DATA FOR A SET OF OVERLAPPING AERIAL PHOTO-GRAPHS FOR USE IN PROBLEMS OF INVESTIGATION

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Synopsis: This study is based on: (1) selecting arbitrary survey coordinates of a number of ground points conveniently spaced; (2) assigning values for the elements of exterior orientation, for a set of fictitious aerial photographs assumed taken over the area; and (3) computing the photographic coordinates of the images of the assumed ground points.

Since Nature does not provide ideal conditions, these data should have wide application in the solution of analytical, and even graphical problems in photogrammetry, and results can always be checked against answers known to be correct.

Important to remember are: (1) here is a set of data which may be used in various investigations in photogrammetry; (2) the formulas are given so that more points can be computed if desired; and (3) the formulas used are quite rigid and the resulting data can be relied upon.—*Publications Committee.*

IN PROBLEMS of investigation in aerial photogrammetry, regardless of whether the solutions are by mathematical analysis, by graphical methods, or any other process, it is often difficult to find aerial photographs which possess all the features needed for test purposes. For example, some problems may require copious ground control which may not have been obtained for any available photographs and which would require a great amount of field work and calculations; or perhaps photographs are not available having the extreme cases of tilts required for the investigation; or perhaps the ground appearing in the available photographs does not have the topographic relief required for the investigation; or some other necessary condition may not be fulfilled.

Furthermore, if a problem solved from measurements on a set of aerial photographs produces unexpected or undesired results, one can never be certain whether the difficulty is caused by errors in image identification, errors in ground control data, lens or film distortion not properly taken into account, or some other error; or whether the difficulty is caused by some faulty reasoning in the method being tested.

It, therefore, seems desirable to have available for test problems a set of over-lapping photographs computed by mathematical means. Such a set of photographs would have known values for the space coordinates of the exposure stations, and known values for the elements of space orientation. They would have any extreme tilts desired. The photographic coordinates would be known for the images of innumerable ground points whose space positions would likewise be known. These ground points would have any desired differences in elevations. These points could be used either as control points or for testing results. In fact, any desired features could be incorporated into such a set of fictitious photographs.

In 1941, a set of fictitious photographs were computed by personnel of the 30th Engineer Topographic Battalion under the supervision of the writer who was a member of that organization at that time. A limited number of mimeographed copies were made and distributed to a few organizations. Since the war, several requests for copies of the data for these photographs have been received by the writer. All available copies, except one, have been given away and some requests are still not fulfilled.

Because of the usefulness of such test photographs, and because of the great amount of work involved in computing data for them, it has been suggested that the set of photographs be submitted to the American Society of Photogrammetry for publication in PHOTOGRAMMETRIC ENGINEERING. Permission has been obtained from the Chief of Engineers for the publication of these data. They are being submitted with the hope that they will be a useful yardstick for testing photogrammetric methods and therefore will aid in the advancement of photogrammetry.

This set consists of two parallel flights of six overlapping photographs each. (See Fig. 2). The space coordinates of a number of ground points were arbitrarily chosen, and the coordinates of the exposure station and the elements of exterior orientation assumed for each photograph. The photographic coordinates for each point in each photograph were computed as shown below:

FIG. 1

In considering the formulas* used in the computation of the photographic coordinates of the images of the ground points, it will be necessary to refer to Figure 1. P is a point on the ground whose photographic coordinates are desired, and X_P, and Y_P, and Z_P are the space coordinates of that point. P_H

* Earl Church, "Analytical Computations in Aerial Photogrammetry."

is a point on the datum plane directly under P. f is the focal length of the camera, o is the principal point and v the nadir point. Then ov will be the principal line of the photograph. p is the image of the ground point P and (x'', y'') its photographic corrdinates with v as the origin and vo the positive y-axis. (x', y') are the photographic coordinates of p, with o as the origin and ov as the negative y-axis. Let (x, y) be the photographic coordinates of p, with respect to the geometric axes of the photograph. They are obtained by rotating the coordinates (x', y') through a certain angle θ. α_{LP} is the survey azimuth of the line LP. The angle ϕ is the horizontal angle, measured clockwise, between the principal plane and the vertical plane through LP and Lv. The angle m is the vertical angle at L between L_H and P. The three elements of exterior orientation of the photograph are: (1) t, or tilt; (2) α_{vo}, or the azimuth of the principal plane; and (3) s, or swing, which is the photographic direction of the line ov. The angle θ is equal to $(s-180°)$. f is equal to 150.000 mm.

	a	b	c	d	e	f	g	h	i	j	k	
	VIII		IX		X		XI		XII			
9	0	200	800	1000	800	600	200	50	700	800	802 700	
8	1000	1800	400	3500	1700	400	500	100	1900	600	400	
7	800	300	0	3000	1200	100	2000	900	2000	100	700	
6	200	-900	400	2000	200	0	4000	700	2500	500	1900	
5	400	300	1000	1600	1800	800	250	300	1050	600	600	
4	150	800	2200	2800	2000	1000	500	900	2000	400	0	
3	1400	2700	3000	1200	600	400	50	750	1100	700	3000	
2	1800	2400	1000	1000	750	150	0	400	800	200	1200	
1	2000	1800	800	900	1000	650	150	0	50	1000	500	
			I		II		III		IV		V	VI

Fig. 2. Index and Elevations of Points.

FORMULAS FOR A TILTED PHOTOGRAPH

SAMPLE OF COMPUTATION

Photo No. III

Exposure Station	Elements of Exterior Orientation
$X_L = 25{,}000$	$\alpha_{vo} = 125°00'$
$Y_L = 15{,}000$	$s = 305°00'$
$H = 20{,}000$	$t = 3°00'$
Ground Coordinates of	$\theta = (s - 180°)$
Point d 4	$= 305°00' - 180°00'$
	$= (125°00')$
$X = 20{,}000$	
$Y = 20{,}000$	
$h = 2{,}800$	

$$\alpha_{LP} = \tan^{-1} \frac{X_P - X_L}{Y_P - Y_L} \qquad (1)$$

$$\phi = \alpha_{LP} - \alpha_{vo} \qquad (2)$$

$$\tan m = \frac{L_H P_H}{Z_P Z_L} \qquad (3)$$

$$= \frac{\sqrt{(X_P - X_L)^2 + (Y_P - Y_L)^2}}{Z_L - Z_P}$$

also

$$\tan m = \frac{Y'' \cos t \sec \phi}{f \sec t - y'' \sin t} \qquad (4)$$

from which

$$y'' \cos t \sec \phi = f \sec t \tan m$$
$$\qquad\qquad - y'' \sin t \tan m$$
$$y''(\sec \phi \cos t + \sin t \tan m) = f \sec t \tan m$$

FORMULAS FOR A TILTED PHOTOGRAPH

$$y'' = \frac{f \sec t \tan m}{(\sec \phi \cos t) + (\sin t \tan m)} \qquad (5)$$

then

$$ov = f \tan t \qquad (6)$$
$$y' = y'' - ov \qquad (7)$$
$$x' = y'' \cos t \tan \phi \qquad (8)$$
$$x = + x' \cos \theta + y' \sin \theta \qquad (9)$$

$$y = - x' \sin \theta + y' \cos \theta \qquad (10)$$

SAMPLE OF COMPUTATION

$$\alpha_{LP} = \tan^{-1} \frac{-5,000}{+5,000} = \tan^{-1} 1.00000 = 315°00'$$

$$\phi = 315°00' - 125°00' = 190°00'$$

$$\tan m = \frac{7071.07}{17,200} = + .41111$$

$$y'' = \frac{(150)(1.00137)(+.41111)}{(-1.01542)(+.99863) + (+.05234)(+.41111)}$$

$$y'' = \frac{+61.75098}{-.99251} = - 62.217$$

$$ov = (150)(.05241) = + 7.862$$
$$y' = - 62.217 - 7.862 = - 70.079$$
$$x' = (-62.217)(.99863)(+.17633) = - 10.956$$
$$x = (-10.956)(-.57358) + (-70.079)(+.81915)$$
$$x = - 51.121$$
$$y = - (-10.956)(+.81915) + (-70.079)(-.57358)$$
$$y = + 49.171$$

VERTICAL PHOTOGRAPHS
Photo 1 Point b2

$$x' = (X_P - X_L)\left[\frac{f}{(H - h)}\right]$$

$$x' = (10,000 - 5,000)\left[\frac{150}{20,000 - 2,400}\right]$$

$$= 5,000 \left(\frac{150}{17,600}\right) = + 42.614$$

$$v' = (Y_P - Y_L)\left[\frac{f}{(H - h)}\right]$$

$$y' = (10,000 - 15,000)\left[\frac{150}{20,000 - 2,400}\right]$$

$$= - 5,000 \left(\frac{150}{17,600}\right) = - 42.614$$

FICTITIOUS PHOTOGRAPHS
Focal Length = 150 mm.

Assumed Data			
Flight 1		**Flight 2**	

Photo I		Photo VII	
X = 5,000'	Tilt = 0°–00.0'	X = 5,000'	Tilt = 2°–00'
Y = 15,000'	Swing = Indeterminate	Y = 35,000'	Swing = 45°–00'
H = 20,000'	α_{vo} = Indeterminate	H = 20,100'	α_{vo} = 225°–00'

Photo II		Photo VIII	
X = 15,000'	Tilt = 1°–00'	X = 15,000'	Tilt = 1°–30'
Y = 15,000'	Swing = 180°–00'	Y = 35,000'	Swing = 0°–00'
H = 20,200'	α_{vo} = 0°–00'	H = 20,000'	α_{vo} = 180°–00'

Photo III		Photo IX	
X = 25,000'	Tilt = 3°–00'	X = 25,000'	Tilt = 3°–00'
Y = 15,000'	Swing = 305°–00'	Y = 35,000'	Swing = 305°–00'
H = 20,000'	α_{vo} = 125°–00'	H = 20,400'	α_{vo} = 125°–00'

Photo IV		Photo X	
X = 35,000'	Tilt = 2°–00'	X = 35,000'	Tilt = 0°–00.0'
Y = 15,000'	Swing = 235°–00'	Y = 35,000'	Swing = Indeterminate
H = 20.000'	α_{vo} = 45°–00'	H = 20,000'	α_{vo} = Indeterminate

Photo V		Photo XI	
X = 45,000'	Tilt = 1°–30'	X = 45,000'	Tilt = 2°–00'
Y = 15,000'	Swing = 102°–00'	Y = 35,000'	Swing = 270°–00'
H = 19,900'	α_{vo} = 270°–00'	H = 19,800'	α_{vo} = 90°–00'

Photo VI		Photo XII	
X = 55,000'	Tilt = 4°–30'	X = 55,000'	Tilt = 0°–30'
Y = 15,000'	Swing = 15°–00'	Y = 35,000'	Swing = 180°–00'
H = 20,000'	α_{vo} = 180°–00'	H = 20,200'	α_{vo} = 0°–00'

X and Y—Survey Coordinates of the Exposure Station.
H —Height of Plane.

FICTITIOUS PHOTOGRAPHS
Focal length, 150 mm.

	GROUND COORDINATES (ft.)			PHOTOGRAPHIC COORDINATES (mm.)					
				Photo 1		Photo 2		Photo 3	
Point	X	Y	h	x	y	x	y	x	y
A-1	5,000	5,000	2,000	00.000	−83.333	−83.229	−85.860		
A-2	5,000	10,000	1,800	00.000	−41.209	−81.922	−43.586		
A-3	5,000	15,000	1,400	00.000	0.000	−79.798	− 2.619		
A-4	5,000	20,000	150	00.000	+37.783	−74.499	+34.636		
A-5	5,000	25,000	400	00.000	+76.531	−75.108	+72.500		
B-1	10,000	5,000	1,800	+41.209	−82.418	−41.158	−84.946		
B-2	10,000	10,000	2,400	+42.614	−42.614	−42.349	−44.974		
B-3	10,000	15,000	2,700	+43.353	0.000	−42.863	− 2.619		
B-4	10,000	20,000	800	+39.063	+39.063	−38.493	+35.880		
B-5	10,000	25,000	300	+38.071	+76.142	−37.367	+72.126		
C-1	15,000	5,000	800	+78.125	−78.125	0.000	−80.663	−85.222	−74.234
C-2	15,000	10,000	1,000	+78.947	−39.474	0.000	−41.872	−86.727	−35.578
C-3	15,000	15,000	3,000	+88.235	0.000	0.000	− 2.619	−97.170	+ 4.568
C-4	15,000	20,000	2,200	+84.270	+42.135	0.000	+38.859	−93.774	+48.199
C-5	15,000	25,000	1,000	+78.947	+78.947	0.000	+74.825	−88.783	+86.815

FICTITIOUS PHOTOGRAPHS (*Continued*)
Focal length, 150 mm.

	GROUND COORDINATES (ft.)			PHOTOGRAPHIC COORDINATES (mm.)					
				Photo 2		Photo 3		Photo 4	
Point	X	Y	h	x	y	x	y	x	y
D-1	20,000	5,000	900	+39.221	−81.071	−45.574	−73.796		
D-2	20,000	10,000	1,000	+39.246	−41.871	−46.111	−35.143		
D-3	20,000	15,000	1,200	+39.480	− 2.619	−46.887	+ 4.536		
D-4	20,000	20,000	2,800	+42.895	+40.282	−51.121	+49.171		
D-5	20,000	25,000	1,600	+39.954	+77.300	−48.526	+88.560		
E-1	25,000	5,000	1,000	+78.853	−81.484	− 6.390	−73.352	−98.303	−68.833
E-2	25,000	10,000	750	+77.478	−41.365	− 6.415	−34.221	−89.527	−28.375
E-3	25,000	15,000	600	+76.541	− 2.619	− 6.440	+ 4.509	−81.491	+10.583
E-4	25,000	20,000	2,000	+82.036	+38.405	− 6.467	+46.605	−79.702	+52.908
E-5	25,000	25,000	1,800	+80.768	+78.161	− 6.494	+88.464	−71.186	+92.531
F-1	30,000	5,000	650			+31.450	−71.163	−57.004	−74.047
F-2	30,000	10,000	150			+30.772	−32.685	−48.667	−34.078
F-3	30,000	15,000	400			+31.498	+ 4.484	−42.249	+ 3.677
F-4	30,000	20,000	1,000			+32.968	+43.897	−36.330	+42.752
F-5	30,000	25,000	800			+31.839	+83.106	−29.037	+80.244
				Photo 3		Photo 4		Photo 5	
G-1	35,000	5,000	150	+66.661	−68.558	−17.613	−78.426	−85.114	−58.565
G-2	35,000	10,000	0	+66.639	−32.049	−10.860	−40.197	−76.716	−21.737
G-3	35,000	15,000	50	+67.332	+ 4.462	− 4.291	− 3.004	−69.160	+14.700
G-4	35,000	20,000	500	+69.542	+42.452	+ 2.364	+34.667	−62.897	+52.379
G-5	35,000	25,000	250	+69.145	+80.058	+ 8.769	+70.932	−54.226	+88.567
H-1	40,000	5,000	0			+19.767	−83.935	−48.380	−66.299
H-2	40,000	10,000	400			+26.768	−47.360	−41.499	−30.251
H-3	40,000	15,000	750			+33.872	− 9.721	−34.232	+ 7.276
H-4	40,000	20,000	900	Photo 6		+40.672	+28.479	−26.377	+45.700
H-5	40,000	25,000	300			+45.557	+64.174	−17.552	+81.478
I-1	45,000	5,000	50	−85.868	−40.121	+56.737	−90.159	−11.874	−74.758
I-2	45,000	10,000	800	−81.088	− 5.924	+65.474	−54.730	− 4.324	−39.239
I-3	45,000	15,000	1,100	−73.842	+32.007	+72.935	−16.597	+ 3.843	− 0.817
I-4	45,000	20,000	2,000	−68.397	+74.921	+83.307	+23.069	+12.557	+40.181
I-5	45,000	25,000	1,050	−55.456	+113.109	+85.186	+59.649	+20.393	+77.046
				Photo 4		Photo 5		Photo 6	
J-1	50,000	5,000	1,000			+26.334	−87.328	−53.412	−52.433
J-2	50,000	10,000	200			+33.386	−46.292	−42.602	−15.052
J-3	50,000	15,000	700			+42.341	− 9.000	−34.597	+21.492
J-4	50,000	20,000	400			+49.797	+29.016	−24.611	+59.493
J-5	50,000	25,000	600			+58.413	+67.610	−14.999	+100.188
K-1	55,000	5,000	500			+64.259	−93.815	−16.200	−60.458
K-2	55,000	10,000	1,200			+75.011	−57.543	− 7.121	−26.575
K-3	55,000	15,000	3,000			+92.088	−19.574	+ 3.055	+11.403
K-4	55,000	20,000	0			+86.550	+20.661	+13.017	+48.581
K-5	55,000	25,000	600			+97.349	+59.885	+24.042	+89.726

FICTITIOUS PHOTOGRAPHS (*Continued*)
Focal length, 150 mm.

Second Flight

Point	GROUND COORDINATES (ft.)			PHOTOGRAPHIC COORDINATES (mm.)					
	X	Y	h	Photo 7		Photo 8		Photo 9	
				x	y	x	y	x	y
A-5	5,000	25,000	400	+ 3.681	−71.563	−75.547	−71.644		
A-6	5,000	30,000	200	+ 3.693	−33.787	−75.284	−33.726		
A-7	5,000	35,000	800	+ 3.704	+ 3.704	−78.151	+ 3.929		
A-8	5,000	40,000	1,000	+ 3.715	+43.263	−79.521	+43.703		
A-9	5,000	45,000	0	+ 3.727	+79.327	−76.021	+79.976		
B-5	10,000	25,000	300	+41.360	−71.642	−37.584	−71.265		
B-6	10,000	30,000	900	+42.790	−35.382	−39.013	−35.097		
B-7	10,000	35,000	300	+41.857	+ 3.715	−38.084	+ 3.929		
B-8	10,000	40,000	1,800	+45.300	+45.300	−41.522	+45.465		
B-9	10,000	45,000	200	+42.165	+80.593	−38.399	+80.753		
C-5	15,000	25,000	1,000	+82.287	−74.879	0	−73.998	−84.404	−73.417
C-6	15,000	30,000	400	+80.384	−34.618	0	−34.108	−82.653	−33.542
C-7	15,000	35,000	0	+79.327	+ 3.727	0	+ 3.929	−81.723	+ 4.558
C-8	15,000	40,000	400	+81.387	+42.563	0	+42.478	−83.880	+43.248
C-9	15,000	45,000	800	+83.562	+83.562	0	+83.188	−86.166	+84.200

Point	X	Y	h	Photo 8		Photo 9		Photo 10	
				x	y	x	y	x	y
D-5	20,000	25,000	1,600	+40.202	−76.503	−46.194	−75.038		
D-6	20,000	30,000	2,000	+41.380	−37.465	−47.413	−36.445		
D-7	20,000	35,000	3,000	+44.133	+ 3.929	−50.183	+ 4.537		
D-8	20,000	40,000	3,500	+45.833	+49.778	−51.933	+49.982		
D-9	20,000	45,000	1,000	+40.039	+84.034	−46.297	+84.109		
E-5	25,000	25,000	1,800	+81.276	−77.375	− 6.389	−74.999	−82.418	−82.418
E-6	25,000	30,000	200	+75.284	−33.726	− 6.417	−32.413	−75.758	−37.879
E-7	25,000	35,000	1,200	+79.813	+ 3.929	− 6.440	+ 4.509	−79.787	0.000
E-8	25,000	40,000	1,700	+82.586	+45.236	− 6.466	+45.016	−81.967	+40.984
E-9	25,000	45,000	800	+79.232	+83.188	− 6.490	+82.374	−78.125	+78.125
F-5	30,000	25,000	800			+30.978	−70.222	−39.062	−78.125
F-6	30,000	30,000	0			+29.784	−31.698	−37.500	−37.500
F-7	30,000	35,000	100			+30.203	+ 4.486	−37.688	0.000
F-8	30,000	40,000	400			+31.002	+41.935	−38.265	+38.265
F-9	30,000	45,000	600			+31.642	+80.711	−38.660	+77.320

Point	X	Y	h	Photo 9		Photo 10		Photo 11	
				x	y	x	y	x	y
G-5	35,000	25,000	250	+65.611	−67.509	0.000	−75.949	−83.455	−78.169
G-6	35,000	30,000	4,000	+82.131	−39.800	0.000	−46.875	−102.439	−48.571
G-7	35,000	35,000	2,000	+73.405	+ 4.457	0.000	0.000	−91.300	0.000
G-8	35,000	40,000	500	+68.041	+41.697	0.000	+38.462	−84.486	+39.600
G-9	35,000	45,000	200	+67.469	+78.383	0.000	+75.758	−83.252	+77.967
H-5	40,000	25,000	300			+38.071	−76.142	−44.094	−77.664
H 6	40,000	30,000	700			+38.860	−38.860	−44.915	−39.653
H 7	40,000	35,000	900			+39.267	0.000	−45.339	0.000
H 8	40,000	40,000	100	Photo 12		+37.688	+37.688	−43.697	+38.435
H-9	40,000	45,000	50			+37.594	+75.188	−43.598	+76.672
I-5	45,000	25,000	1,050	−78.691	−80.004	+79.156	−79.156	− 5.238	−80.048
I-6	45,000	30,000	2,500	−84.958	−43.790	+85.714	−42.857	− 5.238	−43.379
I-7	45,000	35,000	2,000	−82.421	− 1.310	+83.333	0.000	− 5.238	0.000
I-8	45,000	40,000	1,900	−81.775	+39.580	+82.873	+41.436	− 5.238	+41.925
I-9	45,000	45,000	700	−76.584	+75.278	+77.720	+77.720	− 5.238	+78.582

FICTITIOUS PHOTOGRAPHS (*Continued*)
Focal length, 150 mm.

Point	GROUND COORDINATES (ft.)			PHOTOGRAPHIC COORDINATES (mm.)						
	X	Y	h	x	y	x	y	x	y	
				Photo 10		Photo 11		Photo 12		
J-5	50,000	25,000	600			+33.519	−77.467	−38.438	−78.189	
J-6	50,000	30,000	500			+33.321	−38.535	−38.158	−39.469	
J-7	50,000	35,000	100			+32.545	0.000	−37.315	− 1.310	
J-8	50,000	40,000	600			+33.519	+38.734	−38.182	+36.874	
J-9	50,000	45,000	800			+33.924	+78.275	−37.833	+75.670	
K-5	55,000	25,000	600			+71.585	−76.776	0.000	−78.188	
K-6	55,000	30,000	1,900			+77.057	−41.123	0.000	−42.394	
K-7	55,000	35,000	700			+71.980	0.000	0.000	− 1.310	
K-8	35,000	40,000	400			+70.807	+38.000	0.000	+36.490	
K-9	55,000	45,000	1,700			+76.165	+81.353	0.000	+79.397	