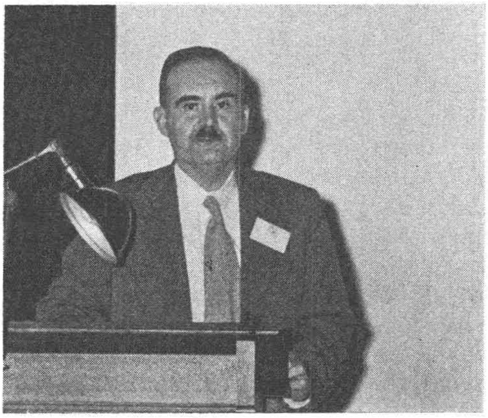
THE ATTENUATION OF CONTRAST BY THE ATMOSPHERE*

W. E. Knowles Middleton, National Research Council of Canada, Ottawa, Canada

1. INTRODUCTION

ONE of the earliest meteorological observations must have been that the atmosphere is sometimes clear and sometimes comparatively obscure. Various people, for instance de *Saussure* (1789) and *Wild* (1868), thought about the matter in a scientific way during the 18th and 19th centuries but it was only 26 years ago that *Koschmieder* (1924) published what may be considered the first reasonably satisfactory theory relating some of the optical properties of the atmosphere with the distance through it at which terrestrial objects can be seen. A quarter of a century is a long time at the pace of modern science and it is very interesting that *Koschmieder's* theory is still considered sound in its general lines and that in fact no very great advance has been made during that time. We can only say that we now understand better some of the implications of the simplifying assumptions that *Koschmieder* made and perhaps also that the theory has been restated in somewhat more elegant terms.



W. E. KNOWLES MIDDLETON

This paper, which is a review paper, will be restricted to a consideration of the reduction of contrast caused by the atmosphere. It will, therefore, not deal with the effect of atmospheric turbidity on the distance to which lights can be seen at night nor will it deal at all with the properties of the eye which are of such great importance in the theory of visual range, since the atmosphere would attenuate the differences in luminance between various objects, even if there were nobody to see what was going on.

2. GENERAL IDEAS OF EXTINCTION

First let us consider a collimated beam of mono-chromatic light having a flux F at a cross-section distant r from any convenient origin. In passing through an additional distance dr of the atmosphere some of this flux may actually be absorbed and converted into heat; more of it may be scattered into other directions so that the incident beam is attenuated both by absorption and by scattering. We may define an absorption coefficient k by the equation

$$dF/dr = -kF \quad (1)$$

and similarly a scattering coefficient b by the equation

$$dF/dr = -bF \quad (2).$$

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If both processes operate simultaneously (as almost always in practice) we may write

$$dF/dr = -(b+k)F \tag{3},$$

which when integrated gives

$$F = F_0 e^{-(b+k)r} \tag{4}.$$

This is an expression of *Bouguer's* law. As the form of the law does not depend on the nature of the process which removes radiation from the incident beam, we may add b and k to produce a quantity σ, and this is spoken of as the *extinction coefficient*. Its dimensions are $[L^{-1}]$.

If we write

$$e^{-\sigma} = \tau, \tag{5},$$

we may define a quantity τ, the *transmissivity*, which is felt by some people to be a more easily understood concept than the extinction coefficient. τ is the fraction of the incident flux which remains in the beam after passing through unit thickness of the medium. However, in this paper we shall make use of the quantity σ.

We have been dealing with monochromatic light and it should be emphasized that *Bouguer's* law is strictly true only where we are dealing with one wave-length. For the common spectral distributions it remains true to the degree of approximation usually required in atmospheric optics, and we frequently speak of an extinction coefficient σ which is tacitly considered to be weighted in accordance with the luminosity function and with the energy distribution of daylight. That this is a reasonable use of the term will be seen if we consider that we do not ordinarily speak of a visual range for red or green light but simply of a visual range for the kind of light which is actually present.

3. ATTENUATION OF CONTRAST

By an argument which now seems unnecessarily involved, *Koschmieder* in 1924 derived the fundamental equation of the theory of visibility, which reads

$$B_b = B_h(1 - e^{-\sigma r}) \tag{6}$$

where B_b is the apparent luminance of a black object at a distance r in an atmosphere of extinction coefficient σ, B_b the luminance of the horizon sky in the same azimuth. We now prefer to think in terms of contrast rather than of luminance. The contrast C of an object of luminance B against a background of luminance B' is given by the expression

$$C = \frac{B - B'}{B'} \tag{7},$$

which is to be considered a definition. It will be seen that C can take any positive value and any negative value not greater than 1. If now we have two adjacent surfaces in a scene, which if viewed from nearby would have an inherent contrast C_O, their contrast when viewed from a distance r will be

$$C_R = (B_O'/B_R')C_O e^{-\sigma r} \tag{8}$$

where B_O' and B_R' are the luminance of the background at distances O and r respectively. For an object seen in a horizontal direction against the horizon sky, which is the case treated by *Koschmieder*, this reduces to

$$C_R = C_O e^{-\sigma r} \tag{9}$$

since the apparent luminance of the background does not change as we recede from it. To this audience the horizontal case will obviously not be the most interesting one. *Duntley*, who developed (1948) the above equation (8) has shown that if we can calculate a quantity called the *optical slant range* $\overline{R}$ where

$$\overline{R} = \int_0^R f(r)dr, \qquad (10),$$

we can use the theory just as if we were dealing with a horizontal line of vision. The quantity $\overline{R}$ "represents the horizontal distance in a homogeneous atmosphere for which the attenuation is the same as that actually encountered along the true path of length R." Unfortunately it is not always easy to put the proper numbers into these equations when we have to deal with non-horizontal vision. Readers who are interested in the possibility of using these equations should refer to *Duntley's* paper, but it should be emphasized that there is as yet no way of determining $f(r)$ from observations made on the ground.

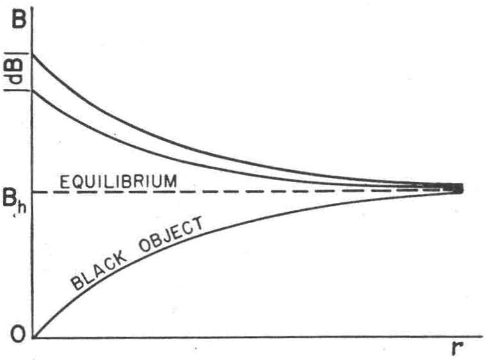

FIG. 1. Showing how the luminance of objects tends to approach an equilibrium value as the distance increases. (After Hugon 1930).

Returning to the horizontal case, it can easily be shown that the luminance of the horizon sky is the "equilibrium luminance" for objects in that direction. Figure 1, due to *Hugon* (1930), shows how the contrast between any object and the sky approaches zero as the distance increases, no matter whether it was positive or negative at a short distance. This means that an object which at any distance appears exactly as bright as the adjacent horizon sky would appear to have this same brightness at any other distance if observed in the same direction. The application of this principle to inclined paths of sight is interesting. *Duntley* has pointed out that there are usually two horizontal directions ON and ON′ (Figure 2) from which sunlight is scattered at the same angle as it is scattered downward along an inclined path. In other words the angle SON = the angle SON′ = the angle SOR. There are also two other directions OM and OM′ for which the angle SOM = the angle SOM′ = 180°—the angle SOR so that sunlight is scattered from these at the same angle as it is scattered upward along the inclined path.

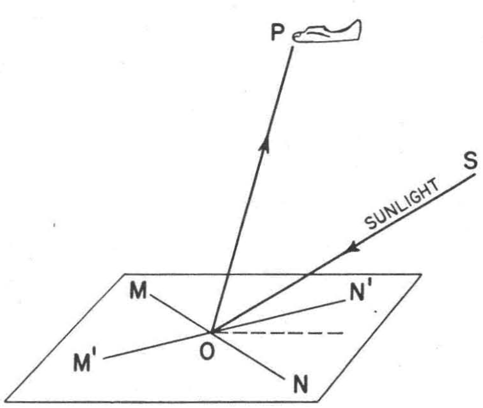

FIG. 2. Geometrical relationships of the oblique path of sight. (After Duntley 1948).

Thus for downward vision the luminance of the horizon sky in the directions OM and OM′ is the equilibrium luminance. The usefulness of this interesting concept is obviously limited to days of almost clear sky.

The work of *Duntley* and of *Hugon* would enable us to calculate the contrast of an object on the ground with its surroundings if we knew the intrinsic contrast and the vertical distribution σ. The extinction coefficient of the atmosphere however, does not vary with height in the same simple way as, for instance, its density. The results of *Waldram* (1945), for example, make it seem likely that the use of equation (10) will be attended by insuperable difficulty. It seems likely that any estimates of the extinction coefficient at various heights, which can be made either from the ground or from the air, will not be any better than an estimate, based on experience, of the final result in terms of aerial photography or reconnaissance.

4. DEPENDENCE OF EXTINCTION COEFFICIENT ON WAVELENGTH

We must now consider the spectral variation of σ. This problem has an enormous literature dating from the time of *Rayleigh* (1871), who dealt with the scattering of light by the molecules of the atmospheric gases, arriving at the well-known law of the inverse fourth power. Unfortunately for our purpose the atmosphere often contains much beside atmospheric gases; for instance, haze, fog, rain, snow, and smoke. The effect of these aerosols on light is much more complicated, because their particles are in a range of sizes from, say, 10^{-6} to 10^{-1} cm., which includes the wave lengths of the radiations of use to the meteorologist and the photographer. The theory of the scattering of light by small spheres in the lower part of this range of sizes was dealt with quite adequately in the classical paper by *Mie* in 1908. A great deal of work has been done on this problem by other workers, with the general result that *Mie*'s theory has been abundantly confirmed.

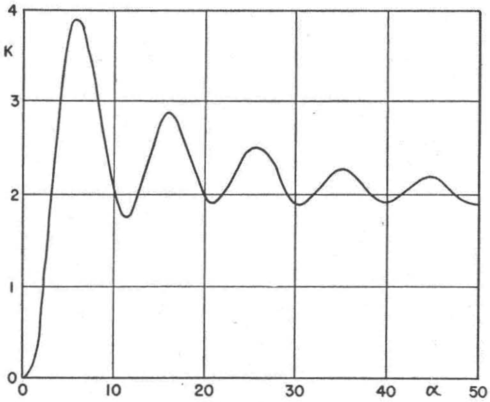

FIG. 3. Ratio of the effective area of a small droplet of water to its actual area. (After Houghton and Chalker 1949).

One of the most interesting results of *Mie*'s theory is that a particle has an effective radius which is in general not the same as its actual physical radius.

The ratio of the effective area of a small droplet of water to its actual area is shown in Figure 3, due to *Houghton* and *Chalker* (1949), in which the abscissae are in units of $2\pi r/\lambda$, where r is the radius of the droplet and λ the wave length of the light. The curve approaches $K=2$ for large values of the radius, a fact which seems rather surprising but can be explained quite adequately by considerations of diffraction. The complexity of the curve is of interest and explains certain anomalous scattering phenomena noted in earlier times by various authors. In visible light the *Mie* theory can reasonably be used for droplets of radius not greater than a few microns. For the larger drops of fog and cloud, geometrical theories, which have been given by *Wiener* (1907, 1910), *Bricard* (1943) and others can be used more easily and provide satisfactory results. Figure 4 shows how these theories can be used to calculate the extinction coefficient of droplets of various sizes throughout the ultraviolet, visible and near infrared spectrum. It will be seen that while for very small particles the extinction in blue light is much greater than in red, this is not true for larger particles. In fact it will be noted that for very large particles, there is practically no change in the extinction coefficient we as pass from one end of the visible spectrum to the other. The question which is most often asked workers in atmospheric optics concerns the possibility of signalling through fog or seeing through it or photographing through it by means of infrared light. If the enquirer is really referring to fog which consists of comparatively large particles of water, the answer must be no, unless he is prepared to use radiation several microns in wave length. The enquirer is always left with the problem of producing such radiation in sufficient quantity.

If on the other hand he is concerned with haze, then the answer is more encouraging. Particles in haze are inclined to be less than a micron in radius, probably on the average 3 or 4 tenths of a micron, and it will be seen from Figure 4 that there is a good deal of change in the extinction coefficient as we go across the visible spectrum and near infrared, so that a considerable increase in range might be expected. Table I, taken from some early work (1935a) by the author, suggests what we might obtain if we could photograph with radiation of wave length 1 micron, and it will immediately appear that the improvement in range by going to a longer wave length is greatest when the visual range is already greatest. In actual fog infrared emulsions give practically no help.

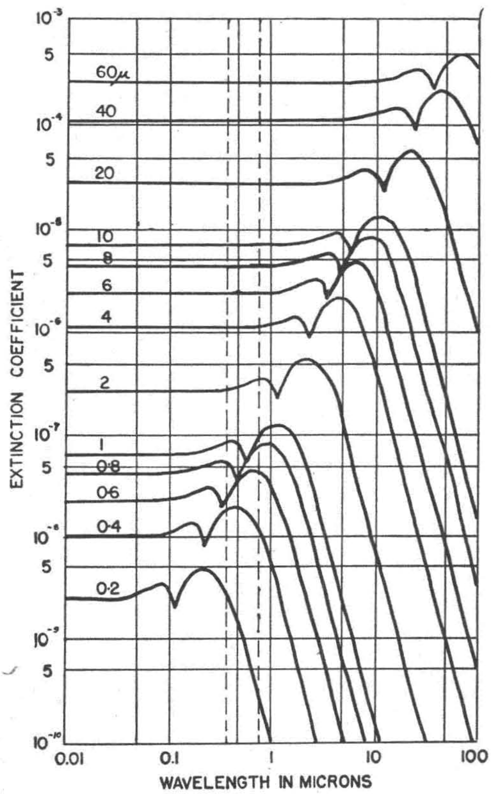

FIG. 4. Extinction coefficient [cm.$^{-1}$] for one particle per cm.^{3} as a function of wave length. The numbers on the curves refer to the radius of the particle in microns. The visible spectrum lies approximately between the two vertical broken lines. (Adapted from Bricard).

TABLE I

σ_{green}	$\sigma_{1.0\mu}$	Visual range for green	"Visual range" for 1.0μ	Ratio
0.0136	0.00103	288 km.	3800 km.	13.6
0.0298	0.00782	131	500	3.82
0.0440	0.0154	89	254	2.86
0.129	0.0541	30.4	72.4	2.38
0.298	0.160	13.1	24.5	1.87
0.461	0.275	8.5	14.2	1.67

There have been a great many experimental investigations of the spectral distribution of σ. Figure 5 summarizes some results obtained by *Foitzik* (1938) and by the writer (1935a) quite independently, the writer's results referring in general to much clearer air than those of *Foitzik*. This diagram, which is due to *Foitzik*, shows that as long as the visual range is less than 1 km. there is little difference in the extinction coefficient for red, green or blue light. At about 1 km.

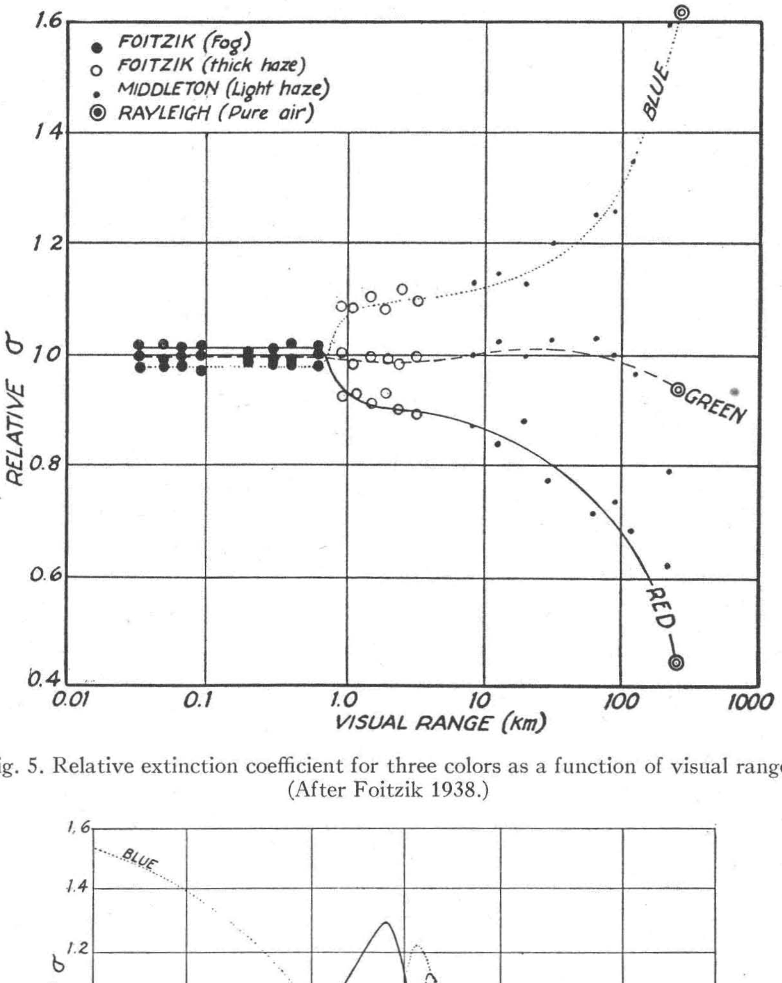

Fig. 5. Relative extinction coefficient for three colors as a function of visual range. (After Foitzik 1938.)

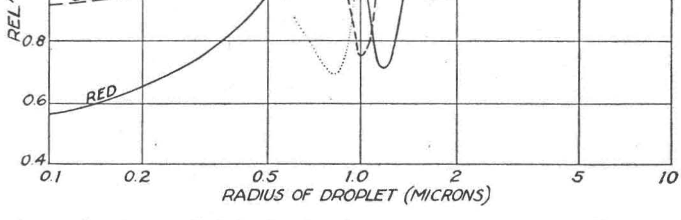

FIG. 6. Relative extinction coefficient for the three colors used by Foitzik, calculated on the basis of the Mie theory. (After Foitzik 1938.)

something happens rather suddenly. The blue begins to be attenuated much more than the red, the difference increasing up to a point where pure air is found and *Rayleigh*'s theory can be applied. *Foitzik* also thought of calculating what ought to happen on the basis of *Mie's* theory, on the supposition that all the drops are of the same radius which, of course, does not occur in practice (Figure 6). The parallelism is quite remarkable except that in the region of the transition from haze to fog there is a very complicated region. This is probably not observed because of the non-homogeneity of the aerosol.

One word of caution is necessary. It does not follow that because the visual range is of the order of half a kilometer we are dealing with an actual fog, especially in industrial regions where very dense haze can sometimes be found.

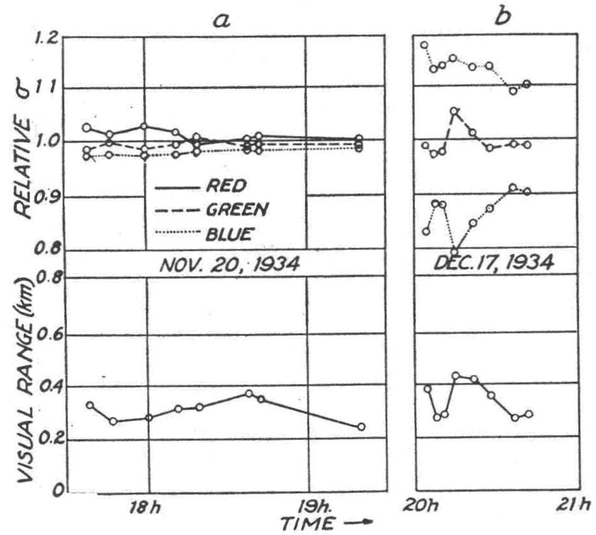

FIG. 7. Relative extinction coefficient on an occasion of fog (a) and one of dense haze (b) with about the same visual range. (After Foitzik 1938.)

The possibility of confusion is illustrated by Figure 7, also from *Foitzik*. On the first of the two occasions dealt with in this figure, there was an actual fog, probably of large water droplets, with a corresponding lack of selectivity. On the second occasion the visual range was much the same, but this was evidently a selective haze. Haze of this sort is almost always either of industrial origin or produced by forest fires.

Because of the complicated results given by the *Mie* theory many attempts have naturally been made to find some simple empirical formula to express the dependence of σ on λ. The most useful kind of formula is of the form

$$\sigma = A\lambda^n + B\lambda^{-4} \qquad (11)$$

where the last term represents the attenuation due to the molecules of the air, and is generally small in comparison to the first term. This reduces the problem to the calculation or determination of the exponent n. It is instructive to apply this sort of formula to the *Mie* theory, as was done by *Götz* (1935). For very small particles $n = -4$ while in the case of water droplets it becomes -2 when their radius is about half a wave length, -1 when it is three quarters of a wave length and zero when it is equal to the wave length, going negative temporarily and approaching zero through a series of oscillations as the radius increases. It will, however, be seen from Figure 4 that it is really impossible to speak of a value of n over the whole visible spectrum especially for particles near 1 micron in radius where the dependence of σ on λ is very complicated.

Probably the most striking demonstration that the atmospheric aerosols are mainly polydisperse is to be found in the absence of the rather vivid colors that would result if all the particles were of the same size. Nevertheless the use of the exponent n is of some value. In general it never seems to be observed much greater than 2 and many authors have found values in the vicinity of 1.3 to 1.6 when the atmosphere is rather clear. It falls, of course, to extremely small values in a fog.

One fact that is perhaps not generally recognized is the extreme transparency of the pure atmospheric gases compared to the comparative turbidity of any ordinary atmosphere. In an atmosphere investigated by *Siedentopf* (1947), with a visual range of 20 km. there would be about 5×10^{-11} gm. cm.$^{-3}$ of water in the form of haze droplets and about 3×10^{-11} gm. cm.$^{-3}$ of dust. In a column of air 1 cm.^{2} in cross-section and 20 km. long there is therefore 1.6×10^{-4} gm. of non-gaseous matter. Now the visual range in pure air at normal temperature and pressure under the same assumptions is 348 km. and the column of air 348 km. long weighs 4.5×104 gm. cm.$^{-2}$. Therefore, the "stopping power" of such a haze, gram for gram, is about 28 million times that of pure gases. This property of aerosols is, of course, extremely useful to the practitioners of chemical warfare.

5. OBSERVATIONS ON THE AEROSOL

The theories and experiments about which we have been speaking are optical and statistical in nature. We shall now refer to some direct observations on the particles of the atmospheric aerosol.

Most of these observations have been concerned with fog droplets, if we exclude the immense amount of work on atmospheric pollution in industrial areas which has been done in various countries. This is concerned chiefly with the number and total weight of the particles deposited from the atmosphere on to the earth but not much with the size distribution of the particles.

There are numerous ways of measuring the size of fog particles and the reader may be referred to a recent monograph by *Gaertner* (1947) or to the memoir of *Vonnegut, Cunningham* and *Katz* (1946) for details of these methods. The difficulties are almost invariably those of sampling, as it is very difficult not to favor particles of some particular range of sizes and almost equally difficult to persuade other workers that one has not done so. One of the most popular methods is that of *Houghton* and *Radford* (1938). Figure 8 shows two of the size distributions that they obtained, but it is thought by many other workers that their method of sampling discriminated against smaller drops. It should be noted that a coastal fog has larger droplets and a wider range of droplets than a cloud. In general it may be stated that the radii of fog and cloud particles extend from less than 1 micron to more than 100 microns. It is, therefore, not surprising that fogs do not appear to be very selective.

For a long time it was thought to be impossible to observe individual particles of haze because these minute bodies leave no visible trace on ordinary surfaces, and indeed would not be far from the limit of microscopic vision even if good contrast were obtainable. A highly imaginative solution of this problem was given by *H. Dessens* (1946, 1947a). After summarizing the various conclusions about haze particles arrived at by various indirect methods, *Dessens* felt that it was necessary at any cost to find a means of direct observation. He succeeded in capturing haze particles on the minute threads spun by very small spiders, very tiny black spiders "gathered in the meadows." He provided a miniature ladder of coarser spider webs for such a little spider to crawl about on and was able to obtain filaments of the order of 10^{-6} cm. in diameter, entirely invisible under the optical microscope but seen later with the aid of the electron microscope. When he exposed this assembly to the breeze, haze particles were captured which could be photographed under the microscope with a magnification of about 1600. These "samples" of haze could be kept indefinitely in a closed box and the measurements repeated months later with identical results, provided the temperature and relative humidity were the same. All his measurements were made well away from sources of industrial pollution. It would take us too far from this present subject to discuss all the remarkable experiments he made with these captive haze particles, but it should be stated that he was able to observe them increase in size with increasing relative humidity, and by repeated measurements on one of the largest he was able to obtain a remarkably clear picture of the behaviour of a salt crystal in the atmosphere, for it indeed became clear that the haze particle was in fact a dilute solution of common salt. This revives the old problem of how enough salt gets into the atmosphere, presumably from the sea, to make all the haze particles in the atmosphere and supply the amount of salt found in rain. Numerous observations by *Dessens* showed a maximum frequency in the region of 0.4 microns radius which agrees well with the general result of optical experiments. *Dessens* then made a further experiment (1947b) in which he rotated his spider's web assembly in a small whirling machine, and calculated the volume of the air swept by each fiber. Assuming that a fiber would collect any droplet that it touched, he calculated the resulting extinction coefficient on the basis of the *Mie* theory and found a perfectly astonishing agreement with simultaneous optical measurements, the discrepancies being less than 20%. The exponent n appeared to vary between 0.3 and 1.3 on the twelve occasions and this comparatively large variation makes *Dessens*' results even more striking.

6. THE COLORS OF DISTANT OBJECTS

The widespread use of color photography makes it of interest to enquire into the effect of the atmosphere on the apparent colors of distant objects. In 1935 the writer worked out a theory of this (1935b), based on a simple extension of *Koschmieder's* theory and making use of the color metric then recently introduced by the *International Commission on Illumination* (*Judd* 1933); this theory has recently been made more general (*Middleton* 1950). It is not feasible to present it here in full; we can only point out that except in certain kinds of artificial pollution the light scattered by the air into the eyes of the observer is blue, having the hue of about 475 $m\mu$. The saturation of this color is greatest when the air is very pure and approaches zero in fog; it is also a function of distance. The color of a dark object at a distance is therefore blue. A very bright object, on the other hand, such as a snowfield in sunlight, looks pale orange. The apparent colors of objects of various actual colors, seen at a distance, are in general displaced more or less towards pale blue. Except in very clear air, all colors approach the white point very rapidly as we recede from them.

When we are interested in the color of small areas, this effect is accentuated by a peculiar property of the eye which makes isolated small patches appear as larger samples would look to an observer with tritanopia, a rare type of "color-blindness" (*Middleton* and *Holmes* 1949). Apart from red and blue-green, all colors suffer apparent hue changes and large decreases in saturation when seen with a subtense of one or two minutes of arc. This property of the eye is not shared by color film, but the visual examination of the picture may bring it in again.

7. CONCLUSION

In conclusion, let us summarize very briefly from the point of view of the photogrammetrist. We have shown that the contrast between two adjacent objects at a distance may be calculated if we have the required information about the atmosphere; but also that this information is not often at hand. The best way to find out whether the weather is "photographic" is to go up and look.

We have shown that there is little chance of seeing or photographing through real fog; and that in general the advantage gained by using infrared film is greatest when the air is very clear. An exception to this rule may occur in industrial haze.

Finally, it has been shown that the possibility of using color-contrast in the identification of details is lessened by the very considerable alteration in color produced by the intervening atmosphere, and by the peculiar behavior of the eye when areas of small subtense are observed.

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