a large amount of ground work will be necessary before any definite statements can be made.

CONCLUSION

The value of air-photographs as an aid to structural geology is amply demonstrated in the areas described. In addition, their use as field maps and aids to traverse planning have been touched upon. Though complete geological surveys are not possible using air photographs alone, a great deal of structural and other information may be obtained from an intensive study of such photographs. In the case of Nova Scotia, where both detailed and reconnaissance maps are available for a large part of the Province, the writer feels that a photographic check would be a most interesting and profitable venture. It is his intention to begin such a project, as soon as time and materials permit.

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PHOTOGRAMMETRY FOR THE NON-PHOTOGRAMMETRIST

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AERIAL photographs perform two valuable functions: (1) they supply voluminous information as to what features are on the surface of the ground, and (2) they present this information in such a way that approximate distances may be measured directly. If this is not sufficiently precise, an accurate map may be made by simple methods. In some instances, accuracy of position is not important and the pictures have served their purpose if they show the general relationship of surrounding objects to the proposed transmission line, expressway, or other improvement. Frequently, however, a more or less accurate determination of ground distances is necessary. This may be done only if the engineer realizes that a photograph is not a map, understands the reasons why this is so, and knows what methods may be used to eliminate or minimize the errors.

It is the purpose of this paper to present the basic concepts of aerial mapping in such a way that they may be readily understood and applied by the engineer or city planner. The refinements necessary when elaborate instruments are employed may well be left to the specialist in photogrammetry. He should be called upon when contour mapping or other high precision work is necessary.

The best map that can be made does not present all details on the surface of the earth with the precision that can be obtained with a careful transit traverse. This is recognized in specifications for standard maps which for scales of 1:20,000 or larger require 90% of all well defined features to be plotted within 1/30 inch of their true horizontal positions. Thus, on a map at a scale of 1:12,000, or 1,000 feet per inch, the ground location (as determined on the map) of 90% of all well identified features should be within 33 feet of the correct location. The same philosophy may well be adopted for many applications of aerial photography. When the engineer has decided what accuracy is required (and this will vary widely depending on the nature of the work), a knowledge of the errors inherent in an aerial photograph will permit him to decide which portions of the pictures may be considered as a map, and in which areas the errors will be excessive and some photogrammetric technique must be used to obtain sufficiently precise horizontal positions.

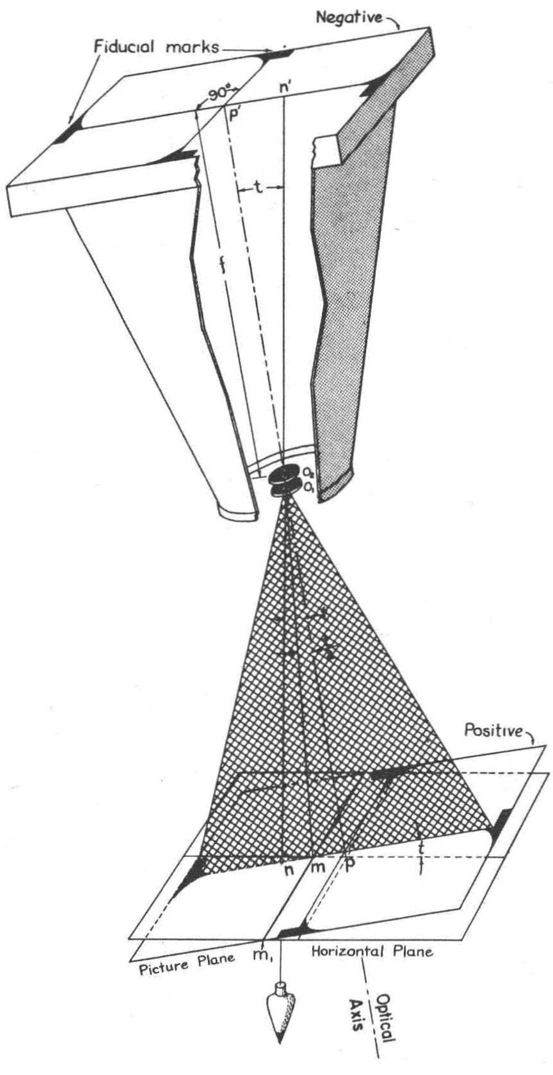

FIG. 1. Diagram illustrating the fundamental terms used in photogrammetry.

An analysis of the errors in aerial photographs will be more intelligible if the terms frequently encountered are thoroughly familiar. The upper part of Figure 1 represents a negative in the aerial camera at the instant of exposure. O_1 and O_2 are the nodal points of the lens since rays from the ground directed toward O_1 leave the lens from O_2 parallel to their original directions so that the cone of incoming rays is exactly reproduced by the lens. While the separation of the nodal points must be considered by the lens and camera designers, they are so close together that the average engineer may treat them as one point, called the *exposure station*, or *air station*, through which rays pass on their way from the ground to the negative.

A perpendicular from O_2 to the negative cuts the latter at p', known as the *principal point*. (In well-constructed cameras this line O_2p' coincides with the axis of the lens.) Since the principal point is used as an origin or reference point in mapping operations, its position must be accurately located. This is done by registering on the film *fiducial marks*, so placed that the intersection of the lines joining them coincides with p'.

Aerial photographs for civilian use are usually taken from such heights that all ground objects will be sharp when the lens is focused for infinity. Accordingly O_2p' is the focal length f of the lens. The *nadir point* n' is of significance since it is the image of the point on the ground directly beneath the air station. The angle $n'O_2p'$ between a truly vertical line and the axis of the lens is known as the *angle of tilt* t. It will be seen from the figure that $n'p'$, the distance between the principal point and the nadir point, is equal to $f \tan t$.

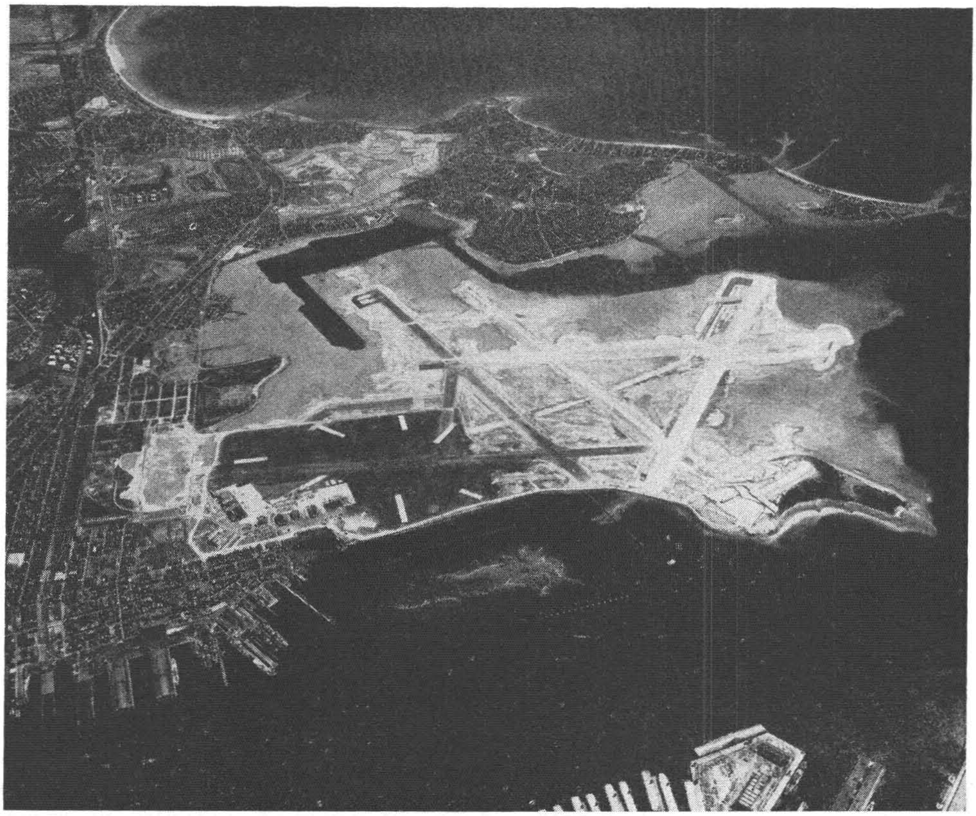

FIG. 2. An oblique has no common scale. The runways on the airport are approximately the same length, although measurements on the picture would not indicate this.—Mass. Dept. Public Works Photo.

A positive rather than a negative would be used under most circumstances. This has been drawn perpendicular to the axis of the lens, a distance f from point O_2 so that all the dimensions are the same as in the camera, but everything is reversed left for right so that ground objects are in their normal relationships. The principal point is at p and the nadir point at n. The vertical plane, shaded in the diagram, containing the vertical line O_1n and the lens axis, is known as the *principal plane* and its intersection with the picture is the *principal line*. A horizontal plane has been drawn at a distance f vertically below O_1 to represent the position of the positive if there had been no tilt. The tilted positive intersects this horizontal plane along m_1m, which is called either the axis of tilt or the isometric parallel, because along this line and this line alone, the tilted picture corresponds exactly with an untilted picture. The intersection m of the isometric parallel and the principal line is called the *isocenter*, a point that is important when studying the effect of tilt.

Aerial photographs are classified according to the inclination of the optical axis, being called *verticals* when the axis points as nearly straight down as possible, and *obliques* when it is deliberately inclined. A *high oblique* shows the horizon, while the horizon is not included in a *low oblique*. Notice that the words high and low merely indicate whether or not the horizon has been recorded, and have nothing at all to do with the altitude of the exposure station. For instance a low oblique can be taken from 20,000 feet and a high oblique from 200 feet. The oblique has certain advantages because it gives a more familiar view, and covers more area from the same air station, but ground distances are very much distorted, as may be seen in Figure 2 where all runways on the airport are approximately of the same length in spite of the impression given in the picture. A vertical, on the other hand, looks quite like a map on casual inspection, permits ready calculation of approximate ground distances, and lends itself to the compilation of accurate maps by simple means, but, as has been pointed out, the area covered is somewhat restricted. To overcome this defect, several verticals may be joined together to form a *mosaic* of an extended area. If the individual photographs are merely fastened together so that their images match, the mosaic is said to be *uncontrolled*, while if numerous points of known position are used to maintain an over-all accuracy, the mosaic is *controlled*.

APPROXIMATE SCALE OF VERTICAL PHOTOGRAPHS

Since an aerial photograph is a conic rather than an orthographic projection it is rarely true to scale; that is, the ratio of a distance on the picture to the corresponding distance on the ground is not constant for all parts of the photograph. Nevertheless, the concept of scale is frequently so convenient that it is common practice to use an average value called the *approximate scale*. This may be computed in either of two ways: (1) from a knowledge of the focal length of the lens used in the camera and the height of the airplane above the ground, or (2) by a direct comparison of the distance between two objects as measured on the picture and on the ground.

Figure 3 illustrates how the two methods may be applied. A lens of focal length f makes a negative from the exposure station O which is a height H above the datum. The optical axis $p'OP$ is assumed to be truly vertical. Perpendicular to this and at a distance f below O, a positive picture has been drawn which has the same dimensions as the negative and which shows objects in their proper relative orientation. As points G_1 and G_2 are a distance L apart on the ground, and their images are separated by a distance l on the positive, the scale of the picture is given by l/L, the ratio between picture distance and ground distance. As triangles G_1OG_2 and g_1Og_2 are similar, it will be seen that

$$\frac{l}{L} = \frac{f}{H} \cdot$$

Thus the scale of the photograph may be determined from the ratio of the focal length of the lens to the flying height. As it is customary to express f in inches and H in feet, difficulty with units will be avoided if the formula is written

$$\text{Scale} = \frac{f \text{ (inches)}}{12H \text{ (feet)}} \cdot \qquad (1)$$

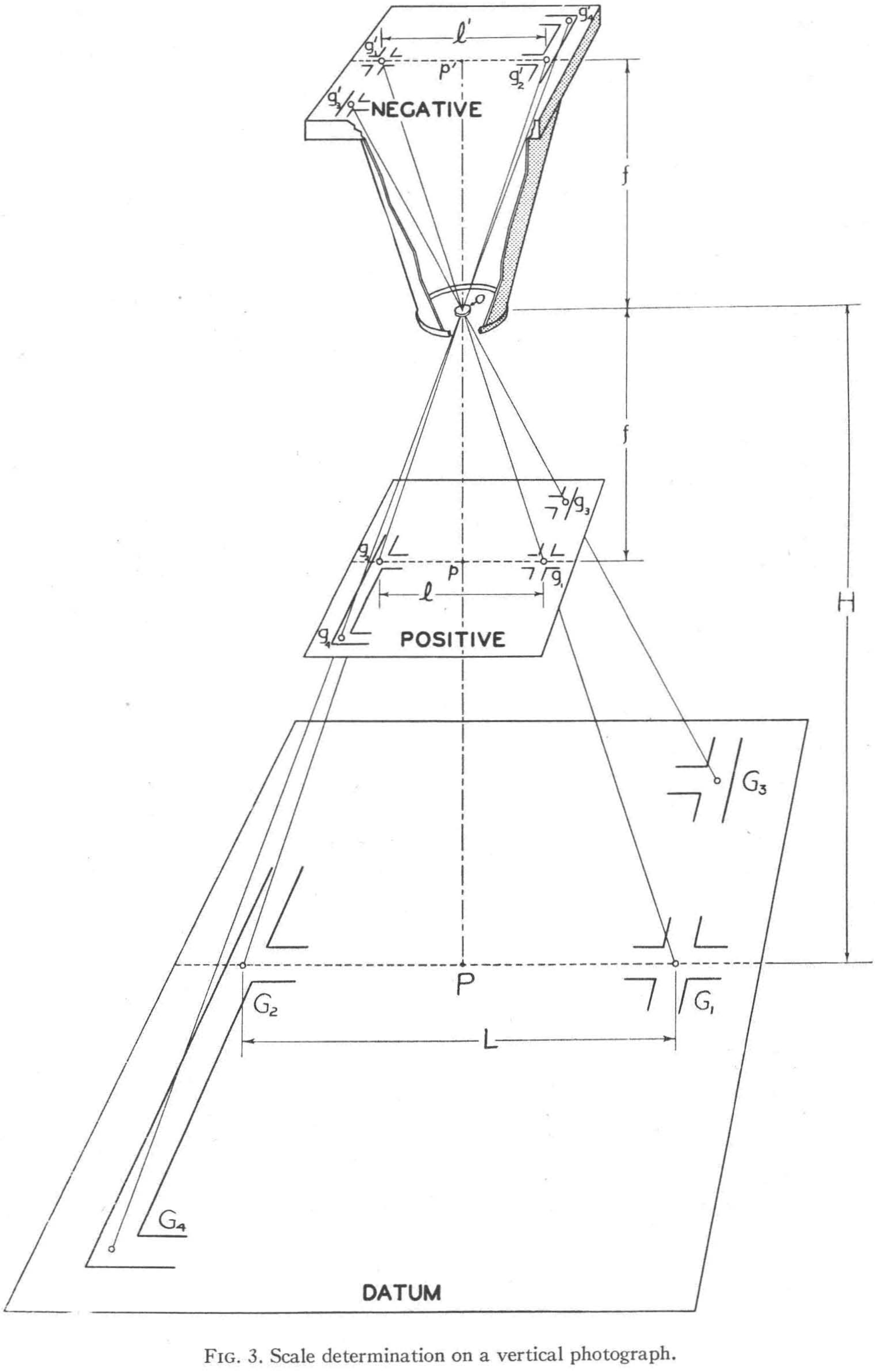

FIG. 3. Scale determination on a vertical photograph.

This formula shows that, for a given flying height, a longer lens will give larger scale pictures. For this reason, the Army is developing 60- and 100-inch lenses to obtain images that are large enough for intelligence purposes, when the pictures are taken from 40,000 feet or higher. Conversely, for many mapping projects, a very small scale is desired and the use of a short lens keeps flying heights within the capability of moderately powered planes.

If the fraction $f/12H$ is so reduced that it has one in the numerator, as 1/12,000, the scale is said to be expressed as a representative fraction. Because such a fraction is non-dimensional, it may be used with English, metric, or any other units of measure. However, the engineer thinks in terms of feet per inch, as 400 feet per inch, and would like his scales expressed in a familiar manner. Using the same triangles G_1OG_2 and g_1Og_2 of Figure 3, it will be seen that

$$\frac{L\text{ (feet)}}{l\text{ (inches)}} = \frac{H\text{ (feet)}}{f\text{ (inches)}}. \tag{2}$$

The scale in feet per inch is equal to the flying height in feet divided by the focal length of the lens in inches. A picture taken at 9,600 feet above the ground with a 24-inch lens would have a scale of 9,600 feet/24 inches or 400 feet per inch.

As the phtographic flights may vary in height by as much as 5% from the prescribed amount, it is often desirable to obtain a more accurate scale determination than that furnished by the altimeter reading. This may be done by comparing the actual distance between two points on the ground, with the distance between the images of the same two points on the photograph. Assume that points g_3 and g_4 are 7.5 inches apart on the picture, and the ground distance between G_3 and G_4 is found (by ground survey or by scaling on a reliable map) to be 4,500 feet, then the approximate scale may be found by substituting in the formula

$$\text{Scale (in feet per inch)} = \frac{\text{Ground Distance (in feet)}}{\text{Photo Distance (in inches)}}$$

and in this case

$$\text{Scale} = \frac{4,500\text{ feet}}{7.5\text{ inches}} = 600\text{ feet per inch.}$$

Results obtained with this method will be more satisfactory if the two points selected are as nearly as possible of the same elevation, and are located equidistant from and on opposite sides of the principal point. Points g_3 and g_4 are arranged in this say, as are g_1 and g_2.

Significance of Scale

The selection of the scale to be used on a project is extremely important as it determines the ease with which objects may be identified, and materially affects the cost of the work. A general rule is that the more detailed the required information, the larger the scale and the higher the cost. Table I gives some idea of scales that have been used successfully in the past, and may serve as a guide until personal experience has been obtained.

Large scale pictures facilitate identification of objects, but they cover less ground. At the present time, the 9×9-inch format is fairly well standardized, so that if a strip of pictures is flown at 800 feet to the inch, a swath 7,200 feet

TABLE I

Purpose	Scale	Comments
Hydroelectric Projects	800 feet per inch	Enlargements to 400 feet per inch may be used as plane table sheets for contouring.
Highway Location[1]	1:45,000 or 3,750 feet per inch	Preliminary investigation of large rural areas.
	1:27,000 or 2,250 feet per inch	Preliminary investigation of large urban areas.
	1:7,200 or 600 feet per inch to 1:4,800 or 400 feet per inch	Rephotography for detailed study of specific line.
City Planning	1:9,600 or 800 feet per inch	Many planners consider a larger scale advisable. Enlargements frequently made to 100 feet per inch for tax assessment purposes.
Forestry	1:15,840 or 1,320 feet per inch	This scale widely used in northeastern United States for type mapping. Larger scales probably needed for volume determinations.
Geology[2]	2,000 feet per inch	Reconnaissance mapping.
	1,000 feet per inch	Regional mapping.
	400 feet per inch	Mining property maps—Original photographs. Controlled mapping at 200 feet per inch. Enlargements at 100 feet per inch for detailed mapping of ore zones.

wide (9″×800′/″) will be photographed. However, if the pictures were at a scale of 400 feet per inch, the resulting strip would be only 3,600 feet wide. When areas are considered, the decreased coverage with larger scale is even more pronounced. Figure 4 shows a photograph whose original scale was 800 feet per inch. The ground that would have been included at 600 feet per inch is indicated by the larger square outlined in white, while the smaller square is the area that would have been covered at 400 feet per inch. Obviously larger scales require a far greater number of pictures for a given project. This materially increases the cost of both the flying and the office compilation, and it is necessary to make a compromise between the advantages of larger scales and the greater expenditure involved.

SOURCES OF ERROR

Errors will result if it is assumed that a photograph is a map. These errors are caused: (1) by the fact that the camera and photographic material fail to satisfy the theoretical requirements completely, thus giving rise to what may be called *instrumental errors;* (2) by the fact that the ground is not perfectly level, thus producing *relief errors;* and (3) by the fact that, in spite of all efforts of the air crew, the camera axis is not truly vertical, thus introducing *tilt errors.*

With modern equipment, the so-called instrumental errors are quite small in relation to the other two, and they may be safely ignored in any but the most

[1] Consensus of data furnished at Public Roads Administration exhibit at 15th Annual Meeting of The American Society of Photogrammetry and findings of Massachusetts Department of Public Works as given in report entitled "Aerial Photographs for Highway Location."

[2] Rooney, G. W. and W. S. Levings, "Advances in the Uses of Air Survey by Mining Geologists," PHOTOGRAMMETRIC ENGINEERING, Vol. XIII, No. 4, Dec. 1947, pp. 570–580.

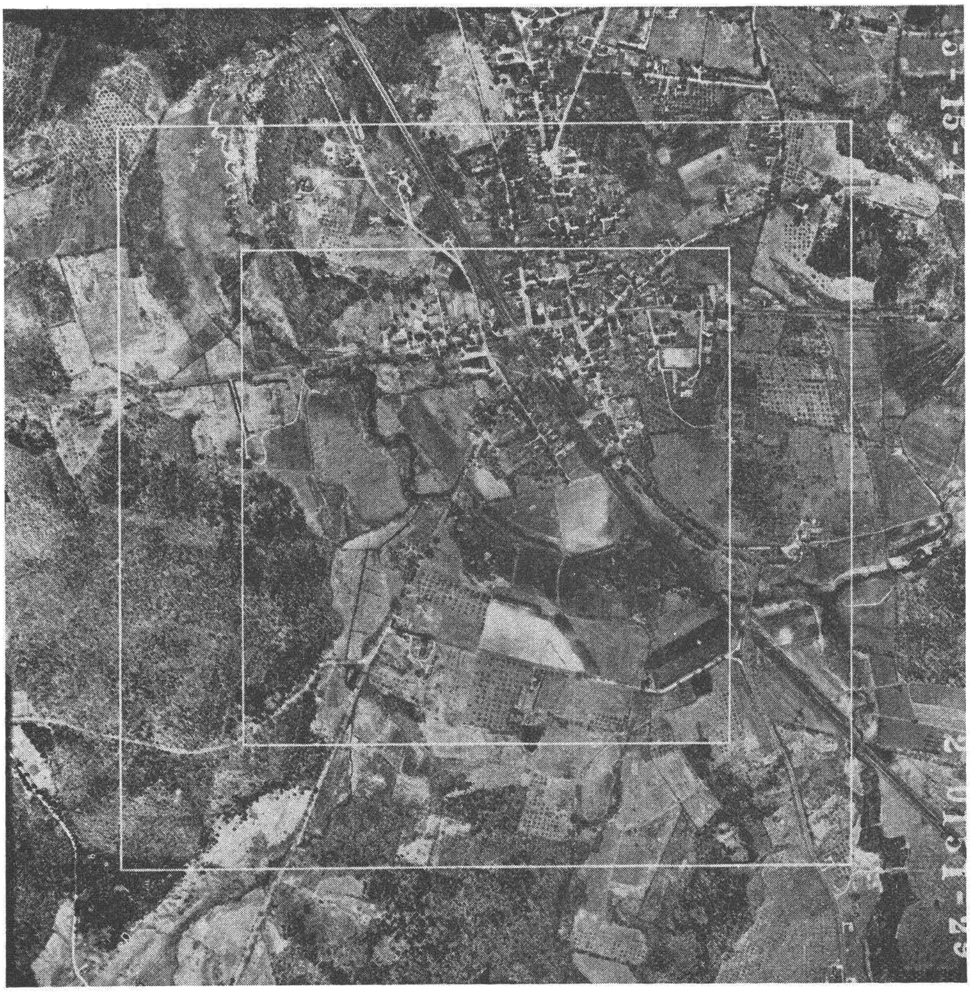

FIG. 4. Areas included on vertical photographs at 800, 600, and 400 feet per inch.—Mass. Dept. Public Works Photo.

exacting work. However, they will be enumerated here with an indication of their magnitude in order that the engineer may satisfy himself that he is not overlooking anything vital. As has been mentioned, this paper is primarily concerned with the determination of horizontal position. When contours are to be drawn by photogrammetric methods, errors of even two or three thousandths of an inch are significant and must be considered.

Instrumental Errors

(1) *Lens distortion*, which causes a displacement (mostly radial from the principal point) of some images in relation to others. While always present, distortion in lenses designed especially for air survey work may be reduced to as little as 0.0008 inch.[3] Because of compensation of radial errors in routine plani-

[3] Report of Commission I to the Sixth International Photogrammetry Congress, PHOTOGRAMMETRIC ENGINEERING, Vol. XIV, No. 2, June 1948, p. 258.

metric mapping operations, the Department of Agriculture has found that a distortion of 0.004 inch at a point 35° from the lens axis is not objectionable. (This is included in its specifications.) A Bureau of Standards certificate stating that the distortion is no greater than this amount, in the particular lens to be used on a given project, should be adequate assurance that the lens will not produce any undue errors.

(2) *Incorrect assembly of the camera and lens* which may produce errors of unpredictable variety and magnitude. A camera of reputable make in the hands of an experienced contractor will prevent trouble from this cause.

(3) *Film negatives not held flat* in the focal plane at the instant of exposure can produce troublesome errors that are hard to track down. In the United States, air pressure or vacuum is commonly used to flatten the negative in the focal plane. However, cameras are not equipped with any device that records the precision with which this is done, but noticeable lack of sharpness in the pictures is one indication of serious malfunction of the vacuum or pressure system. Another clue may be obtained if it is noted that the distance between fiducial marks as measured on the negative departs much from the actual distance as measured in the camera.[4] Neither of these checks will detect minor failures which may be sufficient to produce localized errors. Employment of experienced personnel is the best guarantee of high quality camera performance.

(4) *Use of a glass plate* against which the film is flattened mechanically. While more positive than the use of either air pressure or vacuum, the glass plate produces a distorted image by refracting the light from the lens. The amount of this distortion depends on the angle at which the rays strike the glass, being zero on the optical axis where their angle of incidence is zero, and increasing rapidly with the increased obliquity as the corners are approached. Here the error may be as much as 0.04 inch with an $8\frac{1}{4}$-inch lens,[5] although it is less for lenses of longer focal length. Fortunately, this effect is also radial from the principal point, and may be eliminated for all practical purposes by the standard planimetric mapping techniques, although it may introduce an error of nearly 1% in direct measurements of distance on the picture.

While quite common before the war, film magazines employing a glass plate are little used in the United States at the present time. However, one foreign camera manufacturer is so firmly convinced of the advantage of positive flattening that he employs a special lens in this camera which has been designed to have a distortion that is equal and opposite to that produced by the glass plate, so that the dimensions of his negatives are not affected.

(5) *Use of focal plane shutter* which produces a negative that may be considered to have been taken from a series of exposure stations rather than a single one, as theory requires. Cameras designed for mapping usually have the shutter mounted between two components of the lens, so that all parts of the negative are exposed at the same time. The focal plane shutter, on the other hand, consists of an opaque curtain fitted with a slit. This curtain is placed as close to the film as is feasible, and the exposure is made by moving the slit rapidly across the negative. As a result, one side of the picture is exposed before the other, so that various parts are taken from various places due to the motion of the airplane while the shutter is in operation. If a slit 0.25 inch wide is used to give each portion of the negative an exposure of 1/400 of a second, there is a lapse of 9/100

[4] Subcommittee on Multiplex Test Equipment, "Tests of Aerial Cameras and Multiplex Reduction Printers," PHOTOGRAMMETRIC ENGINEERING, Vol. XIII, No. 4, Dec. 1947, p. 691.

[5] Bagley, J. W., "Aerophotography and Aerosurveying," McGraw-Hill Book Co., New York, 1941, p. 309.

second between the times the opposite sides are exposed. An airplane going 200 miles per hour will cover nearly 30 feet in this period, so there will be an error of this amount in distances measured between points on opposite sides of the picture. This is 0.30 inch on the picture when the scale is 100 feet per inch, but only 0.03 inch if the scale is 1,000 feet per inch, so that the error is far more significant on large-scale pictures.

No matter what type of shutter is used, the airplane will travel some distance while a particular part of the negative is being exposed. If sharp pictures are to be obtained, this motion must be so small that the resultant blurring is not objectionable. Sanders[6] cites tests that indicate that "exclusive of aircraft vibration, photographic quality will be quite usable when the image movement does not exceed 0.01 inch," and he shows that image movement during exposure may be computed from the formula

$$M = \frac{1.467 \times V \times T}{S}$$

when M = image movement in inches
V = ground speed of aircraft in miles per hour
1.647 = factor to convert miles per hour to feet per second
T = exposure time in seconds
S = scale of photograph in feet per inch.

Rewriting this equation to obtain the duration of exposure that gives an image motion of 0.01 inch on the film, we have

$$T = \frac{.01 \times S}{1.467 \times V}.$$

This may be used to determine whether the selected shutter speed is sufficiently fast. Thus, if pictures at a scale of 100 feet per inch are to be used for tax assessment or similar purposes, and the airplane has a ground speed of 180 miles per hour, T is found to be 1/264 second, which is rarely obtainable with the between-the-lens shutters found in cameras having the long focus lens suitable for this type of work. In situations where precision of measurement over the entire width of the picture is not so significant as sharp delineation of detail, the focal plane shutter may well be used to advantage, especially when a wide slit moving at high speed is employed. However, when dimensional accuracy is the prime concern, the between-the-lens shutter should always be specified.

(6) *Changes in dimension of the photographic materials.* Unfortunately photographic materials are not absolutely stable, and dimensional changes occur between the taking of the aerial picture and the use of a positive by the engineer. If glass plates are used throughout the entire process, these changes are kept to a minimum, but even so there are indications that the emulsion may creep slightly and produce minute distortions. With other materials, there are dimensional changes due to processing, ageing, and atmospheric conditions in the workrooms. If these changes were uniform in all directions, they would produce a change in effective focal length for which compensation would be simple. Unfortunately, it will be found that they are quite different when measured in two directions at right angles to each other, producing an effect called *differential shrinkage.* Eastman now reports[7] that for its aerial film, this amounts to

[6] Sanders, R. G., "Design and Construction of Aerial Cameras," MANUAL OF PHOTOGRAMMETRY, Pitman Publishing Corp., New York, 1944, p. 90.

[7] Eastman Kodak Company, "Kodak Materials For Aerial Photography," Rochester, N. Y., 1945, p. 13.

0.05% for permanent shrinkage due to processing and ageing, and in addition there is a differential expansion of 0.005% for each 10°F. increase in temperature or each 10% increase in relative humidity. Thus, the permanent differential shrinkage over the entire width of a negative amounts to 9 inches×0.0005, or 0.0045 inch, which is negligible unless the pictures are to be used for contour determination in elaborate instruments.

The paper used for the positive prints is more subject to shrinkage and expansion than is film, and the differential effect is also more pronounced. Paper is particularly affected by processing methods, the final print being larger than the original negative when dried on ferro-type tins to obtain a high gloss, and smaller than the negative when dried between blotters in a normally steam heated room. Photographic papers are available in two thicknesses, called double weight and single weight, the former being by far the more stable. Tests on prints made on double weight paper showed an average differential shrinkage of 1 part in 800, but cases were noted where it reached 1 part in 400.[8] An error of 1 part in 800 is not considered serious if the prints are to be used for determination of horizontal positions by graphical methods. With single weight paper, the differential shrinkage is more serious, and may reach values as great as 0.5%. Because of this and the fact that it is less durable, single weight paper is not commonly used when measurements are to be made directly on the prints.

Where accuracy is of sufficient importance to justify the extra cost, dimensional stability approaching that of film may be obtained by using "non-shrink" paper which has had its base waterproofed with acetate. Orienting negative and positive materials, so that their directions of greatest change are at right angles to each other, will keep the differential shrinkage to a minimum.[9]

Recognizing the fact that a print is not of the same size as the original image formed by the lens in the camera, a compensation for this variation is made by substituting the *focal length of the print* for that of the lens in any calculations. Since the distances between the fiducial marks in the camera may be accurately measured, the distances between the corresponding marks on the print furnish a convenient method of determining the amount of shrinkage. Assume that the positive is found to be 0.25% small in one direction and 0.35% small in the other; then the average shrinkage would be 0.30% and the focal length of the print would be 99.7% that of the camera lens. The use of the average shrinkage when determining the focal length of a print should prove sufficiently precise for all but the most exacting work.

Summary of "Instrumental" Errors. From the foregoing, it will be seen that with a properly operated aerial camera designed for mapping work, the "instrumental" errors exceed the map maker's 0.025 inch tolerance for horizontal position in only three cases: (1) when single weight paper is used, (2) when the camera has a focal plane shutter, and (3) when the negative is exposed through a glass plate. In practice, prints to be used for mapping are never made on single weight paper; cameras designed for survey work (as opposed to reconnaissance) have a between-the-lens shutter, and the standard magazines in current use employ a vacuum rather than a glass plate for film positioning. Accordingly the photographs taken by reputable aerial survey firms may be considered as being a map so far as instrumental errors are concerned.

[8] Bagley, J. W., ibib. cit., p. 6.

[9] Davis, R., E. J. Stovall, and C. I. Pope, "Dimensional Changes in Aerial Photographic Films and Papers," PHOTOGRAMMETRIC ENGINEERING, Vol. IV, No. 3, July–August–September 1938, p. 196.

Relief Errors

When the photograph is tilted or when the ground is not perfectly level, errors of position will occur. These frequently amount to 0.10 inch or more and are too large to be ignored. While both sources of error occur in the average picture, they will be more easily understood if each is considered separately. A true vertical will be assumed when considering the relief effect, and level ground will be assumed when treating the tilt errors.

Figure 5 shows a truly vertical photograph taken from the exposure station

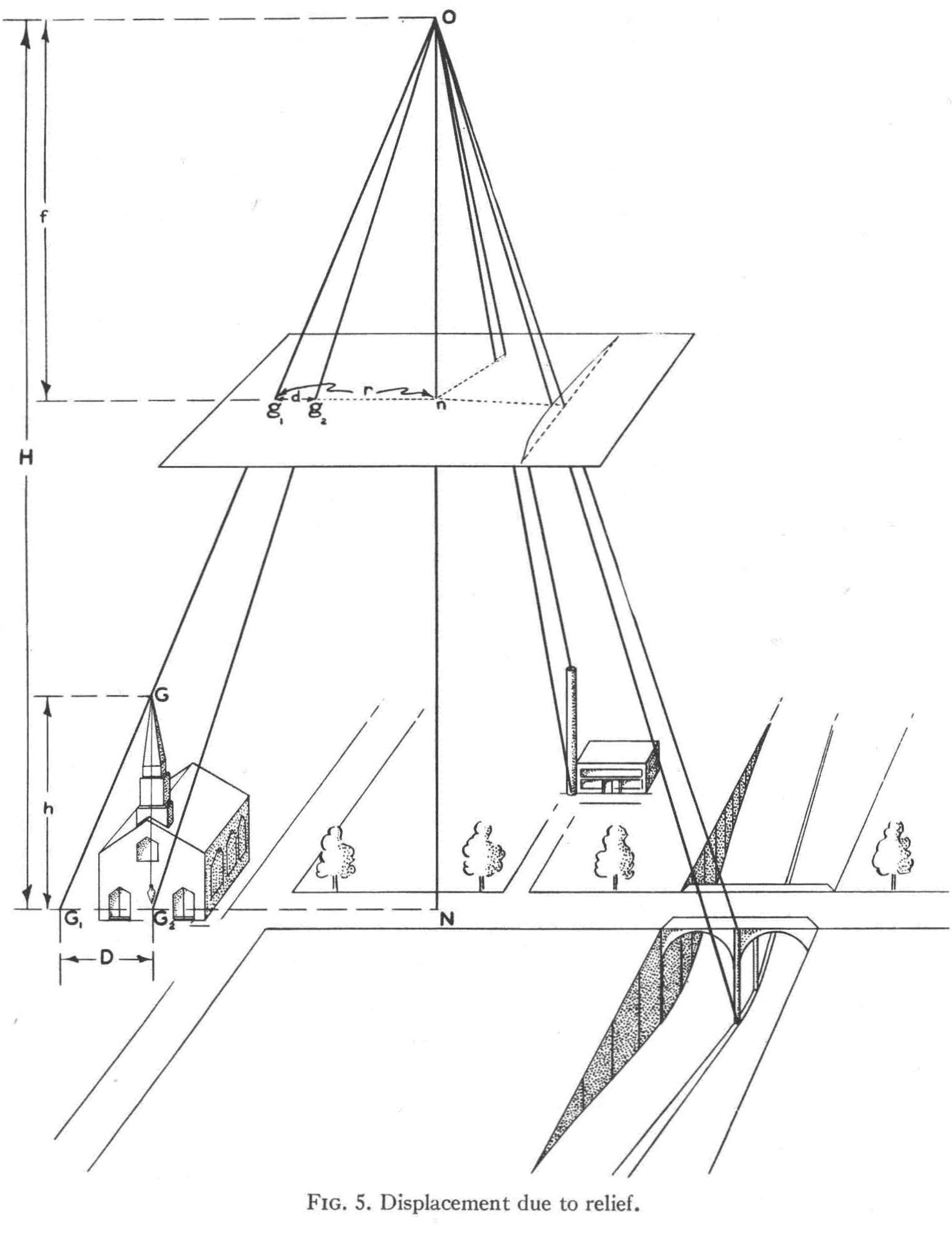

FIG. 5. Displacement due to relief.

O, which is a height H above the datum. (Except for the underpass, all the ground is assumed to be level.) The camera has a focal length f, and since there is no tilt, the optical axis and the plumb line coincide, piercing the picture at n and the datum at N. Because a map is an orthographic projection, the top of the church steeple (G), a height h above the ground, would be plotted vertically beneath itself in the datum at G_2, which would be recorded on the picture at g_2. As the photograph is a conic projection, G would actually appear at g_1, and G_1 would be its datum position. Therefore it will be seen that, because of the height of the steeple, g_1 has been displaced by an amount g_1g_2 from its correct map position.

To calculate the magnitude of the error due to relief, let d equal the displacement g_1g_2 on the photograph, and let D equal G_1G_2 on the ground. Triangles Og_2g_1 and OG_2G_1 are similar. Therefore

$$\frac{d}{D} = \frac{f}{H}$$

or

$$dH = Df.$$

Triangles Ong_1 and GG_2G_1 are also similar. Let r denote distance ng_1 from the nadir point to the image of G, the top of the steeple. It will be seen that

$$\frac{r}{D} = \frac{f}{h}$$

or

$$rh = Df$$

$$\therefore \; dH = rh,$$

since both have been shown to equal Df and

$$d = \frac{rh}{H} \cdot \qquad (3)$$

Thus it is seen that the displacement due to relief is equal to the product of the radial distance r from the nadir point to the photo image and the elevation h of the point (above or below the datum) divided by the flying height H above the datum. It should be particularly noted that the displacement is directly proportional to the radial distance from the nadir point to the image on the picture, and accordingly the error is less pronounced near the center than in the outer portions of the photograph. It should also be observed that the error is inversely proportional to the flying height, and accordingly is less troublesome on pictures taken from high altitudes.

An example may make the above points clearer. Assuming a truly vertical photograph taken with an $8\frac{1}{4}$-inch lens at a scale of 400 feet per inch in the datum, a point at an elevation 200 feet higher and appearing on the picture 3.5 inches from the nadir point will be displaced 0.212 inch.

$$\left[\begin{array}{l} H = 400'/'' \times 8\tfrac{1}{4}'' = 3{,}300 \text{ feet} \\ d = \dfrac{3.5'' \times 200'}{3{,}300'} = 0.212 \text{ inch} \end{array}\right].$$

If a 24-inch lens had been used and the same point had again been 3.5 inches from the nadir, the resulting displacement would have been reduced to 0.073 inch.

$$\left\{\begin{aligned} H &= 400'/'' \times 24'' = 9{,}600 \text{ feet} \\ d &= \frac{3.5'' \times 200'}{9{,}600'} = 0.073 \text{ inch} \end{aligned}\right\}.$$

As may be seen from Figure 5, objects that are above the datum are displaced outward from the nadir point. Conversely, objects below the datum will be displaced inward, as shown by the grass plot in the underpass on the right side of the diagram. It should also be noticed that straight lines on the ground do not photograph as straight lines if their elevation changes. The grass plot is a straight line and should be reproduced as such (dotted line in diagram), whereas actually it will appear curved, as shown by the solid line.

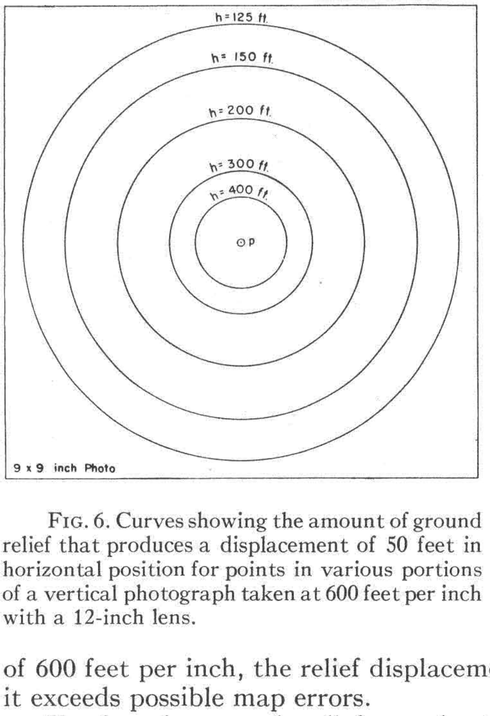

FIG. 6. Curves showing the amount of ground relief that produces a displacement of 50 feet in horizontal position for points in various portions of a vertical photograph taken at 600 feet per inch with a 12-inch lens.

Aerial photographs are frequently used in the preliminary phases of many projects to obtain land utilization data not furnished by existing maps. Under such conditions, is seems logical to ignore any relief displacement that does not produce an error in ground distance that exceeds the tolerance permitted the map maker. In the case of the Geological Survey series at a scale of 1:24,000, positions may be assumed to be correct within 0.025 inch on the map, or 50 feet on the ground. Thus with pictures at a scale of 600 feet per inch, the relief displacement may be 50/600 or 0.083 inch before it exceeds possible map errors.

To show how much relief may be tolerated at various distances from the principal point, Figure 6 has been prepared from Equation (3) for photography at a scale of 600 feet per inch with a 12-inch lens. Each curve represents the locus of points where the displacement (in terms of ground distance) is 50 feet for the indicated value of h. In other words, the circles bound the areas within which relief may be ignored (under the stated conditions) when actual elevation differences do not exceed the given amount. Similar curves are readily prepared for other cases having different accuracy requirements or scale and focal length combinations. When, as is nearly always the case, the photograph is tilted, the nadir point and principal point no longer coincide, being separated by a distance $f \tan t$, as shown in Figure 1. While the displacement formula was derived by use of the nadir point, its location is rarely known in practice. To overcome this difficulty, a tilt of 3° was assumed when preparing Figure 6, and the distance pn for this situation was subtracted from the value of r as computed in Equation (3) before plotting the circles. Accordingly, they include the usable area in relation to the principal point under the most adverse conditions that are likely to occur.

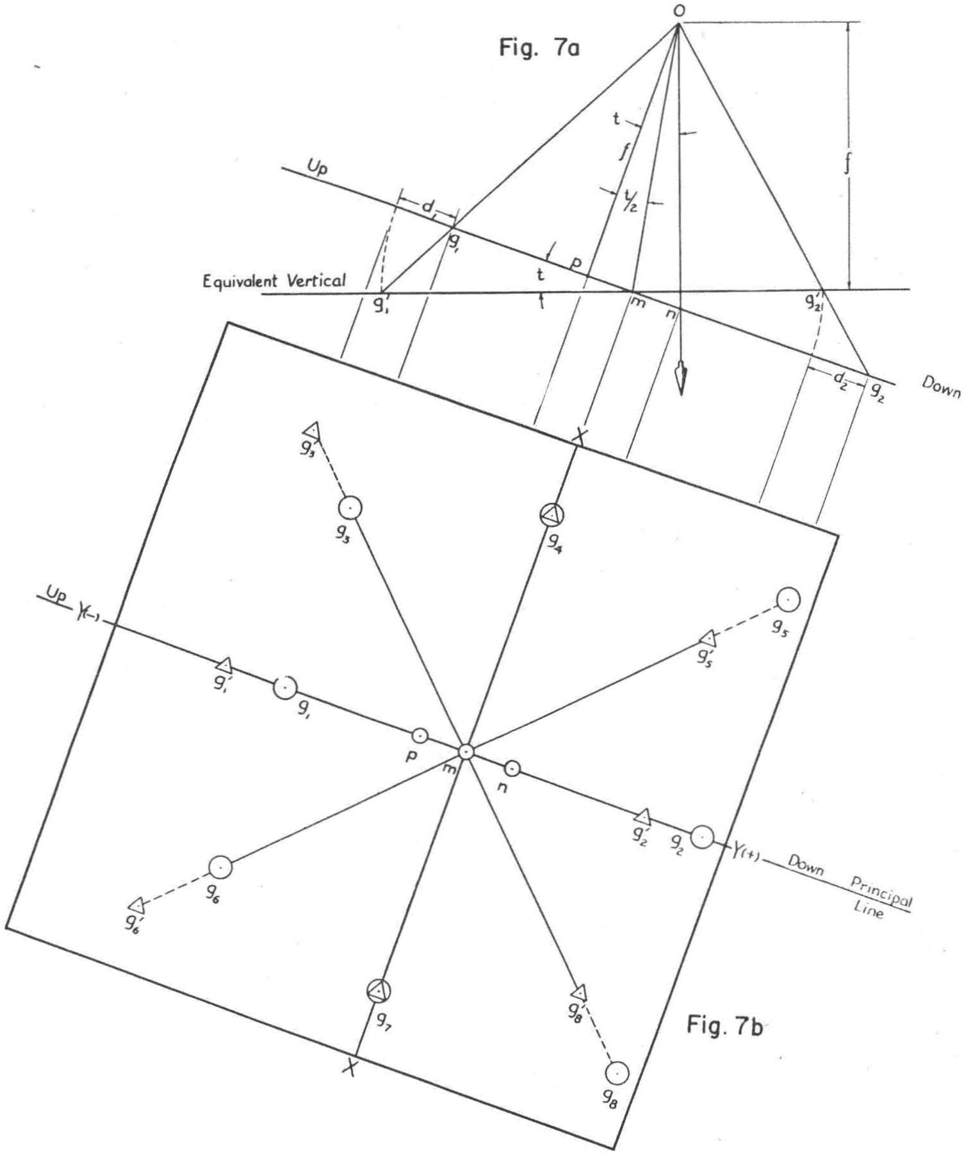

FIG. 7. Displacement due to tilt.

Even though situated far above or below the datum, a point whose elevation is known may be plotted in its approximately correct position by the use of Equation (3). After the radial distance r is measured from the principal point to the photo image in question, the displacement d may be computed since the elevation difference h and the flying height H are known. A radial line is drawn from the principal point through the image on the picture. The displacement d is then laid off along the line from the latter point, inward toward the principal point when the elevation of the ground is greater than that of the datum, and outward when the ground is below the datum. The position thus obtained is absolutely accurate only if the photograph is truly vertical. If there is tilt, the correction will be approximate, but the resulting position will be more accurate than if no allowance had been made.

Tilt Errors

Being drawn in the principal plane, Figure 7a shows the principal line of a photograph tilted the angle t. The focal length is f, the principal point is at p, and the nadir point at n. It is assumed that the ground is level. The trace of the equivalent vertical is represented by the horizontal line a distance f below the exposure station O, so it will be possible to compare distances on the tilted picture with the corresponding distances that would be measured if there had been no tilt. The principal line and the trace of the true vertical intersect at m, the isocenter. Since this point coincides on both the tilted and untilted photographs, it may be used as a reference point when comparing measurements on the two pictures. Ground point G_1 is recorded at g_1 on the up side of the tilted photo, while its correct map position in relation to m is at g_1' on the equivalent vertical. An arc with its center at m and a radius mg_1' has been drawn to show where g_1 would be if it were the correct distance from m on the tilted picture. Inspection will show that the point has been displaced inward an amount d_1 on the up side, and the distance mg_1 is too small. On the down side, point g_2 is displaced outward, and mg_2 is too large by the amount d_2.

Bagley[10] has given formulae for computing the amount of displacement caused by tilt, as have Tewinkel,[11] and Breed and Hosmer.[12] This error may be shown to vary approximately as the square of the distance from the isocenter, thus the central portion of the photograph is more accurate than the edges, as was also true in the case of errors due to relief. For values of tilt normally encountered, the inward displacement on the up side is nearly the same as the outward displacement on the down side, for points equidistant from the isocenter. Accordingly the distance between the two points g_1 and g_2 is very nearly correct, even though the photograph is tilted. It is for this reason that the approximate scale of a picture is best determined by using points equidistant from and on opposite sides of the center, since the effect of tilt is minimized.

In Figure 7b, the tilted photograph shown in Figure 7a has been rotated 90° around its principal line, and is now presented in plan. The circles show where the various points appear on the tilted picture, while the triangles represent their positions on a truly vertical picture. It will be seen that, even with the exaggerated tilt used in the diagram, the distance between photo images on opposite sides of the isocenter is quite close to the true distance measured between corresponding untilted or map positions. It will also be observed that tilt has displaced the points along lines that are radial from the isocenter—inward on the up side and outward on the down side—so that angles measured at this point are true angles when there is no relief. A simple proof of this may be found in Breed and Hosmer.[13]

To determine what portion of a tilted photograph may be considered as a map, Clerc[14] has suggested that curves be drawn representing the loci of points where the tilt displacement is equal to the maximum permissible error. Inside

[10] Bagley, J. W., ibid. cit., pp. 119–123.

[11] Tewinkel, G. C., "Geometry of Vertical Photographs," MANUAL OF PHOTOGRAMMETRY, ibid. cit., pp. 264–268.

[12] Breed, C. B., and G. L. Hosmer, "The Principles and Practice of Surveying, Vol. II, Higher Surveying," Sixth Edition, John Wiley and Sons, Inc., New York, 1947, pp. 374–378.

[13] Breed, C. B., and G. L. Hosmer, ibid. cit., p. 376.

[14] Clerc, L. P., "Applications de la Photographie Aerienne," Octave Doin et Fils, Paris, 1920, pp. 210–216.

these curves the accuracy requirements are satisfied as the errors are less than the allowable amount. Using a coordinate system having its origin at m and the X and Y axes as indicated in Figure 7b, Clerc shows that

$$x^2 = \frac{e^2(f \csc t + y)^2 - y^4}{y^2}$$

where e = displacement due to tilt
f = focal length of the lens
t = angle of tilt
x = perpendicular distance from principal line to point under consideration
y = perpendicular distance from the isometric parallel to the point under consideration,

y being counted positive on the down side of the photograph.

The conchoidal curves in Figure 8 represent the loci of points having displacements of 0.02, 0.05, and 0.10 inch for a 9×9-inch picture taken with an

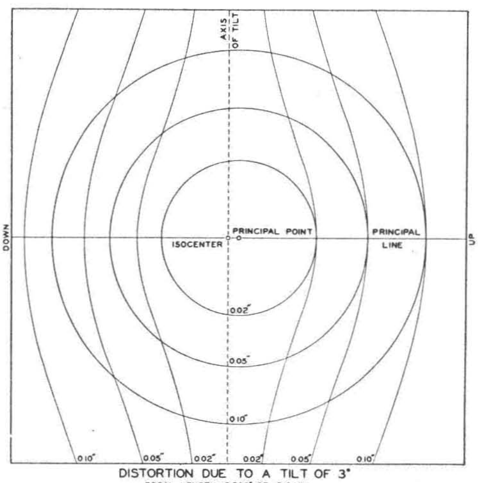

FIG. 8. Curves showing the amount of displacement or distortion caused by tilt.

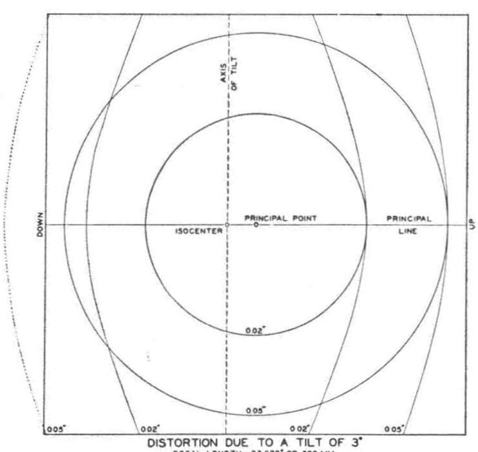

FIG. 9. Curves showing the amount of displacement or distortion caused by tilt.

$8\frac{1}{4}$-inch (210 mm.) lens tilted 3°. These are drawn to scale and are in their correct relative positions even though some reduction has been necessary for reproduction.

The position of the isocenter and the axis of tilt cannot be plotted when the tilt is unknown, so it is impossible to place the conchoidal curves on a random picture to determine the exact area within which the tilt displacement does not exceed a certain specified amount. However, inspection of the curves will show that they are closest to the principal point where they cross the principal line on the up side. Accordingly, a circle drawn with the principal point as center, and a radius equal to the distance from there to the point where the up side curve crosses the principal line, will indicate the area where the tilt displacement never exceeds the indicated amount, regardless of the orientation of the axis of tilt. Such circles have been drawn in Figure 8.

Figure 9 shows the displacement curves when the tilt is 3° and a 600 millimeter lens (nominally 24 inches) is used. Here again, circles have been drawn to show areas within which the tilt displacement never exceeds the indicated amount regardless of the orientation of the axis of tilt. It will be seen that the tilt displacement exceeds 0.05 inch only in the extreme edges of the picture, and the advantage of a long focal length lens becomes quite apparent.

Tests reported by Anderson[15] indicate that on a normal mapping project the tilts will be 3° or less in 90% of the pictures. Accordingly, the diagrams represent the worst conditions that are likely to occur, and the tilt displacement will be materially less (more area could be used) in most cases.

Summary. The previous paragraphs have shown that tilt and ground relief produce large errors in the positions of the images on a photograph. Since the focal length of the lens employed in the aerial camera has an important influence on the magnitude of these displacements, properly drawn specifications will ensure the photographs being as accurate as possible. The engineer using the pictures must then take advantage of the geometry of the picture, to make his measurements in such a way as to minimize residual errors.

With a given taking scale, relief displacement has been shown to be inversely proportional to the focal length of the lens, and the displacement due to tilt was also found to decrease with increasing focal length. Therefore, it is desirable to use as long a lens as is practicable, when approximate determinations of horizontal distance are to be made directly on the pictures.

Having obtained photographs in which judicious selection of focal length has kept the errors as small as possible, the engineer may further improve the accuracy of distance determinations by keeping his measurements as near the principal point as possible, because the displacements due to both tilt and relief increase in the outer portions of the pictures. If the points involved are of the same elevation, relief errors may be eliminated by computing the scale existing at that elevation. Finally, residual tilt displacement may be minimized by using features that are symmetrically located in relation to the principal point.

RADIAL LINE PLOTTING

It sometimes becomes necessary to determine horizontal positions more accurately than is possible by direct measurement on the aerial photographs themselves. Under these conditions, recourse is made to the "radial line assumption" that the displacements due to tilt and relief are radial from the principal point. Actually, of course, errors due to relief are radial from the nadir point, and those due to tilt are radial from the isocenter. However, both these points are quite close to the principal point, the nadir being a distance $f \tan t$ and the isocenter a distance $f \tan t/2$ away. For a 12-inch lens and a tilt of 3°, the principal and nadir points are separated by 0.63 inch, while the isocenter is located 0.31 inch from the principal point. Accordingly, if the tilt and relief are not excessive, a radial line from the principal point will represent a true direction within the tolerances of graphic work. The actual limits within which this radial line assumption is valid have been discussed by Hart,[16] Kowalczyk, Fish, and Dill,[17] and Wood.[18]

[15] Anderson, R. O., "Applied Photogrammetry," Third Edition, Edwards Brothers, Ann Arbor, Mich., 1944, p. 86.

[16] Hart, C. A., "Air Photography Applied to Surveying," Longmans, Green and Co., London, 1940, pp. 146–155.

[17] Kowalczyk, C. E., Fish, L. F. and Dill, A. P., "Radial Plot," MANUAL OF PHOTOGRAMMETRY, ibid. cit., pp. 356–365.

[18] Wood, E. S., Jr., "Maximum Values of Tilt and Relief when Compiling Radial Line Plots Using the Principal Point," PHOTOGRAMMETRIC ENGINEERING, Vol. XII, No. 2, June 1946, pp. 205–211.

If this principle is accepted, then a ray from the principal point to any image represents the correct direction to that particular point, and the angle between two such radial lines will be the true horizontal angle that would have been measured with a transit set up on the ground at the principal point. Since it is possible to use the photographs to determine angles, horizontal positions may be found by intersection, just as is done in surveying by the usual ground methods.

There is another analogy to surveying. Each photograph shows not only its own principal point but also includes the image of the principal points of the two adjacent pictures in the strip, the principal point on one photograph being

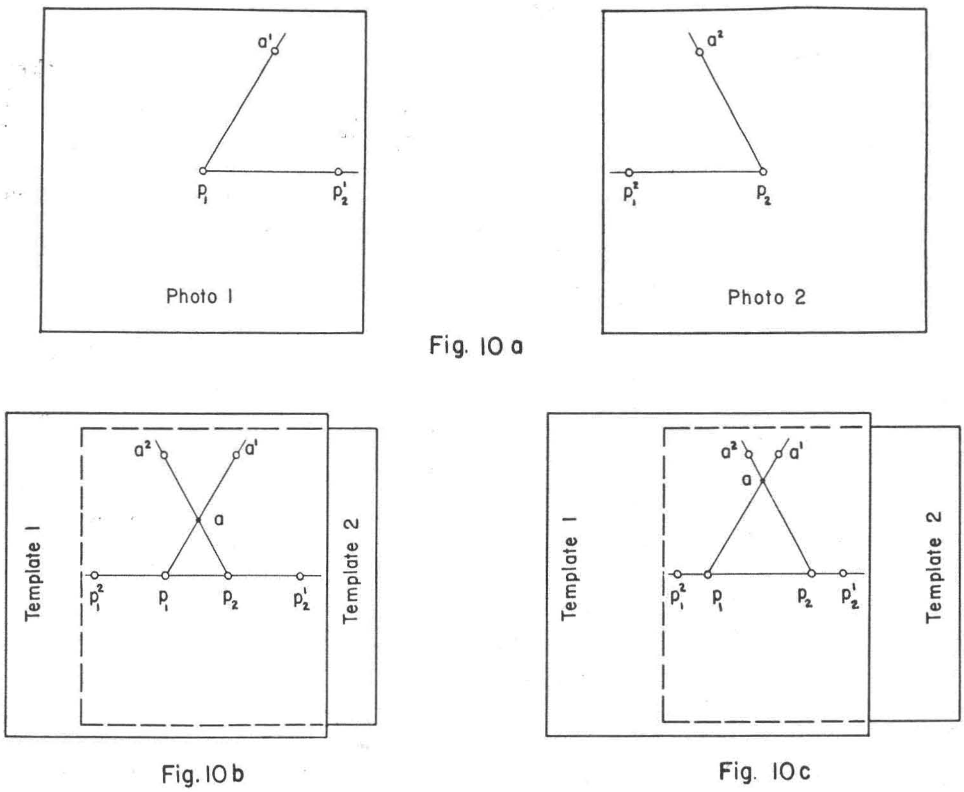

FIG. 10. Determination of true horizontal position by radial line intersection.

called the *conjugate principal point* where it appears on the other picture. The direction on the first picture from its principal point to the conjugate principal point of the second picture is not only the flight line of the airplane, but also corresponds to a foresight. The direction on the second picture from its principal point to the conjugate principal point of the first photo corresponds to a back sight, so that when these two lines are superimposed the two pictures are in their correct mutual orientation, just as a plane table is oriented by back sighting.

Figure 10 shows how the radial line principle may be used to locate the correct horizontal position of point a. On photo 1, the flight line, or foresight, has been drawn from its principal point p_1 to the conjugate principal point p_2^1 and a radial line has been drawn through the image point a^1. Similarly, the flight line, or backsight, has been drawn on photo 2 from its principal point p_2 to the conjugate principal point p_1^2 and the radial to point a^2 has been ruled. The image a^1 on photo 1 has been displaced by tilt and relief, but the radial line assumption states that the true position is somewhere along the ray p_1a^1. On photo 2, the true position of a^2 is somewhere along the radial line p_2a^2. If a piece of transparent material is placed over each photograph, and if the flight line and radial line are traced, the resulting templates may be superimposed as in Figure 10b, in such manner that the flight lines coincide. Having thus obtained the correct mutual orientation, the templates may be slid closer together or farther apart until the distance between p_1 and p_2 represents the actual ground distance at some arbitrary scale. The true position of point a is somewhere along ray p_1a^1 and also somewhere on the ray p_2a^2, hence the intersection of the two must determine its correct horizontal position in relation to p_1 and p_2 at the scale used. If the templates are separated more, so that the principal points are farther apart, as in Figure 10c, the scale of the plotting is larger, but the intersection of the two radial lines will still locate point a in relation to p_1 and p_2 at this larger scale. Thus, the plotting of true horizontal positions may be done by radial line intersection at any convenient scale, whether that is larger, smaller, or the same as the scale of the pictures.

When a map is to be made, this principle is utilized to make a radial line plot which locates the positions of at least nine points on each picture. A few control points, whose survey coordinates have been determined on the ground, are required near the edges of the area to maintain uniform accuracy throughout the project. Planimetric detail is placed on the map in conformity with the radial line points. In rough terrain, the quality of the map is improved if more than the minimum of nine points per picture are plotted. This denser network prevents any cultural feature from being far removed from a well-established point, thus reducing compilation errors. An analogy to this process may be found in the modern skyscraper where the structural steel serves as a frame to hold the walls and windows in place.

The actual steps in preparing a radial line plot are well described by Kowalczyk, Fish and Dill,[19] War Department Manual TM5-230,[20] and Kelsh.[21] Kelsh also cites tests that indicate the amount and distribution of the ground control required. Methods of plotting planimetry with simple instruments are summarized by Spurr.[22]

Controlled mosaics having an accuracy approaching that of a map may be assembled on a radial line plot.[23] Each negative is either tilted or enlarged in a projector until the distance between images on the resulting print is equal to the corresponding distance on the plot, which may be used as a base on which the print is pasted in its correct position. Mosaics compiled in this manner have found wide acceptance with planning commissions, highway engineers, and foresters.

[19] Kowalczyk, C. E., Fish, L. F. and Dill, A. P. ibid. cit., pp. 356–386.

[20] War Department Technical Manual 5-230 "Topographic Drafting," Washington, D. C., 1940, pp. 176–212.

[21] Kelsh, H. T., "The Slotted Templet Method," MANUAL OF PHOTOGRAMMETRY, ibid. cit., pp. 387–422.

[22] Spurr, S. H., "Aerial Photographs in Forestry," The Ronald Press, New York, 1948, pp. 107–115, 149–160.

[23] Meyer, W. H., Jr., "Laying a Controlled Photo Mosaic," MANUAL OF PHOTOGRAMMETRY, ibid. cit., pp. 436–448.

ELEVATION DETERMINATION

Stereoscopic Photography

The discussion so far has dealt with horizontal measurements, but the engineer who confines his use of aerial photographs to this type of work is utilizing only a fraction of the information available. If two overlapping pictures are examined under a stereoscope, the observer has the impression that he is viewing a finely-carved model of the ground with hills, buildings, and trees standing out in bold relief. When the instrument is equipped with what are called *floating marks*, the actual determination of elevation differences becomes possible.

In order to see stereoscopically, it is necessary to view two photographs of the same ground area taken from different exposure stations. Furthermore, the

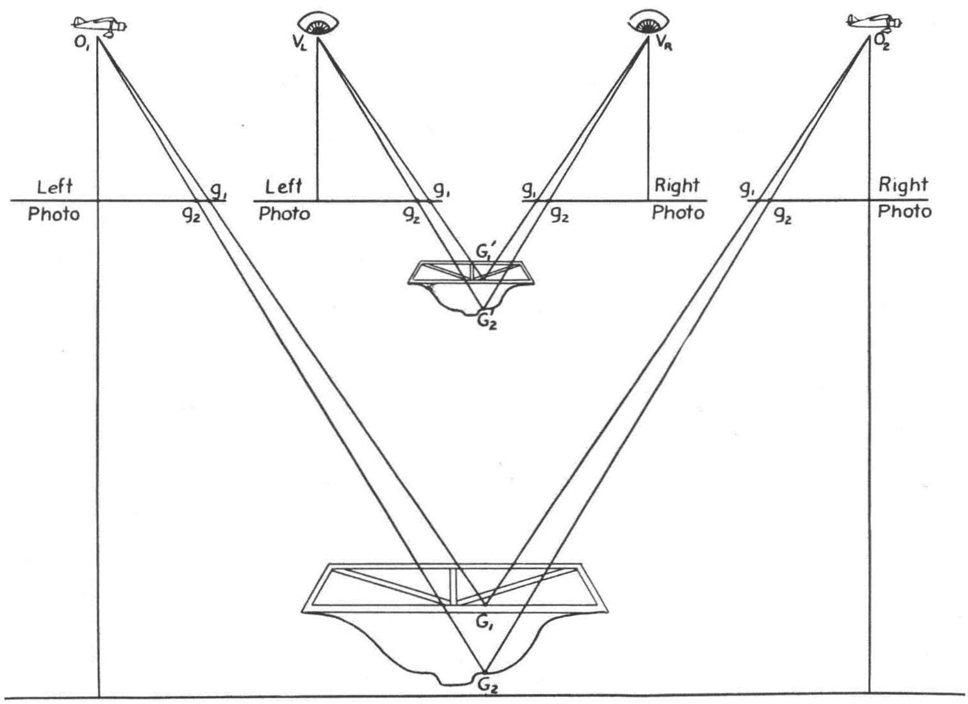

FIG. 11. Principle of stereo vision.

prints must be oriented so that the flight lines form one straight line which is parallel to the observer's eyes. In addition, the left eye must see only the left picture, and the right eye the right-hand one. If the pictures are altered in sequence so that the right eye sees the left picture and vice versa, the relief is inverted and streams appear to flow along ridges, while hills look like depressions. As a final psychological aid to height perception, the pair should be so arranged that shadows fall toward the operator.

Figure 11 illustrates the principles of stereoscopic vision. Vertical photographs are taken of the bridge from points O_1 and O_2. The resulting pictures are then viewed with the left eye at V_L and the right eye at V_R. When looking at the image g_1 on the left picture, the line of sight V_Lg_1 will intersect the line of sight $V_R g_1$ of the right eye directed at g_1 on the right picture. This information is interpreted by the observer's brain in such a way that he sees one point located at G_1'. When the two eyes are directed at their respective images of G_2, the brain again interprets the direction of the lines of sight to mean that there is one point located at G_2'. Thus, when looking at the two pictures simultaneously, the bridge appears to be above the stream.

There are two general types of stereoscope in common use—the mirror and the lens. In the mirror stereoscope, two pairs of parallel mirrors are used to present images of the pictures to their respective eyes, while with the lens stereoscope, the prints are viewed directly through a pair of prisms or magnifying lenses. Each instrument has certain advantages. Because of its larger field of view, the mirror stereoscope is well adapted to the study of the general topogra-

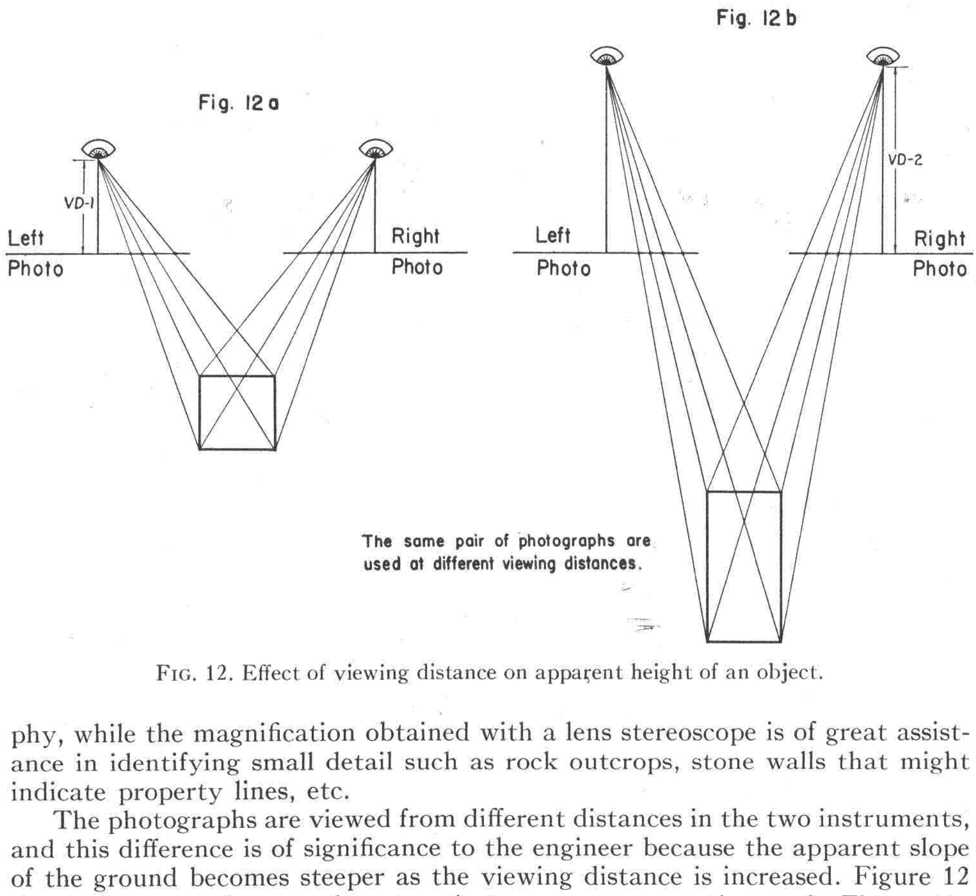

FIG. 12. Effect of viewing distance on apparent height of an object.

phy, while the magnification obtained with a lens stereoscope is of great assistance in identifying small detail such as rock outcrops, stone walls that might indicate property lines, etc.

The photographs are viewed from different distances in the two instruments, and this difference is of significance to the engineer because the apparent slope of the ground becomes steeper as the viewing distance is increased. Figure 12 shows the same photographs set up in two stereoscopes, the one in Figure 12a corresponding to the lens type, and the one in Figure 12b representing the mirror type with a longer viewing distance. In Figure 12a the pair is examined with the observer's eyes a distance *VD-1* from the prints and, as the lines of sight show, the stereoscopic model appears to be as high as it is wide. In Figure 12b the viewing distance *VD-2* is twice as great as *VD-1* and the model now appears to be twice as high as it is wide. This increase in the apparent height of an object in relation to its base is known as exaggeration. Methods of calculating its amount are quite controversial, so the simplest procedure is to study selected pairs from each project right on the ground, in order to compare slopes as they appear under the stereoscope with actual conditions.

Computation of Parallax Values

Relative heights of adjacent objects are readily observed with the aid of a stereoscope. Actual elevations may be determined if the instrument is fitted with a suitable measuring device. Figure 13 represents two truly vertical photographs taken at a height H above the datum from stations O_1 and O_2 which are a distance B apart. A lens of focal length f was used. The nadir points on the ground are at N_1 and N_2 respectively, while the corresponding images are at n_1 and n_2 on the pictures. The vertical plane containing the two air stations cuts photo 1 along the line $n_1n_2^1$ and photo 2 along the line $n_2n_1^2$, which represent

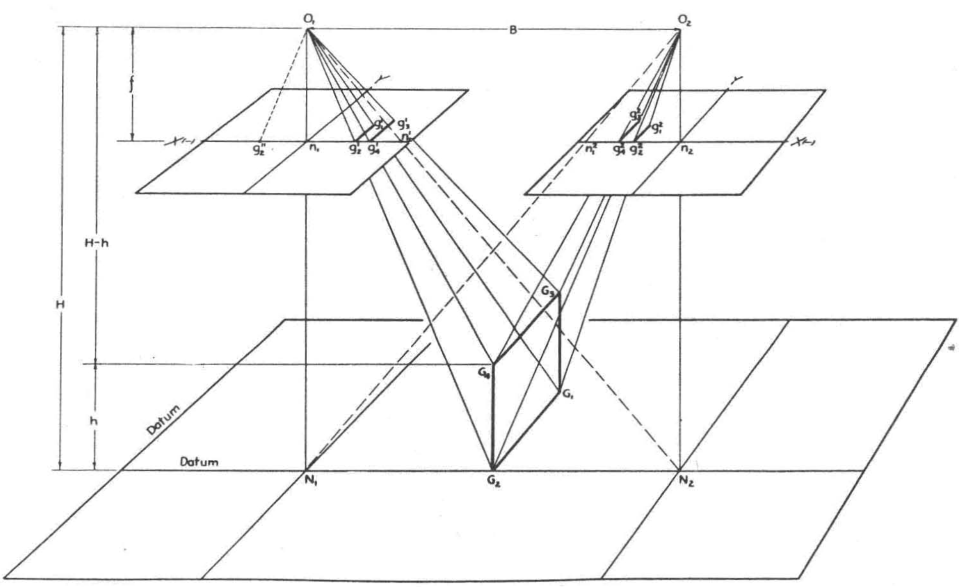

FIG. 13. Geometry of elevation determination on a stereo pair.

the lines of flight and will be used as the X axes, the nadir point serving as origin on each photograph. X will be counted positive in the direction of the nadir point of the adjacent picture of the pair. Lines through n_1 and n_2 will be used as Y axes. A perpendicular dropped from any point G_1 in the datum to the line N_1N_2 will intersect it at G_2 which is recorded at g_2^1 and g_2^2 on pictures 1 and 2, so that $n_1g_2^1$ represents the X value on photo 1, and $n_2g_2^2$ is the X value on photo 2.

Texts on photogrammetry show that for all points in a given plane, the sum of the X distances on the two pictures is a constant called the *absolute parallax*. Letting X_1^1 and X_1^2 be the X values of point G_1 on pictures 1 and 2 respectively, it will be found that

$$X_1^1 + X_1^2 = \frac{Bf}{H} \cdot \tag{4}$$

Expressed in words, the absolute parallax of a point is equal to the product of the air base B and the focal length f, divided by the height H of the exposure stations above the point. As the exposure stations will be a height $H-h$ above the point G_3 (which is a height h above the datum), the absolute parallax of this point may be expressed by

$$X_3^1 + X_3^2 = \frac{Bf}{H - h}.$$

The difference between the absolute parallex of point G_3 and of point G_1 is known as the *differential parallax*, or merely the *parallax* corresponding to the difference in elevation h between the two points. Using P_X to represent the parallax, we have

$$P_X = (X_3^1 + X_3^2) - (X_1^1 + X_1^2) = \frac{Bf}{H - h} - \frac{Bf}{H}$$

or

$$P_X = \frac{Bfh}{H(H - h)}.$$

On photo 1, $n_1 n_2^1$ is called the *measured base*, b, as it represents the actual air base as reproduced on the photograph where it is easily measured. Therefore

$$b = \frac{Bf}{H}$$

which, when substituted in the previous equation, gives

$$P_X = \frac{bh}{H - h}. \tag{5}$$

It will be seen that the parallax corresponding to a given difference of elevation h increases directly with the measured base, and that it will increase as the flying height H decreases. If it is desired to increase the parallax corresponding to a given value of h in order to make more accurate determinations of elevation differences, this may be done by increasing b or by decreasing H. In practice, b cannot be altered if the standard 9×9-inch picture is used with the necessary 60% overlap, so the only recourse is to reduce H. Since the scale of the photographs is fixed by the requirements of the particular project, H can be made small only by using a short focal length lens. It is for this reason that photographs to be used for contour mapping are taken with short lenses.

For example, if pictures are to be taken at a scale of 800 feet per inch in the datum, H will be 4,160 feet with a 5.2-inch lens, and 9,600 feet with a 12-inch lens. Assuming normal 60% overlap on 9×9-inch pictures, the measured b is 3.6 inches. For an elevation difference h of 200 feet, the parallax P_X in the case of the 5.2-inch lens is given by

$$P_X = \frac{3.6'' \times 200'}{4{,}160' - 200'} = 0.182''$$

while for the 12-inch lens

$$P_X = \frac{3.6'' \times 200'}{9{,}600' - 200'} = 0.077''.$$

Thus the parallax with the 5.2-inch lens is approximately 2.4 times that obtained with the 12-inch lens when the pictures are at the same scale.

Equation (5) is adapted to use when drawing contours. If elevation differences are to be measured for tree height determination or similar purposes, a more convenient form is

$$h = \frac{P_X H}{b + P_X} \tag{6}$$

where h may be computed directly from a parallax measurement. Another application of Equation (6) is to find a minimum amount of relief that can be detected on a given pair of pictures. Using Bagley's[24] value of 0.001 inch as the smallest parallax difference that the average person can detect with moderate magnification, the least difference in elevation that could be noticed on the pictures referred to above, taken with a 5.2-inch lens, would be approximately 1.2 feet, while if the 12-inch lens were used this would amount to 2.7 feet.

Figure 13 shows one other point in regard to a pair of truly vertical pictures taken from the same height. The Y coordinates of a given point are the same on the two photographs, or, as it is more commonly expressed, there is no Y parallax. From the figure it will be seen that

$$\frac{g_3{}^1 g_4{}^1}{G_3 G_4} = \frac{f}{H - h}$$

and

$$\frac{g_3{}^2 g_4{}^2}{G_3 G_4} = \frac{f}{H - h}$$

Therefore the images of G_3 are the same distance from the flight line on both photographs. If there is Y parallax in a stereoscopic pair, then there is tilt in one or both of the pictures, or they have been taken from different heights. This knowledge is sometimes used to make a rough check to see whether specifications in regard to the maximum allowable tilt have been complied with.

Methods of Measuring Parallax

A simple method of determining elevation differences requires only an engineer's scale. A perpendicular to the flight line is erected at each nadir point of the pair, and the distance from this line to one of the points, say G_3 in Figure 13, is measured on each picture, care being taken to keep the scale parallel to the flight line. The sum of these two measurements in the X direction represents the absolute parallax of the point. The X values for the other point, say G_1, are measured in the same way and its absolute parallax obtained. The difference between these two values represents the parallax P_X, which when substituted in Equation (6) permits an evaluation of the difference in elevation h between the two points.

While having the advantage of not requiring special equipment, the above method of measuring parallax is quite crude. A more precise way makes use of *wandering marks* or *floating marks* which are placed on the prints and viewed through a stereoscope. These may be in the form of a dot, cross, or other convenient shape, and are so mounted that their separation in the X direction may be changed, an indicator or micrometer being used to measure the amount of this change. Figure 14 represents a pair set up under a stereoscope with ground points A and D being recorded at a^1 and d^1 on the left picture, and a^2 and d^2

[24] Bagley, J. W., ibid. cit., p. 162.

on the right photo. The left eye looking in the direction O_1d^1 sees D along this line, while according to the right eye, the point lies somewhere along the line O_2d^2, the brain interpreting this to mean that there is one point at D. If one floating mark is placed exactly over d^1 on the left picture, and the other is set on d^2 on the right picture, the observer believes he sees a single mark touching the ground at D. Similarly, if the marks are set on a^1 and a^2, the observer now feels that there is a single mark touching the ground at A. A movement of the right mark to c^2 causes the right eye to believe it is somewhere along the line O_2c^2 extended, while the left mark is apparently along the line O_1c^1 extended. When this information is fused in the observer's brain, he sees one mark floating in space at C, hence the name *floating mark.*

To determine the elevation difference between points A and D, the floating marks are made to apparently touch the ground at A, the amount of their sep-

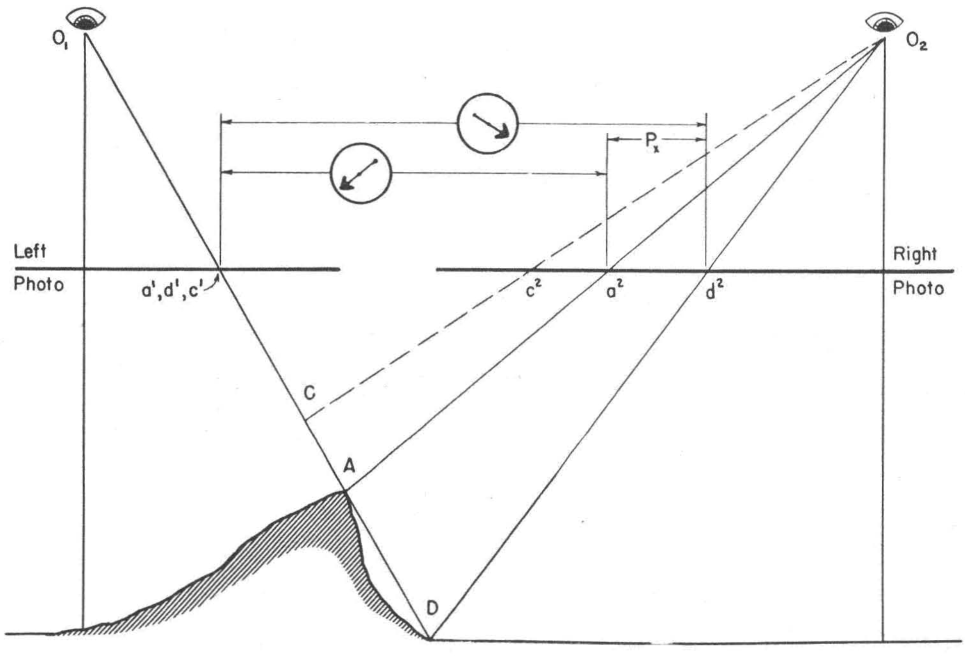

FIG. 14. Use of floating marks to measure elevation difference.

aration being read on an indicator or micrometer. The marks are then set so that they apparently touch the ground at D and the indicator is again read. As shown in the figure, the difference between the two readings is a measure of the parallax P_X, which, when substituted in Equation (6), permits the calculation of the difference in elevation h between the two points.

It is essential to remember that the parallax equation was derived by assuming that both pictures were truly vertical, and exposed from the same height H above the datum. Failure to satisfy both these conditions results in false parallax readings and incorrect values of h. These errors will be much greater if the points are widely separated. In the discussion of tilt, it was shown that the displacement increased nearly as the square of the distance from the isocenter. Accordingly, if the right photo of Figure 14 had been tilted, point a^2 would be displaced much more than point d^2 which is nearer the center. It is this difference in tilt displacement in various parts of the picture that produces the false parallax values which may result in errors of as much as 150 or 200 feet in elevation.

When two points are quite close together, as the top and bottom of a tree, they are both displaced practically the same amount of tilt, and elevation differences may be determined by parallax measurements with sufficient accuracy for foresters or city planners. The method should be applied with caution in rough country, where the relief may introduce errors in the value of the measured base. Reports indicate, however, that if the corrections outlined by Spurr[25] are applied, forest stands may be classified into 5-foot height classes on pictures at a scale of 1,000 feet per inch.

Contouring or accurate elevation determination over the entire area of overlap may be done only when provision is made to overcome the errors produced by tilt and different heights of the exposure stations. This is done in the precise equipment designed for contour mapping. Four systems are currently used by commercial operators—the Brock Process, the Stereoplanigraph, the Multiplex, and the Wild A-6 plotter. If standard mapping accuracy is necessary, one of these processes should be used. If, however, the project is of a reconnaissance nature, the method developed by the 29th Engineers and described in TM5-230[26] will produce far better results than if run of the mill verticals had been used without any attempt at allowing for tilt errors.

Summary. From the foregoing it should be clear that when elevations are to be determined from aerial photographs, it is desirable to use a short lens to keep the flying height as low as possible, thus increasing the parallax values. In addition, unless precise instruments are available to eliminate the effects of tilt, elevation differences will be accurate only when the points are quite close together, while contouring should not be attempted unless adequate equipment and trained operators are available.

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[25] Spurr, S. H., ibid. cit., pp. 131–137, 241.
[26] War Department Technical Manual 5-230, ibid. cit., pp. 248–254.