SCALE-POINT METHOD OF TILT DETERMINATION

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THE purpose of writing this article is two-fold (1) to explain the complete operational procedures of the scale-point method of tilt determination, and (2) to present a standardized form to facilitate the solution.

This form is developed in progressive stages to permit the operator to work with a high degree of efficiency, as the recorded results at the close of each stage serve as indicators of the necessity of further computations. In addition, they also indicate what degree of refinement is necessary.

As an operational and teaching expedient, all mathematical derivations are cast aside and replaced with a system which was used to the hilt with great success during the last World War, in highly improving the teaching efficiency. This was indeed necessary as the students same from all walks of life, and the scientific matter was generally quite involved. And last, but not least, time was at a premium.

A future publication, treating this subject in detail, is now in preparation. Therefore, only a brief general outline of the procedures will be presented in this article.

For the purpose of demonstrating this mannner of tilt determination, a fictitious example* is employed. The scale-point method of solution is recorded on *FORM B* together with footnotes thereon describing the procedures. Mr. Chow treats this problem in considerable detail covering both spatial resection derivations and the actual solution. Professor Earl Church of Syracuse University also worked this problem. The results of his solution are quoted in Mr. Chow's article. The writer's, Church's, and Chow's results are in close agreement. They are as follows:

	H	t	s
Church	20,201′	0°59.7′	180°29′
Chow	20,198′	1°0.6′	179°39′
Anderson	20,200′	1°0.1′	180°45′

The scale-point method, which is mainly graphical, is accompanied by only simple mathematics. Mathematics is replaced with *item-values* which are determined by means of previously established *item-values*, pyramiding from the known ground survey data to the required tilt and flying height of the photograph.

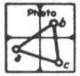
The ground survey data required for tilt determination consists of three check-lines of known horizontal ground distances (items *5*, *6*, and *7* as shown on *FORM B*) together with the elevations of their extremities or terminals (items *2*, *3*, and *4*). In addition, the check-line terminal images must be sharply defined on the photograph so that precise lengths of the three lines (items *8*, *9*, and *10*) may be measured on the photograph.

For convenience the photo-datum is selected as the lowest of the three terminal elevations. Accordingly, h_a equals the sea-level elevation of image point a minus the photo-datum, or elevation of image c. Therefore,

$$\text{Item } 2 = 1800 - 900 = 900'$$
$$\text{Item } 3 = 1000 - 900 = 100'$$
$$\text{Item } 4 = 900 - 900 = 00'$$

as shown on *FORM B*.

* Carl Chow, *ASCE Proceedings*, Feb. 1949, pp. 209–233.



h_a, h_b & h_c are (sea-level minus photo-datum) elevations of points a, b and c, respectively.

[1]SCALE-POINT METHOD OF TILT DETERMINATION

FORM B

FOR TILTS UP TO 5°

Analogies are given for line ab but they also apply to lines bc and ca

Items 2, 3, 4, 5, 6 and 7 are measured on the ground and items 8, 9 and 10 are measured on the photograph.

With a few minor revisions this form is valid for check-lines that do not form a triangle. Thus, 17 = low elev. plus (elev. difference)(14/8) − [4]h″ − [10]h‴.

All slant numerals are item numbers or item values.

Raised prefixes indicate footnotes.

I & II denote 1st and 2nd Tilt Determinations

Focal length f 5.9055/″	Photo-Point Designation	1 Photo-datum Elev. c = 900′	Line			
Date	a High Elev.	2 High Elev ($h_a - {}^8h'_a$)	ab	5 Ground Dist.	8 Photo Dist.	11 f(5/8)
3-7-49	Elev. 1800′ Pt. a2	900.0′ − 0.5′		18027.76′	5.6687″	18780.9′
Photo No.	b Med Elev.	3 Med. Elev. ($h_b - {}^8h'_b$)	bc	6 Ground Dist.	9 Photo Dist.	12 f(6/9)
*	Elev. 1000′ Pt. C5	100.0 − 1.9		20 615.53	6.3287	19237.0
Comp. / Chk'd.	c Low Elev.	4 Low Elev ($h_c - {}^8h'_c$)	ca	7 Ground Dist.	10 Photo Dist.	13 f(7/10)
RA / GM	Elev 900′ Pt. d1	0.0 − 2.2		15811.39	4.9924	18,703.3

14 [3,9]J = [2]DP to a		17 3 + (2−3)(14/8) − [4]h''_{ab} − [10]h'''_{ab}		20 11 + 17	
I 3.250″	II 3.120″	I 558.7′ − 1.5′	II 539.2′ − 1.5′ − 0.1′	I 19338.1′	II 19318.5′
15 [3,9]J = [2]DP to b		18 4 + (3−4)(15/9) − [4]h''_{bc} − [10]h'''_{bc}		21 12 + 18	
I 2.845	II 3.010	I 45.0 − 0.0	II 45.5 − 0.0 − 0.2	I 19282.0	II 19282.3
16 [3,9]J = [2]DP to a		19 4 + (2−4)(16/10) − [4]h''_{ca} − [10]h'''_{ca}		22 13 + 19	
I 2.570	II 2.590	I 463.3 − 5.9	II 465.6 − 5.9 − 0.1	I 19160.7	II 19162.9

23 High Value of (20, 21, 22)		Effective at	Line	26 [5]a′c′ (Scaled)		29 (23−25) − 28		32 (29)(f/31)		[12]35 $K_a = 1 \pm {}^{10}M_a \tau$	38 H = 23 ÷ 35
I 19338.1′	II 19318.5′	[2,11]a′	ab	I 2.860″	II 2.880″	I 64.28	II 57.10	I .01965	II .01747	M_a = 0.330 1.00098	19299.6′
24 Medium Value of (20, 21, 22)		Effective at		27 [5]a′d = (26)((23−24)/(23−25))		30 [6]go	[6]gi	33 (32)[3440] min		36 $K_b = 1 \pm {}^{10}M_b \tau$	39 H = 24 ÷ 36
I 19282.0	II 19282.3	[2,11]b′	bc	I 0.904″	II 0.670″	I 0.380″	II 0.330″	I 68	II 60.1	M_b = −0.310 0.99908	19300.1
25 Low Value of (20, 21, 22)		Effective at		28 [6]ge (Scaled)		31 23 [7]± (29)(30) = H		34 [6]ov = (f)(32)		37 $K_c = 1 \pm {}^{10}M_c \tau$	40 H = 25 ÷ 37
I 19160.7	II 19162.9	[2,11]c′	ca	I 2.760″	II 2.725″	I 19314′	II 19300′	I 0.116″	II 0.103″	M_c = −2.390 0.99292	19299.5

SUMMARY	
H = Item 31	H = Avg. of items (38, 39, 40)
I 19314′	II 19299.7′
tilt = Item 33 (in degrees & minutes)	
I 1° 08′	II 1° 00′
τ = Item 32 ÷ f	
I .00333	II .00296
s = Swing Angles (Scaled)	
I 176° 30′	II 180° 45′

[1]Reference: R. O. Anderson "Applied Photogrammetry" 4th ed., Edwards Bros., Inc., Ann Arbor Michigan.

STAGE A: Test for Tilt

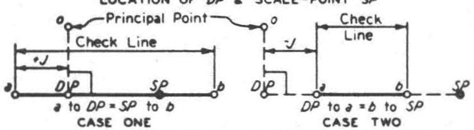
LEGEND

⊥ Perpendicular; ∥ Parallel; ⊾ 90°

DP Intersection of dropped ⊥ [5](from o, i or v) and the check line

● Position of Scale-Point SP (designated as a′, b′ or c′)

[2]LOCATION OF DP & SCALE-POINT SP



For any line the SP position ● may be designated as a′, b′ or c′.

[3]J is negative in items 14, 15 & 16 only when the [2]DP falls upon the line extended from the high elevation.

[4]VALUES OF h″

Q	50	100	200	300	400	500	600	700
W	0.1	0.5	2.0	4.5	8.0	12.5	18.0	24.5

Line ab:
Enter tabulation with $Q = (N/P)(h_a - h_b)$ to find W;
finally, $h''_{ab} = \frac{10{,}000}{\text{item } 11}(W)$.

See [5]GENERAL DIAGRAM OF PHOTOGRAPH for N & P.

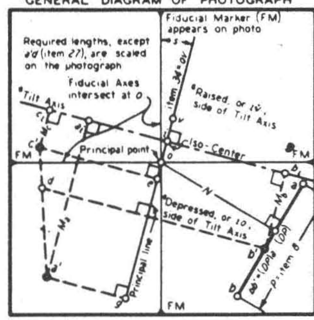
Example (line ab):
Given Q = 231 & Item 11 = 12,570′.
Find h''_{ab}. By Interpolation,
$W = 2.0 + \frac{231-200}{300-200}(4.5-2.0) = 2.8'$.
Finally, $h''_{ab} = \frac{10{,}000}{12{,}570}(2.8) = 2.2'$

Note (line ab):
The value of item 11 equals f multiplied by the value of item 5 and thence divided by the value of item 8.

Note: When items 20, 21, & 22 are equal to each other, the photograph is not tilted.

STAGE B: First Tilt Determination

[5]GENERAL DIAGRAM OF PHOTOGRAPH



Items 23, 24 & 25 prevail at scale-points a′, b′ & c′ respectively.

[6]LOCATION OF [5]d, g, e, i, v & tilt axis (See Legend)
Draw ⊥'s from o to [5]ab, bc & ca. Locate SP's, a′, [5]b′ & c′. a′d is laid off from a′ toward c′. a′g and c′e are ∥ to b′d. The principal line gov is ⊥ to b′d. The direction (o to v) is identical to the direction (g to e). The tilt axis is ⊥ to ov and passes through the mid-point of ov which is i, the iso-center. Measure the swing angle s with a protractor.

[7]When e and o are on the same side of [5]a′g, use the minus sign in item 31 and when they are on opposite sides use the positive sign.

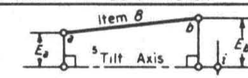
H (item 31) represents the flying height above the datum elevation. The datum scale is equal to: S = H ÷ f.

Complete computations up to item 34 and list values in summary as indicated. List the tilt (item 33) in degrees and minutes.

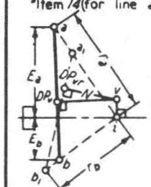
STAGE C: Second Tilt Determination

$^8h'_a = H(E_a \tau)^2$

τ = (item 32) ÷ f



[9]Item 14 (for line ab) = $J_{vr} = (DP_{vr}$ to $a)_I$, or $J_v = (DP_v$ to $a)$ approx.



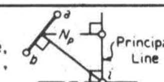
($aa_r = E_a\ r_a\ \tau$) is laid off toward i as a is on the [5]raised side.

Item 14 is always measured to terminal of highest elevation.

Tilt Axis — Raised Side / Depressed Side

($bb_r = E_b\ r_b\ \tau$) is laid off away from i as b is on the [5]depressed side.

[10]$h'''_{ab} = (N/2)(N_p\ \tau)^2 - (3)(\pm {}^5M\ \tau)$:
+ sign when SP is on [5]depressed side,
− " " " " [5]raised ",
3 is low elevation of line.



[11]Revise [2]SP positions (a′, b′ & c′) to conform with DP's from [6]i.

[12]Items (35 to 40) should conform to the 2nd tilt value.

GROUND DISTANCE & ELEVATION DETERMINATIONS WHEN TILT & H ARE KNOWN

5 = [8/f][HK − 17] = Ground dist. (line ab) on Form B.

Determination of h_a or h_b using line ab on Form B.
$h'_a = HK - 11 + h''_{ab} + h'''_{ab}$
$2 = {}^8/_{14}(h'_a - 3) + 3$ or $3 = \frac{(h'_a)(8) - (14)(2)}{8 - 14}$
$h_a = 2 + h'_a$
$h_b = 3 + h'_b$

Item *2* becomes $(h_a - {}^8h_a')$ only in the second (and subsequent) determinations if needed. In other words, foot notes *8* to *12* cannot be applied until a tilt value has been established. In the first tilt determination they must be assumed as equal to zero.

While stated clearly on *FORM B*, it may be wise to show in writing how an *item-value* is computed. Thus, the value of item *11* equals the value of f multiplied by the value of item *5* and thence divided by the value of item *8*. Consider item *20*. The value of item *20* is equal to the sum of the values of item *11* and *17*. The value of item *20* in the first determination as shown on *FORM B* is equal to

$$18{,}780.9 + 558.7 - 1.5 = 19{,}338.1'.$$

Also note that raised vertical prefixes denote foot notes, thus${}^{3,9}J$ indicates two foot notes. The first foot note, which is *3*, is found in the lower left column under the heading of *STAGE A*. This foot note indicates J is negative in items *14*, *15*, and *16* only when the ${}^2DP$ falls upon the line extended from the highest of its two terminal elevations. In the preceding sentence ${}^2DP$ means that foot note *2* shows how the position of the dropped perpendicular DP is located. Note that J is minus in CASE TWO because the line was extended from a, the high elevation, to receive its dropped perpendicular DP. In all other cases J is positive. Further detailed description of determining item values would be a repetition of the operational procedures indicated on *FORM B*.

Considerable time and study have been given to the preparation of *FORM B* with the hopes that it will be comprehensible without reference to other sources. But even so, the writer is at a loss to know just how comprehensible *FORM B* is and also how *far-reaching* it may be with respect to satisfying a wide range of diversified personnel. To attain the *ultimate* in *far-reaching* comprehensibility will require the cooperation of many individuals acting in the role of critical examiners as they read this article and trace through the indicated procedures of *FORM B*. It will be observed that only a few of the procedures, in connection with the fictitious example on *FORM B*, are discussed in this article. This was intentional. This manner of treatment will permit the reader to test the degree of comprehensibility of *FORM B*.

No doubt, upon the readers' completion of following through the procedures of *FORM B*, he will have encountered some issues which may not be clear to him, or quite possibly, he may have suggestions to offer that will materially assist another less-informed person.

The readers' summary of confusing issues and remedial suggestions directed to P.O. Box 18, Benjamin Franklin Station, Washington, D. C., will reach the writer of this article who will proceed to examine each report and subsequently revise *FORM B* wherever necessary. Eventually, this revised *FORM B* will be submitted for publication in the proposed revised edition of the American Society of Photogrammetry's MANUAL OF PHOTOGRAMMETRY.

General Remarks

It is believed that it will be found expedient to apply all foot note corrections when the second determination is required, as it will consume less time to apply these corrections than to deliberate their usage. After acquiring a general knowledge of this method of tilt determination, certain liberties may be taken. For instance, suppose the allowable working tolerance for a certain project was set at five feet of elevation or flying height. Knowing this, the operator would make the second determination accordingly. He would revise only items *14*, *15*, and *16* by using the approximate J_v value as indicated in foot note *9* on *FORM*

B. He would disregard foot notes *8*, *10*, *11*, and *12*. However, the (foot note 9) revisions should be carried forward up to item *34*. Under these conditions the following calibration would result:

$$H = 19{,}304' \quad \text{error} \quad 4'$$
$$t = 1^\circ\ 01' \quad \text{error} \quad 01'$$
$$s = 182^\circ\ 00' \quad \text{error} \ 2^\circ\ 00'$$

Note that the flying height H is the elevation above the photo-datum which is 900 feet. Therefore, the flying height above sea-level is $19{,}304+900=20{,}204'$. The above errors would not be considered excessive for a great many types of photogrammetric work.

The fact that the absence or very small degree of tilt can be detected at an early stage in the computations (when items *20*, *21*, and *22* are equal to each other) is quite noteworthy. Mechanical aids now in current use make possible the execution of aerial photography of very low tilts and in rare cases the negative may accidentally be tilt-free at the instant of exposure. When such is the case, the values of items *20*, *21*, and *22* will be equal to each other which is proof of the absence of tilt. The datum flying height H is equal to items *20*, *21*, and *22* (which are one and the same value) when the photograph is not tilted. When the three terminal elevations (a, b, and c) are equal to each other and items *11*, *12*, and *13* are also found to be equal to each other, the photograph is not tilted. Under these conditions, item *11* is the flying height above the photo-datum elevation. It must be remembered that proving the non-existence of tilt, or that the tilt is zero degrees, is in effect a solution. Many other methods of tilt determination require considerably more work in the course of proving the non-existence of tilt.

The scale-point method readily furnishes calibrations (determination of H and tilt) by means of three check-lines that do not form a triangle, or do not have common terminals. These lines may be defined as random lines. Quite frequently, ground distances and elevations may be secured from railroad or highway alignment plans and profiles for the purpose of calibrating the aerial photograph in which case random lines must be used. The use of random lines permits a practical method of surveying up a flight of overlapping photographs without resorting to ground measurements except at *flight-ends*. This procedure is known as *Control Extension** as it serves to compute progressively the tilt and flying height of each photograph comprising the flight. In addition, elevations and horizontal ground survey coordinates of selected image points that appear on the photographs comprising the flight may also be computed as an adjunct to mapping of the terrain by means of mechanical equipment such as the multiplex, etc.

At the present time the accuracy of this method of control extension is limited to a large extent by distortions of the camera lens and resolution of images. Research on the latter topics is currently being rigorously pursued.

* Photogrammetric Control Extension, R. O. Anderson, Edwards Bros., Inc., Ann Arbor, Michigan. 50 pp., $1.00.