GRAPHICAL INTERPOLATION-ADJUSTMENT OF A "DOUBLE-STRIP"*

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SPACIAL air-triangulation, which, due to the results attained, is receiving increased consideration and which already has been used very often, requires a simple, thorough, quick, and at the same time sufficiently accurate method of adjustment. The practical results of air triangulation show that an absolutely rigorous adjustment requiring a considerable amount of time is not suitable.

A corresponding analytical consideration of the problem for practical purposes has been published in "Beispiel für die Ausgleichung eines Doppelstreifens mit Statoskopangaben" by Prof. Dr. M. Zeller and Dr. A. Brandenberger. (See "Mitteilungen aus dem geodätischen Institut der E.T.H." Vol. 4.)

As a result of my work at the Photogrammetric Institute, I have devised and applied a graphical interpolation-adjustment which enables the various corrections to be determined very quickly and accurately. Prof. Zeller was kind enough to advise me and to revise the following work; for this I wish to express my thanks.

If, in a double-strip, there are three groups of points, then it is known that for each of the x-, y- and height errors, an error surface may be constructed in which it is assumed that the longitudinal sections (parallel to the x-axes) along these error surfaces are of parabolic shape. The fundamental difference between the graphical method and the analytical adjustment consists only of the fact that in the former the deformation of the whole strip is not calculated from approximate formulae, but that the errors are interpolated, longitudinally and laterally.

If the height differences in a single model are relatively small, then the influence of the height differences on the errors and correspondingly the corrections Δx, Δy, ΔH, can be neglected. The latter can then be regarded as functions of the autograph-co-ordinates and we may write:

$$\Delta x = f_x(x, y) \tag{1a}$$

$$\Delta y = f_y(x, y) \tag{1b}$$

$$\Delta H = f_h(x, y). \tag{1c}$$

Each of the three equations (1) represents an error-surface from which the corrections Δx, Δy, and ΔH may be obtained in the Z-direction. Any point P_i, of the surface (1a) has therefore the co-ordinate x_i, y_i, Δx_i, a point on the surface (1b) has the coordinates x_i, y_i, Δy_i, and such a point on the surface (1c) has the co-ordinates x_i, y_i, ΔH_i. As soon as the function f_x, f_y, and f_H are known, one can calculate the individual corrections of any desired point.

In practice, we are concerned with the given point groups at the start, in the middle, and at the end of the strip. Therefore, we can calculate the value of the function (I) for these groups of points. The corrections are:

$$\begin{aligned} \Delta x_i &= (x_i) - x_i \\ \Delta y_i &= (y_i) - y_i \\ \Delta H_i &= (H_i) - H_i \end{aligned} \tag{2}$$

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where (x_i) (y_i), (H_i) are the final values for x_i, y_i, and H_i.

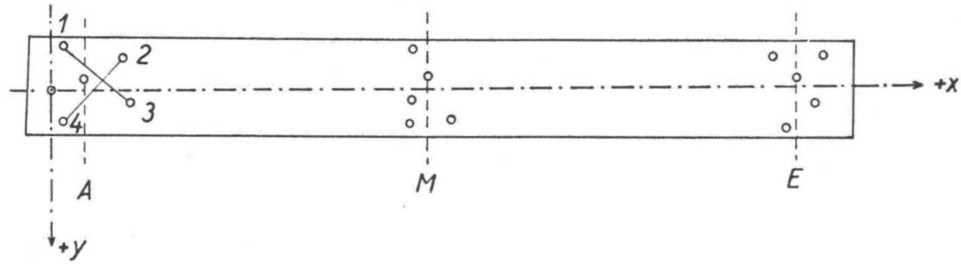

FIG. 1.

If we wish to carry out the complete adjustment in autograph co-ordinates, then, to obtain the final values, we must also convert the given points into machine co-ordinates. For this purpose we use, as usual, the azimuths of the straight lines between points in the first group, eg. 1–3 and 2–4 (Figure 1). The known values Δx, Δy, ΔH for the group of points at the beginning, in the middle, and at the end of the strip, i.e. the values of the function (1) in these groups of points, enables us to construct the three error surfaces and, therefore, to determine the values Δx_i, Δy_i, ΔH_i for any desired point of the strip.

The analytical solution of this problem by calculation is fairly tiresome, and almost outweighs the advantages of speed in the photogrammetric methods. However this problem can also be solved in a simpler fashion and more quickly with the aid of transversal and longitudinal sections of the error surfaces. This procedure has, in addition, the advantage that the non-linear course of the error is also taken into account in the transverse section.

With the supposed parabolic form of the longitudinal sections of the error surfaces, the formulae below suffice for this purpose:

$$\Delta x_i = a_0 + a_1 x_i + a_2 x_i^2 \text{ for the } x\text{-error surface} \quad (3a)$$

$$\Delta y_i = b_0 + b_1 x_i + b_2 x_i^2 \text{ for the } y\text{-error surface} \quad (3b)$$

$$\Delta H_i = c_0 + c_1 x_i + c_2 x_i^2 \text{ for the } H\text{-error surface.} \quad (3c)$$

Consider, for example, the H-error surface (Figure 2). We intersect this surface with three vertical planes parallel to the yz-plane, i.e. at right angles to the x-axis, at distances x_A, x_M, x_E, where these distances correspond to the abscissae

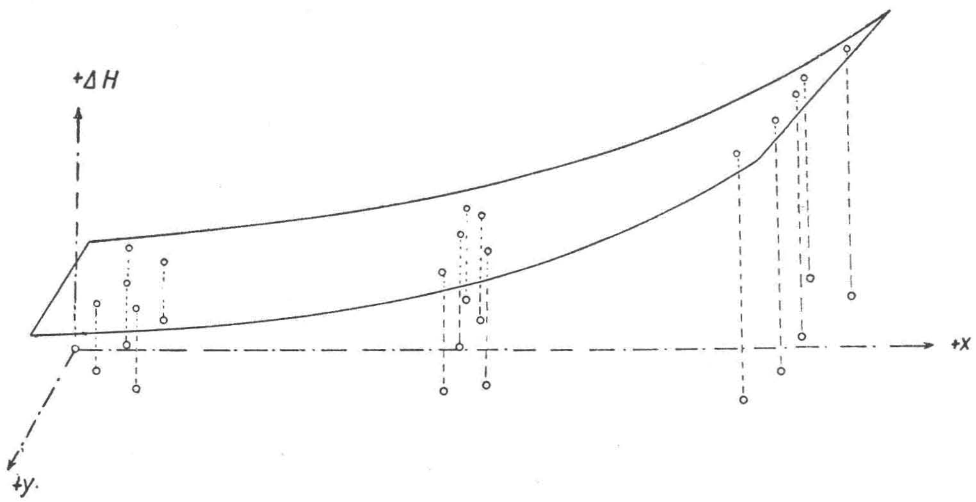

FIG. 2.

of points which lie as far as possible in the center of the point group respectively at the beginning, middle, and end of the strip. Since the corrections ΔH_i increase as x increases, to be able to draw the transverse section, the ΔH_i values must be reduced at the beginning to x_A, in the middle to x_M and at the end of the strip to x_E. For this purpose a linear reduction to x will suffice. When we have these values we can draw the three transverse sections of the error surface. (Figure 3). The individual points of the transverse sections are joined by means of straight lines. Theoretically, these points ought to be joined by a curve; however, experience has shown that the residual error resulting from the straight line junctions are so small, that a more accurate construction of the transverse section is unnecessary.

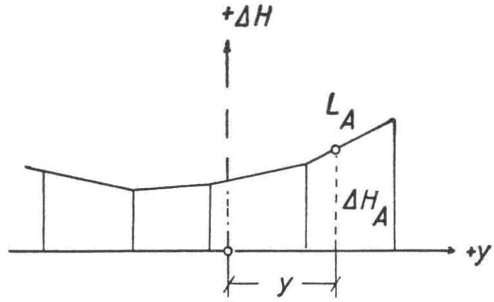

FIG. 3a.

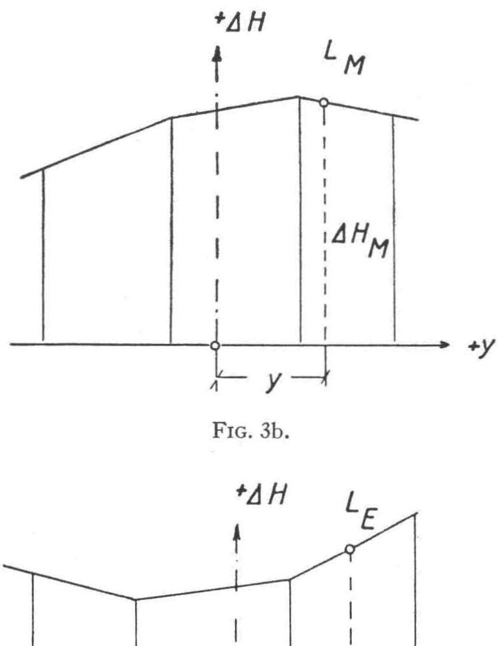

FIG. 3b.

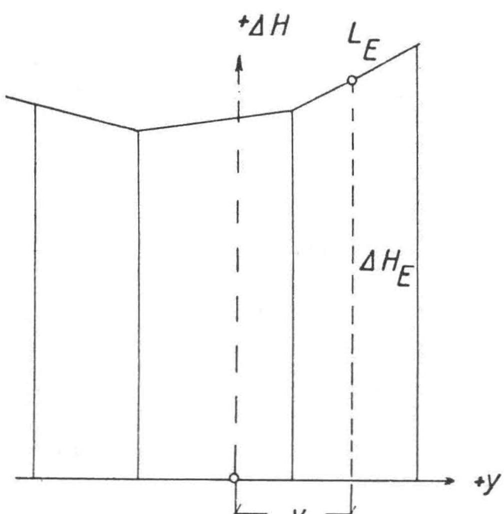

FIG. 3c.

The scale of ΔH must be chosen such that the desired accuracy (e.g. 0, 1 m) may be read conveniently in the transverse sections. The scale for y depends on the machine scale and the curvature of the transverse sections. The number and the positions of the longitudinal sections depend on the number of fixed points to be used for the adjustment, on the disposition of the passpoints, and on the relative complications of the forms of the transverse sections. In practice, we use at least three, but not more than five, longitudinal sections, which are to be constructed in correspondingly chosen distances from the x-axis.

For the construction of one longitudinal (Figure 4) section, we read from the three transverse sections, the corrections ΔH_A, ΔH_M, ΔH_E for the chosen y, plot these values in x_A, x_M, and x_E, and place a parabola through these ordinate end points L_A, L_M, L_E, according to equation (3c). If we put

$$x_M' = x_M - x_A$$

and

$$x_E' = x_E - x_A$$

we get from the equations:

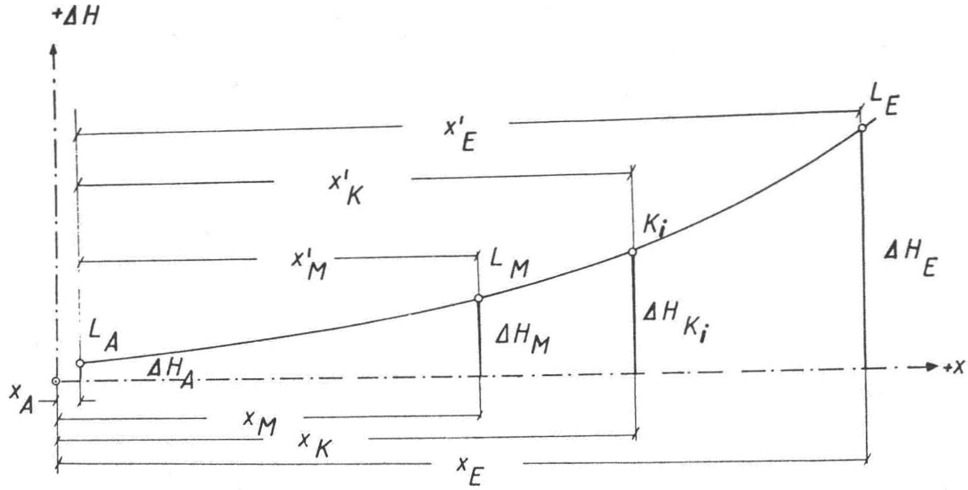

FIG. 4.

$$\begin{aligned} \Delta H_A &= c_0 \\ \Delta H_M &= c_0 + c_1 x_M' + c_2 x_M'^2 \\ \Delta H_E &= c_0 + c_1 x_E' + c_2 x_E'^2. \end{aligned} \tag{4}$$

The co-efficients c_0, c_1 and c_2 according to the formulae:

$$\begin{aligned} c_0 &= \Delta H_A \\ c_1 &= \frac{x_E'^2(\Delta H_M - c_0) - x_M'^2(\Delta H_E - c_0)}{x_E' \cdot x_M'(x_E' - x_M')} \\ c_2 &= \frac{x_M'(\Delta H_E - c_0) - x_E'(\Delta H_M - c_0)}{x_E' \cdot x_M' \cdot (x_E' - x_M')} . \end{aligned} \tag{5}$$

To be able to draw the parabola as accurately as possible, we must also calculate the individual points K_i in between. For this purpose we use the formula:

$$\Delta H_i = c_0 + c_1 x_i' + c_2 x_i'^2. \tag{6}$$

The calculation (6) may be carried out with the sliderule.

In similar fashion, we draw the remaining longitudinal sections of the H-error surface and those for the x- and y-error surfaces.

The scale to be chosen for x is given from the length of the strip in the machine scale. If this is for example 7,000 mm then one chooses for x the scale 1:10. It is more suitable to choose the same scale for all longitudinal sections.

By means of the transverse and longitudinal sections of the error surface, one obtains in a simple way, the correction ΔH_i for any particular point P_i of the strip. Let the point P_i have the coordinate x_i, y_i choosing two longitudinal sections constructed at the distances y_1 and y_2 from the x-axis, and such that $y_1 < y_i < y_2$, then P_i is found between the two longitudinal sections. If one draws the transverse section at a distance x_i between the two chosen longitudinal sections, then the ordinates ΔH_i at distance y_i can be read straight away, or simply interpolated (Figure 5). In similar fashion we obtain the corresponding corrections Δx_i and Δy_i.

Since the parabolae for the various longitudinal section of an error surface

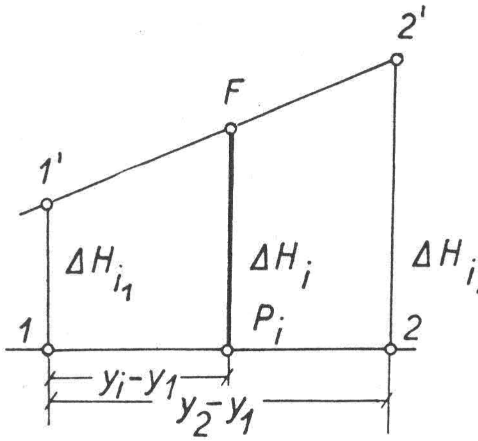

FIG. 5.

deviate only very little from each other, it is not convenient to represent all the parabolae for an error surface in the same drawing. It is more convenient to include the longitudinal sections for a certain y for all three error surfaces in one drawing. It is possible then to read at the same time the three corrections Δx, Δy and ΔH in the particular longitudinal sections concerned.

The corrections Δx, Δy, and ΔH are, in general, fairly large at the ends of the strip, and the representation of the longitudinal section would become too large with normally chosen scales. To be able to draw the parabolae on a narrow strip of paper, we divide them as shown in Figure 6. In this way we obtain a "ladder-parabola" in which, for the various x-sections, simply the scale for the corrections Δx, Δy, ΔH respectively was altered.

According to this method, the author has carried out the adjustment for the 100 km long strip "Payerne-Schönenwerd" (wide-angle Wild Film Camera, aircraft 4,600 m above ground) 70% overlap, independent image pairs with statoscope readings carried out with the Wild Autograph A5. The mean square height error m_H was found to be ± 6.8 m, the mean square error in X reached ± 4.6 m and the mean square error in Y was $\pm$ 4.4 m against $m_H - \pm 7.3$ m, $m_X = \pm$ 7.6 mm and $m_Y = \pm 4.7$ m in the analytical adjustment.

The graphical interpolation adjustment gives, therefore, smaller mean square errors than the analytical method. The reason for this is that in the former method, the transverse bending of the error surface is taken into account, whereas in the analytical adjustment, the error in the transverse direction is regarded as being linear.

In similar fashion Dipl. Ing. Schucany has adjusted the 100 km long strip "Payerne-Aarau" graphically. (Wide-angle film-camera Wild, height of aircraft 4,600 m above ground, 60% overlap, the image series connected by statoscope readings, carried out with the Wild Autograph A5.)

The adjustment gave the mean square errors:

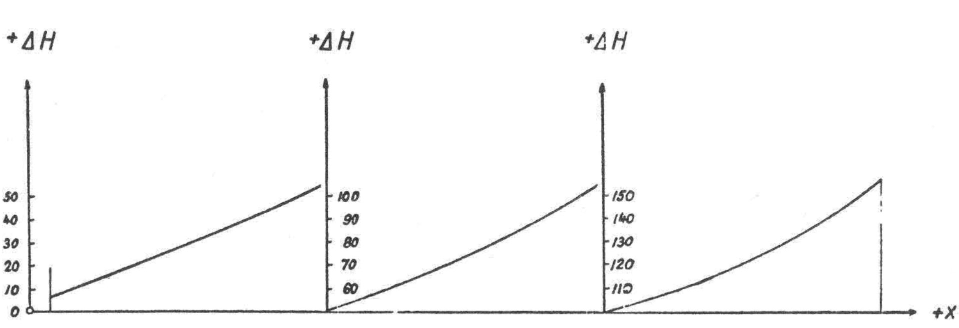

FIG. 6.

$$\left.\begin{aligned} m_X &= \pm\ 10.9m \\ m_Y &= \pm\ 5.6m \end{aligned}\right\}\ \text{in terrestrial co-ordinates}$$

$$\left.\begin{aligned} m_X &= \pm\ 8.4m \\ m_Y &= \pm\ 8.9m \end{aligned}\right\}\ \text{in strip co-ordinates.}$$

For the adjustment, it is also possible to use the values of the errors in the terrestrial co-ordinates (ΔX and ΔY). The values read off as x_i, y_i are converted into terrestrial co-ordinates (X_i, Y_i) and the corrections below are formed:

$$\begin{aligned} \Delta X_i &= (X_i) - X_i \\ \Delta Y_i &= (Y_i) - Y_i \end{aligned} \tag{2a}$$

(X_i) and (Y_i) are the final values. In the transformation formulae, putting in the values of Δx and Δy, viz;

$$\begin{aligned} \Delta X &= \Delta x \cos \delta - \Delta y \sin \delta \\ \Delta Y &= \Delta x \sin \delta + \Delta y \cos \delta \end{aligned}$$

one obtains:

$$\begin{aligned} \Delta X &= (a_0 + a_1 x + a_2 x^2) \cos \delta - (b_0 + b_1 x + b_2 x^2) \sin \delta = A_0 + A_1 x + A_2 x^2 \\ \Delta Y &= (a_0 + a_1 x + a_2 x^2) \sin \delta + (b_0 + b_1 x + b_2 x^2) \cos \delta = B_0 + B_1 x + B_2 x^2 \end{aligned}$$

i.e. the same parabolic shape as for Δx and Δy with correspondingly altered co-efficients. These are obtained from the formulae:

$$\begin{aligned} A_0 &= \Delta X_A \\ A_1 &= \frac{x_E'^2(\Delta X_M - A_0) - x_M'^2(\Delta X_E - A_0)}{x_E' \cdot x_M' \ (x_E' - x_M')} \\ A_2 &= \frac{x_M' \ (\Delta X_E - A_0) - x_E'(\Delta X_M - A_0)}{x_E' \cdot x_M'(x_E' - x_M')} \end{aligned} \tag{5a}$$

and the corresponding formulae for the coefficients B. It is, therefore, more convenient to use for the construction of the parabolae and the transverse sections, the errors ΔX and ΔY in terrestrial co-ordinates instead of Δx and Δy, whereby for the abscissae, the machine co-ordinates x and y respectively are to be used. For the rest, the procedure is the same as for the graphical adjustment in machine co-ordinates.

The advantages of this process are: (1) the conversion of the given points into machine co-ordinates falls away; (2) the corrections ΔX and ΔY of any particular point of the strip may be read directly in terrestrial co-ordinates.

The author has also adjusted the foregoing strip "Payerne-Schönenwerd," according to this method, in terrestrial co-ordinates, and has obtained the same mean square errors for X and Y.