EXPERIMENTS IN THE USE OF SUBTENSE TACHOMETRY FOR ESTABLISHING FOURTH ORDER HORIZONTAL CONTROL

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Synopsis: This article presents a worthwhile and interesting method of locating fourth order horizontal control, by direct observations into otherwise inaccessible places, with precise subtended angle measurements from points of known geographic position to a portable base of known length. By use of azimuth and computed distances, these positions can be plotted, according to test results, on the manuscript base to the limit of plottable accuracy at 1:24,000. The method will be of interest to those operating in rough country, and in any area where fourth order control is essential, but points cannot be located by conventional triangulation or traverse procedures, without excessive costs.—*Publication Committee.*

FIELD work for mapping in the Forest Service presents difficulties not generally found elsewhere in the country. Transportation problems in the more remote areas increase costs, and visibility is frequently restricted by timber, particular in low lying areas where planimetric detail may be very important. Many canyons and other secluded formations cannot be controlled economically by ordinary triangulation. As a means of overcoming this situation, it was decided to experiment with a new method of locating fourth order photo control. No claims are made that the method is original, or that it will furnish all the answers to map control problems. It is merely a new application of an old idea. To date, results obtained with it are encouraging, and although the preference is for a longer period of use before making any statements as to its practical value, it is believed that others may also like to test the method. At any rate, they will no doubt be interested in what has been found out so far.

For a long time, there has been dissatisfaction with the usual procedure of locating map control on high points only, when most of the map detail is in the bottoms. Yet to establish triangulation or traverse control down in the canyons and pockets, is either slow and costly, or even out of the question entirely. Frequently, however, a lookout or some other triangulation station overlooks a considerable area of lower country, in which many good photo control points along roads, railroads, etc., could be selected, and a single direction shot read to each from the one station. If the true distances to those points were known, they could be scaled along these direction lines and the points located.

Several methods for measuring distances were considered and later discarded for one reason or another. The stadia principle was one of them, but it involves a slope correction for inclined sights and has other deficiencies. Likewise, the idea of a modified range-finder, with angles read at both ends of a known base, had some merit but also some serious weaknesses. Measurements of elapsed time of radio waves between points could not be financed. It finally boiled down to one method—a sort of reversal of the range-finder principle. Why not establish the base at the station to be located and simply read the angle which this base subtends, in order to compute the distance between the instrument on the established station and the base at the new station? This is the same method Zeiss and Wild use for distance measurements, in which an invar subtense bar of known length with targets at either end is used as a base. This and the angle subtended by the targets, make possible the computation of distance between instrument and targets. Unlike stadia, there is no slope correction involved, as a simple geometric analysis will prove. With this method, then, the problem resolves itself into three parts—the length of base needed to provide the desired accuracy, the accuracy of measuring this base, and the accuracy of measuring the subtended angle.

The first step is to state the maximum error in position that is permissible. A plotting accuracy of greater than 1/100 inch, seemed unnecessary for fourth order points. At a manuscript scale of 1:24,000, 1/100 inch equals 20 feet. Therefore, the maximum allowable error in the computed distance was set at plus or minus 20 feet. It appeared possible that with a precision instrument, such as the Wild or Zeiss Theodolite reading to one second, a base short enough to be practical might be used.

Even with the finest instruments made, angular accuracy to single seconds is not easy to obtain. At ten miles, one second is only about 0.25 feet. However, to see what could be done, there was used a Wild Model T-2 Theodolite, reading to single seconds, with a micrometer which permitted estimating to tenths. A set of 12″×12″ and 24″×24″ masonite targets were prepared, with white triangles painted on backgrounds of various colors. These were individually mounted on Johnson-head tripods provided with target carriers which were laterally adjustable. A special 500-ft. band-chain tape was rated against an invar tape, and used under carefully controlled tension and temperature corrections to position the targets exactly the desired distance apart. Special tape supports were used where needed.

To provide a known distance against which the computed distance could be compared, the targets were set up at a Coast and Geodetic Survey first order station, and the instrument at a similar station about four miles distant. The two stations were "Richardson East" and "Ring"; see Coast and Geodetci Survey Special Publication No. 202. Most of the instrumental work was done at the north station "Ring" with shots directly into the sun, but this procedure was reversed on occasion.

It was found that, with sun on the targets, excellent results could be obtained but that, with the same targets, shots into the sun were not practical. A better all around solution appeared to lie in the use of lights, and in subsequent work in the field, the use of masonite targets was discontinued. Therefore two sealed-beam head-light lamps in special boxes equipped with two-vane shutters to provide triangular apertures of variable width, were mounted on the target carriers in place of the targets. These made better signals but there was a flare which was difficult to control. Next a pair of Navy Aldis Multipurpose signal lamps were used, with the directional honeycombs in the sleeves removed. These lamps have a rheostat knob on the back. By radio (two SX sets were used) between the two stations, the observer was able to direct the men at the targets so that the illumination was adjusted to exactly the right brilliance. When so adjusted, they no longer appeared as lights, but as fine pinpoints of white, which made excellent signals. Red and amber screens are also used under certain light conditions and against certain backgrounds. These will take care of nearly all conditions under which the use of this method is practical.

It was found that, under average atmospheric conditions and with careful work, it was possible to obtain an angular measurement with an error of less than a second. By angular measurement, is not meant single angles but the mean of several angles. With a Wild or Zeiss Theodolite, angles can be turned so rapidly that there is little point in reading fewer than six angles and twelve would be better. A set of twelve can easily be read in from twenty to twenty-five minutes. Not over thirty to forty minutes should be required for the instrumental work required to locate a point, and the targets can be set up by two trained men in half an hour or less. Surely this represents a modest expenditure for a well located control point.

An example of the results obtained on one particular day may be of interest. The instrument was at Station Ring and the lights at Richardson East. The base was 100 feet. A gusty wind of up to 35 and 40 miles per hour was blowing. Humidity was rather high—a fruitful source of error due, it is believed, to a certain amount of lateral refraction. Pointing was difficult on account of the wind. No attempt would be made to do other than experimental work of this kind under those field conditions. The true distance between Ring and Richardson East is 21,586.7 feet. The results were:

Angles in Set	Computed Distance	Errors
6	21,609.3 ft.	+22.6
9	21,604.4 ft.	+17.7
12	21,598.9 ft.	+12.2
6	21,564.5 ft.	−22.2
9	21,586.5 ft.	− 0.2
12	21,595.0 ft.	+ 8.3
6	21,591.8 ft.	+ 5.1
9	21,586.9 ft.	+ 0.2
12	21,603.1 ft.	+16.4
6	21,572.4 ft.	−14.3
9	21,573.7 ft.	−13.0
12	21,575.8 ft.	−10.9

The third series of twelve angles differ from most other series that were read, in that the mean of twelve angles shows a greater error than the means of six and nine angles. Almost always, the error becomes less as the number of angles increases. Probably, if enough angles are read with a one-second instrument, the desired accuracy can be obtained as long as the targets are visible, provided, of course, that the base is of such length that the subtended angle is not less than sixteen minutes. In actual use, there would be required four readings from the initial station to the "point" light and either 12 or 18 repeats on the subtended angle. The bases used were of from 100 to 150 feet at four miles; of course better results were obtained with a longer base. It was possible to reach out eight miles with a 300 foot base. In actual practice, bases up to 900 feet have been used. The greatest distance measured so far, was slightly over 9 miles.

It is obvious that accuracies would be questioned, when operating under normal field conditions. From the start, it was known that some method of obtaining a reliable check on a significant percentage of the observations was absolutely necessary. It was decided that, tentatively at least, the following procedure would afford a fair and adequate check. In selecting the locations for the subtense stations (this was done at the stations from which they were to be located, and by inspection of the photos and the surrounding terrain), about 10% of these were to be purposely selected so that other triangulation stations would be visible. Then these check stations could later be occupied, and their positions determined along the with triangulation computation, as well as by the subtense method. To eliminate any chance that extra pains would be taken in setting up the bases, etc., the base line crew was not told that these were check stations. In other words, in this as well as in all other phases of the work, there was sought facts for information and guidance, not pretty figures that would look well in a report. The adequacy of a 10% check may be open to question, but to date, no one with whom this has been discussed, has seemed to feel that a larger percentage of check stations is necessary.

In all, eighty-eight subtense stations were established on one project, of which nine stations were later occupied as check stations.

The results were:

Station	Distance by Triangulation	Distance by Subtense	Difference
Weed #2	12,039.9 ft.	12,034.3 ft.	− 5.6 ft.
Weed #5	13,142.8 ft.	13,143.5 ft.	+ 0.7 ft.
Beacon 26-3	16,182.7 ft.	16,187.0 ft.	+ 4.3 ft.
Espee Tank	6,710.5 ft.	6,711.1 ft.	+ 0.6 ft.
P.B.M. K-118 (Horn Peak #2)	12,970.7 ft.	12,974.5 ft.	+ 3.8 ft.
North Gazelle #2	10,997.0 ft.	10,991.3 ft.	− 5.7 ft.
North Gazelle #3	16,382.2 ft.	16,387.9 ft.	+ 5.7 ft.
Mt. Bradley #1	22,606.2 ft.	22,611.1 ft.	+ 4.9 ft.
Mt. Bradley #2	23,627.7 ft.	23,643.0 ft.	+15.3 ft.

Since all of these differences fall within the allowable limits of plus or minus 20 feet, it is believed that, unless unforeseen complications arise, subtense stations can be accepted wilh full confidence that they will meet all fourth-order accuracy requirements for any type of mapping, standard or substandard. Obviously, extra care will be required in plotting them on the laydown or manuscript sheets, but adequate checking should insure accuracy in this phase of the work.

The success of the method is quite dependent on adequate radio communication between the base crew and the observer on the station. Considerable use is made of signal mirrors. These, when flashed, are a signal to turn on the radio and much battery drain on standby is thus avoided.

The base crew's pickup is equipped to carry two six-volt storage batteries for the signal lights, in a rack on the running board. Leads to the car generator permit charging the batteries between setups. In additions there were three midget gasoline-driven, portable generators weighing around 60 lbs. each. No trouble was experienced in keeping batteries charged under almost any condition. Each light draws one ampere, and of course, is rheostat-controlled.

A brief description of the way in which these stations are plotted for templet laydown use, may be advisable. There was used a very rigid laydown board with $\frac{3}{4}''$ plywood top, 8 ft. by 12 ft., with one extra section, 4 ft. by 8 ft., that can be butted up against either the side or end of the larger unit, to take care of odd shaped laydowns. These boards are so constructed that a special transit unit can be used on them, without fear that the operator's weight will introduce errors due to movement of the base.

The transit is a 1-minute instrument mounted on two lateral adjusters, 90° opposed, which in turn are threaded on a standard trivet which has been mounted on a heavy steel ring. The transit spindle has been drilled so that the telescope can be directed straight down at the plotted position of an existing control point. A supplementary lens is of course required so that the glass will focus sharply in the plane of the surface on which the transit assembly rests, or in other words, the laydown board. This assembly is similar in most respects to the Photo Transit as described in PHOTOGRAMMETRIC ENGINEERING, Vol. XII, Number 1, pp. 106 by J. E. King. However, for this purpose, no need exists for the photo holder as generally illustrated.

Fourth-order control, secured by ordinary intersections, is plotted as follows: The transit is set up over a previously plotted third or higher order stations. Using the list of directions for that station, and with the plates at zero, the instrument is turned to the initial station. Then each successive direction is laid off on the laydown base, by sighting first on a needle and then on the needle hole, after the shot has been pricked. As successive stations are occupied, the positions of the fourth order points are determined by actual intersections on the laydown board. With careful work, excellent intersections are obtained.

It will be seen that, with the subtense method described above and this system of plotting the photo control, the location of the new points becomes a simple matter. However, since there is no check on the plotting, great care must be taken both in turning the angles and scaling the distance. An accurately graduated steel scale should of course be used.

Other mapping projects may present conditions in which this method would not be as suitable as in California. For example, the target lamps and accessory equipment cannot be taken too far from a road, because of the weight and bulk. Perhaps later on, refinements may result in increased portability.

It now appears that the cost of a subtense station will not ordinarily run over 25% of that for fourth-order triangulation stations which are flagged and occupied. This should not be taken to mean that substitution of triangulation by subtense is advocated. The two go hand in hand, with subtense the accessory method. But it is nice to know that it is now possible to reach down into holes that heretofore would have been beyond reach except at prohibitive cost.