SELECTIONS OF CAMERA STATIONS IN TERRESTRIAL PHOTOGRAMMETRY

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Synopsis: This paper points out that in terrestrial photographic surveying it is frequently advantageous to occupy camera stations which are not control stations and presents two methods for solving the resulting "inaccessible base" or "two-point" problem. The first method using the law of tangents is mathematically interesting, but the second method using the law of cosines is the least laborious. The second method is perhaps best computed by adopting a machine computation form similar to that given in Mr. R. O. Anderson's *Applied Photogrammetry* (4th Edition), page 440.—*Publication Committee.*

TERRESTRIAL photographic surveying is a map-making method employed where conventional plane table methods are neither feasible nor economical, where speed is at a premium, where vertical photography is impractical because of a lack of ground control, where differences in elevation are excessive,[1] where the detail requirements are greater than that obtainable from near verticals, and where it is required to establish fourth order control for maps constructed from aerial photographs.[2] Those situations, then, where terrestrial photographic surveying is the logical map-making solution, are those situations where control is entirely lacking, or control is inadequate in density, or control exists in sufficient quantity but is not located geometrically and topographically for the maximum exploitation of the terrestrial photograph.

It is essential that the photographic surveyor select those sights that will give panoramic views, that will give views that satisfy certain geometric conditions with respect to other camera stations, that will utilize existing control, that will yield the maximum common coverage with respect to related camera stations, and that will image the greatest amount of detail with the sharpest definition and clarity.[1] In order to accomplish the above prerequisites, considerable ingenuity must be used. Frequently, two control points may be mutually imaged much more feasibly on photographs from two or three related camera stations than to serve as the camera stations themselves. In many cases, trigonometric interpolation and modified triangulation methods may be used.[3] This paper describes two methods of locating the camera, occupying two unknown points, when the resulting photographs have two or more control points mutually imaged. Moreover the distance, bearing and coordinates of all other ground objects, whose images appear on the photographs of both camera stations, as well as the distance separating the two camera stations, may be accurately calculated.

Method I

This method is described as being somewhat simpler than the conventional inaccessible base solution,[3] or, more accurately stated, is another solution to the

[1] Photographic Surveying, by M. P. Bridgland, Canada, Dept. Interior, Bulletin no. 56, 1924, pp. 17–19.

[2] Terrestrial Photogrammetry for Supplementary Control, by J. E. King. In PHOTOGRAMMETRIC ENGINEERING, vol. XII, no. 1, March 1946, pp. 106–114.

[3] Textbook of Topographical and Geographical Surveying; ed. by C. Close and J. L. Winterbotham; 3d ed., London, H.M. Stationery Office, 1925, pp. 56–58.

inaccessible base problem involving nothing more than elementary trigonometry. Two camera stations, A and B, having common coverage are established on unknown points. The cameras are set up on unknown points, because ordinarily none of the existing control points satisfy the conditions required to obtain the maximum accuracy and economy in the use of the terrestrial photographs. However several of these control point signals are imaged on the photographs of both camera stations. The straight line distance separating the two camera stations is assumed (AB_A). The angles at each station, between the opposite camera station and the ground control points, are observed or derived by analytical photogrammetry. At camera station A, angle α is derived from measure-

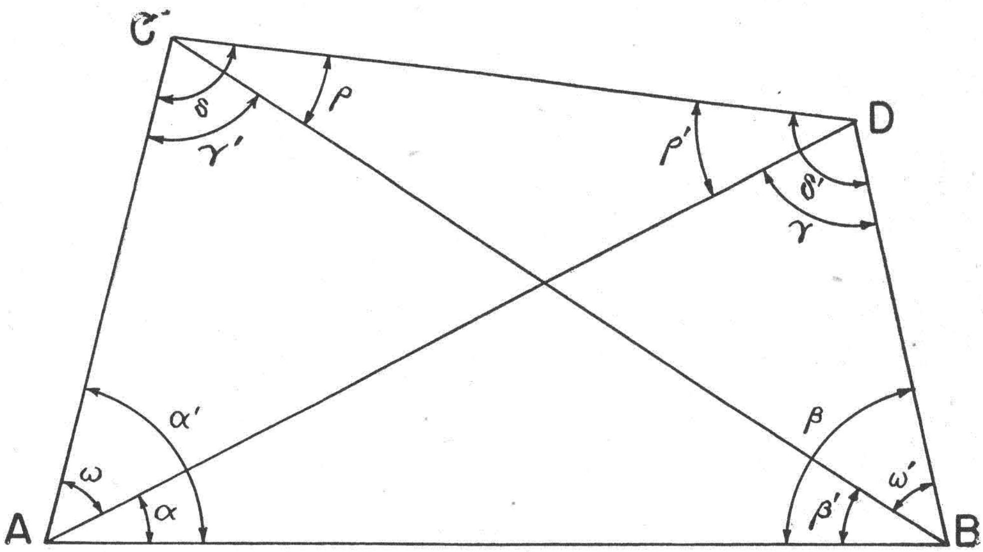

FIG. 1

ments made on images B and D, and angle α' from measurements made on images B and C. Similarly, at camera station B, angles β and β' are derived. Angles γ and γ' are deduced:

$$\gamma = 180° - (\alpha + \beta)$$

$$\gamma' = 180° - (\alpha' + \beta').$$

Thus, we have two triangles: 1) ΔABC consisting of assumed distance AB_A and angles α, β, and γ; 2) ΔABD consisting of assumed distance AB_A and angles α', β' and γ'. From the law of sines, distances AC_A, AD_A, BC_A, and BD_A can be computed.

$$AC_A = \frac{\sin \beta' \, AB_A}{\sin \gamma'} \qquad AD_A = \frac{\sin \beta \cdot AB_A}{\sin \gamma}$$

$$BC_A = \frac{\sin \alpha' AB_A}{\sin \gamma'} \qquad BD_A = \frac{\sin \alpha \cdot AB_A}{\sin \gamma}.$$

Subscript $_A$—approximate or assumed distance
Subscript $_T$—true or known distance

The ratio of each of the computed lengths to its corresponding true length will be exactly equal to the ratio of assumed length AB_A to true length AB_T. The angular values are unaffected as the shape of the figure is unchanged. δ, δ', ρ and ρ' are readily computed by the law of tangents.

$$\frac{\delta + \rho'}{2} = \frac{180° - \omega}{2}$$

$$\frac{\delta' + \rho}{2} = \frac{180° - \omega'}{2}$$

$$\tan \frac{(\delta - \rho')}{2} = \frac{AD_A - AC_A}{AD_A + AC_A} \tan \frac{(\delta + \rho')}{2}$$

$$\tan \frac{(\delta' - \rho)}{2} = \frac{BC_A - BD_A}{BC_A + BD_A} \tan \frac{(\delta' + \rho)}{2}$$

$$\frac{(\delta + \rho')}{2} + \frac{(\delta - \rho')}{2} = \delta \qquad \frac{(\delta + \rho')}{2} - \frac{(\delta - \rho')}{2} = \rho'$$

$$\frac{(\delta' + \rho)}{2} + \frac{(\delta' - \rho)}{2} = \delta' \qquad \frac{(\delta' + \rho)}{2} - \frac{(\delta' - \rho)}{2} = \rho.$$

Since C and D are known points, and distance CD_T is precisely known, the true lengths are easily computed.

$$AC_T = \frac{\sin \beta' \cdot CD_T}{\sin \omega}, \qquad AD_T = \frac{\sin \delta \cdot CD_T}{\sin \omega}$$

$$BC_T = \frac{\sin \delta' \cdot CD_T}{\sin \omega'}, \qquad BD_T = \frac{\sin \rho \cdot CD_T}{\sin \omega'}.$$

And distance AB_T separating the two camera stations, then is computed by the law of cosines.

$$AB_T = \sqrt{(BD_T)^2 + (AD_T)^2 - 2(BD_T)\cdot(AD_T)\cdot \cos \gamma}$$

$$AB_T = \sqrt{(BC_T)^2 + (AC_T)^2 - 2(BC_T)\cdot(AC_T)\cdot \cos \gamma'}$$

or

$$AB_T = \frac{\sin \gamma' AC_T}{\sin \beta'} = \frac{\sin \gamma' BC_T}{\sin \alpha'} = \frac{\sin \gamma AD_T}{\sin \beta} = \frac{\sin \gamma BD_T}{\sin \alpha}.$$

The bearing of line CD_T and coordinates of C and D being known, the bearing and coordinates of A and B are quickly computed.

Method II

A simpler solution, however, may be used. After the distances AD_A, AC_A, BD_A and BC_A have been computed from assumed length AB_A and derived angles α, β, γ, α', β' and γ', the length CD_A is computed.

$$CD_A = \sqrt{(AC_A)^2 + (AD_A)^2 - 2(AC_A)\cdot(AD_A)\cdot \cos \omega}$$

$$CD_A = \sqrt{(BC_A)^2 + (BD_A)^2 - 2(BC)\cdot(BD)\cdot \cos \omega'}.$$

Since all the angles are true angles—that is, angles as they exist in nature—all the lengths are in error an exactly equal ratio. In other words,

$$\frac{CD_T}{CD_A} = \frac{AB_T}{AB_A} = \frac{AC_T}{AC_A} = \frac{AD_T}{AD_A} = \frac{BC_T}{BC_A} = \frac{BD_T}{BD_A} = k.$$

Since CD_T is known, k is known.

$$\frac{CD_T}{CD_A} = k$$

and obviously

$$AB_T = k \cdot AB_A, \qquad AC_T = k \cdot AC_A, \qquad AD_T = k \cdot AD_A$$

$$BC_T = k \cdot BC_A \quad \text{and} \quad BD_T = k \cdot BD_A.$$