
- p — fiducial axes intersection
- pp' — principal line
- vv' — horizon line
- x,y — photo coordinates of image a
- L — perspective center of taking camera
- L' — intersection of horizontal and vertical rotation axes of photogoniometer

$$\tan o = \frac{x}{f} \qquad \tan o' = \frac{x}{f'} \qquad k = \frac{f'}{f}$$

$$\tan o = \tan o' \cdot k = \frac{x}{f} \cdot \frac{f'}{f} = \frac{x}{f} \cdot$$

[5] *Ibid.*, Fig. 5, p. 305.

than that for which it was designed. Nevertheless, the tests were executed in order to secure some data on the relative precision of these instruments. One more correction had to be made before starting to tabulate survey values inasmuch as the maximum focal length range of the Photogoniometer was not equal to that of the taking camera. Therefore, a false f was used. The observed horizontal and vertical angles were corrected by multiplying the tangent of the observed values by a constant whose derivation is illustrated in figure 12.

The tangent of any observed horizontal angle multiplied by constant k is equal to the tangent of the true horizontal angle at the perspective center of the taking camera.

$$\tan v = \frac{y}{La'}$$

$$\tan v'' = \tan v' \cdot k$$

$$\frac{y}{f} = \frac{y}{f'} \cdot \frac{f'}{f}$$

$$\cos O = \frac{f}{La'}$$

$$\tan v = \tan v' \cdot \cos O \cdot k$$

$$\text{or} \quad \frac{y}{La'} = \frac{y}{f'} \cdot \frac{f'}{f} \cdot \frac{f}{La'} .$$

Since both horizontal and vertical angles at the perspective center of the camera may be easily derived by multiplying the tangents of observed horizontal and vertical angles, respectively, by a common constant, no great inconvenience results from not being able to observe at the focal length of the taking camera.

In Tables 1 and 2 are listed the survey values derived from angles observed with the Wilson Photoalidade and Heyde Photogoniometer respectively. These values may be compared station for station with the true survey values listed

TABLE 1 —Tabulation of Survey Values Derived from Angles Observed with the Wilson Photoalidade—in Feet

Station	Distance from		Coordinates		Elevation
	A	*B*	*X*	*Y*	*Z*
C	———	———	———	———	605.96
D	18,043.10	12,096.99	12,010.78	5,706.09	371.02
E	22,474.56	27,081.41	23,161.40	16,261.29	349.57
F	22,909.67	27,341.64	23,578.14	16,116.65	353.04
G	22,596.78	27,039.32	23,250.57	16,059.61	348.98
H	30,870.41	30,839.08	30,306.20	10,297.97	417.53
I	8,725.59	18,393.16	9,243.12	17,138.72	138.90
J	29,047.48	28,677.80	28,195.97	9,795.31	385.94
K	16,741.49	20,050.05	16,428.55	13,500.97	267.14
L	23,661.76	20,360.08	20,490.33	6,583.37	256.52
M	14,469.12	15,609.79	12,513.50	11,236.74	326.18
N	20,928.43	16,135.36	16,324.55	5,746.67	275.82
O	19,008.02	11,457.38	11,845.64	4,389.85	287.73
P	14,143.60	10,947.40	8,903.71	8,270.86	209.28
Q	18,895.03	10,426.69	10,916.88	3,916.55	195.62
R	13,043.30	10,500.13	7,680.20	8,797.20	147.00
S	14,762.44	5,604.76	3,858.74	5,517.00	143.58

in Table 7, Part I of this paper.[6] Tables 3 and 4 have been used to tabulate the errors of survey values derived from Wilson Photoalidade angles and Photogoniometer angles. It is readily noted that the Heyde Photogoniometer is not only much more accurate than the Wilson Photoalidade, but it has an absolute position accuracy exceeding the specifications of fourth order control when the intersecting perspective rays are no longer than five miles. This is obviously not true for the Wilson Photoalidade. The time required to make a series of measurements on a single exposure is approximately the same for both instruments.

TABLE 2 —Tabulation of Survey Values Derived from Photogoniometer Angles—in Feet

Station	Distance from		Coordinates		Elevation
	A	*B*	*X*	*Y*	*Z*
C	11,651.11	15,401.38	10,295.70	12,975.88	588.75
D	18,029.01	12,102.48	12,009.25	5,722.68	367.76
E	22,466.68	27,074.66	23,153.37	16,260.99	347.36
F	22,874.62	27,323.82	23,545.39	16,132.97	354.95
G	22,600.92	27,047.11	23,255.84	16,065.68	346.86
H	—	—	—	—	—
I	8,726.05	18,391.37	9,242.94	17,136.81	125.02
J	28,957.09	28,576.09	28,093.74	9,780.29	405.80
K	16,761.42	20,062.06	16,448.15	13,496.15	257.58
L	23,633.42	20,330.74	20,458.53	6,587.17	246.69
M	14,481.29	15,615.07	12,525.10	11,231.88	309.81
N	20,955.90	16,152.54	16,347.70	5,731.24	298.35
O	19,008.11	11,494.61	11,877.46	4,411.89	280.78
P	14,147.38	10,972.04	8,927.09	8,282.08	203.03
Q	18,901.68	10,462.70	10,950.54	3,929.52	192.29
R	13,025.73	10,524.67	7,688.16	8,822.43	142.50
S	14,753.70	5,613.47	3,860.27	5,526.21	137.62

TABLE 3 —Tabulation of Error in Survey Values Derived from Wilson Photoalidade Angles—in Feet

Station	Coordinate Error			Position Error	Ratio of Precision
	ΔX	ΔY	ΔZ	$\sqrt{(\Delta X)^2+(\Delta Y)^2}$	$\dfrac{D_A+D_B}{2\sqrt{(\Delta X)^2+(\Delta Y)^2}}$
C	0	0	11.93	—	—
D	2.93	−15.96	− 2.65	16.23	1: 930
E	17.92	1.69	3.21	18.00	1:1,380
F	19.56	−15.94	9.96	25.23	1:1,000
G	11.05	− 2.42	6.09	11.31	1:2,200
H	−78.43	3.30	− 7.58	78.50	1: 390
I	9.27	1.61	2.44	9.41	1:1,440
J	119.57	20.42	− 4.94	121.30	1: 240
K	−22.41	2.48	5.54	22.55	1: 820
L	39.88	− 0.95	7.51	39.89	1: 550
M	−10.10	6.15	13.00	11.83	1:1,270
N	−17.81	14.91	−24.42	23.23	1: 800
O	−27.69	−22.02	4.71	35.38	1: 430
P	−21.64	−11.68	− 1.13	24.59	1: 510
Q	−23.81	−10.29	− 4.52	25.94	1: 570
R	− 7.33	−22.88	6.10	24.03	1: 490
S	0.90	− 3.48	3.15	3.59	1:2,840
			±6.96	±27.50	1:1,031

[6] PHOTOGRAMMETRIC ENGINEERING, Vol. XIII, No. 2, p. 309.

TABLE 4 —Tabulation of Error in Survey Values Derived from Photogoniometer Angles—in Feet

Station	Coordinate Error			Position Error	Ratio of Precision
	ΔX	ΔY	ΔZ	$\sqrt{(\Delta X)^2+(\Delta Y)^2}$	$\frac{D_A+D_B}{2\sqrt{(\Delta X)^2+(\Delta Y)^2}}$
C	0	0	− 5.28	——	——
D	1.40	0.63	− 5.91	1.50	1:10,040
E	99.89	1.39	1.00	9.99	1: 2,480
F	−13.19	0.38	11.87	13.20	1: 1,900
G	16.32	3.65	3.97	16.72	1: 1,480
H	——	——	——	——	——
I	9.09	−0.30	−11.44	9.10	1: 1,490
J	17.34	5.40	14.92	18.16	1: 1,580
K	− 2.19	−2.34	− 4.02	3.20	1: 5,740
L	8.08	2.85	− 2.32	8.57	1: 2,560
M	1.50	1.29	− 3.37	1.98	1: 7,600
N	5.34	−0.52	− 1.89	5.36	1: 3,460
O	4.13	0.02	− 2.24	4.13	1: 3,690
P	1.74	−0.46	7.38	1.80	1: 6,980
Q	9.85	2.68	− 7.85	10.21	1: 1,440
R	0.63	2.35	1.60	2.43	1: 4,850
S	2.43	5.73	− 2.81	6.22	1: 1,640
			±5.49	±7.51	1: 3,800

B. CONCLUSIONS: (*See note below*).

It appears safe to conclude that the Heyde Photogoniometer is far more suitable than the Wilson Photoalidade for extension of fourth order control, since the accuracy of the former is greater and no noteworthy difference in time of operation of the two instruments exists. However, in view of previously stated objections, the Photogoniometer is not adapted to field use because of the dependence on a rigid stable support and numerous tedious adjustments.

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NOTE: The Heyde Photogoniometer is designed to accommodate the original negative plate in a frame set before a light source which illuminates the negative from behind. Observation of negatives so illuminated suffers least loss of original exposure resolution. The Wilson Photoalidade has an opaque easel which prevents illumination of negatives from behind and correspondingly requires that paper prints be used. It must be noted then in comparing the two instruments that some of the difference in precision is due to the loss of detail on paper prints. The authors attempted to reduce this difference by using the original negative backed by a sheet of white paper in the Wilson Photoalidade. This represents an improvement over use of paper prints but does not eliminate as much of the ray deviation as is done by use of negative illumination from behind. Mr. R. M. Wilson has expressed the desire to have some Photoalidades modified to accommodate negatives for those operations requiring more precision than that of the usual tri-metrogon mapping techniques.

Though the results of this comparison indicate the Photogoniometer would be more accurate, no discredit is reflected on the Applicability of the Photoalidade to the use for which designed. As mentioned in the article, the Photoalidade was selected for comparison because of its similar capabilities and many are familiar with the instrument. Furthermore, the tests were not made with the latest model Photoalidade presently available. It is expected that the new model, incorporating Wild transit, would compare more favorably. Certainly, of the two, the Photogoniometer would be much more expensive to build.