EFFECTS ON MAP PRODUCTION OF DISTORTIONS IN PHOTOGRAMMETRIC SYSTEMS

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I. Introduction

THIS report concerns the results of nearly two years of investigation of the problem of correlation of known distortions in photogrammetric systems of mapping with observed effects in map production. To begin with, distortion is defined as the displacement of a point from its true position in any plane image formed in a photogrammetric system. By photogrammetric systems of mapping is meant every major type (1) of photogrammetric instruments. The type of distortions investigated are of a magnitude which is considered by many photogrammetrists to be negligible, but unfortunately in their combined effects this is not always true.

To buyers of finished maps, you need not be alarmed by open discussion of these effects of distortion on photogrammetric systems for producing maps. The effect of these distortions does not limit the accuracy of the map you buy; but rather it restricts those who use photogrammetric systems to make maps to limits established by experience. Thus the users are limited to proper choice of flying heights for aerial photography and control of other related factors (2) in producing a map of the accuracy you specify. You, the buyers of maps are safe behind your contract specifications. The real difference that distortions cause is the final cost of the map to you, which is often established by competitive bidding.

II. Problem

In examining the problem of correlating photogrammetric instruments with their resultant effects on map production, it is natural to ask how large these distortions are, whose effects are being considered. We believe that it is generally agreed that the magnitude of these distortions is approximately of the order of one part in ten thousand of the distance from the principal point of a photograph or projected image to the image point which is displaced. For example, this means that a point at the corner of a 9″×9″ aerial photograph having a displacement of one part in ten thousand is displaced about 15 microns or 6 /10,000 of an inch. A "micron," however, is a much easier unit to talk about, write about, and use in distortion measurements than the unit "one-twenty-fifth of a thousandth of an inch." How big is this micron? Well, there is a big white cat we know, who incidently depends on Photogrammetry to keep alive—poor cat!—He is minus one whisker donated to the cause used to illustrate the size of a micron. This whisker if split in about 100 equally thick parts would give one micron. In considering this problem, each of the various types of distortion in a photogrammetric system comes from several sources and accumulate to give the total distortion of a system. Improvement of one type of distortion at one point in a system, such as radial distortion in the lens, without consideration of the sum of all other sources leads to questionable hope of improvement in the near future in map production.

In order to correlate these distortions with their effects on map production and consequently with their effects on the final projected or viewed stereoscopic model, it was first necessary to find practical ways and means to measure or to compute these separate point distortions to a micron or two. This required measuring displacement patterns which occur on a photograph or in a projected image to at least one part in 10,000 or more. The problem then was to demonstrate simply the effects of each type of distortion pattern on the final model, and to correlate them by type with reported map production effects. Consequently, a method was required to show distortion effects in vertical as well as horizontal positions of points in all regions in a three dimensional model and not just in one plane. Also required was a method in which "*y*" parallax is cleared at six points *not* in the same horizontal plane, simply because this condition exists in the stereoscopic terrain models used in map production. Others have approached this problem mathematically, but such three dimensional results are too time consuming for interpretation by those who produce maps, particularly when attempting to compute the effects of other than radial distortion.

III. Method Developed

The particular method used to observe the effects of distortion that proved most practical was one which met the above requirements. This method, in one sense an outgrowth of a method developed by Paul Blake (6), can be used with any photogrammetric system of instruments. The procedure is accomplished by drawing in pencil a coordinate grid on a sheet of paper of the same size as the particular aerial film being used such as a 9″×9″ film. This is the "true square grid." The center of the grid becomes the principal point. Its lines in both directions are preferably spaced apart at a distance equal to one tenth of the camera focal length of the system being used to observe effects. The reason for this latter condition becomes evident in use, as it allows studying distortion effects at base-height ratios of values ranging in tenths from 5 tenths to 9 tenths. The intersections of the grid lines represent true points on an aerial photograph in their undistorted position. The new points are marked on the drawing displaced from the grid line intersections by 100 times the actual magnitude and in the direction of the displacements of the particular distortion pattern to be studied (Figure 2). Thus, the distortion pattern studied becomes large relative to the other distortions inherent in the particular system investigated.

Since no essential difference resulted in the study of these effects at 50 times as compared with 100 times increase, it is evident no essential differences exist at true scale. And as these displacements referred to the perspective center of the aerial photograph are angular ones, increased from an order of 1 minute to 2 degrees, such increase does not materially effect the "sine" and "tangent" relationships of the various angles involved. Thus, this method is sound mathematically. The drafting errors in making this film distortion grid pattern and the other lens and photographic distortions of a particular system of instruments becomes small in relation to the increased distortion pattern studied.

These displaced points are linked as crosses and grid lines are removed (see Figure 1). A contact copy on 9″×9″ film is then made of this distortion pattern. In the Multiplex system of instruments, the procedure, which is similar in all other systems of photogrammetric instruments, is to process this film as if it were an aerial photograph. A series of diapositives are made of this aerial film grid in the reduction printer and placed in a group of projectors.

A "true grid" is then placed under the series of projectors which is an enlarged copy at projected model scale of the original "true grid" made on the film scale drawing. This grid, however, is extended in length to pass under the number of projectors used (three or four) and is analogous to the ground control plot used in map production. We then adjust the distorted grid model points to their related true "ground control" grid points. One of several base height ratios is selected and parallax cleared at any six selected points. Because the distortion is exaggerated, the parallaxes "x" and "y" are cleared by coincidence, rather than stereoscopically, using the tracing table in the conventional way in map production. Thus, the vertical and horizontal displacements of a point due to the distortion of the two intersecting points (rays) are found to be exaggerated by approximately 100 times. Also important, but not readily observable by other previous methods is the residual "y" parallax observed when the "x" parallax is measured. Also observed is the "x" and "y" horizontal displacement of each point in the model. If you raise the level of the tracing table up by one or more tenths of the projection distance, you will find other projected planes containing intersection points in a grid pattern. Thus, you will discover you have a model in three dimensions as is observed with an ordinary set of diapositive grids. Consequently "y" parallax can be cleared at six points *not* at the same elevation, which is the condition which occurs in map production; and thus in spite of its accuracy limitations makes this a powerful and practical tool for understanding and correlating known distortions with map production effects, by non-mathematical methods. The usual procedures used in map production bridging are likewise used here, and the semi-quantitative effects on extended models in all three dimensions can be observed, and correlated with reported effects in map production. Use of four projectors is usually sufficient because of the distortion exaggeration in studying the effects on "bridging" and extension of control.

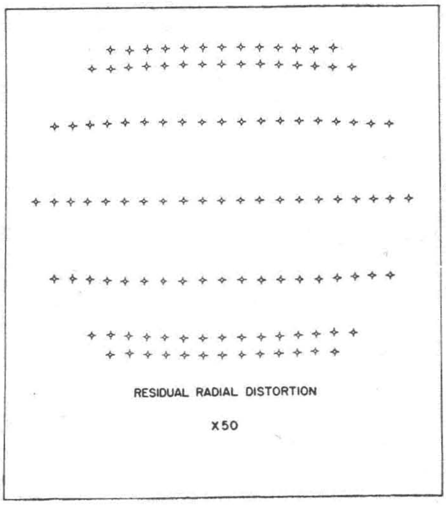

FIG. 1

DISTORTION PATTERNS INVESTIGATED

Recorded in this paper are the major types of distortion patterns (Figure 2) which can be measured, computed and demonstrated. They are listed by type, causes, effects on map production, and methods of compensation particularly as used in the Multiplex System. These distortions are in every system, and in amounts which can be specified by a manufacturer. These distortion patterns have been studied by this method and results are as listed.

1. RADIAL DISTORTION PATTERN

Definition

This pattern is one in which all points at the same radial distance from the center of the image are displaced by the same amount radially, but the displacement varies at different points along a radius.

Caused by

(1) Variation in lenses of same design particularly important in the stereoplanigraph system, where camera lenses are used in reverse.
(2) Lens design limitations.
(3) Uniform concave or convex shape of an image supporting base or plate such as observed on glass plates or film due to low humidity in operation, or processing (11).

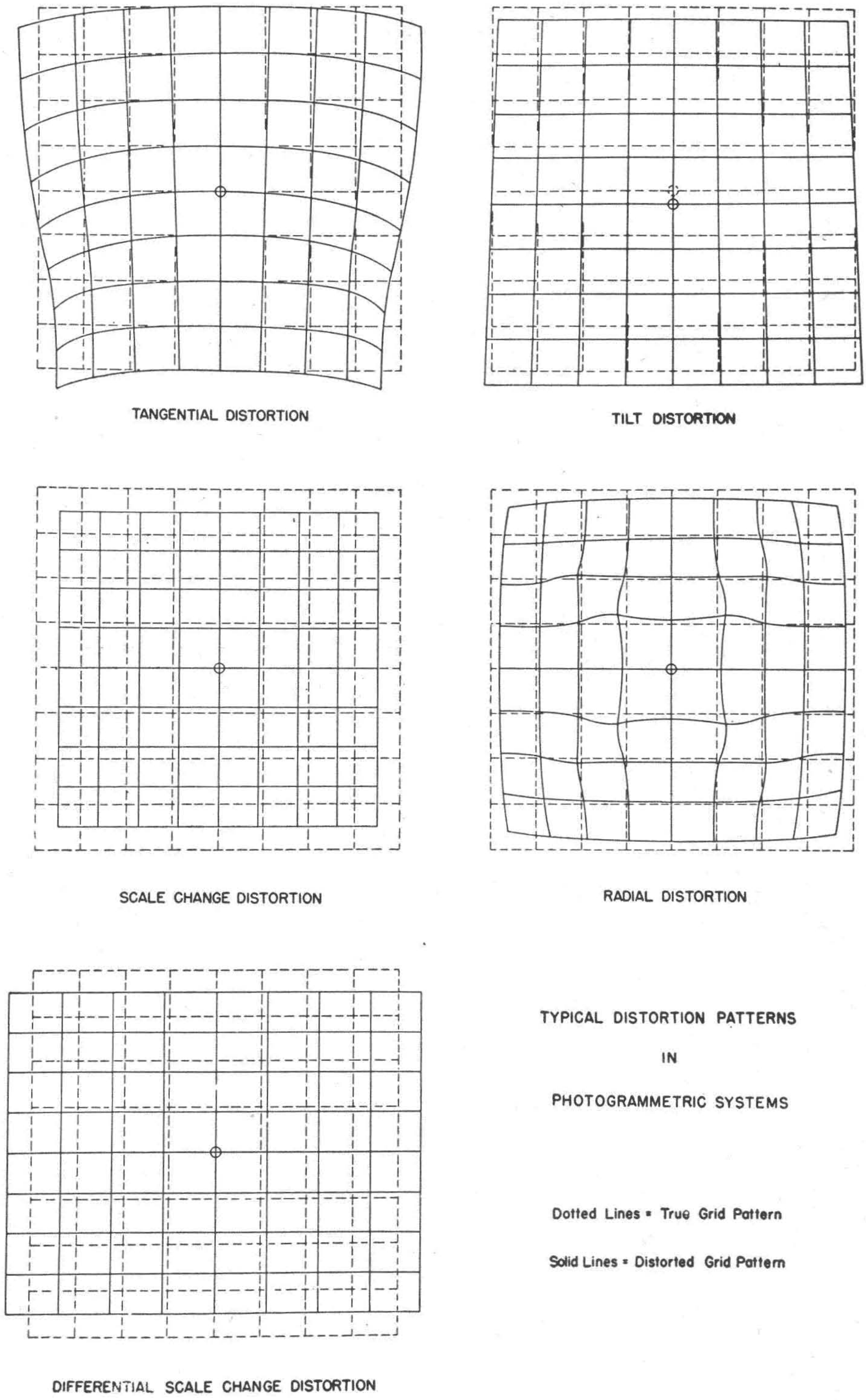
TANGENTIAL DISTORTION
TILT DISTORTION
SCALE CHANGE DISTORTION
RADIAL DISTORTION
TYPICAL DISTORTION PATTERNS
IN
PHOTOGRAMMETRIC SYSTEMS
Dotted Lines = True Grid Pattern
Solid Lines = Distorted Grid Pattern
DIFFERENTIAL SCALE CHANGE DISTORTION

FIG. 2

(4) Concave or convex warping due to aging operation or temperature effects on film supporting planes.
(5) Atmospheric refraction varying particularly with altitude and humidity.
(6) Curvature of the earth (4).

Effects on Individual Model

Model is warped, with its shape dependent on the choice of the six points at the same or different elevations at which "*y*" parallax is cleared. In practice, these six points are usually at different elevations. The method of investigation indicates that choice of points lying in selected regions produce better model uniformity than others.

Effects in Extended Models

(1) Horizontal Plane of Model bends either up or down depending on the type of pattern of residual radial distortion of the entire system. The center line of this plane is known as the "*bz*" curve.
(2) Discontinuities between models unless points at which parallax is cleared are carefully chosen.

Method of Compensation

Design of lenses and instruments of a system to reduce or compensate such distortion. This is another reason why a compensating projection printer is advisable for accurate mapping. Changing the distortion characteristics of one lens without due consideration of the magnitude of all other sources of radial distortion may lead one to question expectation of success of such a project (2). Reduction of the radial distortion by a factor of about ten times appears necessary to obtain considerable improvement in map production.

2. UNBALANCED RADIAL DISTORTION PATTERN

Definition

This pattern is one in which all points at the same radial distance from the center of an image are displaced radially, but *not* by the same amount; likewise along a radius they are displaced by varying amounts. They repeat a definite pattern, however, in each photograph, or projected image, and are not random.

Caused by

(1) Decentration of lens elements, but effect is a fraction of the magnitude of the tangential distortion.
(2) Non-uniform film support plane.
(3) Glass plates including those made of "selected" plate glass whether coated on one side or both.

Effects

Similar to Model Warpage due to radial distortion, except effects are unsymmetrical both in the model and in extension of control.

Method of Compensation

No method of control in individual model is known; but the unsymmetrical effects on the extended models can be effectively compensated by printing diapositives in a projection printer with every other film reversed in direction. Due consideration must be given tangential distortion effects in the use of this method. Reprinting a new set of plate glass diapositives may help a particular model.

3. SCALE CHANGE PATTERN

Definition

Uniform change of scale of a photograph or projected image.

Caused by

(1) Film shrinkage due to uniform temperature and humidity effects in camera operation and processing (8) (7) (11).
(2) Variation in focal length of lenses in manufacture and operation due to temperature.

Effects

Produces a vertical scale change from the horizontal scale by ratio of final projection distance to the distance of the perspective center from the final image plane.

Method of Compensation

Compensated for by using a projection printer or an equivalent instrument.

4. DIFFERENTIAL SCALE CHANGE PATTERN

Definition

Change in the scale in the film along the length of the film roll with respect to the scale across the film.

Caused by

Humidity and processing effects on the film in all phases of mapping including camera operation (7) (8) (11).

Effects

Difference in transverse (y) scale in the projected model along the line of flight as compared with the longitudinal (x) scale in a horizontal plane.

Method of Compensation

Only use of operational techniques are available to counteract this effect in producing the final map (5).

5. TILT DISTORTION PATTERN

Definitions

Displacement of image points such that the image of a square centered and perpendicularly oriented to the direction of tilt is imaged as a trapezoid.

Caused by

(1) Tilting the camera at time of exposure
(2) Tilting the film or diapositive planes with respect to the optical axes.
(3) Decentration of the principal point in camera, or projector.
(4) Decentration of elements of a lens—this also causes tangential distortion at the same time.
(5) Film shrinkage of one side of a $9'' \times 9''$ section of the film more than the other causes a tilt pattern, apparently due to the effects of low humidity shrinkage of one side more than the other in the unwinding process of the film in the camera just prior to exposure (8) (7) (11).

Effects

No effect when compensated by the tilt of a projector, except from some causes, a slight displacement of the perspective center from the nodal point results and consequently a horizontal shift of points of an image in a projected model. This shift increases with increased elevation of the plane of the points in question, but has not been reported as yet in map production.

Method of Compensation

The amount is controlled in instruments by manufacture to specification, but residuals can be compensated to practical limits by tilting projectors.

6. TANGENTIAL DISTORTION

Displacement of points in an optical image from their true position perpendicular to a radial line from the image center through their true positions. Along one radius it is a maximum, and along a radius perpendicular to this radius, it is practically zero. The maximum sagitta of the curve measured out to 45° half angle is the magnitude of the tangential distortion for a 90° wide angle lens. While this curve may take several forms, its limitating shape closely approximates a parabolic curve (see Figure 2).

Caused by

The lens elements decentered in manufacture or subsequent disturbance in use by a few microns bends the optical axis as well as other rays to produce a variety of patterns. A camera filter with excessive wedge could cause it. Two multi-element lenses of the same design with the same amount of "bent" axis do not necessarily produce the same pattern of tangential distortion. Consequently, two such lenses would not have the same effect on map production. Furthermore, a multi-element lens may have zero bent axis, and yet might have measurable amounts of tangential distortion. Therefore, to refer to the effect as solely a prismatic effect (12), is somewhat misleading, and attempts to correct the effect by use of a prism normally does not eliminate the effect of this type of distortion.

Effects

On a single model:

(1) "Skew" effect, defined as the warp of a model such that one or two corners deviate from a flat stereoscopic model, produced when each projector has tangential distortion in different directions.

(2) "Trapezoidal" effect produced in a relatively flat model whose horizontal "*y*" scale changes with respect to the "*x*" scale. Consequently, a true square with one side perpendicular to the line of flight in the model would become a trapezoid. This occurs when the tangential distortions in each projected image forming the model are in the same magnitude and shape in same direction as the line of flight.

(3) Other irregular effects are noted with tangential distortion in other orientation and a variety of shapes result for any one combination. Also there are several ways to partially clear the "*y*" parallax from the model, and to distribute the resultant residual amounts. Consequently, an infinite variety of model shapes, within model flatness limits, can be obtained. This parallels the production report that two operators setting up the same stereoscopic model independently do not arrive at the same model shape. This effect also parallels the report that several independent orientations of the same pair of diapositives in a stereoplanigraph resulted in a variety of tilt solutions of the projectors of about 3 minutes (13).

(4) When the tangential distortion patterns in the pair of projected images

are oriented parallel to the line of flight, but in opposite directions, the front and rear edges of the true model planes are at the same elevation, but above or below the center line. This also accounts for discontinuities between models, and results when using the technique of alternating the direction of each photograph in a projection printer, when attempting to compensate for other types of distortion patterns.

Effect on an Extension of Models

(1) When the tangential distortion pattern in all projected models of an extension is oriented parallel to the line of flight, the horizontal plane of the image is bent upward (and vice-versa) which accounts in part for the occasional reversal of the direction of the "*bz*" curve in bridging control. The "*y*" scale also reduces continuously as the "*x*" scale is held constant in such an extension. This is undoubtedly the so-called trapezoidal scale effect reported in extensions in map production.

(2) When the tangential distortion pattern of all models is oriented in one direction perpendicular to the line of flight, the extended vertical planes of the model are bent in the same direction. This is the effect published by J. T. Pennington (3).

(3) When the tangential pattern of all models is oriented at 45° to the line of flight both of the above effects are observed. This likewise has been observed in map production.

Method of Compensation

This distortion pattern is now being more effectively controlled in manufacture and can be controlled in use only by careful handling of the lens as one would any precision instrument from jar, abuse, or careless disassembly for repairing a camera shutter. The control of centering in manufacture requires centering to a degree of accuracy bordering on the impossible, and causing excessive costs except when manufacturing instruments in quantity.

The effects of tangential distortion can also be controlled by adjusting the tangential distortion of all the lenses of a system in a preferred orientation. In so doing, consideration of all the instruments of a system, and of the various other distortion patterns, and of the methods of compensating their effects, is important. Taking all factors into consideration, it appears that one preferred orientation of the tangential distortion of each instrument of the Multiplex System is perpendicular to the line of flight for camera, printer and projector. The film with tangential distortion in this orientation can then be compensated in the reduction printer and at the same time, compensate to a degree the unbalanced radial distortion pattern. Thus, the curvature of the extended vertical plane is prevented, and at the same time, the trapezoidal horizontal scale effect, and the excessive discontinuities between models are avoided. Another preferred orientation of this distortion is parallel to the line of flight, in which trapezoidal scale effects are recognized and accounted for in ground control plot, and the "*bz*" curve is used in part to compensate the "*bz*" curve due to radial. This does not allow however for compensation of unbalanced radial distortion.

7. RANDOM DISTORTION

Definition

Random displacement of points from true position in a purely chance basis, and no repetitive pattern is observable.

Caused by

(1) Warpage of glass plates of a random nature.
(2) Film support plane warpage of a random nature.
(3) Point to point image shifts due to slight shifts of silver grains in processing (11).
(4) Failure of film flattening devices to operate.

Effects

Random only, except occasionally when film flattening devices fail completely, sufficient to destroy the stereoscopic effect in the model.

Method of Compensation

Except for film flattening failure, drawing of the contour line in the final model is an averaging process, and in all probability this distortion has purely random effects and practically no effect on the resulting map accuracy.

Combinations of Distortion Patterns

It is obvious in studying all these known patterns of any particular system of photogrammetric instruments by the method used, that they could be grouped together and added together point by point in the same manner as vectors are added. The accumulated pattern which results could be also studied and the collective effects observed. Such patterns have been very recently reported, secured by excellent experimental techniques under operational conditions (10) (8) (9).

8. OTHER DISTORTION PATTERNS

No attempt was made to investigate other measurable distortion patterns of the order of one part in ten thousand introduced in semi-mechanical systems of mapping instruments. Included in the group are those instruments whose accuracy is effected by cams, gears, slides, etc., not only in manufacture, but also in operation due to wear or failure to properly maintain these instruments. Systems such as the Multiplex System depending primarily on lenses for accuracy are not subject to these types of distortion patterns. It is unfortunate that more data are not available in regard to the effects of wear on the distortions introduced by such instruments, but reference to operation and maintenance manuals supplied with these instruments indicate considerable maintenance is required, to maintain accuracy.

CONCLUDING COMMENTS

Without actually using this direct method of observed map production effects and measured distortion patterns, a full understanding can not be obtained from this brief presentation of such a comprehensive subject. This method does lead to a useful correlation of distortions with map production in instrument manufacture. It leads also to a balanced economic control of these distortions to a practical minimum in cameras, as well as in other photogrammetric instruments. Consequently, since every system uses a camera, the accuracy of the system can be no greater than the accuracy limitations imposed by that camera. So use of instruments of a system not economically geared to camera accuracy is open to question economically.

This study results in re-affirmation of the fact that the Multiplex printer becomes a useful tool in compensating the effects of at least four types of residual distortion patterns, namely:

Radial Distortion
Tangential Distortion
Unbalanced Radial Distortion
Scale Change Distortion

It is also noted that many mysterious difficulties reported in the past from various users of various systems of map production, can be explained, in part, by the effects of tangential distortion. Exceptions are the well known effects of radial distortion, humidity effects on film and glass plates, and effects of occasional film flattening failure in the camera. The effects of tangential distortion are relisted in summary form as follows:

(1) Variation in the center line "*bz*" curve of an extended series of models usually bending vertically in one direction, but at times reversing itself.
(2) Bending of this center line ("*bz*" curve) in a horizontal direction.
(3) Warping of single models at the corners, often referred to as "skew" associated in part with an operating phenomena called cross-tilt.
(4) Discontinuities between models, particularly at the four corners.
(5) Variation in a single model shape when adjusted by separate operators clearing "*y*" parallax and distributing the residual amounts in as many ways as there are operators. This effect has been, we believe, erroneously attributed to partial loss in resolved detail in the final stereoscopic model compared with the detail existing in the film.
(6) Trapezoidal scale effect in an extension of models which is when "*x*" scale is held constant and the "*y*" scale changes progressively.

Consequently, as a result of this study, control of tangential distortion in lenses by controlling, centering of the lens elements, to previously impractical tolerances is in effect at Bausch and Lomb. In the past, centering of lenses has been accurately controlled by observing the behavior of a star image. Additional control was also effected by controlling the unbalanced radial distortion, also an effect of decentering (9). Concurrent with this correlation of distortion with map production has been a concerted effort to find instruments and methods for measuring to microns for use in production for controlling tangential distortion to a degree of accuracy not previously considered practical in manufacture on a production basis. Several practical methods of measuring tangential distortion in production to microns were evolved and perfected at a high degree of accuracy necessary to establish realistic specifications. These specifications result from a balance between cost, quantity and demand for accurate mapping instruments. Also the effects of other types of distortions that exist was a factor. In view of this work, it appears practical to set a specification for tangential distortion for each instrument. Consequently, by proper orientation of each instrument of the system, the maximum tangential distortion of any one instrument becomes practically the maximum for the entire system if due regard to direction of tangential is observed in operation of the system in map production.

It can be shown that the tangential distortion associated with a bent axis of one minute which is equivalent to 18 microns of tangential on the $9'' \times 9''$ film is that caused by displacing one surface of one element of the camera lens 15 microns, or sixteen-thousandths of an inch. But even with this tight tolerance, the tangential displacement of points is of the same order of magnitude as the residual radial distortion of the Multiplex System.

It is well to note that instruments not made to strict specification can vary widely in their effect on map production and organizational economies. The differences between operating prototype instruments and a subsequent instrument of the same type manufactured for sale, should be required to meet specifications as to distortion limits. Also relying on accuracy, reports published years ago about accuracy of a particular type of instrument, like the U. S. War Dept. Technical Manual 5–244 on the subject of Multiplex Equipment is considered inapplicable to Multiplex equipment manufactured to present day specifications.

It may be of interest to note in this study, that we measured tangential distortion to ±2 microns, lack of flatness of glass plates to ¼ micron, bent axis of a lens to ±15 seconds, and grid lines to ±1.5 microns, using equipment which requires considerable capital investment.

Distortions of the type we have discussed, obviously will never be completely eliminated in any photogrammetric systems. It follows then that those operators who come to understand the behavior of residual distortion patterns which exist can improve their own map production techniques to the point where competitive advantages in map production are significant.

Results of this correlation have been applied to production and improvement of Multiplex equipment. Reflecting these results a more comprehensive specification will be announced shortly including specifying tangential distortion. The Fairchild Corporation is cooperating in maintaining a coordinated specification in their Cartographic Camera.

It is noticed that a few years ago enthusiasm grew for low-cost less accurate instruments; now a trend appears toward the more cumbersome type of instrument. But no matter which way the trend, each system needs at least to account for the following distortions:

Radial	6 Causes
Unbalanced Radial	3 "
Tangential	1 "
Scale Change	2 "
Differential Scale Change	1 "
Tilt	5 "
Random	4 "
Mechanical	? "

Thus with 22 causes of distortion, it is important for users of equipment to ask to what extent these distortions are compensated or controlled in manufacture and operation.

We wish to thank the many individuals and organizations which have made available to us their map production problems, and observed distortion effects. This has made possible this correlation between types of measurable distortions and their effect on actual map production. Finally it becomes evident that photogrammetric systems of instruments are rapidly approaching the day when we can speak of such systems, with specified methods of compensation of distortion and operating techniques, in terms of accuracy specified in minutes and seconds. To the average civil engineer who uses these maps, this is the language he speaks, in referring to the accuracy of surveying instruments.

Particular credit goes to many individuals associated with this work, both at Bausch & Lomb and in Commercial and Government Organizations.

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