FIELD CALIBRATION OF AERIAL MAPPING CAMERAS

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A. INTRODUCTION

1. The calibration of aerial mapping cameras has always been considered essential when precise ground measurements were to be determined from measurements on films or plates exposed in these cameras. It has long been realized that certain constant relationships must exist between the lens, the focal plane and the fiducial marks so that once the camera has been calibrated, the calibration data will remain unchanged for normal uses of the camera. With the advent of wide angle lenses and allied precision stereoscopic plotting instruments as well as rigorous analytical solutions of photogrammetric problems, it has become increasingly important to determine accurately camera calibration data.

2. The purpose of this paper is to give a step by step description of the procedure now used by the Aerial Photographic Branch, Engineer Research & Development Laboratories, Wright Field, in the field calibration of aerial cameras. No special laboratory calibration equipment was available at that office so it was necessary to establish an outdoor calibration range and to develop methods from which a complete calibration could be accomplished from measurements with a comparator, a dial indicator and a theodolite, all standard precise measuring instruments.

3. Since the majority of mapping cameras in use today are equipped with Metrogon (or Topogon) lenses, most emphasis will be placed on the calibration of cameras having these types of wide angle lenses. It will be shown how to (1) construct a camera calibration range, (2) accomplish the field work, (3) measure the exposed film or plate, (4) compute the camera's equivalent focal length, (5) determine the Point of Symmetry (point about which the radial distortions are symmetrical), (6) determine the calibrated focal length, (7) compute the distortion along the two diagonals, (8) construct the distortion curves, (9) measure the angles between lines connecting opposite fiducial marks, and (10) measure the flatness of the locating back (vacuum or pressure back) of the camera.

B. GENERAL

1. Data derived from camera calibrations generally include the determination of the calibrated focal length of the lens in place in the camera, the error in the indicated position of the principal point, the radial distortion of the lens, the resolution of the lens, and the degree of flatness of the focal plane (locating back). The determination of the resolution of the lens does not come within the scope of this paper. The calibrated focal length of a camera is usually defined as an adjusted value of the equivalent focal length so computed as to distribute the effect of lens distortion over the entire field used in the aerial camera. (The equivalent focal length referred to here is the perpendicular distance from the rear nodal point of the lens to the camera's focal plane.) The *calibrated focal length of a Metrogon lens* is an adjusted value of the camera's equivalent focal length so computed as to cause the distortion curve to assume most nearly the shape of the nominal Metrogon curve, in which the negative distortion at 45° from the lens axis equals the positive maximum distortion of the lens. The point of maximum positive distortion is usually about 33 degrees from the lens axis. (See Fig. 1.)

Radial distortion is said to occur when the measured distance of an image from the principal point is not equal to the computed theoretically distortion free distance. If the measured distance is greater than the computed distortion free distance, the distortion is positive and, if the measured distance is less than the computed distortion free distance, the distortion is negative. The *principal point* of a camera is the foot of the perpendicular from the interior perspective center to the plane of the photograph. This is a point easy to define and extremely difficult to find. For an ideal lens accurately mounted, a principal ray from a point on the optic axis (i.e., the optic axis) would intersect the focal plane of the camera at the principal point. However, the optic axis of the lens system is seldom an unbroken line and an axial ray may actually bend at each surface of each lens element and the ray which finally emerges may not be parallel to the incident ray.[1] This effect is presumably produced by slight decentering of the components of the lens system. Also the lens barrel may not be mounted with its axis perpendicular to the focal plane which would cause another error in the point where the axial ray intersects the focal plane. In view of these

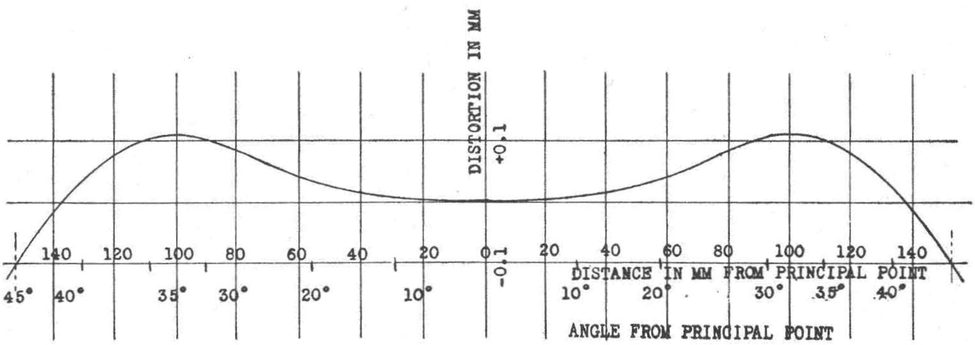


FIG. 1. Nominal distortion curve for 6″ Metrogon lens.

facts, it is obvious that the principal point cannot be determined by merely locating the point where the axial ray intersects the focal plane. An indirect method for accurately determining the position of the principal point has been described by Dr. Francis E. Washer, National Bureau of Standards, U. S. Dept. of Commerce, in Research Paper RP 1428. Since special equipment is required for finding the position of the principal point by this method, it was not practicable to adopt this method for field calibrations. Therefore, it was necessary to devise a substitute method. The one recommended here determines the location of the point in the focal plane about which all radial distortions are symmetrical. This point will be referred to as the Point of Symmetry or Ps. It is not known just exactly what this point is since its position is influenced by the various factors which cause asymmetrical distortions. It has been found, however, that it is sufficiently close to the principal point to assume that a line from the interior perspective center of the lens to the Point of Symmetry equals the equivalent focal length of the camera and also that this line is perpendicular to the focal plane. It is the opinion of the writer that, regardless of whether or not the Point of Symmetry is the same as the principal point, it is the point of greatest interest to the photogrammetrist since all radial distortions about it are approximately symmetrical.

2. In general, the procedure for calibrating an aerial camera is as follows: A

[1] NBS Research Paper RP 1428 by Dr. F. E. Washer.

row of easily identifiable targets is selected so that its distance from the camera station is greater than the hyperfocal distance of the camera to be calibrated. A theodolite is set over the camera station and the angles between the targets and a target centrally located (central target) are accurately measured. The aerial camera is placed horizontally over the camera station so that one diagonal of the focal plane frame is parallel to the ground and the camera is pointing at the central target. An exposure is made on a low shrink base film or a sensitized glass plate. By means of a comparator, the distance of each of the target images from the central target image is measured. The equivalent focal length is computed by using the central target and two other targets not more than twelve degrees off axis (the distortion of a Metrogon lens is negligible to 12°). The Point of Symmetry is then computed by finding the point about which the distortions on the two sides of the curve are equal at approximately 33 degrees off axis. The calibrated focal length is determined and the lens distortion at each target is found by computing the theoretically distortion free distance to each target from the Point of Symmetry and comparing it with the measured distance. Distortion curves are then drawn. This procedure is repeated for the other diagonal and an average calibrated focal length and the resultant Point of Symmetry are found. The angle between lines connecting opposite fiducial marks is found and the flatness of the vacuum back is measured in a manner to be described later.

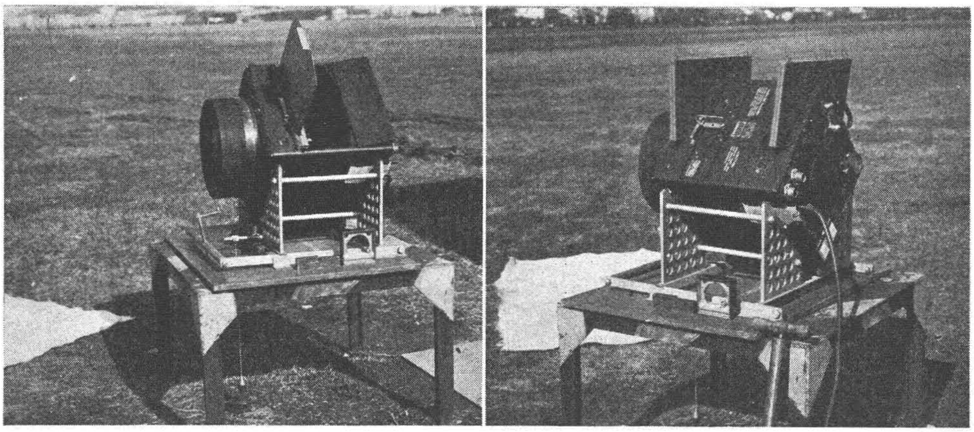
C. DESCRIPTION OF CAMERA CALIBRATION RANGE

1. Great care should be taken in selecting a site for a calibration range. By meeting the following simple requirements when selecting the site, a lot of time can be saved, errors will be reduced and computations will be simplified: (1) camera station and targets should be near the same elevation, (2) the targets should be about the same size and shape and easily identified, (3) the direction of the line of targets should be approximately east-west with the camera station to the south, (4) the camera station should be easily accessible.

2. Any stationary object which can be easily and accurately identified and which is correctly located can be used as a target. A group of buildings, a row of telephone poles, a picket fence or a group of trees usually make satisfactory targets. A camera calibration range recently constructed by the Aerial Photographic Branch, ERDL, at Wright Field is considered very satisfactory and is described here in detail.

3. This calibration range is located at the northwest edge of Wright Field inside of the steel wire fence surrounding the field. The camera station, consisting of a reference point and a camera platform, is located approximately 400 feet southeast of the line of targets. The reference point is a cross (+) marked on the cap of a two inch pipe driven firmly into the ground. The camera platform, located directly over the reference point, is 30 inches square and 18 inches high. (See Fig. 2.) The four legs are made of 2″ steel angles set in concrete. The top of the platform is made of $\frac{1}{4}$ inch sheet steel and is welded to the four legs. A hole four inches in diameter is cut in the top directly over the reference point. The targets are selected posts of a high fence that borders Wright Field. The posts, made of steel and set in concrete, are two inches wide across the surface facing the reference point and are spaced approximately ten feet apart. (See Fig. 3.) A 20 inch section of each target post is painted black and a white target background made of two $8'' \times 20'' \times \frac{1}{2}''$ sheets of Masonite is fastened to the fence, one sheet on each side of the painted area of the post. The target number is painted on one of the background sheets in black numerals 8″ high and 1″ thick. (These

numbers are not necessary as long as the central target can be identified but they are very convenient and are recommended for a permanent calibration range.) A total of 125 posts have been prepared as targets and the targets are numbered consecutively from left to right. (See Fig. 3a.) (Generally a smaller number of targets will suffice but this range was designed to be capable of calibrating the Pleon lens which has an angle of coverage of 138°.) A one (1) second theodolite was stationed over the reference point and readings of the



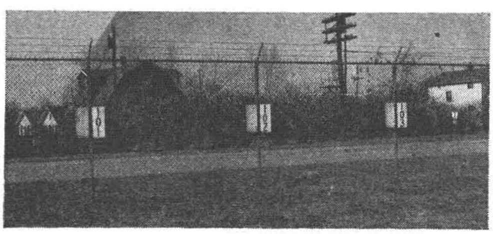
T-5 camera on camera platform. A glass plate is in place in the camera. Note plate holder with dark slide partially removed.

T-5 camera on camera platform. Film magazine in place. Note camera sights, vacuum pump, and cloth on ground.

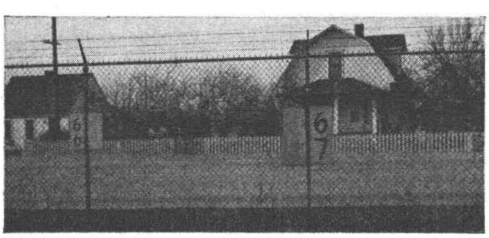
FIG. 2

horizontal angles were made on each target. A complete set of readings was made on each of three days and an average was taken of the three readings for each target. In the few cases where the angles failed to agree within six seconds, additional readings were made.

4. The calibration used prior to the one described above was set up at Huffman Dam about one half mile from Wright Field. This range consisted of a camera station and a line of guard rail posts on the road over the dam. The posts, which were about 570 feet from the camera station, were approximately 8 feet apart and were all in a straight line approximately at right angles to the axis of the camera lens. The intersection of the post with the top rail was the reference point for all measurements. Post number 107 was chosen as the central target. (See Fig. 4.)



Typical targets on Wright Field calibration range.



Target No. 67 is central target.

FIG. 3

FIG. 3a. A portion of the camera calibration range, Wright Field, Dayton, Ohio.

5. If a calibration range is set up for calibrating a certain type lens (Metrogon, for example), the targets should be selected so as to be properly spaced along the distortion curve. A suggested spacing of the targets is given below. The angle shown is the angle at the camera station between the central target and the targets to the right and left. Target No. 1 is on the extreme left and 51 is on the extreme right. Twenty-six is the central target. (See Figs. 5 and 6.)

D. DESCRIPTION OF FIELD WORK

1. Sensitized glass plates should be used for camera calibrations whenever possible. The plates should be thick enough to eliminate errors due to bending. (The plates used at Wright Field are ¼ inch thick and are coated with Super-Panchro Press emulsion.) If film is used, rough and careless handling should be avoided so that no unusual stresses are caused.

2. The camera to be calibrated is set on the camera platform in such a manner that the axis of the lens is pointing towards the central target and the front node of the lens is directly over the reference point. The camera should be positioned so that the image of the row of targets will fall along one diagonal of the focal plane frame. The front node can be located by measuring from the

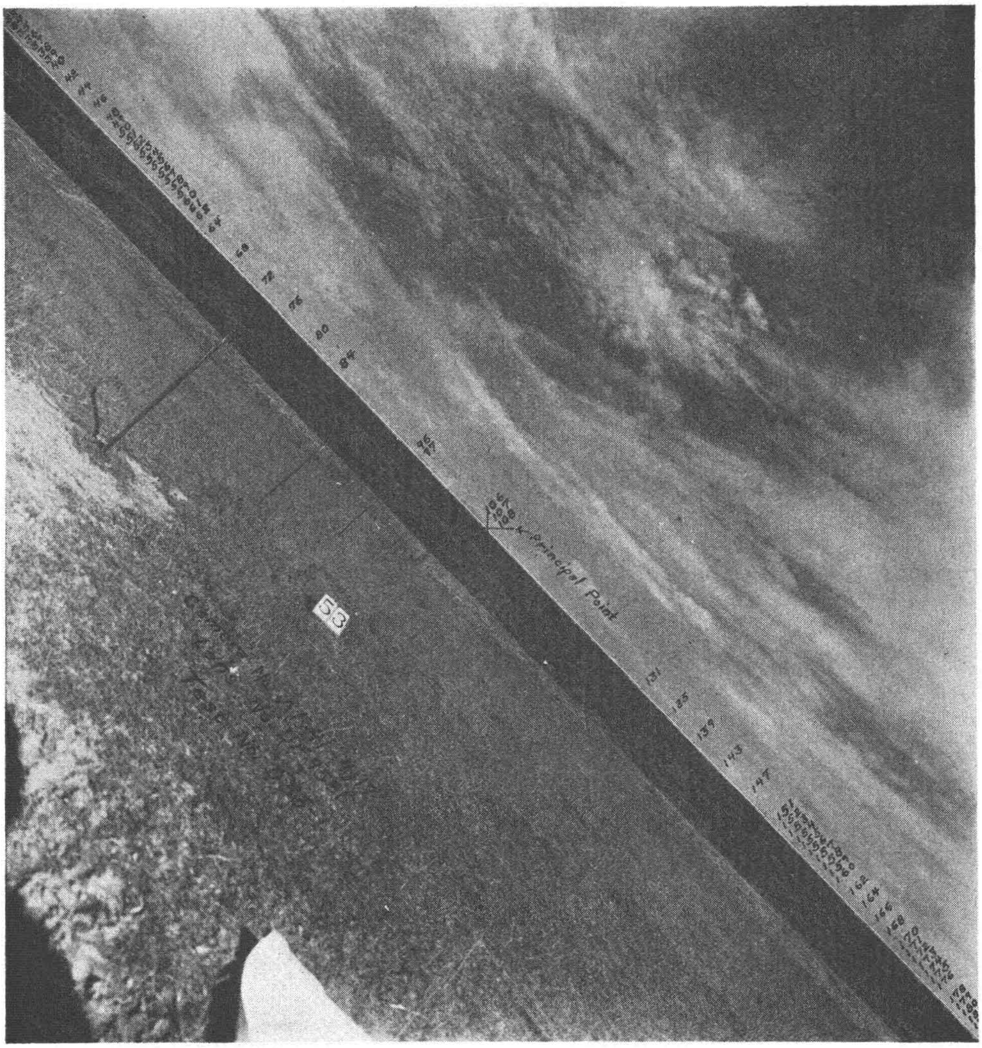


FIG. 4. Camera calibration range at Huffman Dam near Wright Field, this photograph was made with a T-5 camera and has been reduced slightly.

focal plane a distance equal to the focal length (stamped on the lens holder) less the nodal separation of the lens. (The average separation of the nodes of the 6″ Metrogon lens is approximately 13.4 mm.) The front node should be positioned over the reference point within $\pm\frac{1}{8}$ inch. It should be remembered that the nodes of a Metrogon lens are reversed. (See Fig. 6.) The lens axis can be directed accurately enough towards the central target by sighting down a straight edge of the camera or by means of very simple sights placed on the camera body. (See Fig. 2.) The lens axis can generally be directed towards the central target to within 20 minutes of the correct direction although an error of several times that amount will cause no harm.

3. Two of the fiducial marks always appear along the edges of the film which photographs the ground directly in front of the camera and are generally very difficult to see on the negative, especially if the ground is dark. In order to assure

Targets No.	Angle from Central Target	Targets No.	Angle from Central Target
1 and 51	46°	14 and 38	30°
2 50	45	15 37	28
3 49	44	16 36	26
4 48	43	17 35	24
5 47	42	18 34	21
6 46	40	19 33	18
7 45	38	20 32	14
8 44	36	21 31	12
9 43	35	22 30	11
10 42	34	23 29	10
11 41	33	24 28	9
12 40	32	25 27	4
13 39	31	26	0

FIG. 5

that sharp fiducial marks appear on every negative, two pieces of white cloth are placed on the ground in front of the camera so they will be photographed along the edges at the fiducial marks.

4. If the exposure is to be made on film, then a source of pressure differential must be provided for holding the film flat in the focal plane unless, of course, the

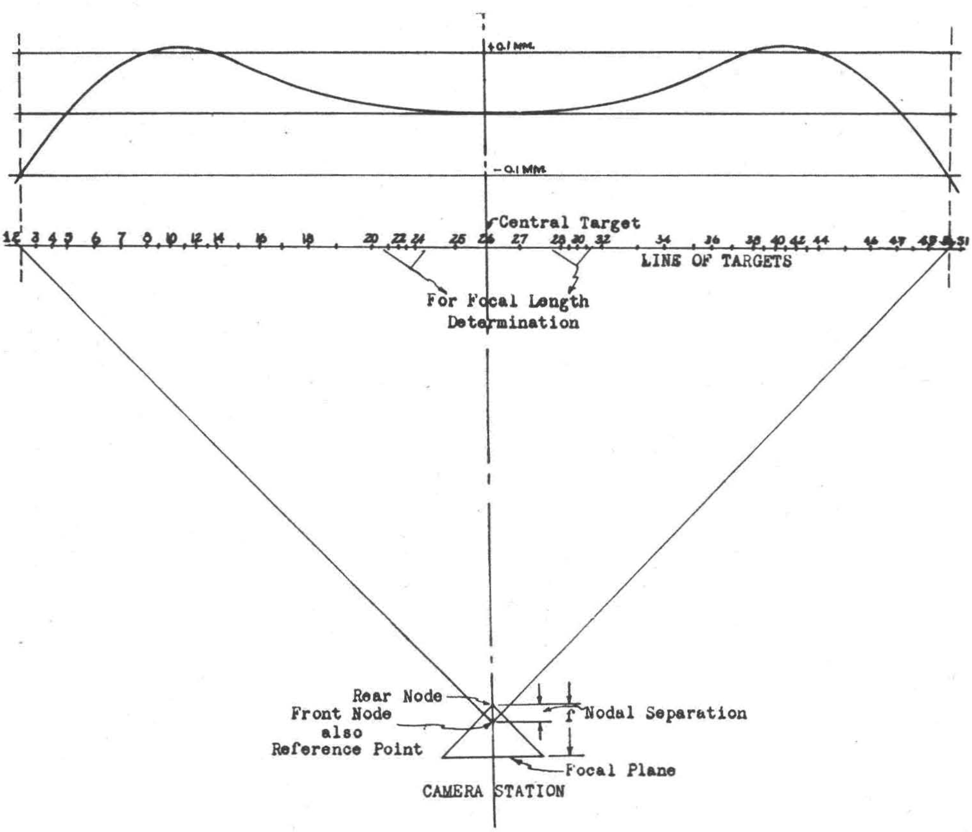


FIG. 6

camera is provided with a glass pressure plate. A satisfactory pressure differential can be supplied by an ordinary tire pump which has been modified slightly so a vacuum is created by an upward stroke. The exposure is made during the latter part of the stroke.

5. After the exposure is made along one diagonal, the camera is rotated 90°, realigned and another exposure is made on a new plate. The plates are processed in the usual manner using a fairly high contrast developer.

E. MEASURING THE NEGATIVE

1. After the film or plate has been processed, the next step is to make comparator measurements along the diagonal between target images. A comparator capable of reading directly to .001 mm. is recommended although suitable results can be obtained by using a comparator which reads to .01 mm. The precision to which a point should be measured will be discussed in a later section of this paper. The indicated position of the principal point is first located by one of several possible methods. The fastest method and the one most commonly used is to scratch very fine lines in the negative emulsion by laying a straight edge along opposite fiducial marks and scribing with a sharp needle. If great care is taken, this method might be sufficiently accurate but it is not recommended since there are other methods more suitable.

2. A simple and accurate method which can be used with any standard comparator is to position the negative so that the horizontal cross hair in the comparator telescope will be aligned with opposite fiducial marks as the negative, or telescope, is moved by the main screw of the comparator. The cross hairs are then moved in this manner until the vertical hair is approximately in line with the two vertical fiducial marks. A small piece of acetate with a fine straight line on the lower surface is then positioned at this point so that the line corresponds exactly with the horizontal cross hair. It is fastened to the negative with scotch tape. The negative is rotated 90° and the opposite fiducial marks are again aligned to the horizontal cross hair. The cross hairs are then moved as before until the vertical hair is positioned over the line on the acetate. The intersection of the cross hairs now indicate the intersection of lines between opposite fiducial marks and the acetate can be removed. The remaining problem is to prick the negative at the exact intersection of the cross hairs. This can be done quite accurately by using a piece of acetate with a very small dot or hole. The dot or hole is positioned at the intersection of the cross hairs and the acetate is fastened to the negative with scotch tape. The plate can be peimanently marked by pricking through the dot or hole with a small needle. This pricked point should be checked for accuracy before the plate is removed from the comparator. Of all the methods tried by the writer, the one just described is the most satisfactory.

3. After the indicated position of the principal point is marked, the plate is reoriented in the comparator so the diagonal to be measured is parallel to the principal motion of the comparator. Readings are taken on each target and on the indicated principal point. The angle between the line joining opposite fiducial marks and the line of targets is then measured.

4. Measuements should also be made of the focal plane frame between opposite fiducial marks. This information, which will be used for correcting mapping film for shrinkage, should always be included as part of the camera calibration data. In order to be consistent on each plate the measurements are usually made 3.0 mm. from the face of the fiducial marks as shown in Fig. 7.

5. If film is used for calibrating the camera then some means must be em-

ployed for getting the image of the fiducial marks photographed on non-shrink base. One method is to lay a sensitized glass plate across the focal plane and expose it through the camera lens over a light box in the dark room. Sensitized aluminum foil has also been used successfully by slipping it in the magazine in place of the film and using vacuum to hold it flat during exposure. It is not necessary to correct each individual measurement for film shrinkage but instead a correction can be applied to the calibrated focal length as computed along each diagonal.

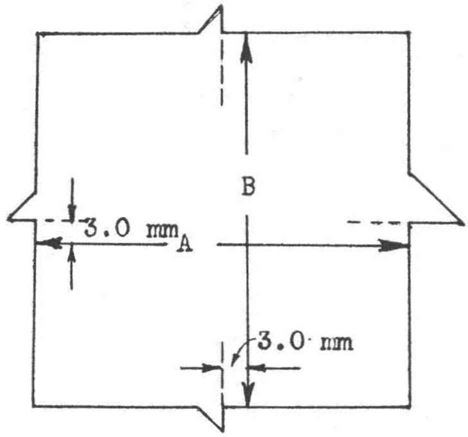


FIG. 7. Focal plane of T-5 camera.

F. DETERMINING THE EQUIVALENT FOCAL LENGTH

1. The method shown below for computing the equivalent focal length is contained in Brigadier M. Hotine's book, "Surveying From Air Photographs." Figure 8 is a section containing the rear node of a wide angle lens L and the

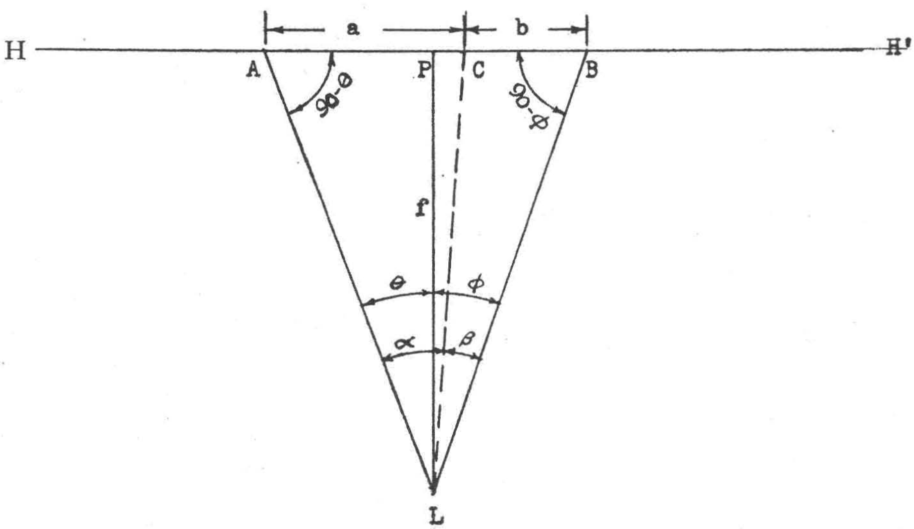


FIG. 8

horizon trace HH'. A and B are two targets whose angles, α and β, are less than 12° and greater than 4°. The principal point is assumed to be a small distance away from the horizon trace and P is the foot of the perpendicular from the principal point to the horizon trace. Therefore PL is perpendicular to HH'. Point C is the image of the central target and α and β are the angles measured with the theodolite between the central target and targets A and B respectively. a and b are the measurements obtained by the comparator. Angles θ and ϕ are unknown and are to be determined. By applying the rule of Sines to triangles ALC and CLB:

$$\frac{a}{\sin \alpha} = \frac{CL}{\sin (90 - \theta)} \quad \text{and} \quad \frac{b}{\sin \beta} = \frac{CL}{\sin (90 - \phi)}$$

$$a \cos \theta = CL \sin \alpha \qquad b \cos \phi = CL \sin \beta$$

$$\frac{b \cos \phi}{a \cos \theta} = \frac{CL \sin \beta}{CL \sin \alpha} \qquad \frac{\cos \phi}{\cos \theta} = \frac{a \csc \alpha}{b \csc \beta}$$

Now choose an auxiliary angle λ such that

$$\tan (45° + \lambda) = \frac{a \csc \alpha}{b \csc \beta} = \frac{\cos \phi}{\cos \theta} \tag{1}$$

By expanding and rearranging the last equation

$$\frac{\tan 45° + \tan \lambda}{1 - \tan 45° \tan \lambda} = \frac{\cos \phi}{\cos \theta} \qquad \frac{1 + \tan \lambda}{1 - \tan \lambda} = \frac{\cos \phi}{\cos \theta}$$

$$\cos \theta + \cos \theta \tan \lambda = \cos \phi - \cos \phi \tan \lambda$$

$$\tan \lambda(\cos \phi + \cos \theta) = \cos \phi - \cos \theta$$

$$\tan \lambda = \frac{\cos \phi - \cos \theta}{\cos \phi + \cos \theta} = -\frac{2 \sin \frac{1}{2}(\phi + \theta) \sin \frac{1}{2}(\phi - \theta)}{2 \cos \frac{1}{2}(\phi + \theta) \cos \frac{1}{2}(\phi - \theta)}$$

$$\tan \lambda = \tan \tfrac{1}{2}(\theta - \phi) \tan \tfrac{1}{2}(\theta + \phi)$$

or

$$\tan \tfrac{1}{2}(\theta - \phi) = \tan \lambda \cot \tfrac{1}{2}(\theta + \phi) = \tan \lambda \cot \tfrac{1}{2}(\alpha + \beta) \tag{2}$$

2. Referring now to the focal length computation form (See Fig. 20, Sample Computations). λ is computed from equation (1) and $\frac{1}{2}(\theta-\phi)$ is computed from equation (2). Since $\frac{1}{2}(\theta+\phi)$ is equal to $\frac{1}{2}(\alpha+\beta)$, θ and ϕ are readily found by adding $\frac{1}{2}(\theta-\phi)$ and $\frac{1}{2}(\theta+\phi)$.
Referring to Fig. 8

$$PL = f = AL \cos \theta$$

$$\frac{AL}{\cos \phi} = \frac{a + b}{\sin (\theta + \phi)}$$

$$\therefore \quad f = \frac{(a + b) \cos \phi \cos \theta}{\sin (\theta + \phi)}. \tag{3}$$

3. The focal length is usually computed for three combinations of targets A and B and an average is taken. However, if the first two computations agree within .02 mm., the third computation is unnecessary. A set of sample computations are contained in a later section of this article.

4. When determining the equivalent focal length of a camera having a *normal angle* lens in which the distortion is negligible, it is recommended that targets A and B be selected so that angles α and β are greater than 22°. This will give a stronger solution for the equivalent focal length.

5. When complete calibration data are to be determined for a camera (i.e., the P_s, the C.F.L. and the distortion curves), it is not necessary to compute the camera's equivalent focal length if an approximate value for it is known. An approximate value for the focal length can be quickly determined by assuming that PL and CL are equal and are perpendicular to HH'. The angle at L between

these two lines is always small and the two distances are always near the same value. Then

$$f = \frac{a}{\tan\alpha} \quad \text{and} \quad f = \frac{b}{\tan\beta}$$

adding

$$2f = \frac{a}{\tan\alpha} + \frac{b}{\tan\beta}$$

$$f = \frac{a}{2\tan\alpha} + \frac{b}{2\tan\beta}.$$

Also the E.F.L. stamped on the lens holder is sufficiently accurate to use for the value of f in the Point of Symmetry and Calibrated Focal Length computations shown in sections G and H, which follow. Since the calibrated focal length is normally found by adding to or subtracting from the camera's equivalent focal length a value which will cause the maximum positive distortion to equal the negative distortion at 45°, it is immaterial whether the computed equivalent focal length is used or a value near to it as long as the conditions just stated are met by the final values of the C.F.L.

G. DETERMINING THE POINT OF SYMMETRY

1. The Point of Symmetry determined for a single diagonal should probably be called an Axis of Symmetry since its location is defined by a single measurement—a distance along the row of targets from the central target. For convenience, however, the term "Point of Symmetry" will be used. Reference is made to Fig. 9.

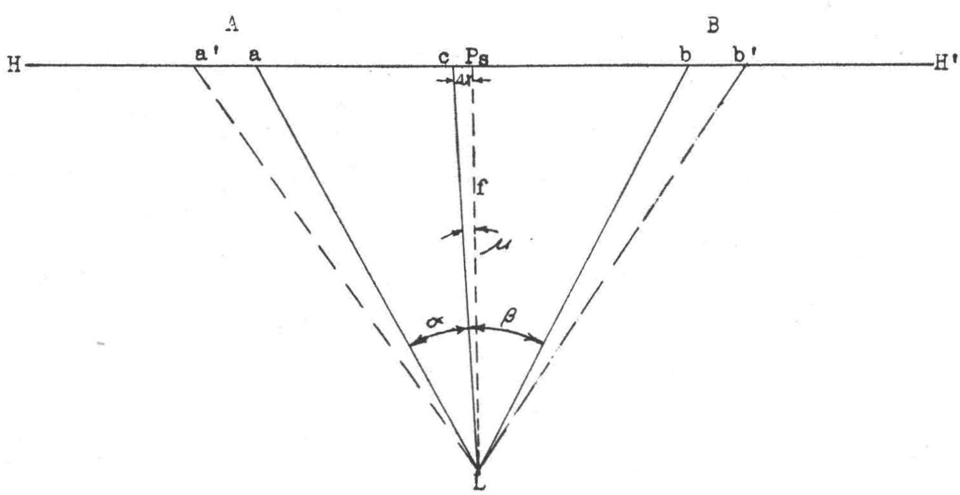


FIG. 9

f = Equivalent Focal Length. (This is not necessarily the true equivalent focal length since that would be the perpendicular distance to the focal plane and not the distance to the Point of Symmetry. However, the difference in the two lengths will always be so small that it can be neglected.)

c = image of central target

P_s = Point of Symmetry
a and b = theoretically distortion free positions of images of targets A and B
a' and b' = positions of distorted photographic images of targets A and B
α and β = measured angles to targets A and B
L = Rear nodal point of camera lens
Δx = distance from central target to P_s
μ = angle at L between central target and P_s
$k = (a'c - cb')$

2. Two targets A and B whose angles α and β, respectively, are between 33° and 35°, were selected because their images fall in the zone of maximum positive distortion of a Metrogon lens. An equation was written so that about a point a distance Δx from the central target, the distortions at A and B would be equal. Since the change in the distortion in the zone between 32°30′ and 35°30′ is less than .005 mm., any targets selected in that zone can be safely used.

$$a'P_s - aP_s = P_sb' - P_sb$$

$$(a'c + \Delta x) - (ac + \Delta x) = (cb' - \Delta x) - (cb - \Delta x)$$

$$(a'c + \Delta x) - f \tan(\alpha + \mu) = (cb' - \Delta x) - f \tan(\beta - \mu).$$

Transposing and substituting k for $(a'c - cb')$

$$k + 2\Delta x - f[\tan(\alpha + \mu) - \tan(\beta - \mu)] = 0. \tag{4}$$

Equation (4), being transcendental, is not solvable explicitly for μ. Both α and β in the equation above are large and known; μ is small, of the order of several minutes. It therefore seems desirable to expand the coefficient of f in equation (4) in a Taylor's series with the remainder, and thus reduce the equation to a solvable algebraic form. The expansion will be made around α and β for the two terms in the bracket. Taylor's theorem is:

$$\phi(x + y) = \phi(x) + y\phi'(x) + \frac{y^2\phi''(x)^2}{2!} + \cdots + \frac{y^{n-1}\phi^{n-1}(x)}{(n-1)!} + y^n\phi^n(x + \theta y)$$

where $0 \leqq \theta \leqq 1$ and where $\phi^n(x)$

represents the nth derivative of the function ϕ, evaluated at x. This expansion is readily performed and stopping with the fourth term in the expansion, the two terms of the coefficient of f in equation (4) becomes

$$\tan(\alpha + \mu) = \tan\alpha + \mu\sec^2\alpha + \mu^2\sec^2\alpha\tan\alpha$$
$$+ \frac{\mu^3}{3}[\sec^4(\alpha + \theta\mu) + 2\tan^2(\alpha + \theta\mu)\sec^2(\alpha + \theta\mu)]$$

and

$$\tan(\beta - \mu) = \tan\beta - \mu\sec^2\beta + \mu^2\sec^2\beta\tan\beta$$
$$- \frac{\mu^3}{3}[\sec^4(\beta - \theta\mu) + 2\tan^2(\beta - \theta\mu)\sec^2(\beta - \theta\mu)]$$

where $0 \leqq \theta \leqq 1$

3. These two third order remainders can be evaluated approximately to determine whether they can be dropped. Because α and β are approximately equal and because μ is so small, it is seen that both remainder terms are approximately equal numerically. The total error incurred in dropping the two remainders will therefore be equal to twice the remainder in α.

Let R_α be the remainder term in α

$$R_\alpha = \frac{\mu^3}{3} [\sec^4 (\alpha + \theta\mu) + 2 \tan^2 (\alpha + \theta\mu) \sec^2 (\alpha + \theta\mu).$$

Since μ is of the order of several minutes, it is safe to assume $\mu < 0.01$ radians. Taking $\theta = 1$, $\alpha = 35°$ and $\mu = .01$ radians

$$2R_\alpha = \frac{2 \times 10^{-6}}{3} (3.68)_t \cong 1.2 \times 10^{-6}.$$

Thus, 1.2×10^{-6} is the largest possible value of the remainder term.

Substituting $2R_\alpha$ for the remainder terms in the expansion of equation (4), and substituting C for the rest of the expansion (through the second order terms),

$$k + 2\Delta x - f(C + 2R_\alpha) = 0$$

$$\Delta x = f \frac{(C + 2R_\alpha) - k}{2} = \frac{fC - k}{2} = fR_\alpha.$$

The maximum error in Δx incurred by dropping the remainder is therefore $f R_\alpha$

$$f \cong 154\text{mm}. \qquad R_\alpha \cong 1.2 \times 10^{-6}.$$

Therefore, max error in $\Delta x \cong 10^{-4}$ mm. This error, 10^{-4} mm., resulting from the 2nd order approximations, is of higher order than the precision of plate measurement and therefore the third order remainders can be dropped.

4. A test will now be made to determine how much error in Δx will result from substituting μ for $\tan \mu$. Let $\mu = .01$ Radians $= .573°$ $\tan .573° = .0100020$

$$\begin{aligned} f\mu &= 1.5400000 \\ f \tan \mu &= 1.5403080 \\ \text{difference} &= 0.0003080 \text{ mm.} \end{aligned}$$

This is also negligible so for values of μ up to .01 radians, $f\mu$ can be substituted for $f \tan \mu$.

5. Now rewriting equation (4), in its expanded form but omitting the remainders:

$$k + 2\Delta x - f[\tan \alpha + \mu \sec^2 \alpha + \mu^2 \sec^2 \alpha \tan \alpha - \tan \beta + \mu \sec^2 \beta - \mu^2 \sec^2 \beta \tan \beta] = 0.$$

Collecting like terms and substituting $f\mu$ for Δx

$$k + 2f\mu - f[(\tan \alpha - \tan \beta) + \mu(\sec^2 \alpha + \sec^2 \beta) + \mu^2(\sec^2 \alpha \tan \alpha - \sec^2 \beta \tan \beta)] = 0$$

$$\mu^2 f[\sec^2 \beta \tan \beta - \sec^2 \alpha \tan \alpha] + \mu f[2 - \sec^2 \alpha - \sec^2 \beta] + [k - f(\tan \alpha - \tan \beta)] = 0. \quad (6)$$

The equation is now in the standard quadratic form, $A\mu^2 + B\mu + C = 0$ and can be solved in the usual manner;

Where

$$\begin{aligned} A &= f[\sec^2 \beta \tan \beta - \sec^2 \alpha \tan \alpha] \\ B &= f[2 - \sec^2 \alpha - \sec^2 \beta] \\ C &= [k - f(\tan \alpha - \tan \beta)] \end{aligned}$$

$$\therefore \quad \mu = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \text{ Radians}$$

and

$$\Delta x = f\mu. \quad (7)$$

6. It should be noted that, for a given calibration range, two targets could be selected and used for determining the Point of Symmetry of all cameras with similar lenses to be calibrated on that range. Thus all of the elements of equation (6) would remain the same except μ, f, and k. Since f is determined for each camera and since k is easily found, the computations involved in determining the Point of Symmetry become very simple and rapid. The computations can be simplified further by setting up a table as shown below. This table is computed for targets 47 and 92 of the Wright Field Calibration Range for cameras having Metrogon lenses whose equivalent focal lengths fall between 151.9 and 155.1 mm. $\alpha=34°16'32''$ and $\beta=34°39'44''$

E.F.L.	A	B	Part of C*
151.9	3.6405873	−143.1802868	1.5082151
152.0	3.6429840	.2745464	1.5092080
2.1	3.6453807	.3688060	1.5102009
2.2	3.6477774	.4630655	1.5111938
2.3	3.6501741	.5573251	1.5121867
2.4	3.6525708	.6515847	1.5131796
2.5	3.6549675	.7458443	1.5141725
2.6	3.6573642	.8401038	1.5151654
2.7	3.6597609	.9343634	1.5161583
2.8	3.6621576	−144.0286230	1.5171512
2.9	3.6645543	.1228825	1.5181441
153.0	3.6669510	.2171421	1.5191370
3.1	3.6693477	.3114017	1.5201299
3.2	3.6717444	.4056612	1.5211228
3.3	3.6741411	.4999208	1.5221157
3.4	3.6765378	.5941804	1.5231086
3.5	3.6789345	.6884400	1.5241015
3.6	3.6813312	.7826995	1.5250944
3.7	3.6837279	.8769591	1.5260873
3.8	3.6861246	.9712187	1.5270802
3.9	3.6885213	−145.0654782	1.5280731
154.0	3.6909180	.1597378	1.5290660
4.1	3.6933147	.2539974	1.5300589
4.2	3.6957114	.3482569	1.5310518
4.3	3.6981081	.4425165	1.5320447
4.4	3.7005048	.5367761	1.5330376
4.5	3.7029015	.6310357	1.5340305
4.6	3.7052982	.7252952	1.5350234
4.7	3.7076949	.8195548	1.5360163
4.8	3.7100916	.9138144	1.5370092
4.9	3.7124883	−146.0080739	1.5380021
155.0	3.7148850	.1023335	1.5389950
5.1	3.7172817	.1965931	1.5399879

* $f(\tan\alpha-\tan\beta)$

FIG. 10

7. It is suggested that two sets of targets and two separate computations be used in determining the Point of Symmetry. An average value for Δx and μ would then be used in the computations which follow. Since it requires only a few minutes to make a Point of Symmetry computation, and since the complete calibration depends on a correctly located P_s, it is believed to be good practice always to make two independent solutions for Δx and μ.

H. DETERMINING THE CALIBRATED FOCAL LENGTH

1. After the average Point of Symmetry has been located, the value of the maximum distortion is found. Since the targets which were used to find the Point of Symmetry are located very close to the points of maximum distortion, it will

be sufficiently accurate to determine the distortion at any one of those points and call it the maximum positive distortion of the lens. Reference is made to Fig. 9. It is again assumed that the line from L to P_s is perpendicular to the horizon trace HH'. The distortions at targets A and B can be found by the expressions $a'a=(a'c+\Delta x)-f\tan(\alpha+\mu)$ and $bb'=(cb'-\Delta x)\,f\tan(\beta-\mu)$. The values of μ and Δx have already been determined from equations (6) and (7), and the values $a'c$ and cb' are measured directly from the film or plate. α and β are known horizontal angles at the camera station to the two targets A and B respectively. If the values of Δx and μ have been accurately determined, the distortion at target A (aa') will equal the distortion at target B (bb') in each Point of Symmetry computation. Although it is not necessary to determine both aa' and bb', it is suggested that the two values be found as a check on the computations used for determining μ and Δx.

2. As previously stated, the calibrated focal length of a Metrogon lens is an adjusted value of the camera's equivalent focal length for which the negative distortion at 45° equals the maximum positive distortion. Therefore the next step is to determine the negative distortion at 45° out from the Point of Symmetry. If a calibration range target were always located at exactly 45° out from the P_s, then the distortion could be computed directly. Since there is no way of causing this to occur, an indirect method is employed. Two or three targets, which fall between 40° and 45° out on each side of the P_s, are selected and their distortions are found in the same manner as described for targets A and B above. Using ordinary graph paper, the distortions are plotted as ordinates against the distances from the P_s as abscissae. Since an average value of the distortions at 45° on each side of the center is desired, all of the distortions just computed are plotted as a common curve. A curve is then drawn through the plotted points (it is practically a straight line) and the value on this curve at 45°, or f distance from the P_s is the value used for the average unadjusted negative distortion (d_n). (See Fig. 231 Sample Computations.)

3. Reference is now made to Fig. 11.

f = equivalent focal length

Δf = difference between f and calibrated focal length.

p = average of measured distances to targets A and B from P_s

n = distance to imaginary target at 45° from P_s

d_p = maximum positive distortion of lens which occurs at p distance out.

d_n = negative distortion at 45°

α_p = average of horizontal angles at camera station between P_s and targets A and B.

$\alpha_n = 45°$

$d_p = p - f\tan\alpha_p$

$d_n = n - f\tan\alpha_n$

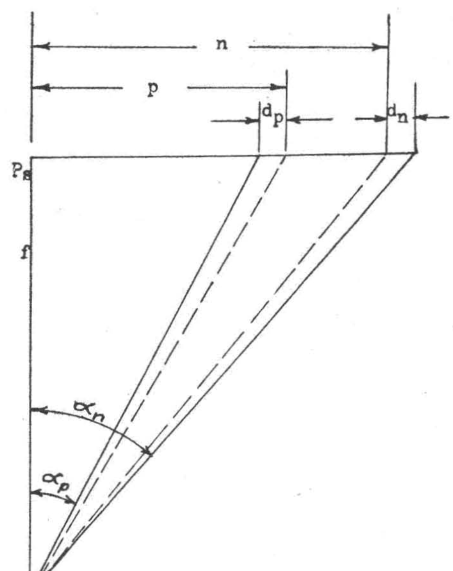


FIG. 11

Select Δf so that d_p will equal $(-d_n)$

$$p - (f + \Delta f) \tan \alpha_p = -n + (f + \Delta f) \tan \alpha_n$$

$$p + n = (f + \Delta f)(\tan \alpha_n + \tan \alpha_p)$$

or

$$\Delta f = \frac{p + n}{\tan \alpha_n + \tan \alpha_p} - f = \frac{p + n - f \tan \alpha_n - f \tan \alpha_p}{\tan \alpha_n + \tan \alpha_p}. \quad (8)$$

Since for a Metrogon lens, $\alpha_n = 45°$

$$\tan \alpha_n = 1 \qquad f \tan \alpha_n = f$$

also

$$n = f \tan \alpha_n - d_n = f - d_n$$

and

$$p - f \tan \alpha_p = d_p.$$

Substituting in equation (8)

$$\Delta f = \frac{d_p + f - d_n - f}{1 + \tan \alpha_p} = \frac{d_p - d_n}{1 + \tan \alpha_p} \quad (9)$$

$$CFL = f \pm \Delta f \quad (10)$$

If d_p is smaller numerically than d_n, then Δf is negative. It should be noted that in equation (9), d_p and d_n are subtracted without regard to signs. $\tan \alpha_p$ is the average of $\tan \alpha$ and $\tan \beta$ in the Point of Symmetry computation.

4. If film is used for making the exposure instead of glass plates, then a correction must be applied to the calibrated focal length to compensate for film shrinkage. Reference is made to Fig. 12.

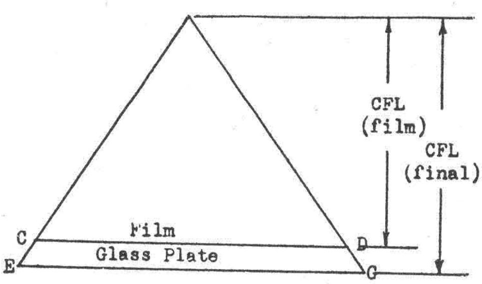


FIG. 12

CD = Average of measurements on film across fiducial marks $= \frac{A+B}{2}$ (See Fig. 7.)

EG = Average of measurements on non shrink base across same fiducial marks.

CFL (film)—calibrated focal length uncorrected for film shrinkage.

CFL (final)—corrected Calibrated Focal Length.

$$\frac{CFL \text{ (final)}}{CFL \text{ (film)}} = \frac{EG}{CD}$$

$$CFL \text{ (final)} = \frac{EG \times CFL \text{ (film)}}{CD}. \quad (11)$$

I. PLOTTING THE DISTORTION CURVE

1. The Point of Symmetry has been defined earlier as the point about which all radial distortions are symmetrical. The method just described for determining the calibrated focal length is based on average values of distortions on opposite sides of the P_s. It is therefore logical that the distortion curve plotted should be an average of the two sides of the complete distortion curve. The most direct method of obtaining this is to plot, as a common curve, values of distortion for both sides of the curve as ordinates against their respective distances from the P_s as abscissae. Since the maximum positive distortions are equal, and also the negative distortions at 45° are equal, the two halves of the curves should combine into a single average curve very readily.

2. Any type of standard graph paper with parallel lines closely spaced can be used for plotting the curve but that most commonly used by the writer has 20 lines to the inch. Along the abscissa, each line (.05 inch) represents one millimeter distance from the P_s, and along the ordinate, each line represents .005 mm. distortion. A typical distortion curve is shown in Fig. 23 Sample Computation.

3. The calibration methods described so far have been for one diagonal of the focal plane. It was mentioned earlier that plates were exposed on the calibration range for both diagonals of the focal plane. The location of the Point of Symmetry, the determination of the calibrated focal length, and the plotting of the average distortion curve must also be accomplished for the other diagonal. The calibrated focal lengths for the two diagonals should have the same value but, if there is a slight difference, an average is taken for the camera's calibrated focal length.

4. To go a step further in arriving at an average distortion curve for the camera lens, it has been found that time can be saved and results improved if the two halves of the two diagonals are plotted as a single curve. In plotting this curve, only every fourth target on each half diagonal is used. The targets are carefully selected so there is an even distribution of points along the curve. Occasionally a few extra points are plotted if the location of the curve appears questionable.

5. In order to determine the resultant P_s, the angles between a set of fiducial marks and the row of targets across the two diagonals must be known. These angles can be measured to the required accuracy on the comparator or with any good protractor. The images of the targets will seldom be in a straight line so an "average" line through the targets is used. Reference is made to Fig. 13.

r = distance from indicated PP to P_s on diagonal A

s = distance from indicated PP to P_s on diagonal B

γ, ψ = Angles between line joining fiducial marks and lines of targets across diagonals A and B respectively

x_{P_s} = x coordinate of resultant P_s

y_{P_s} = y coordinate of resultant P_s

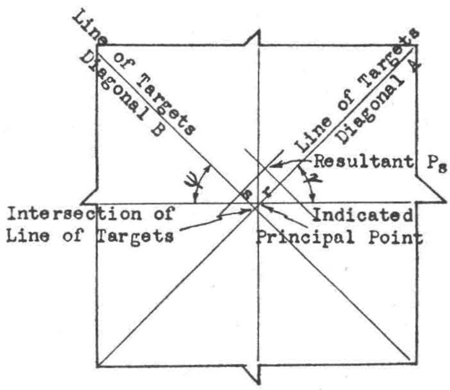


FIG. 13

From analytics the Normal Form is

$$x_{P_s} \cos \gamma + y_{P_s} \sin \gamma = r$$

and

$$x_{P_s} \cos \psi + y_{P_s} \sin \psi = s. \qquad (12)$$

By substituting the known values of r, s, γ, and ψ and solving the two equations simultaneously, the coordinates of the Point of Symmetry are found.

6. The values of x_{P_s} and y_{P_s} can also be found very readily by a graphical solution. (See Fig. 25-IV, Sample Computations.)

J. MEASURING ANGLE BETWEEN LINES CONNECTING OPPOSITE FIDUCIAL MARKS

1. Air Force specifications for precision mapping cameras (both the T-5 and T-9 cameras) state that "lines connecting opposite pairs of fiducial marks shall intersect at 90° plus or minus one minute of arc." The measuring of the angle between these lines can be accomplished in a number of ways but only two will be described here. By far the simpler method is to use a comparator having a rotating plate holder with a circle graduated to the required accuracy. The calibration plate is then fastened securely in the comparator, rotated and translated until a pair of opposite fiducial marks are accurately aligned to the longitudinal cross hair. The graduated circle is read. The plate is then rotated until the other pair of fiducial marks is aligned and the circle is read again. In order to successfully use the method just described, the angles should be read to an accuracy of 10 seconds. If a comparator having a circle graduated to the required accuracy is not available, the following secondary method is suggested.

2. This method requires the use of a plane-parallel glass plate with two finely etched lines 90° apart. This template should be as large as the negative size of the camera to be calibrated, and the lines must be positioned to an accuracy of a few seconds of arc. The negative (film or plate) is placed in the comparator with the emulsion side up and the template is placed over the negative with the surface containing the lines in contact with the negative emulsion. The template is then moved over the negative until one line passes through opposite fiducial marks and the other line passes through one of the remaining fiducial marks and close to the other. The perpendicular distance, d, (see Fig. 14) from the face of the fiducial mark to the template line and the distance, R, between this fiducial mark and the one opposite are measured. The error in alignment of the fiducial marks is the angle, ϵ, whose tangent is d/R. This value can be checked by shifting the template until the lines pass through another set of three fiducial marks.

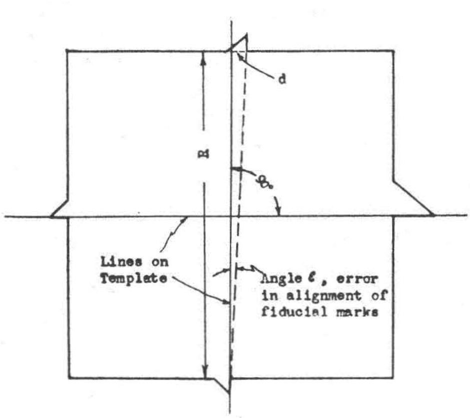


FIG. 14

An alternate, and perhaps easier, method of using the glass template is to center the intersection of the template lines over the indicated position of the principal point which has previously been marked on the negative. The template is then rotated until lines are on or near the four fiducial marks. The distances aa', bb', cc' and

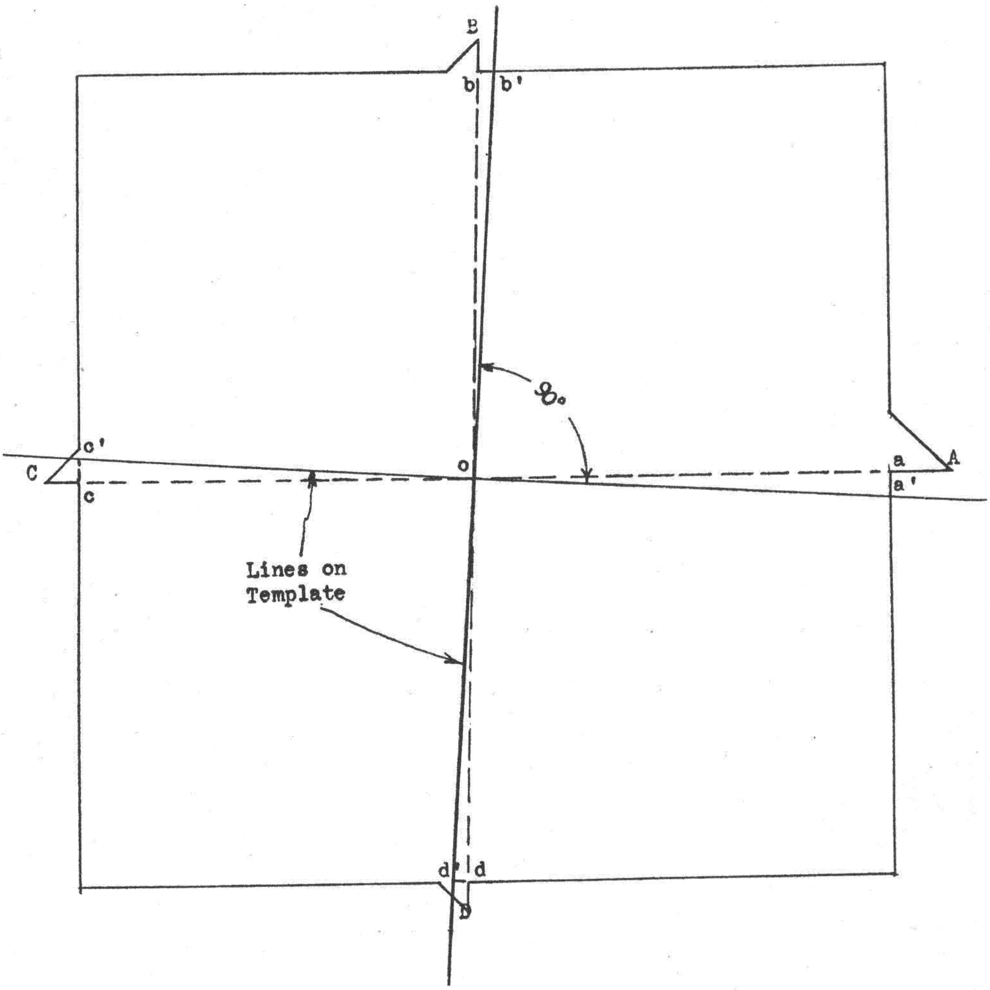


FIG. 15

dd' (Fig. 15) are measured as before. Distances *ao, bo, co, do* are also measured. Then

$$\tan \measuredangle aoa' = \frac{aa'}{oa'} \qquad \tan \measuredangle coc' = \frac{cc'}{oc'}$$

$$\tan \measuredangle bob' = \frac{bb'}{ob'} \qquad \tan \measuredangle dod' = \frac{dd'}{oa'}$$

$$\measuredangle aob = 90° - \measuredangle aoa' + \measuredangle bob' \qquad \measuredangle cod = 90° - \measuredangle coc' + \measuredangle dod'$$

$$\measuredangle boc = 90° - \measuredangle bob' + \measuredangle coc' \qquad \measuredangle doa = 90° + \measuredangle dod' + \measuredangle aoa'.$$

3. The average of the differences between 90° and the four angles *aob, boc, cod,* and *doa*, is the error in the alignment of the fiducial marks. If this error is greater than one minute of *arc*, the fiducial marks should be re-set. This adjustment should be made and the results checked before the rest of the calibration is accomplished.

4. In measuring the distances described above, it is important that a systematic method of recording them be used so the small angles will be given proper signs. One method is to letter the fiducial marks, *A*, *B*, *C*, *D*, in a counter clockwise direction starting with the large fiducial mark as shown in Fig. 15. The inside corners of the fiducial marks are named *a*, *b*, *c*, *d*, and the points on the template lines adjacent to the corners, *a'*, *b'*, *c'*, *d'*. The intersection of the lines is called *o*. The small distance between the fiducial mark and the line is then recorded by listing first the fiducial mark 90° counter clockwise to the fiducial mark being measured. For example, in Fig. 15 the small distance at *A* is recorded *Baa'*. The distance at *B* would be recorded *Cbb'*, at *C*, the distance would be *Dcc'* and at *D* it would be *Add'*. From this information, it is easy to reconstruct the location of the template with respect to the fiducial marks.

K. MEASURING THE FLATNESS OF THE LOCATING BACK

1. The most recent Air Force specifications for precision mapping cameras (type T-9) read as follows: "The surface or surfaces against which the film is held, shall, when in position for making an exposure, be such that the overall range of flatness over the entire area is not greater than .001 of an inch, and will remain within that value for a period of not less than one year." Army Map Service's (Corps of Engineers) specifications for aerial photography which accompany contracts state that, "The film shall be held flat in the focal plane to plus or minus .0005 inch either by an adequate vacuum behind the film or air pressure in front. No glass will be permitted between the lens and the film."

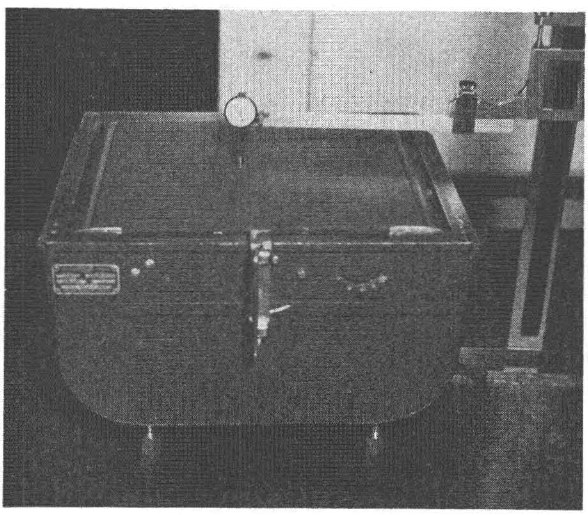


FIG. 16. Magazine from T-5 camera. The vacuum back is being measured for flatness with height gage and dial indicator. Magazine rests on small jacks which rest on surface plate.

2. A simple and accurate way of measuring the flatness of the locating back is to mount it face up on top of a surface plate and to level it by means of a height gage and dial indicator. (See Fig. 16.) The locating back's position is adjusted until readings on three corners are exactly equal. Readings are then made to an accuracy of .0001 inch on several other points across the frame as shown in Fig. 25, Sample Computations. By subtracting the value of the lowest reading from the other readings, the actual variations in the focal plane are found. The orientation of the locating back should be referenced to the fiducial marks of the camera.

3. The flatness of focal plane frames should be checked periodically since there is always danger of the frame warping from age and from improper handling.

L. CALIBRATING SHORT FOCAL LENGTH CAMERAS

1. The methods described earlier for calibrating aerial mapping cameras can be used for calibrating cameras of any focal length. Changes may have to be made in the size and the spacing of the targets and in the distance of the camera station from the row of targets, for cameras having very short or very long focal lengths.

2. A number of cameras having 35 mm. focal length lenses have recently been calibrated by the writer on an indoor calibration range. This range consisted of a camera station and reference point located about 22 feet from a wall along which the targets were located. (See Fig. 17A.) The angles to each target from the central target were carefully measured and the front node of the camera was positioned over the reference point as before. Fiducial marks had been previously located in the camera so all exposures, measurements, computations, and adjustments were made exactly as described earlier.

M. ACCURACYOFMEASUREMENTS REQUIRED

1. In 1941, Mr. Gilbert G. Lorenz made a study of the tolerances of measurement required in the calibration of a camera equipped with a 6″ Metrogon lens. The study was based on data computed for a plate exposed on a calibration range whose camera station was approximately 600 feet from the central target. The angles from the camera station to the various posts were measured with a 10 second theodolite and the distances between images of the targets on the plates were read with a comparator reading to .001 mm. The following is copied directly from the report:

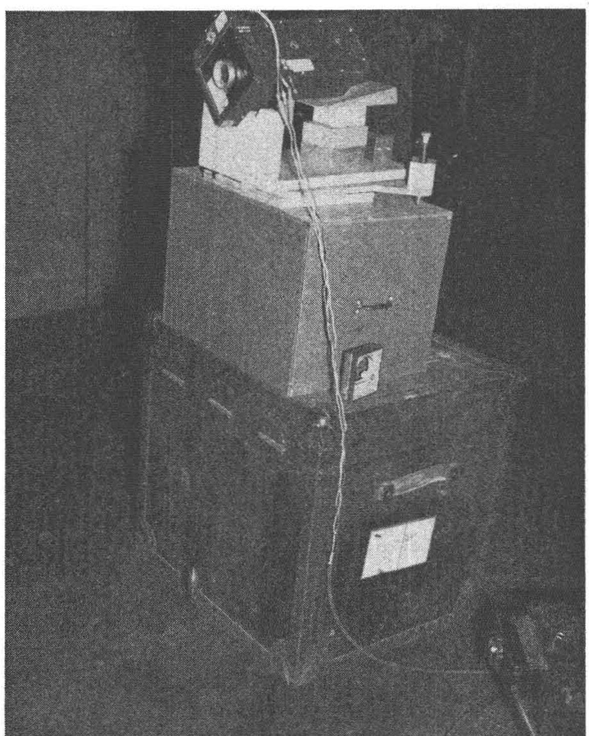


FIG. 17. 35 mm. type O-15 radar recording camera over reference point on indoor calibration range.

"B. 6. From the results of several calibrations the following errors in the factors mentioned earlier (location of node over reference point, measurements on the comparator, measurements of angles) may be expected:

a. The positioning of the front node over the station depends upon the accuracy the node can be located. The maximum errors of locating the front node of the camera should be $\pm\frac{1}{8}''$ both along the axis and perpendicular to the axis. The probable errors of location should be approximately half the maximum values.

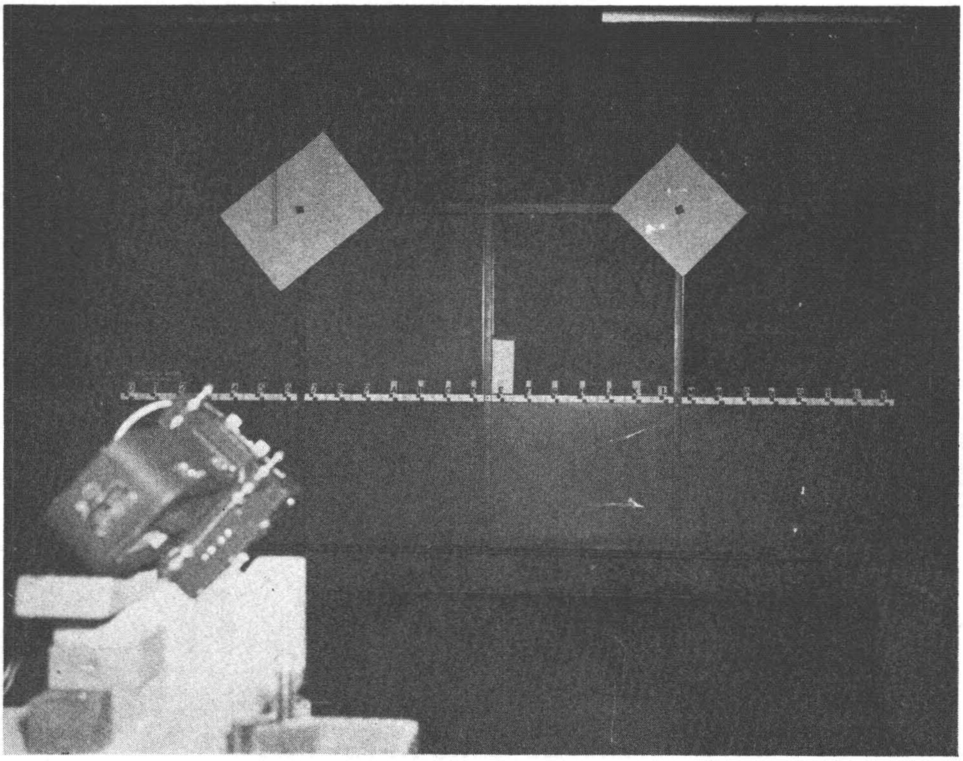


FIG. 17A. Indoor calibration range. Large white sheets photograph along edges at fiducial marks.

b. Readings of angles turned with the 10 second theodolite were estimated to 5 seconds of arc. The maximum departure of any angle from its mean in the measurement of these angles was 5 seconds. The probable error in any one angle is 1.5 seconds.

c. Although the comparator measures accurately to .001 mm., fuzziness of image makes it impossible to attain this accuracy when measuring distances on the plate. The maximum departure of any recording from its mean in one calibration was .005 mm. The probable error in any one reading for this calibration was .003 mm.

C. CONCLUSIONS:

1. In the computation of the focal length the following errors could result:

Factor	Maximum Tolerance	Probable Tolerance	Effect on focal length Maximum	Effect on focal length Probable
Positioning of node along axis	$\frac{1}{8}$ inch	$\frac{1}{16}$ inch	.002 mm.	.001 mm.
Position of node perpendicular to axis	$\frac{1}{8}$ inch	$\frac{1}{16}$ inch	none	none
Measurement of angles	5 seconds	$1\frac{1}{2}$ seconds	.014 mm.	.003 mm.
Measurement on comparator	.005 mm.	.003 mm.	.025 mm.	.015 mm.
Total			.041 mm.	.019 mm.

Since the final computed focal length is the average of four independent calculations, the probable error of the result should not be in error by more than .01 mm.

2. The positioning of the node of the lens over the station is the only factor that directly affects the distortion curve since errors in the measurement of angles and measurements on the comparator affect individual points only are averaged out in the final distortion curve. However, care must be exercised in all measurements since it is difficult to arrive at the correct curve from a series of irregular points. Since the front node of the camera can be positioned with a maximum error of $\frac{1}{8}$ inch both along the principal axis and perpendicular thereto, the effect of a displacement of this magnitude on the distortion curve will be negligible. The camera node must be located accurately from the available data on the lens design and the node must be positioned accurately over the station to maintain the accuracy of the distortion curve."

N. MISCELLANEOUS

1. Camera calibration data are generally used either (a) to correct image positions or (b) to determine whether or not a camera meets certain specifications. If the data are to be used for correcting image positions, then the corrections for distortion should be referenced to the Point of Symmetry. Positive distortion is subtracted from the measured distance between the P_s and the image in question, and negative distortion is added. In making diapositives for multiplex use, no corrections are presently made for error in the indicated position of the principal point if that error is less than a specified amount. It is the opinion of the writer that wide angle diapositive printers should be designed so that the Point of Symmetry on the aerial negative can be centered exactly over the "Point of Symmetry" on the lower pressure plate of the printer. By placing the film in the printer as just described, the resulting diapositive would be corrected for distortion by the optimum provided by the printer lens.

2. If the distortion curves of diagonals A and B do not have the same maximum distortion, it may be necessary to determine distortions also between opposite fiducial marks if precise data are desired. These distortions can be determined in the same manner as for the distortion across the diagonals except that the values already found for μ, Δx and the *CFL* can be used. After the new distortion curves are drawn, it might be advantageous to draw "Iso-Distortion" lines. This can be done by reading the distortion from the curves at predetermined intervals (say each .01 mm. of distortion) and then reading the distance from the P_s of that particular distortion. These distortions can then be plotted on polar coordinates and lines drawn through points of equal distortion.

3. Sufficient data are not available to indicate with any degree of accuracy the time required to make a complete calibration of, say, a T-5 camera. The break down given below is a fair estimate of the time required to calibrate a T-5 camera on the Wright Field Calibration Range. The time will vary with different ranges and different personnel.

	Hours	Minutes
Field Work (2 diagonals exposed)	0	30
Comparator Measurements (2 diagonals)	2	30
Computations & Curves (2 diagonals)	3	30
Totals	6	30

4. The writer wishes to express his appreciation to the following persons for

their assistance in the preparation of this paper; To: Mr. A. H. Katz, Photographic Laboratory, AMC, Wright Field for mathematical assistance in the Point of Symmetry computations; To: Mr. R. G. Hickson, Aerial Photographic Branch, ERDL, for comparator measurements and sample computations; To: Mr. George E. Whiting, Aerial Photographic Branch, ERDL, for the field work and the preparation of enclosures; and To: Miss Ruth E. Hutchins, Aerial Photographic Branch, ERDL, for typing and general clerical assistance.

APPENDIX I

1. A few important general rules are repeated here for quick reference.

a. Angles on the calibration range should be accurate to plus or minus five (5) seconds.

b. Nodes of a Metrogon lens are reversed.

c. The front node should be located within $\pm\frac{1}{8}$ inch of the camera station reference point.

d. When computing the equivalent focal length of a wide angle lens, targets should be selected which are less than 12° out from the central target.

e. When computing the equivalent focal length of a normal angle lens in which the distortion is negligible, targets should be selected which are at least 30° out from the central target.

f. When solving the equation

$$\mu = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

the negative value of $\sqrt{B^2-4AC}$ is always used.

g. If μ and Δx are positive, they are added to the values of angles and distances to the left of the central target and subtracted from the angles and distances to the right of the central target in the distortion computations.

h. Regarding the calibrated focal length computation, the value used for tan α_p is not too critical. Since the point of maximum distortion usually falls between 32°30′ and 35°30′, any α_p selected in that range will give usable results. However, an average of the four values of tan α_p used in the maximum positive distortion computation is recommended here.

i. Referring again to the calibrated focal length computation, Δf is subtracted from the equivalent focal length if d_p is smaller numerically than d_n, and added if d_p is larger than d_n. In the equation

$$\Delta f = \frac{d_p - d_n}{1 + \tan \alpha_p},$$

d_p and d_n are subtracted, without regard to signs.

j. Regarding the resultant Point of Symmetry computation, r and s represent distance measurements, and always have positive signs.

k. It is again pointed out that generally only one distortion curve is plotted for each calibration performed. There may be special cases, however, where two curves are required as in Fig. 21, and there may even be instances where a curve is required for each half of each diagonal.

l. The calibrated focal length is usually shown to the nearest hundredth of a millimeter.

APPENDIX II—SAMPLE COMPUTATIONS

1. The calibration of aerial mapping cameras by the methods described in this paper is very simple and rapid if the work is organized properly and if computation forms are used. No suggestions will be made here concerning forms to be used since each office using such forms usually prefers to design its own.

2. A list of data and a set of sample computations for calibrating type T-5 Aerial Camera No. 41–4172 are shown on the pages which follow. Detailed computations are shown for one diagonal only, although the results for the complete calibration are given.

a. Fig. 18 is a list of horizontal angles measured on the Wright Field Calibration Range.

b. Fig. 19 shows the comparator readings on the negative.

c. Fig. 20 is the form used for computing the equivalent focal length by use of a computing machine.

Target No.	Angle from Target 67 ° ′ ″	Target No.	Angle from Target 67 ° ′ ″	Target No.	Angle from Target 67 ° ′ ″
30	49 31 57	55	25 32 57	81	21 51 01
31	48 49 58	56	24 23 36	82	23 02 01
32	48 07 34	57	23 11 38	83	24 11 38
33	47 22 54	58	19 31 48	84	25 19 30
34	46 39 02	59	18 17 22	85	26 26 38
35	45 07 20	60	17 02 40	86	27 32 32
36	44 19 26	61	11 56 25	87	28 37 13
37	43 30 18	62	10 37 03	88	30 42 57
38	42 38 39	63	9 19 02	89	31 39 14
39	41 47 34	64	7 59 43	90	32 44 33
40	40 55 25	65	2 40 48	91	33 42 55
41	40 02 20	66	1 20 18	92	34 39 44
42	39 07 27	67	0 00 00	93	35 36 14
43	38 12 09	68	3 59 55	94	36 31 05
44	37 15 40	69	5 19 40	95	37 24 52
45	36 17 02	70	6 40 31	96	38 17 48
46	35 17 25	71	7 59 35	97	39 08 50
47	34 16 32	72	9 18 28	98	39 59 02
48	33 14 40	73	10 37 09	99	40 48 26
49	32 12 52	74	11 55 55	100	41 36 03
50	31 09 31	75	13 12 52	101	42 22 57
51	30 03 39	76	14 28 52	102	43 08 46
52	28 58 38	77	16 59 29	103	43 53 51
53	27 50 20	78	18 14 26	104	46 43 19
54	26 42 18	79	19 27 20	105	47 23 58
		80	20 39 43		

FIG. 18. Horizontal Angles on Wright Field Calibration Range

d. Fig. 21 is the form used for computing the equivalent focal length by logarithms.

e. If the method suggested in Par. F-5 is used the results will be as shown below:

Targets 62, 67, 73

$$f = \frac{a}{2 \tan \alpha} + \frac{b}{2 \tan \beta} \qquad f = \frac{28.936}{2 \times .1874611} + \frac{28.910}{2 \times .1874912}$$

$$= 77.179 + 77.097 = 154.276$$

Targets 63, 67, 72

$$f = \frac{25.308}{2 \times .1640649} + \frac{25.272}{2 \times .1638957}$$

$$= 77.128 + 77.098 = 154.226$$

Targets 61, 67, 74

$$f = \frac{32.633}{2 \times .2114674} + \frac{32.588}{2 \times .2113154}$$

$$= 77.158 + 77.111 = 154.269$$

$$\text{Aver. } f = 154.257$$

Target	Reading mm.	Target	Reading mm.	Target	Reading mm.
35	1.436	59	105.569	90	57.541
36	5.668	60	109.278	89	61.627
37	9.907	61	123.995	88	65.071
38	14.236	62	127.692	87	72.532
39	18.386	63	131.320	86	76.262
40	22.543	64	134.952	85	79.983
41	26.650	65	149.409	84	83.681
42	30.802	66	153.024	83	87.393
43	34.893	67	156.628	82	91.110
44	38.933			81	94.825
45	43.042	Plate Rotated 180°		80	98.510
46	47.126	103	8.727	79	102.185
47	51.183	102	12.523	78	105.840
48	55.226	101	16.288	77	109.534
49	59.152	100	20.041	76	116.826
50	63.093	99	23.775	75	120.446
51	67.108	98	27.551	74	124.070
52	70.973	97	31.296	73	127.748
53	74.969	96	35.041	72	131.386
54	78.837	95	38.829	71	135.007
55	82.719	94	42.592	70	138.599
56	86.525	93	46.326	69	142.270
57	90.391	92	50.107	68	145.874
58	101.818	91	53.816	67	156.658

FIG. 19. Comparator Readings for Diagonal A—Camera No. *AF41-4172*

f. *Locating the Point of Symmetry.* See par. G-5. Targets 47 and 92 were chosen since they are sufficiently close to the points of maximum distortion on the curve.

$$\mu = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Where

$$A = f[\sec^2 \beta \tan \beta - \sec^2 \alpha \tan \alpha]$$

$$B = f[2 - \sec^2 \alpha - \sec^2 \beta]$$

$$C = [k - f(\tan \alpha - \tan \beta)]$$

and

$$k = a'c - cb'$$

$f = 154.255$ $\qquad\qquad$ $\sec \beta = 1.2157771$

Camera No. AF-41-4127 Exposure No. A Date 5/3/48 Computer RGH

	Computation 1	Computation 2	Computation 3
a = distance from	Tar 62 to Tar 67 = 28.936	Tar 63 to Tar 67 = 25.308	Tar 61 to Tar 67 = 32.633
b = distance from	Tar 67 to Tar 73 = 28.910	Tar 67 to Tar 72 = 25.272	Tar 67 to Tar 74 = 32.588
a + b	57.846	50.580	65.221
α	10°37'03'	9°19'02"	11°56'25"
sin α	.1842516	.1619005	.2068920
β	10°37'09"	9°18"28"	11°55'55"
sin β	.1842801	.1617378	.2067497
$(\alpha+\beta) = (\theta+\phi)$ . .	21°14'12"	18°37'30"	23°52'20°
sin $(\alpha+\beta)$ . . .	.3622211	.3193728	.4046983
$1/2(\alpha+\beta) = 1/2(\theta+\phi)$ =	10°37'06"	9°18'45"	11°56'10"
1. b x sin α . . .	5.3267138	4.0915494	6.7421965
2. a x sin β . . .	5.3323290	4.0932602	6.7468630
3. $2 \div 1 = \tan(45° + \lambda)$	1.0010542	1.0004181	1.0006921
4. $45° + \lambda$	45°01'49"	45°00'43"	45°01'12"
5. λ	0°01'49"	0°00'43"	0°01'12"
6. tan λ	0005285	.0002085	.0003491
7. cot $1/2(\alpha+\beta)$.	5.3340129	6.0982935	4.7305617
8. 6x7 = tan $1/2(\theta-\phi)$	.0028190	.0012715	.0016514
9. $1/2(\theta - \phi)$.	0°09'41"	0°04'22"	0°05'41"
10. $1/2(\theta + \phi)$.	10°37'06"	9°18'45"	11°56'10"
11. 9 + 10 = θ . .	10°46'47"	9°23'07"	12°01'51"
12. $(\theta+\phi) - \theta = \phi$.	10°27'25"	9°14"23"	11°50'29"
13. cos ϕ	.9833916	.9870252	.9787194
14. (a + b) x cos ϕ	56.8852705	49.9237346	63.8330580
15. $14 \div \sin(\alpha+\beta)$	157.0457118	156.3180541	157.7299880
16. cos θ	.9823535	.9866141	.9780356
17. 15 x cos θ = f	154.274	154.226	154.266

Average f = 154.255

FIG. 20. Focal length computation sheet (machine computation).

$k = 105.445 - 106.551 = -1.106$ $\quad$ $\sec^2 \beta = 1.4781140$

$\alpha = 34°16'32''$ $\quad$ $\tan \alpha = .6815288$

$\beta = 34°39'44''$ $\quad$ $\tan \beta = .6914578$

$\sec \alpha = 1.2101577$ $\quad$ $\sec^2 \alpha \tan \alpha = .9980865$

$\sec^2 \alpha = 1.4644817$ $\quad$ $\sec^2 \beta \tan \beta = 1.0220535$

$$A = 154.255(1.0220535 - .9980865) = +3.6970296$$

$$B = 154.255(2 - 1.4644817 - 1.4781140) = -145.4000997$$

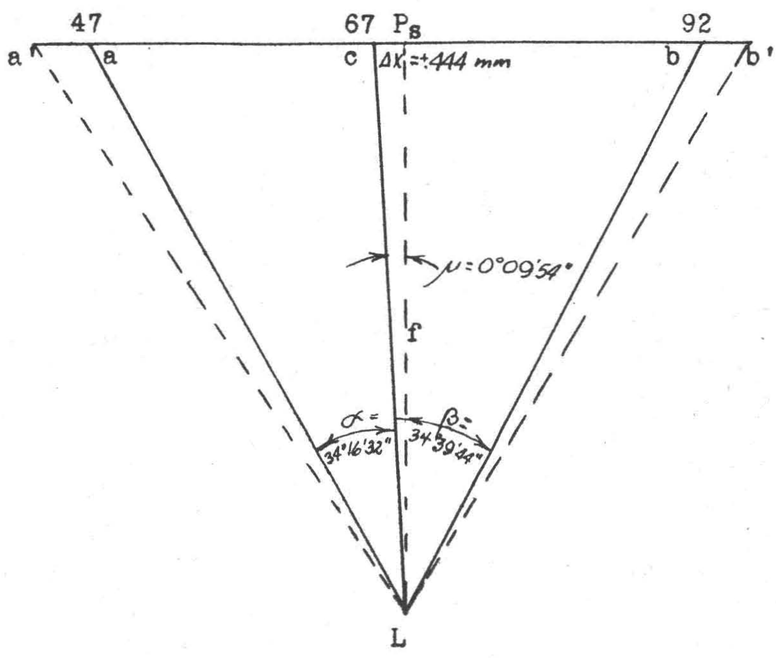
$$C = [-1.106 - 154.255(.6815288 - .6914578)] = +.4255979$$

$$\mu = \frac{145.4000997 \pm \sqrt{(-145.4000997)^2 - 4(+3.6970296)(+.4255979)}}{2(+3.6970296}$$

$$\mu = .00292743 \text{ Radians} = 0°10'04''$$

$$\Delta x = f\mu = (154.255)(00292743) = +.452 \text{ mm.}$$

g. *Determining Maximum Positive Distortions.* See par. H-2. These computations offer a check on the computations in Par. f.



$$a'a = (a'c + \Delta x) - f \tan(\alpha + \mu)$$

$$bb' = (cb' - \Delta x) - f \tan(\beta - \mu).$$

Angle at L between target 47 and $P_s = \alpha + \mu = 34°26'36''$
Angle at L between target 92 and $P_s = \beta - \mu = 34°29'40''$
Distance from target 47 to $P_s = a'c + \Delta x = 105.445 + .452 = 105.897$
Distance from target 92 to $P_s = cb' - \Delta x = 106.551 - .452 = 106.099$

Tar	∢ at *L*	Tan ∢	*f* tan ∢	Measured Distance	Distortion
47	34°26′36″	6858258	105.792	105.897	+.105
92	34 29 40	6871582	105.995	106.099	+.104

h. In making a second solution for the Point of Symmetry, targets 48 and 91 are used.

$$A = +\ .0273145f = +\ 4.2133982$$

$$B = -\ .8749597f = -\ 134.9669085$$

$$C = k + f(.01181231) = -\ 1.440 + (1.8221079) = +\ .3821079$$

$$\mu = \frac{134.9669085 - 134.9430490}{+8.4267964} = .00283137 \text{ Radians} = +\ 0°09'44''$$

The values for $a'a$ and bb' are again computed

Tar	∢ at *L*	tan ∢	*f* tan ∢	Measured Distance	Distortion
48	33°24′24″	6595455	101.738	101.839	+.101
91	33 33 11	6632180	102.305	102.405	+.100

Since the values of the distortions are slightly different from those computed above, when targets 47 and 92 were used, an average of the four values will be used in the calibrated focal length computations in par. *j* below.

Camera No. AF 41-4172

a = Target 62 to Target 67

b = Target 67 to Target 73

a = 28.936 mm $\qquad \alpha = 10°37'03''$

b = 28.910 mm $\qquad \beta = 10°37'09''$

a + b = 57.846 mm $\qquad \alpha+\beta = 21°14'12''$

$1/2(\theta + \phi) = 1/2(\alpha + \beta) = 10°37'06''$

(1)	log a	1.4614385	(13)	$1/2(\theta - \phi)$	0°09′41″
(2)	log sin	9.2654783	(14)	$1/2(\theta + \phi)$	10°37′06″
(3)	(1)+(2)	10.7269168	(15)	(13)+(14)	10°46′47″
			(16)	2(14)-(15)	10°27′25″
(4)	log b	1.4610481			
(5)	log sin( )	9.2654111	(17)	log(a + b)	1.7622733
(6)	(4)+(5)	10.7264592	(18)	log cos ϕ	9.9927264
			(19)	(17)+(18)	11.7549997
(7)	(3)-(6)	0.0004576	(20)	log sin $(\alpha+\beta)$	9.5589738
	log tan $(45 + \lambda)$		(21)	(19)-(20)	2.1960259
(8)	$45° + \lambda$	45°01′49″	(22)	log cos θ	9.9922678
			(23)	(21)+(22)	2.1882937
(9)		+0°01′49″			
(10)	log tan	6.7228283		f = 154.274	
(11)	log cot 1/2(+)	0.7270560			
(12)	(10)+(11)	7.4498843			
	log tan $1/2(\theta - \phi)$ →				

FIG. 21. Principal distance computation sheet (logarithmic).

$$\text{Average } d_p = \frac{+.104 + .105 + .101 + .100}{4} = +.102$$

$$\text{Average } \tan\alpha = \frac{.6858258 + .6871582 + .6595455 + .6632180}{4} = .6739369$$

i. *Determining Negative Distortion at 45° From the Lens Axis.* See par. H-2. From pars. g and h above the average value of

$$\mu = +00°09'54'' \quad \text{and} \quad \Delta x = +.444 \text{ mm.}$$

Tar.	∢ at L	tan ∢	f tan ∢ mm.	Measured Distance mm.	Distortion mm.
35	45°17′14″	1.0100765	155.809	155.636	−.173
37	43 40 12	.9546195	147.255	147.165	−.090
39	41 57 28	.8990706	138.686	138.686	−.000
100	41 26 09	.8827307	136.166	136.173	+.007
102	42 58 52	.9318989	143.750	143.691	−.059
103	43 43 57	.9567066	147.577	147.487	−.090

The distortions just determined for the six points are plotted as ordinates against their respective distance from the P_s as abscissa, as shown in Fig. 23—*LA* curve, is drawn through these points and the average value of the distortion at 45° is found. For this example $d_n = -.160$ mm.

j. *Determining the Calibrated Focal Length.*

$$\Delta f = \frac{d_p - d_n}{1 + \tan\alpha_p}$$

$$d_p = +.102 \qquad d_n = -.160 \qquad \tan\alpha = .6739369$$

$$\Delta f = \frac{.102 - .160}{1 + .6739369} = .035 \qquad CFL = 154.255 - .035 = 154.220 \text{ mm.}$$

k. *Plotting the Distortion Curve.* As in Par. g above, the distortion at each target is found by comparing the measured distance from the P_s, with the computed theoretically distortion free distance. The measured distance is found by adding the average value of Δx(+.444 mm.) to the distances between the P_s and the target images to the left of P_s, and by subtracting the average Δx from the distances between the P_s and the target images to the right of P_s. The computed distances are found by the expression f tan ∢ where the angles to the left of P_s are equal to the measured angles shown in Fig. 18 plus the average value of μ(0°09′54″). The angles to the right of P_s are equal to the measured angles minus the value of μ. f is equal to the *CFL* just determined (154.220 mm.). The distortion computations for diagonal *A* are shown in Fig. 22. The computations for diagonal *B* are not shown. The distortion curves for diagonals *A* and *B* are shown in Fig. 23. However, it is not recommended that these curves be plotted separately. In Par. I-4 it is suggested that the two halves of the curves for the two diagonals be plotted as a single curve by using selected targets from each half diagonal to provide an even distribution of points along the final curve. Fig. 24 shows the computations for the selected points along this curve. An average of the *CFL*'s for diagonals *A* and *B* is used. *CFL* (diagonal *B*) = 154.200

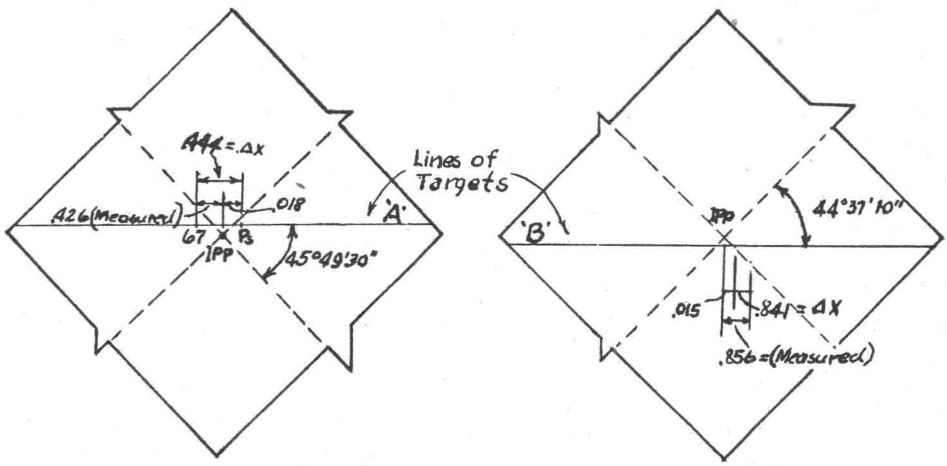
$$\text{Average } CFJ = \frac{154.22 + 154.20}{2} = 154.21 \text{ mm.}$$

$$\Delta x(\text{diagonal } B) = -.841 \text{ mm.} \qquad \mu(\text{diagonal } B) = -.0°18'45''$$

Target	∢ at L	tan ∢	f tan ∢	Measured Distance	Distortion
35	45°17′14″	1.0100765	155.774	155.636	−.138
36	44 29 20	.9823161	151.493	151.404	−.089
37	43 40 12	.9546195	147.221	147.165	−.056
38	42 48 33	.9263074	142.855	142.836	−.019
39	41 57 28	.8990706	138.655	138.686	+.031
40	41 05 19	.8720056	134.481	134.529	+.048
41	40 12 14	.8451819	130.344	130.422	+.078
42	39 17 21	.8181748	126.179	126.270	+.091
43	38 22 03	.7916671	122.091	122.179	+.088
44	37 25 34	.7652801	118.021	118.139	+.118
45	36 26 56	.7385815	113.904	114.030	+.126
46	35 27 19	.7121160	109.823	109.946	+.123
47	34 26 26	.6857545	105.757	105.889	+.132
48	33 24 34	.6596150	101.726	101.846	+.120
49	32 22 46	.6341162	97.793	97.920	+.127
50	31 19 25	.6085741	93.854	93.979	+.125
51	30 13 33	.5826177	89.851	89.964	+.113
52	29 08 32	.5575585	85.987	86.099	+.112
53	28 00 14	.5317965	82.014	82.103	+.089
54	26 52 12	.5066708	78.139	78.235	+.096
55	25 42 51	.4815721	74.268	74.353	+.085
56	24 33 30	.4569564	70.472	70.547	+.075
57	23 21 32	.4318870	66.606	66.681	+.075
58	19 41 42	.3579534	55.204	55.254	+.050
59	18 27 16	.3337115	51.465	51.503	+.038
60	17 12 34	.3097324	47.767	47.794	+.027
61	12 06 19	.2144778	33.077	33.077	0
62	10 46 57	.1904437	29.370	29.380	+.010
73	10 27 15	.1845118	28.455	28.466	+.011
74	11 46 01	.2083089	32.125	32.144	+.019
75	13 02 58	.2317773	35.745	35.768	+.023
76	14 18 58	.2551963	39.356	39.388	+.032
77	16 49 35	.3024204	46.639	46.680	+.041
78	18 04 32	.3263782	50.334	50.374	+.040
79	19 17 26	.3500100	53.979	54.029	+.050
80	20 29 49	.3738239	57.651	57.704	+.053
81	21 41 07	.3976507	61.326	61.389	+.063
82	22 52 07	.4217711	65.046	65.104	+.058
83	24 01 44	.4458330	68.756	68.821	+.065
84	25 09 36	.4697118	72.439	72.533	+.094
85	26 16 44	.4937724	76.150	76.231	+.081
86	27 22 38	.5178465	79.862	79.952	+.090
87	28 27 19	.5419455	83.579	83.682	+.103
88	30 33 03	.5902407	91.027	91.143	+.116
89	31 29 20	.6125341	94.465	94.587	+.122
90	32 34 39	.6389735	98.542	98.673	+.131
91	33 33 01	.6631482	102.271	102.398	+.127
92	34 29 50	.6872096	105.981	106.107	+.126
93	35 26 20	.7116850	109.756	109.888	+.132
94	36 21 11	.7359997	113.506	113.622	+.116
95	37 14 58	.7604024	117.269	117.385	+.114
96	38 07 54	.7849930	121.062	121.173	+.111
97	38 58 56	.8092704	124.806	124.918	+.112
98	39 49 08	.8337273	128.577	128.663	+.086
99	40 38 32	.8583827	132.380	132.439	+.059
100	41 26 09	.8827307	136.135	136.173	+.038
101	42 13 03	.9073013	139.924	139.926	+.002
102	42 58 52	.9318989	143.717	143.691	−.026
103	43 43 57	.9567066	147.543	147.487	−.056

FIG. 22. Distortion Computation Sheet for Diagonal *A*—Camera No. *AF 41-4172*

1. *Determining the Resultant Point of Symmetry.* Shown below are the relative locations of the central target (67), the P_s and the IPP for diagonals *A* and *B*.



$r = .018$ Angle $\gamma = 45°50'$
$s = .015$ Angle $\psi = 44°37'$
$x_{P_s} \cos \gamma + y_{P_s} \sin \gamma = r$
$x_{P_s} \cos \psi + y_{P_s} \sin \psi = s$

$\sin \gamma = +.71732 \quad \cos \gamma = +.69675$
$\sin \psi = -.70236 \quad \cos \psi = +.71182$
$+.69675\, x_{P_s} + .71732\, y_{P_s} = +.018$
$+.71182\, x_{P_s} - .70236\, y_{P_s} = +.015$

$+\ .72698\, x_{P_s} - .71732\, y_{P_s} = +.015$
$+1.42373\, x_{P_s} \quad = \quad +.035$

$$\left.\begin{array}{l} x_{P_s} = +.023 \\ y_{P_s} = +.002 \end{array}\right\} \text{(Coords. of } P_s\text{)}$$

For graphical solution see Fig. 25-IV

m. A set of sample computations are shown below in which the value of *f* in the Point of Symmetry and *CFL* computations was taken as 154.1 (as stamped on lens holder) instead of 154.255 as computed by the method given in Par. F. These computations are for the purpose of showing that the value of the calibrated focal length is practically unaffected by a relatively large change in the equivalent focal length.

Determination of Point of Symmetry (See par. G-5)
From table in par. G-6

$$A = +\ 3.6933147 \qquad B = -\ 145.2539974$$

$$C = -\ 1.106 + 1.5300589 = +\ 0.4240589$$

$$\mu = \frac{+\ 145.2539974 \pm \sqrt{(-\ 145.2539974)^2 - 4(3.6933147)(.4240589)}}{+\ 7.3866294}$$

$$\mu = \frac{+\ 145.2539974 - 145.2324308}{+\ 7.3866294} = +\ .002919667 \text{ Radians}$$

$$= .16728460° = 0°10'02''$$

$$\Delta x = \mu f = +\ .450 \text{ mm.} \qquad (\Delta x = .452 \text{ mm. when } f = 154.255).$$

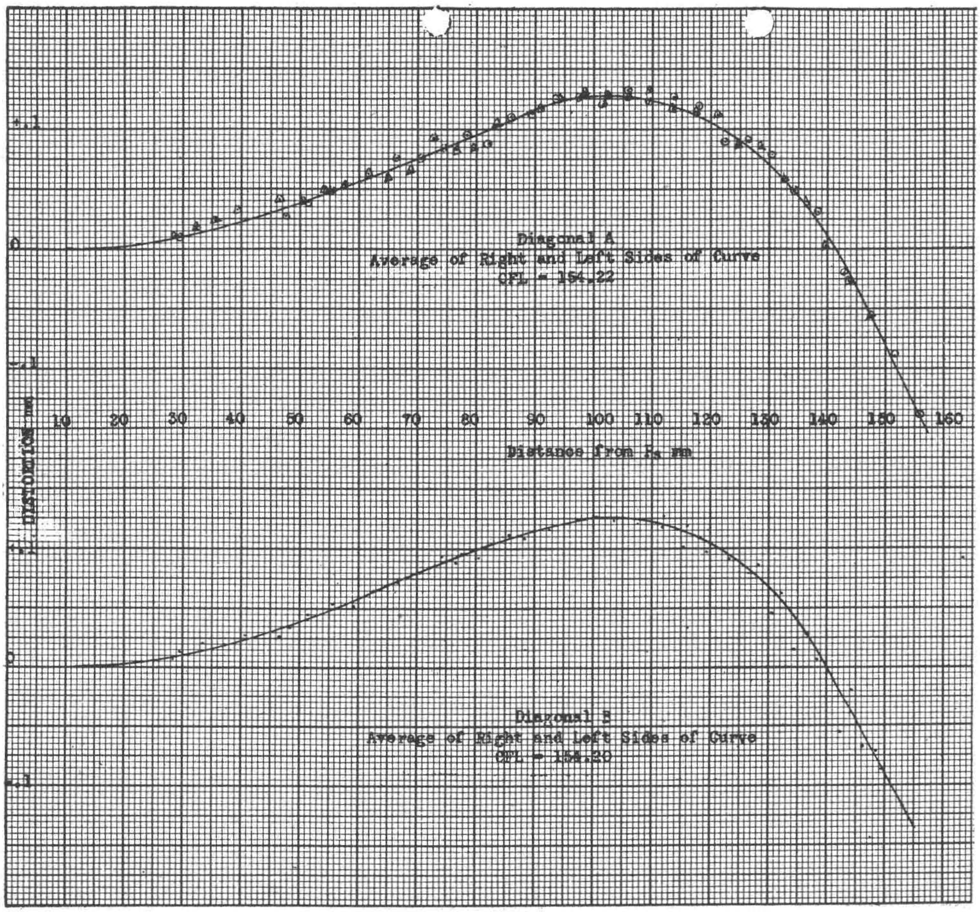


FIG. 23

Determination of maximum positive distortion (See par. H-2)

Target	∢ at *L*	tan ∢	f tan ∢	Measured Distance	Distortion (d_p)
47	34°26′34″	.6858115	105.684	105.895	−.211
92	34 29 42	.6871525	105.890	106.101	+.211
		Determination of negative distortion at 45° (See par. H-3)			
35	45 17 22	1.0101549	155.665	155.642	+.023
37	43 40 20	.9546936	147.118	147.171	+.053
39	41 57 36	.8991407	138.558	138.692	+.134
100	41 26 01	.8826617	136.018	136.163	+.145
102	42 58 44	.9318265	143.594	143.681	+.087
103	43 43 49	.9566323	147.417	147.477	+.060

Target No.	$\sphericalangle$ at L	tan $\sphericalangle$	f tan $\sphericalangle$	Measured Distance	Distortion
35	45°17′14″	1.0100765	155.764	155.636	−.128
39	41 57 28	.8990706	138.646	138.686	+.040
43	38 22 03	.7916671	122.083	122.179	+.096
47	34 26 26	.6857545	105.750	105.889	+.139
52	29 08 32	.5575585	85.981	86.099	+.118
54	26 52 12	.5066708	78.134	78.235	+.101
58	19 41 42	.3579534	55.200	55.254	+.054
61	12 06 19	.2144778	33.075	33.077	+.002
73	10 27 15	.1845118	28.454	28.468	+.014
78	18 04 32	.3263782	50.331	50.374	+.043
81	21 41 07	.3976507	61.322	61.389	+.067
84	25 09 36	.4697118	72.434	72.533	+.099
88	30 33 03	.5902407	91.021	91.143	+.122
91	33 33 01	.6631482	102.264	102.398	+.134
97	38 58 56	.8092704	124.798	124.918	+.120
101	42 13 03	.9073013	139.915	139.926	+.011
103	43 43 57	.9567066	147.534	147.487	−.047

Diagonal B

Target No.	$\sphericalangle$ at L	tan $\sphericalangle$	f tan $\sphericalangle$	Measured Distance	Distortion
36	44 00 41	.9660730	148.978	148.897	−.081
41	39 43 35	.8309943	128.148	128.224	+.076
45	35 58 17	.7257799	111.923	112.041	+.118
48	32 55 55	.6477202	99.885	99.999	+.114
50	30 50 46	.5972109	92.096	92.200	+.104
56	24 04 51	.4469202	68.920	68.989	+.069
60	16 43 55	.3006222	46.359	46.382	+.023
76	14 47 37	.2640921	40.726	40.751	+.025
80	20 58 28	.3833524	59.117	59.165	+.048
82	23 20 46	.4316224	66.560	66.630	+.070
86	27 51 17	.5284614	81.494	81.590	+.096
90	33 03 18	.6507732	100.356	100.476	+.120
92	34 58 29	.6995502	107.878	107.994	+.116
95	37 43 37	.7736393	119.303	119.393	+.090
99	41 07 11	.8729620	134.619	134.626	+.007
102	43 27 31	.9475926	146.128	146.052	−.076

FIG. 24. Diagonal A

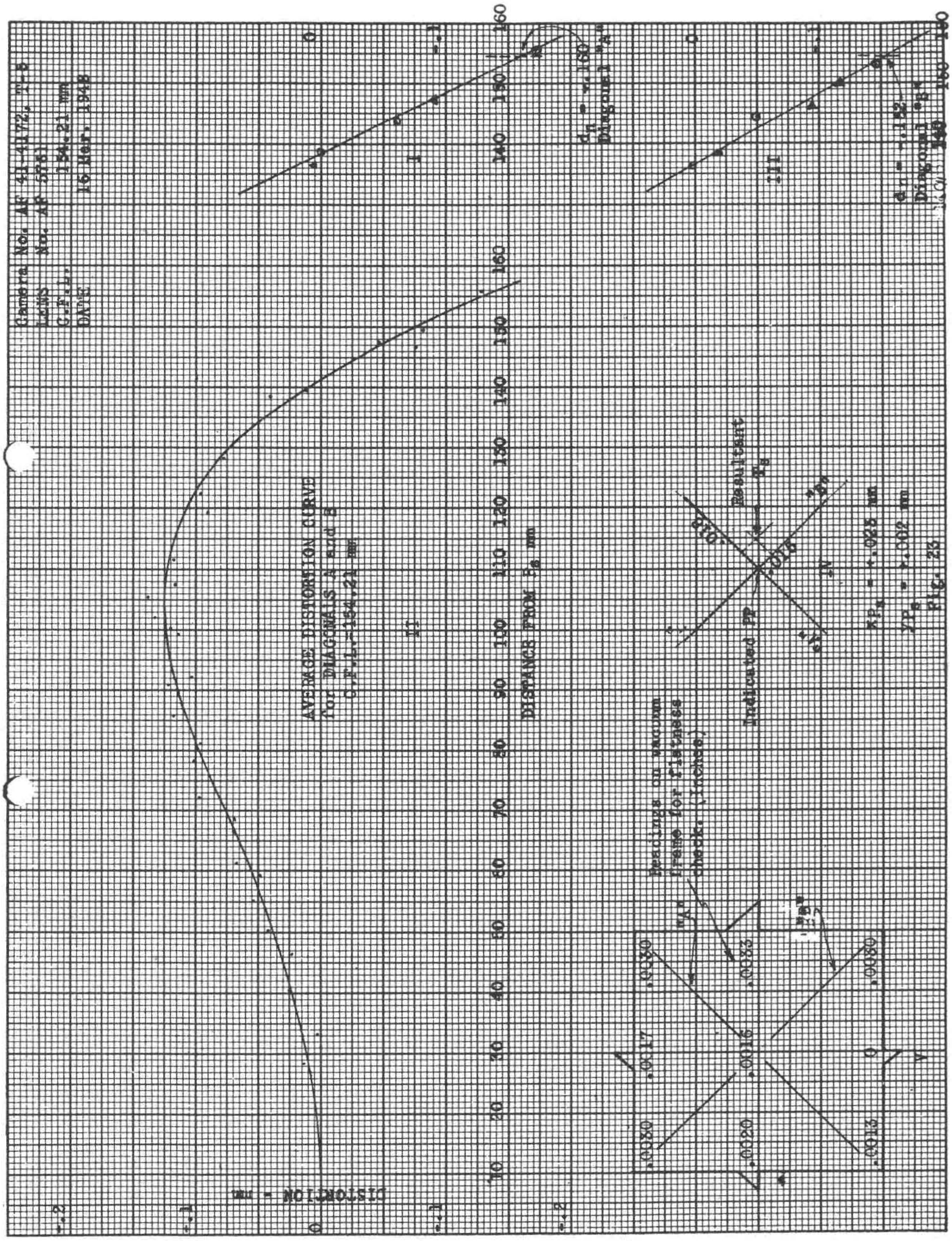
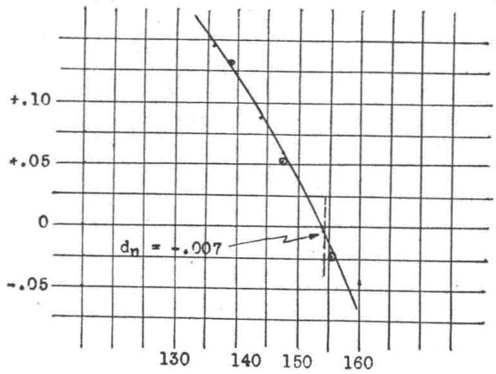

Camera No. AF 41-4172, T-5
LENS No. AF 5761
C.F.L. 154.21 mm
DATE 16 Mar. 1948
AVERAGE DISTORTION CURVE
for DIAGONALS A and B
C.F.L.=154.21 mm
DISTANCE FROM r mm
DISTORTION - mm
Readings on vacuum frame for flatness check. (inches)
Indicated PP
Resultant
Fig. 23

FIG. 25



Determination of Calibrated Focal Length (See par. H-3)

$$\Delta f = \frac{.211 - .007}{1 + .6864820} = .121.$$

Since d_p is larger numerically than d_n, Δf is positive.

$CFL = 154.1 + .121 = 154.221$ mm. $(CFL = 154.220$ when $f = 154.255)$.

n. Measuring the vacuum back for flatness. (See par. K-2) The measurements found for the magazine used with the camera being calibrated are shown in Fig. 23-V.