FIELD CAMERA CALIBRATION

E. L. Merritt, Topographic Engineer, U. S. Naval Photographic Interpretation Center

Synopsis: The following article describes a method of calibrating surveying cameras under field conditions. Use of this procedure can be advantageous in saving valuable time for a field party operating in a remote area and finding its surveying cameras to be out of calibration.

The method involves: (1) Taking a terrestrial exposure with the camera to be calibrated, and selecting one well defined point in each of the four quadrants of the negative plate; (2) With a theodolite at the camera station, observing the horizontal and vertical angles subtended by all possible pairs of the four selected objects; (3) Measuring the photo coordinates of the four image points, and the length of the lines connecting each pair of these points.

With these angles, photo coordinates and distances, the exact principal distance and the exact coordinates of the plate perpendicular can be computed by the mathematical procedure outlined.—*Publication Committee.*

Introduction

After completing the calibration of a surveying camera under one set of conditions, it is often observed at the time of camera use, that the calibration data have changed. Temperatures radically different from those at the time of calibration, and unavoidable rough treatment after calibration, are some of the factors causing this variance. For these reasons, calibration of the camera in the field is often desirable. This is particularly true if the area being photographed is a great distance from laboratory calibration facilities. This paper describes a rigorous method of determining the principal distance and the photograph coordinates of the plate perpendicular in the field.

M. P. Bridgland[1] describes a method of determining the principal distance of a surveying camera by the solution of a quadratic equation, the known elements of which are (1) the observed horizontal angle subtended by two point objects of zero elevation with respect to the camera lens and (2) the exact X ordinates of the corresponding point images when referred to the plate perpendicular. Bridgland's method assumes that the optical axis is perpendicular to the negative plate and that the photograph coordinates of the plate perpendicular are known. Recently, manufactured surveying cameras are so designed that a preliminary determination of the plate perpendicular and adjustment of the optical axis perpendicularity to the negative plate are not possible. These preliminary values are not required in the method to be described.

Field Procedure

Four point objects are so selected that their point images will define an irregular rectangle on the negative plate. It is desirable that the point images be widely separated and that a point object be imaged in each of the four quadrants of the negative plate. The point objects need not be coplanar. Since the coordinates of the point objects are not required, any four natural features which define sharp images may be used. The angular orientation of the camera is not

[1] Bridgland, M. P., "Photographic Surveying," Canada, Department of the Interior, Bulletin no. 56, 1924.

required. However, in calibrating the terrestrial camera, the horizontal and vertical arcs should be read and recorded, for the purpose of subsequent determination of the index error of the camera's optical axis with respect to the horizontal and vertical plate axes. The horizontal and vertical angles, subtended by all possible pairs of the four point objects, are observed with a theodolite at the camera station. An exposure is made, at the camera station, of these same four point objects. Figure 1 shows a plate with ground objects A, B, C, and D imaged as a, b, c, and d.

The following angles are observed and recorded:

Horizontal	*Vertical*
$\angle A'LB'$	$\angle VA$
$\angle A'LC'$	$\angle VB$
$\angle A'LD'$	$\angle VC$
$\angle D'LB'$	$\angle VD$
$\angle B'LC'$	
$\angle D'LC'$	

Obviously the last three angles can be deduced from the first three.

$$\angle D'LB' = \angle A'LB' - \angle A'LD'$$

$$\angle B'LC' = \angle A'LC' - \angle A'LB'$$

$$\angle D'LC' = \angle A'LC' - \angle A'LD'$$

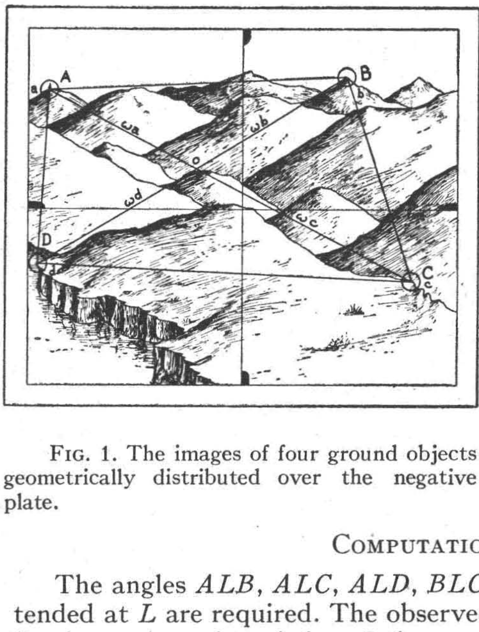

FIG. 1. The images of four ground objects geometrically distributed over the negative plate.

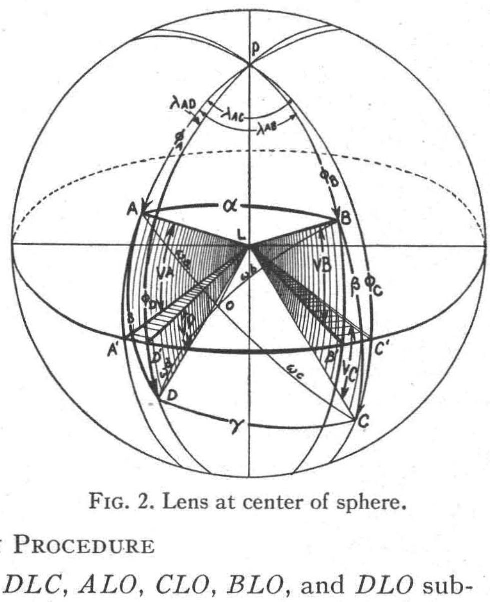

FIG. 2. Lens at center of sphere.

Computation Procedure

The angles ALB, ALC, ALD, BLC, DLC, ALO, CLO, BLO, and DLO subtended at L are required. The observed angles are not the angles subtended at L; the angles subtended at L, however, may be computed from the observed angles by spherical trigonometry. The horizontal angles become arcs of a great circle passing through the lens, corresponding to differences in longitude, while the vertical angles become meridianal arcs above or below the horizon plane bounding the horizontal arcs. These angular relations are illustrated in Figure 2. For convenience the following angular notation is adopted:

$$\angle A'LB' = \lambda AB \qquad 90° + \angle VC = \phi_C$$
$$\angle A'LC' = \lambda AC \qquad 90° + \angle VD = \phi_D$$
$$\angle A'LD' = \lambda AD \qquad \angle AB = \alpha$$
$$\angle D'LB' = \lambda DB \qquad \angle BC = \beta$$
$$\angle B'LC' = \lambda BC \qquad \angle CD = \gamma$$
$$\angle D'LC' = \lambda DC \qquad \angle AD = \delta$$
$$90° - \angle VA = \phi_A \qquad \angle AC = \omega a + \omega c$$
$$90° - \angle VB = \phi_B \qquad \angle BD = \omega b + \omega d$$

The angular sides and diagonals of the quadrilateral are computed with the law of cosines of spherical trigonometry.

$$\begin{aligned}
\cos\alpha &= \cos\phi_A \cos\phi_B + \sin\phi_A \sin\phi_B \cos\lambda AB \qquad (1)\\
\cos\beta &= \cos\phi_B \cos\phi_C + \sin\phi_B \sin\phi_C \cos\lambda BC\\
\cos\gamma &= \cos\phi_C \cos\phi_D + \sin\phi_D \sin\phi_C \cos\lambda DC\\
\cos\delta &= \cos\phi_D \cos\phi_A + \sin\phi_D \sin\phi_A \cos\lambda DA\\
\cos(\omega a + \omega c) &= \cos\phi_A \cos\phi_C + \sin\phi_A \sin\phi_C \cos\lambda AC\\
\cos(\omega b + \omega d) &= \cos\phi_B \cos\phi_D + \sin\phi_B \sin\phi_D \cos\lambda BD
\end{aligned}$$

And

$$\cos ABD = \frac{\cos\delta - \cos\alpha\cos(\omega b + \omega d)}{\sin\alpha \sin(\omega b + \omega d)} \qquad (2)$$

$$\cos BAC = \frac{\cos\beta - \cos\alpha\cos(\omega a + \omega c)}{\sin\alpha\sin(\omega a + \omega c)}$$

$$\tan\tfrac{1}{2}(\omega a + \omega b) = \frac{\cos\frac{1}{2}(ABD - BAC)}{\cos\frac{1}{2}(ABD + BAC)}\tan\frac{\alpha}{2} \qquad (3)$$

$$\tan\tfrac{1}{2}(\omega a - \omega b) = \frac{\sin\frac{1}{2}(ABD - BAC)}{\sin\frac{1}{2}(ABD + BAC)}\tan\frac{\alpha}{2}$$

$$\tfrac{1}{2}(\omega a + \omega b) + \tfrac{1}{2}(\omega a - \omega b) = \omega a$$
$$\tfrac{1}{2}(\omega a + \omega b) - \tfrac{1}{2}(\omega a - \omega b) = \omega b$$
$$\omega c = (\omega a + \omega c) - \omega a$$
$$\omega d = (\omega d + \omega d) - \omega b$$

Thus angles α, β, γ, δ, ωa, ωb, ωc, and ωd have been computed. With these angles the length of the photo lines, and the photo coordinates, the exact principal distance and the exact coordinates of the plate perpendicular can be computed. The lengths of the photo lines

$$ab = \sqrt{(xa - xb)^2 + (ya - yb)^2} \qquad (4)$$
$$bc = \sqrt{(xb - xc)^2 + (yb - yc)^2}$$

$$cd = \sqrt{(xc - xd)^2 + (yc - yd)^2}$$
$$da = \sqrt{(xd - xa)^2 + (yc - ya)^2}$$
$$ac = \sqrt{(xa - xc)^2 + (ya - yc)^2}$$
$$bd = \sqrt{(sb - xd)^2 + (yb - yd)^2}$$

Slope of the photo lines

$$mab = \frac{ya - yb}{xa - xb} \tag{5}$$

$$mac = \frac{ya - yc}{xa - xc}$$

$$mbd = \frac{yb - yd}{xb - xc}$$

$$\tan bac = \frac{mab - mac}{1 + mab \cdot mac} \tag{6}$$

$$\tan abd = \frac{mbd - mab}{1 + mbd \cdot mab}$$

Radials from 0

$$0 = 180° - (bac + abd) \tag{7}$$

$$ao = \frac{\sin abd \cdot ab}{\sin o} \qquad bo = \frac{\sin bac \cdot ab}{\sin o} \tag{8}$$

$$co = (ac - ao) \qquad do = (bd - bo)$$

Diameters of ⊙ aLc and ⊙ bLd

$$d_{ac} = \frac{ac}{\sin(\omega a + \omega c)} \qquad d_{bd} = \frac{bd}{\sin(\omega b + \omega d)} \tag{9}$$

Chords of angles ωa, ωb, ωc, ωd

$$ao' = \sin \omega a \cdot d_{ac} \qquad co' = \sin \omega c \cdot d_{ac}$$
$$bo' = \sin \omega b \cdot d_{bd} \qquad do' = \sin \omega d \cdot d_{bd}$$

Bearing of Lo′ with respect to ac and bd

$$\tan \frac{(\rho - \rho')}{2} = \frac{ao' - ao}{ao' + ao} \cdot \tan \frac{(180° - \omega c)}{2} \tag{10}$$

$$\tan \frac{(\rho_1 - \rho_1')}{2} = \frac{bo' - bo}{bo' + bo} \cdot \tan \frac{(180° - \omega d)}{2}$$

$$\frac{(\rho - \rho')}{2} + \frac{180° - \omega c}{2} = \rho$$

$$\frac{(\rho_1 - \rho_1')}{2} + \frac{180° - \omega d}{2} = \rho_1.$$

Perspective ray lengths

$$La = \frac{\sin \rho \cdot ao}{\sin \omega a} \qquad Lc = \frac{\sin \rho \cdot co}{\sin \omega c} \tag{11}$$

$$Lo = \frac{\sin (\rho - \omega a) \cdot ao}{\sin \omega a} = \frac{\sin (\omega c + \rho) \cdot co}{\sin \omega c}$$

$$Lb = \frac{\sin \rho_1 \cdot bo}{\sin \omega b} \qquad Ld = \frac{\sin \rho_1 \cdot do}{\sin \omega d}$$

$$Lo = \frac{\sin (\rho_1 - \omega b) \cdot bo}{\sin \omega b} = \frac{\sin (\omega d + \rho_1) \cdot do}{\sin \omega d}$$

At this point, sufficient values are known to compute the principal distance and the photograph coordinates of the plate perpendicular. This is more clearly visualized by examining Figure 3.

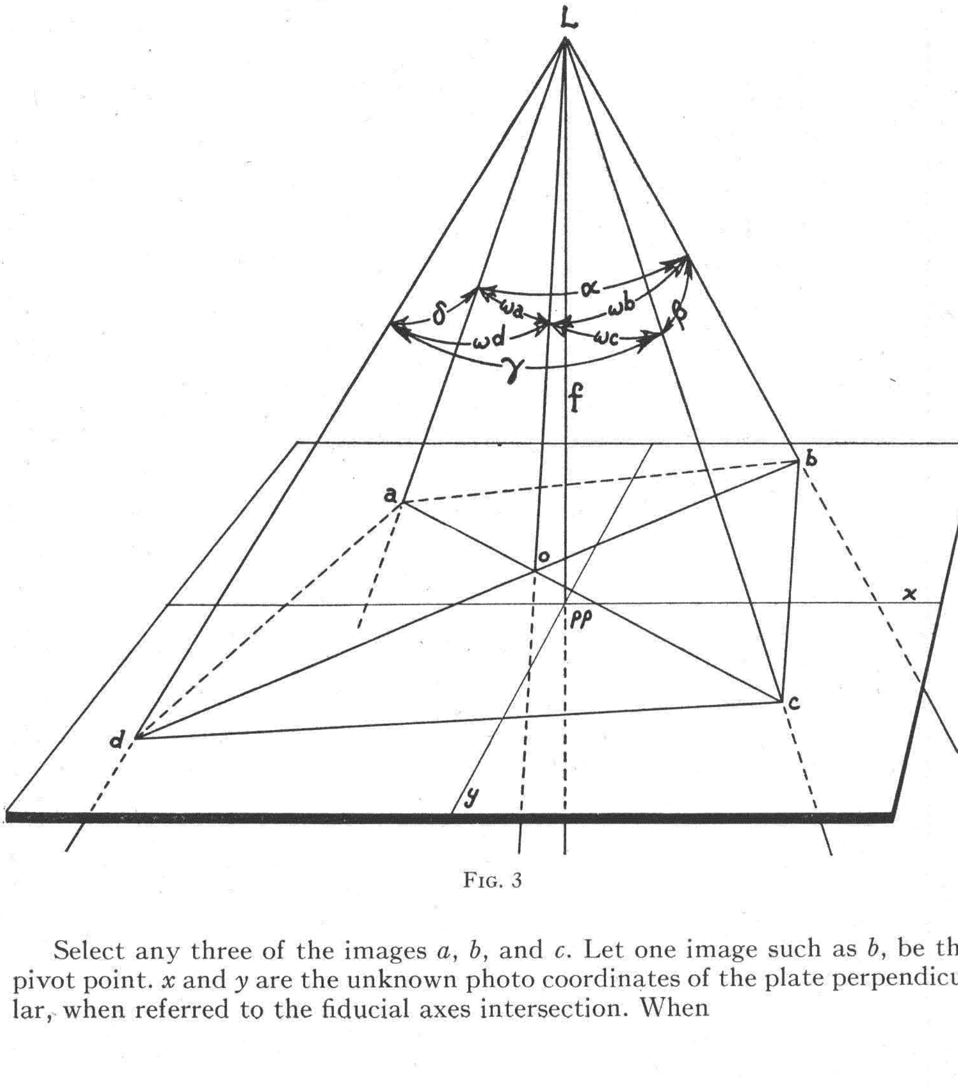

FIG. 3

Select any three of the images a, b, and c. Let one image such as b, be the pivot point. x and y are the unknown photo coordinates of the plate perpendicular, when referred to the fiducial axes intersection. When

$$xb - x = \Delta x$$
$$yb - y = \Delta y$$
$$z = f$$
$$Lb^2 = \Delta x^2 + \Delta y^2 + f^2$$

Since the abscissa of a relative to the plate perpendicular is

$$xa - x = xa - (xb - \Delta x) = xa - xb + \Delta x$$

And letting

$$xab = xa - xb$$
$$xa - x = xab - \Delta x$$

Then

$$La^2 = (xab - \Delta x)^2 + (yab - \Delta y)^2 + f^2 \qquad (1)$$
$$Lc^2 = (xcb - \Delta x)^2 + (ycb - \Delta y)^2 + f^2 \qquad (2)$$

Expand (1) and (2).

$$La^2 = xab^2 - 2xab \cdot \Delta x + \Delta x^2 + yab^2 - 2yab \cdot \Delta y + \Delta y^2 + f^2$$
$$Lc^2 = xcb^2 - 2xcb \cdot \Delta x + \Delta x^2 + ycb^2 - 2ycb \cdot \Delta y + \Delta y^2 + f^2.$$

Collect, transpose, and make the following substitutions.

$$xab^2 + yab^2 = ab^2$$
$$xcb^2 + ycb^2 = cb^2$$

Letting

$$\tfrac{1}{2}(Lb^2 - La^2 + ab^2) = Q_1$$
$$\tfrac{1}{2}(Lb^2 - Lc^2 + cb^2) = Q_2$$

Whence

$$xab \cdot \Delta x + yab \cdot \Delta y = Q_1 \qquad (1)$$
$$xcb \cdot \Delta x + ycb \cdot \Delta y = Q_2 \qquad (2)$$

The solution of these simple simultaneous equations gives Δx and Δy.

$$f = \sqrt{Lb^2 - (\Delta x^2 + \Delta y^2)} \qquad (3)$$
$$x = xb - \Delta x$$
$$y = yb - \Delta y \qquad (4)$$

This solution may be checked by using other combinations of three images, such as b, c, d, or a, b, d.

Conclusion

The method described is evaluated as follows:

Advantages

(1) It is applicable to any surveying camera.
(2) It requires no preliminary calibration or adjustments.
(3) It requires no special set-up, such as two targets of the same elevation.

(4) The principal distance, coordinates of the plate perpendicular, and horizontal and vertical index error of the optical axis may be determined from a single exposure.
(5) A camera transit can be calibrated simultaneously with the extension of control, since the same field angles and negative plates are used.

Disadvantages

(1) The accuracy of calibration falls off, if paper prints are used.
(2) The images must be very sharply defined.
(3) A coordinate measuring machine, comparable in precision to the Mann comparator, is required.

Acknowledgment

The writer expresses his grateful acknowledgment to Christie G. Mares for drafting the figures; to Arthur C. Lundahl for helpful suggestions in the composition of the manuscript; and to Elaine D. Merritt for preparation of the manuscript. The writer also wishes to thank G. C. Tewinkel of the U. S. Coast and Geodetic Survey for his careful check of the equations for mathematical validity and his evaluation of the practicality of the method.

Note: In order not to overcrowd the journal with wholly technical material a numerical example has been omitted. The interested reader may obtain one by writing to the author.

ANNOUNCEMENT OF SEMI-ANNUAL MEETING OF AMERICAN SOCIETY OF PHOTOGRAMMETRY

Philadelphia, the home of the Liberty Bell and the First Continental Congress, will be the scene of the semi-annual meeting of the American Society of Photogrammetry, on October 7 and 8, 1948.

As a result of inquiries made by the Board of Direction, Aero Service Corporation, in cooperation with Franklin Institute, has offered their facilities for sponsorship of the meeting.

Franklin Institute, where the technical sessions will convene on October 7, is known as the "Wonderland of Science." The Institute contains, besides a spacious lecture hall, 4,000 exhibits of modern science and engineering.

Plans now underway insure an interesting and informative meeting with technical papers and discussions from experts in the commercial and industrial fields of photogrammetry and mapping. Reports from Delegates to the 1948 Congress, International Society for Photogrammetry, to be held at The Hague, Holland, will be given on the state of photogrammetric industry outside of the United States and on proceedings of the International Congress.

A conducted tour of Aero Service, home of the Brock and Weymouth process, is one of the features now planned for the second day.

The Benjamin Franklin Hotel, largest in Philadelphia, has reserved a block of rooms for out-of-town members and their guests. Social events are planned to make the meeting a pleasant and memorable occasion.

A cordial invitation is extended to all members and their friends to attend. Look for a later announcement from Philadelphia on registration and further details.